XMUT220 Signals and Systems

Demo 5.2: Time-Domain Analysis

Exercise 1 – First Order Systems

You are given the following first-order systems:

i. System (i):

$$G(j\omega) = \frac{5}{j\omega + 5}$$

ii. System (ii):

$$G(j\omega) = \frac{20}{j\omega + 20}$$

a. Perform simulation of the step responses in MATLAB.

b. Plot of the step responses in MATLAB.

c. Discuss the differences between step response of the first system as compared with the step response of the second system.

d. Calculate the transient response parameters of the systems.

Solution

a. Simulation of the step responses in MATLAB

MATLAB code for System (i):

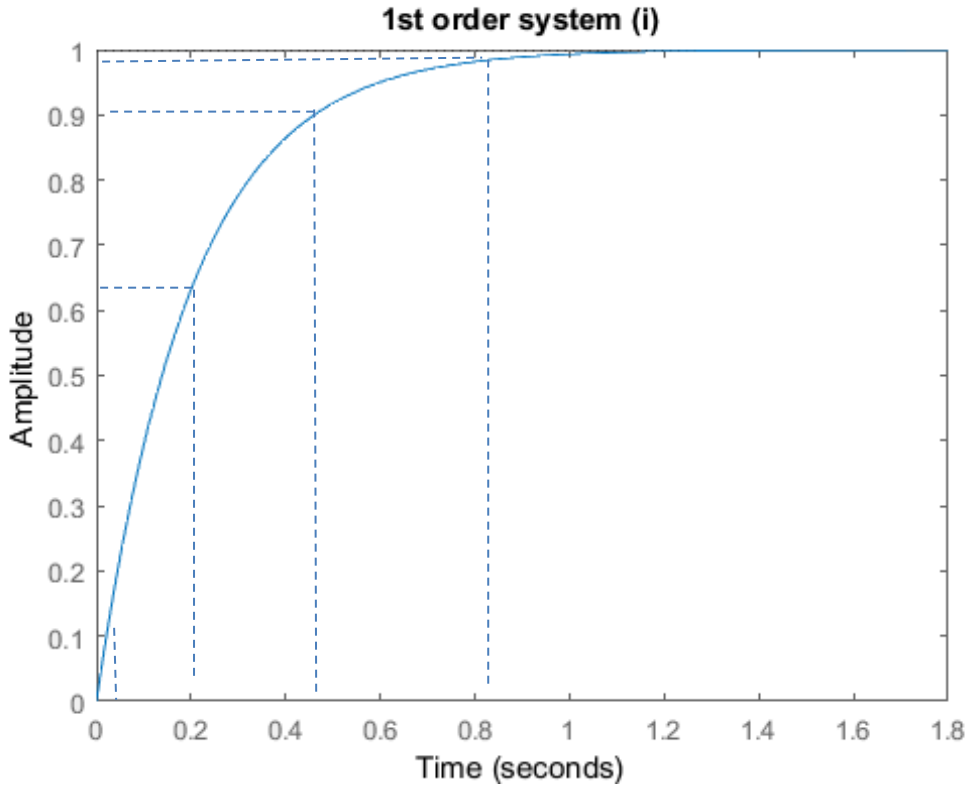
```
num=5;
den=[1 5];
```

```
Gi=tf(num, den);

step(Gi);

title('1st order system (i)');
```

MATLAB code for System (ii):

```
num=20;
den=[1 20];

Gii=tf(num, den);

step(Gii);

title('1st order system (ii)');
```

b. Plot of the step responses in MATLAB.

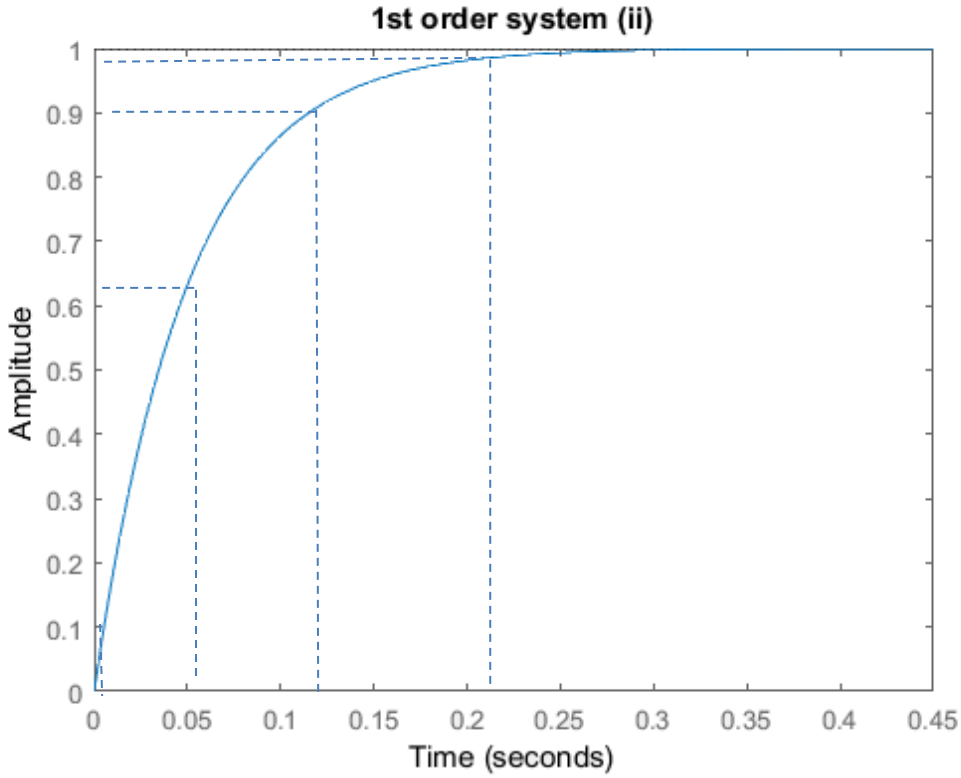


From the simulation results, the transient response parameters of the system (i) are:

iii. Time constant, $\tau \approx 0.2$ second

iv. Rise time, $T_r \approx 0.46 - 0.03 = 0.43$ second

v. Settling time, $T_s \approx 0.82$ second



From the simulation results, the transient response parameters of the system (ii) are:

- Time constant, $\tau \approx 0.052$ second
- Rise time, $T_r \approx 0.12 - 0.002 = 0.118$ second
- Settling time, $T_s \approx 0.21$ second

c. It seems System (ii) has considerable less time constant, rise time and settling time compared with System (i) when both systems are subjected to a step input.

As a result the overall response of System (ii) is less sluggish (i.e. more responsive) compared with System (i).

Exercise 2 – Second-Order Systems

You are given the following second-order systems:

i. System (i) (overdamped response)

$$G(j\omega) = \frac{16}{(j\omega)^2 + 10j\omega + 16}$$

ii. System (ii) (critically damped response)

$$G(j\omega) = \frac{16}{(j\omega)^2 + 8j\omega + 16}$$

iii. System (iii) (underdamped response)

$$G(j\omega) = \frac{16}{(j\omega)^2 + 3j\omega + 16}$$

iv. System (iv) (undamped oscillatory response)

$$G(j\omega) = \frac{16}{(j\omega)^2 + 16}$$

v. System (v) (growing oscillation response, unstable system)

$$G(j\omega) = \frac{16}{(j\omega)^2 - 16}$$

a. Simulate the step responses of the systems in MATLAB.

b. Plot the step responses of the systems in MATLAB and comment on the plot results.

c. From your plots of the systems in MATLAB, as in part (b), measure their transient response parameters: rise time (T_r), peak time (T_p), percent overshoot ($\%OS$), and the settling time (T_s).

d. Comment on the results obtained in part (c).

Solution

a. Step response simulation of the systems in MATLAB.

MATLAB code for System (i):

```
clf;

% Define transfer function of system
numi=16;
deni=[1 10 16];
Ti=tf(numi,deni);
```

```
% Calculate transient response parameters
omegai=sqrt(deni(3));
zetai=deni(2)/(2*omegai);
Tsi=4/(zetai*omegai);
Tpi=pi/(omegai*sqrt(1-zetai^2));
Tri=(1.76*zetai^3-0.417*zetai^2+1.039*zetai+1)/omegai;
percenti=exp(-zetai*pi/sqrt(1-zetai^2))*100;

% Plot transient response
step(Ti);
title('2nd order system (i)');
```

MATLAB code for System (ii):

```
clf;

% Define transfer function of system
numii=16;
denii=[1 8 16];
Tii=tf(numii,denii);

% Calculate transient response parameters
omegaii=sqrt(denii(3));
zetaii=denii(2)/(2*omegaii);
Tsii=4/(zetaii*omegaii);
Tpii=pi/(omegaii*sqrt(1-zetaii^2));
Trii=(1.76*zetaii^3-0.417*zetaii^2+1.039*zetaii+1)/omegaii;
percentii=exp(-zetaii*pi/sqrt(1-zetaii^2))*100;

% Plot transient response
step(Tii);
title('2nd order system (ii)');
```

MATLAB code for System (iii):

```
clf;

% Define transfer function of system
numiii=16;
deniii=[1 3 16];
Tiii=tf(numiii,deniii);

% Calculate transient response parameters
omegaiii=sqrt(deniii(3));
zetaiii=deniii(2)/(2*omegaiii);
Tsiii=4/(zetaiii*omegaiii);
Tpiii=pi/(omegaiii*sqrt(1-zetaiii^2));
Triii=(1.76*zetaiii^3-0.417*zetaiii^2+1.039*zetaiii+1)/omegaiii;
percentiii=exp(-zetaiii*pi/sqrt(1-zetaiii^2))*100;

% Plot transient response
step(Tiii);
title('2nd order system (iii)');
```

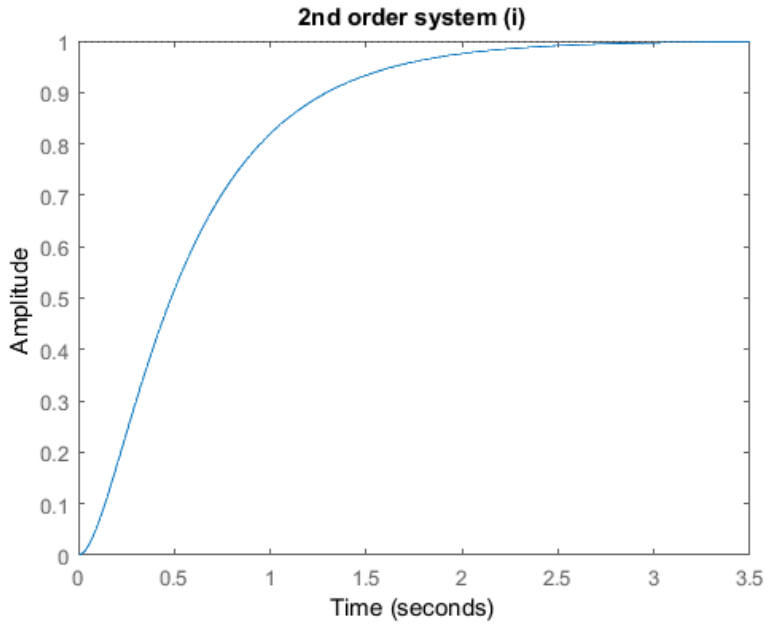
MATLAB code for System (iv):

```
clf;

% Define transfer function of system
numiv=16;
deniv=[1 0 16];
Tiv=tf(numiv,deniv);

% Calculate transient response parameters
omegaiv=sqrt(deniv(3));
zetaiv=deniv(2)/(2*omegaiv);
Tsiv=4/(zetaiv*omegaiv);
Tpiv=pi/(omegaiv*sqrt(1-zetaiv^2));
Triv=(1.76*zetaiv^3-0.417*zetaiv^2+1.039*zetaiv+1)/omegaiv;
percentiv=exp(-zetaiv*pi/sqrt(1-zetaiv^2))*100;

% Plot transient response
step(Tiv);
axis([0 10 -0.5 2.5]);
title('2nd order system (iv)');
```

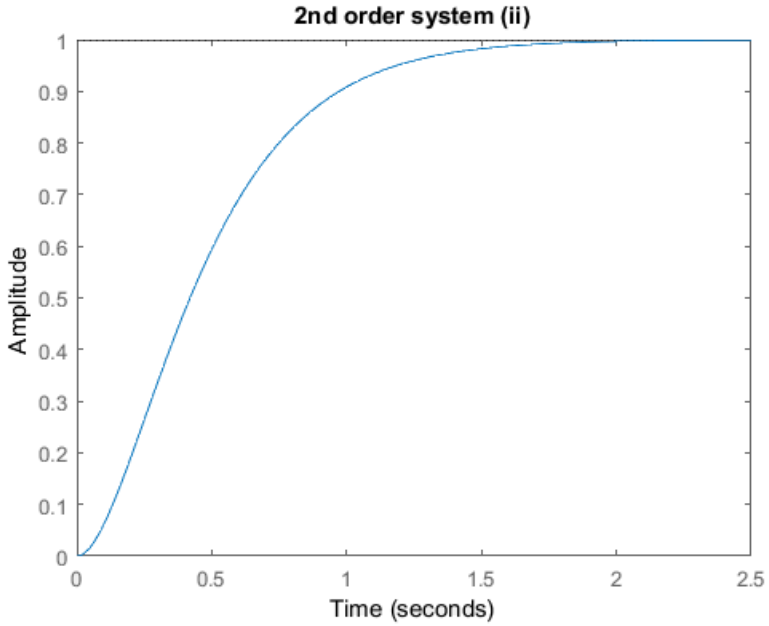
MATLAB code for System (v):

```
clf;

% Define transfer function of system
numv=16;
denv=[1 0 -16];
Tv=tf(numv,denv);

% Calculate transient response parameters
omegav=sqrt(denv(3));
zetav=denv(2)/(2*omegav);
Tsv=4/(zetav*omegav);
Tpv=pi/(omegav*sqrt(1-zetav^2));
Trv=(1.76*zetav^3-0.417*zetav^2+1.039*zetav+1)/omegav;
percentv=exp(-zetav*pi/sqrt(1-zetav^2))*100;

% Plot transient response
step(Tv);
title('2nd order system (v)');
```

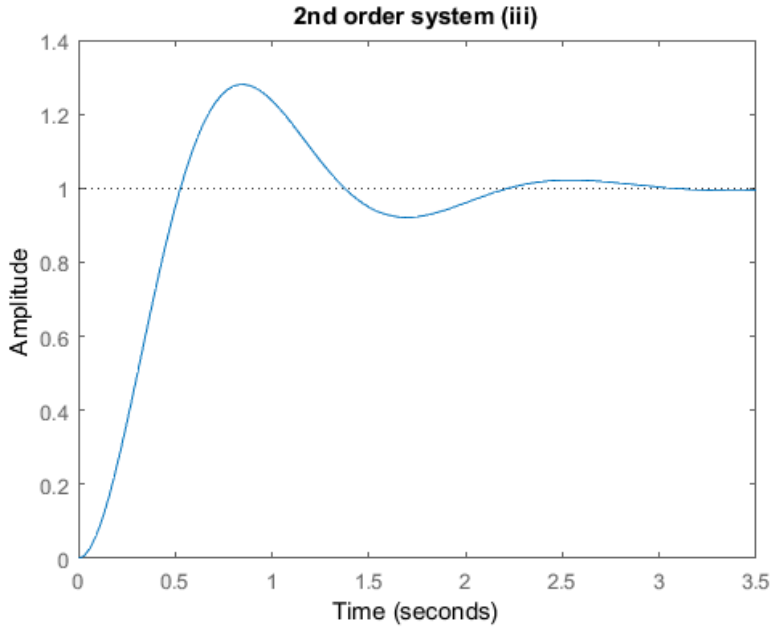
b. Plot of step response simulations of the systems in MATLAB and comment on the step responses of the systems.



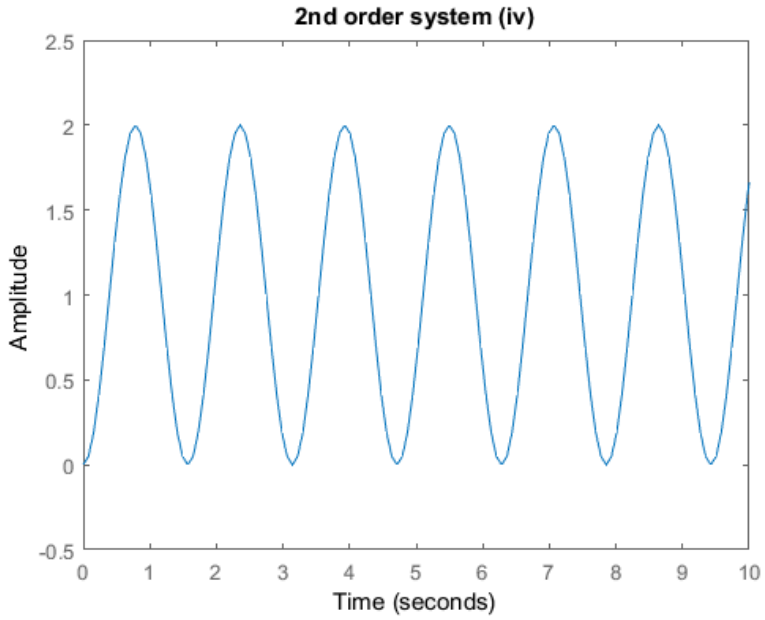
For system (i), the step response of the system is a heavily damped response (overdamped response). Its response is increasing exponential response that settles down to the systems' steady-state value and condition. With this overdamped response, it implies that the system is stable as well. It is not a growing oscillation e.g. which is the characteristic of an unstable system.



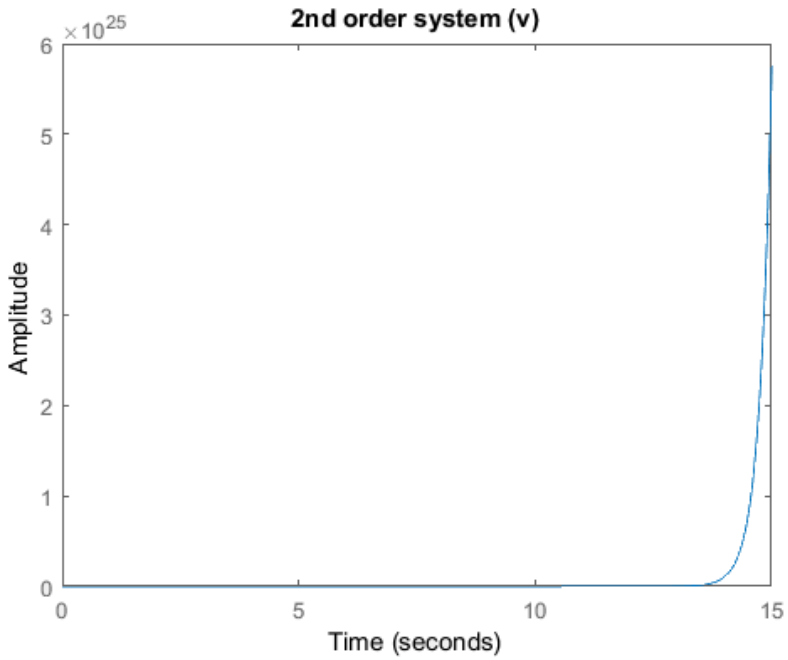
For system (ii), the step response of the system is also a critical damped response that settles down to the systems' steady-state value and condition. This response means a stable system as well.



For system (iii), the step response of the system is also lightly damped (underdamped) response. Like the other responses, the transient response settles down to the systems' steady-state value and condition. Due to this type of response the system is deemed to be stable.

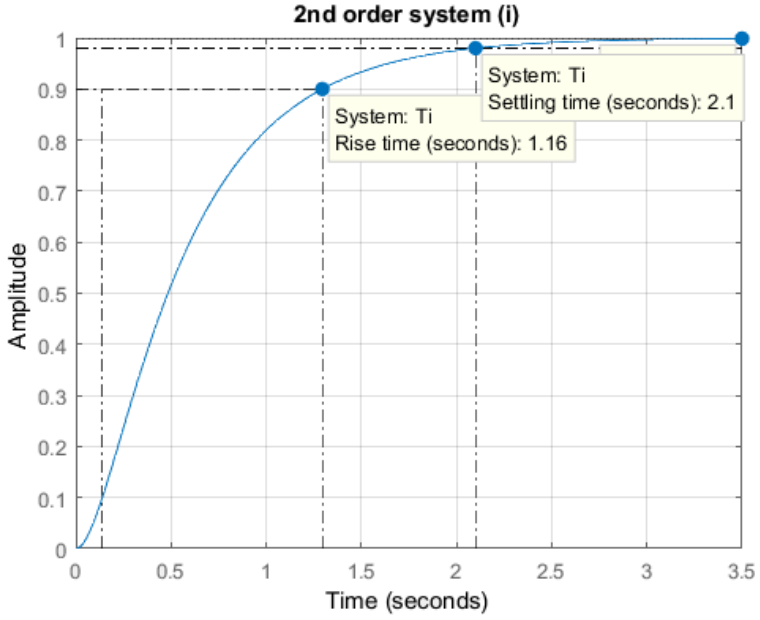


For system (iv), the step response of the system is a constant amplitude oscillation (undamped) response. Unlike the previous responses, the transient response does not settle down to the systems' steady-state value and condition. Due to this reason, the system is deemed to be marginally stable (e.g. in between stable and unstable conditions).



For system (v), the step response of the system is growing oscillation response. Unlike the other responses, the transient response does not settle down to the systems' steady-state value and condition. Due to this reason, the system is deemed to be unstable.

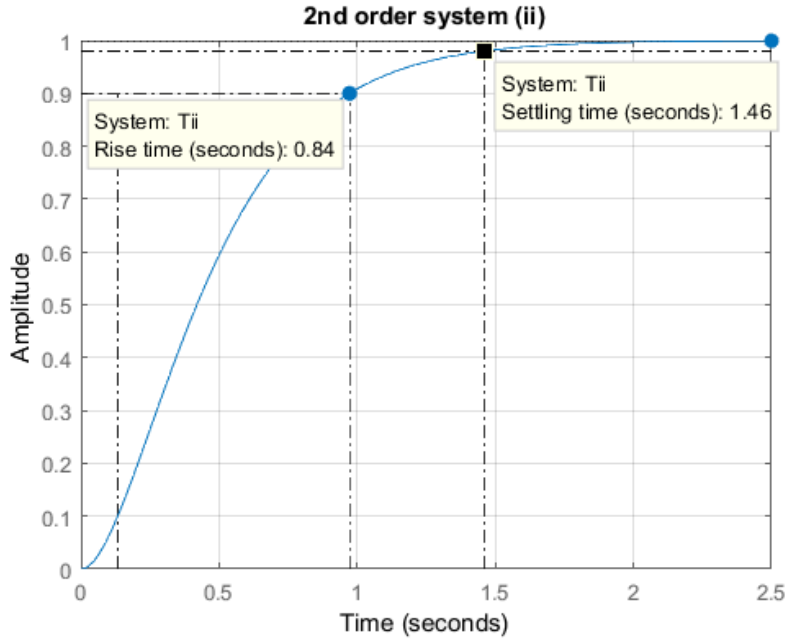
c. Measure of the rise time (T_r), peak time (T_p), percent overshoot ($\%OS$) and settling time ((T_s) from the simulation plots.



The transient response parameters for System (i) are:

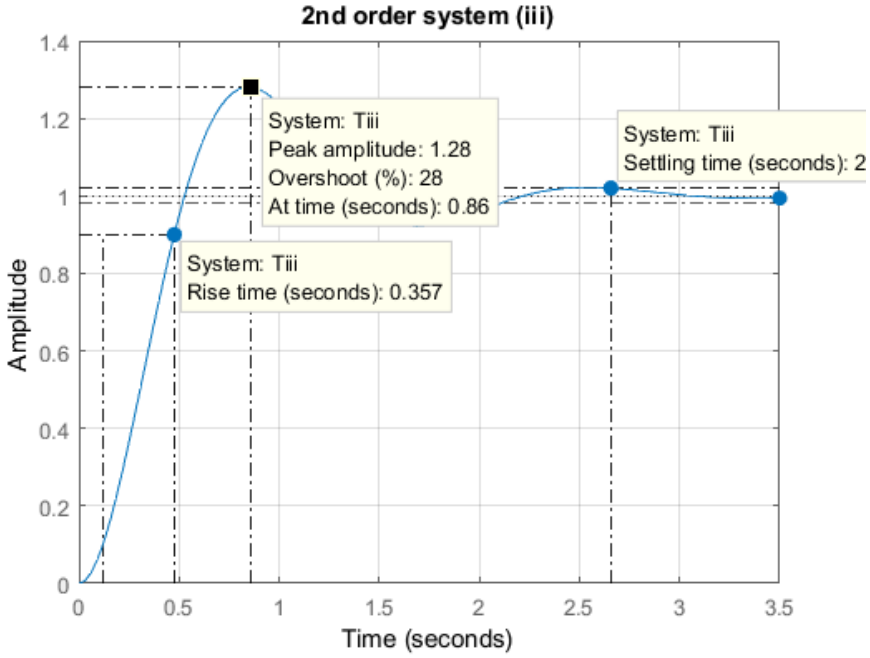
- Rise time (T_r) = 1.16 seconds

- Peak time (T_p) = N/A
- Percent overshoot (%OS) = N/A
- Settling time (T_s) = 2.1 seconds



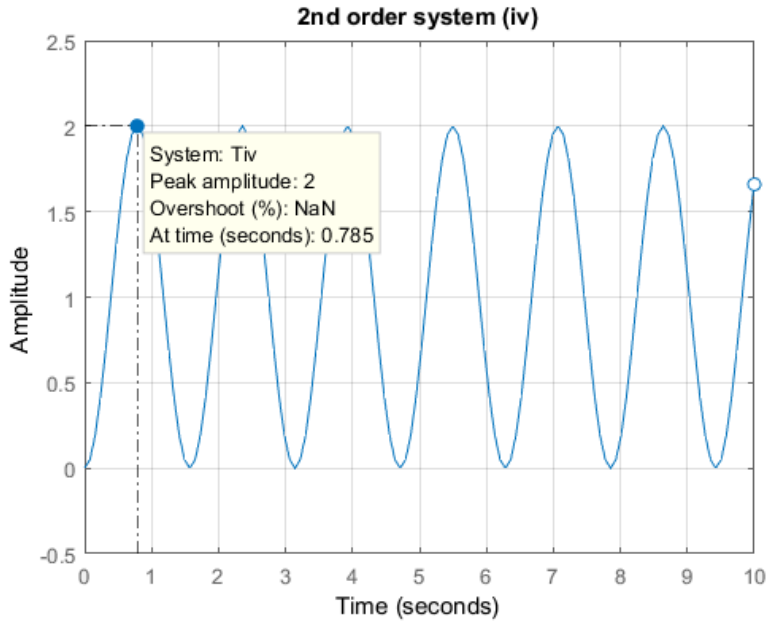
The transient response parameters for System (ii) are:

- Rise time (T_r) = 0.84 second
- Peak time (T_p) = N/A
- Percent overshoot (%OS) = N/A
- Settling time (T_s) = 1.46 seconds



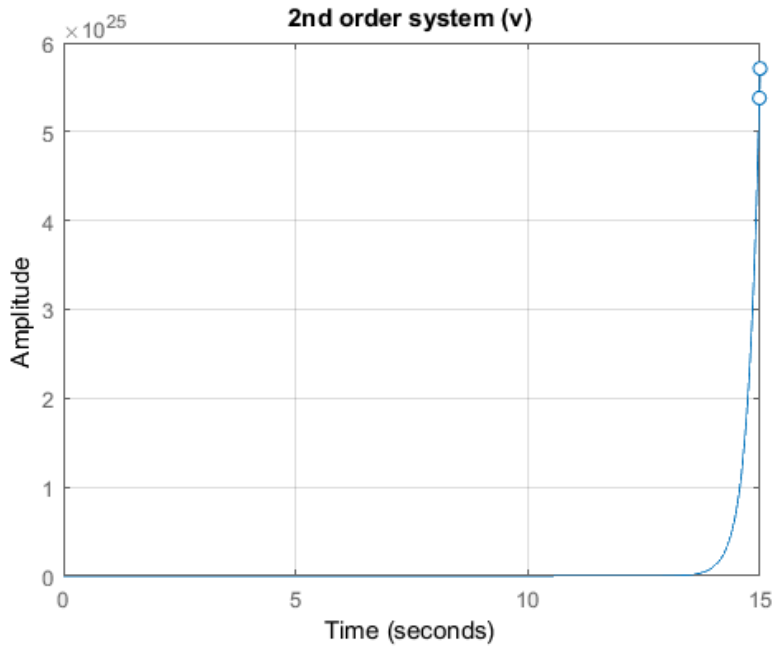
The transient response parameters for System (iii) are:

- Rise time (T_r) = 0.357 second
- Peak time (T_p) = 0.86 second
- Percent overshoot (%OS) = 28%
- Settling time (T_s) = 2 seconds



The transient response parameters for System (iv) are:

- Rise time (T_r) = N/A
- Peak time (T_p) = 0.785 second
- Percent overshoot (%OS) = N/A
- Settling time (T_s) = N/A



The transient response parameters for System (v) are:

- Rise time (T_r) = N/A
- Peak time (T_p) = N/A
- Percent overshoot (%OS) = N/A
- Settling time (T_s) = N/A

d. From the results of simulation in part (c), the transient response characteristics of the systems are analysed as follows:

- Type of Response of the System:

  it looks as if the transient response of Systems (i) is an overdamped response, System (ii) is critical damping (e.g. its damping ratio is 0.707), System (iii) is an underdamped response (e.g. lightly oscillate initially before settling down to its steady state value and condition),

System (iv) is an undamped response with a constant amplitude oscillation, and System (v) is a growing oscillation (e.g. its amplitude is getting larger).

- Time Periods of the System (e.g. Rise Time (T_r), Time-to-Peak (T_p), and Settling Time (T_s)):

  The fastest rise time is observed in System (iii) and the slowest is in System (i). This relative faster rise time is at the expense of the having oscillatory response and longer settling time. Rise time is not observed in Systems (iv) and (v) due to oscillatory and unstable natures of the response of the systems respectively.

  The characteristics of the time-to-peak and settling time of the systems follow similar profile of the rise time of the system e.g. systems with longer time-to-peak and settling time are due to longer rise time. Moreover, time-to-peak is only observed on the systems that have underdamped transient responses. Settling time is not observed in Systems (iv) and (v) due to oscillatory and unstable natures of the response of the systems respectively.

- Percentage Overshoot ($\%OS$):

  The biggest percentage overshoot is experienced by System (iii). Whereas for the other responses, there are no overshoots recorded due to damping (e.g. Systems (i) and (ii)) or oscillation nature of the response of the systems (e.g. systems (iv) and (v)).

- Stability of the System:

  The Systems (i), (ii) and (iii) are stable systems as their transient responses are either a growing exponential or decaying oscillation, whereas System (iv) is marginally as its response is a constant amplitude oscillation and System (v) is unstable system as its response is growing oscillation.