

Demo 5.3 – Frequency Domain Analysis

XMUT220 Signals and Systems

Overview

1. Bode plot fundamentals.
2. Construction of Bode plots.
3. Analysis of frequency response with Bode plots.

1. Draw manually, simulate in MATLAB, and describe frequency response at critical frequencies of a first-order continuous system with the following transfer function (effect of a pole):

$$G(j\omega) = \frac{1}{j\omega + 2}$$

Solution

Manual Drawing:

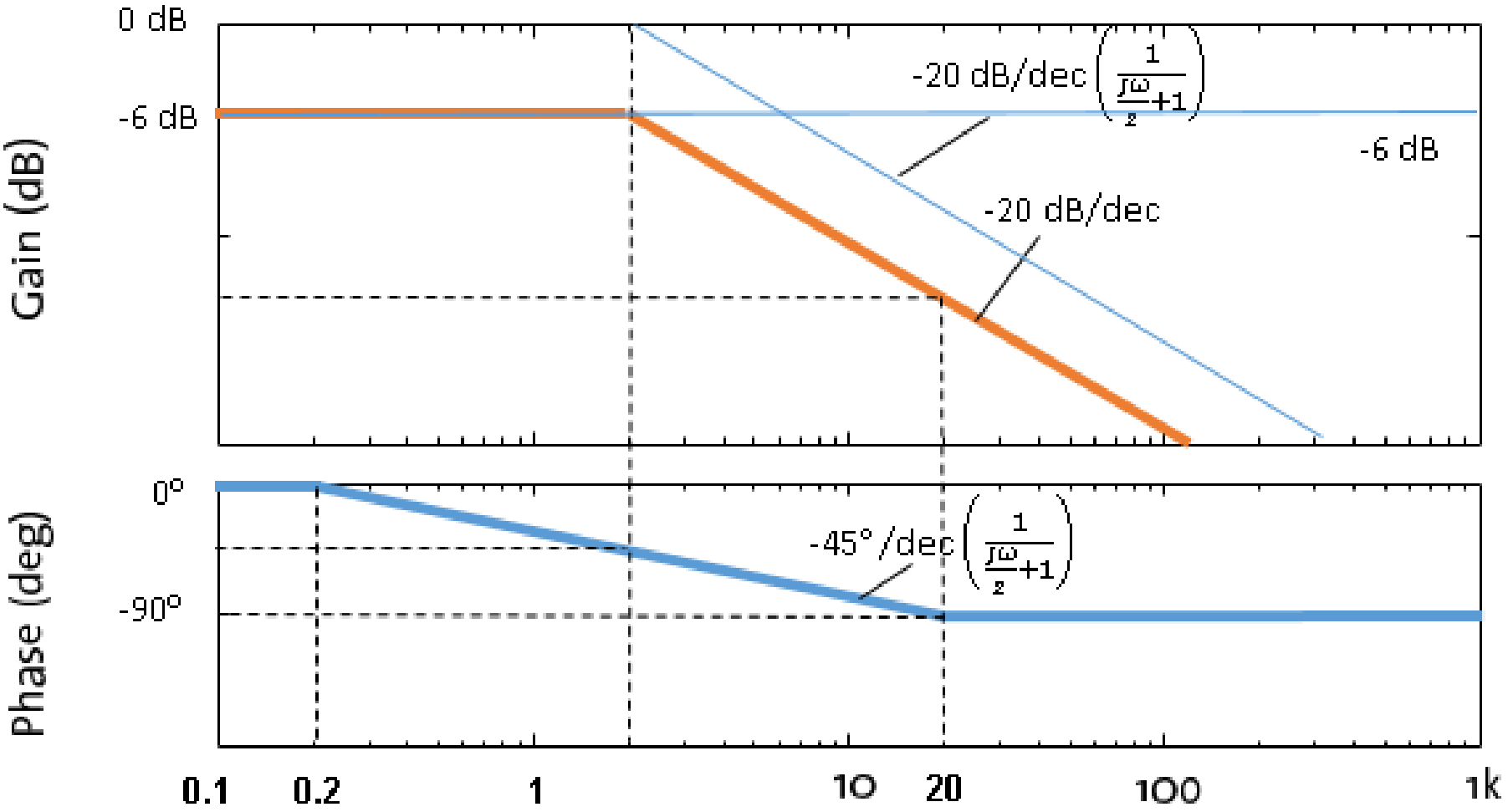
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/2)\left[\frac{1}{j\omega/2 + 1}\right]$$

As a result, DC gain is: 20 log (1/2) = -6 dB.

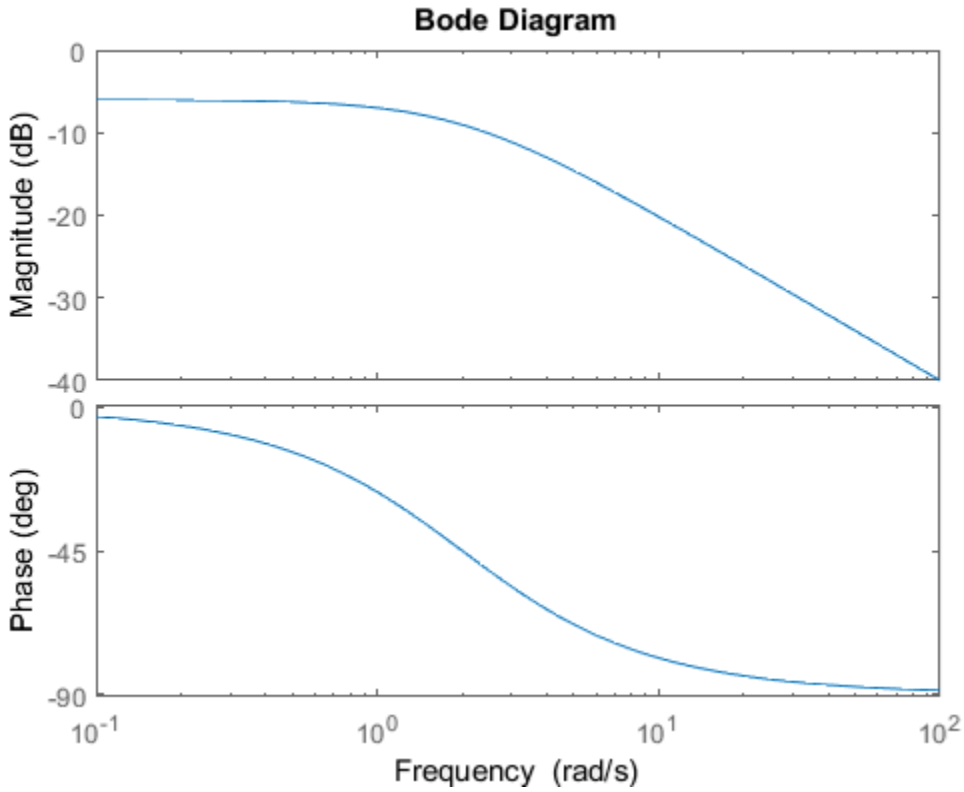
Overall phase shift of the system is: -90° (e.g. one pole).

Determine the break point on the graph i.e. a pole at $j\omega$ = 2.


0 dB
-6 dB
Gain (dB)
-20 dB/dec
$\frac{1}{\frac{j\omega}{2}+1}$
-6 dB
-20 dB/dec
0°
-90°
Phase (deg)
-45°/dec
$\frac{1}{\frac{j\omega}{2}+1}$
0.1
0.2
1
10
20
100
1k

MATLAB Code:

```
clear, clc;
% Transfer function
num=[1];
den=[1,2];
H = tf(num, den);
% Bode Diagram
bode(H);
```

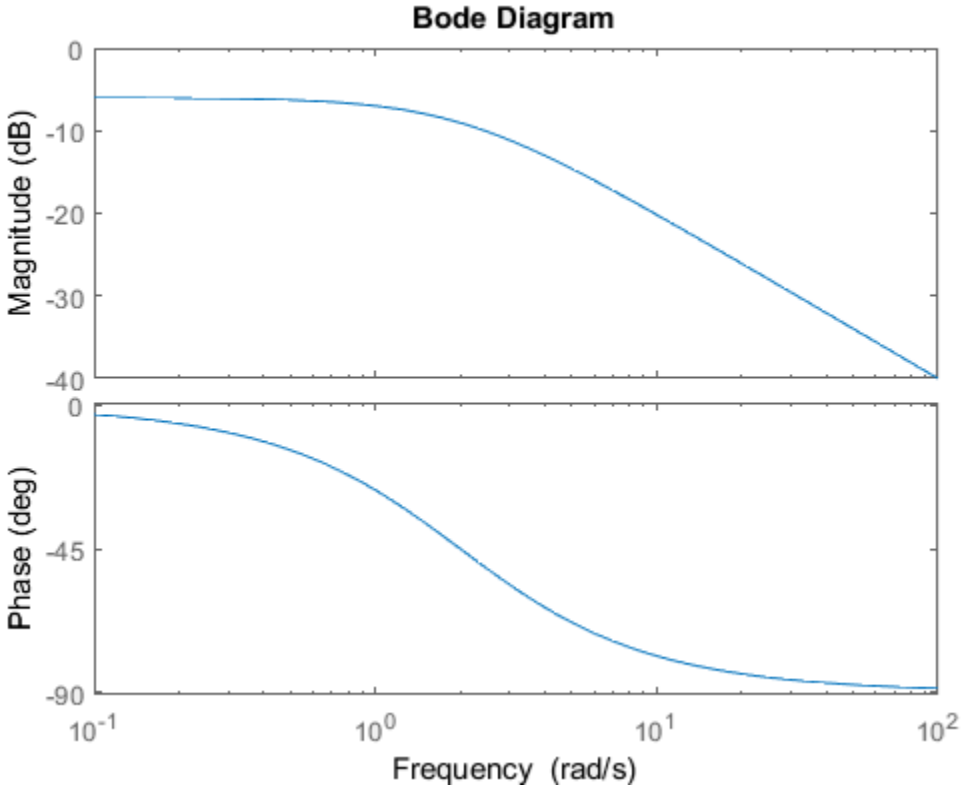


The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{1}{j\omega + 2}$$

For the magnitude plot, at low frequency the gain is -6 dB, a break point at 2 rad/s and after the break point is -20 dB/decade slope.

For the phase plot, at low frequency the phase is 0°, a decade before the break point the phase is at -45°/decade slope and at a decade after the break point the phase settles at -90°.



2. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a second-order continuous system with the following transfer function (effect of double poles):

$$G(j\omega) = \frac{1}{(j\omega + 5)(j\omega + 50)}$$

Solution

Manual Drawing:

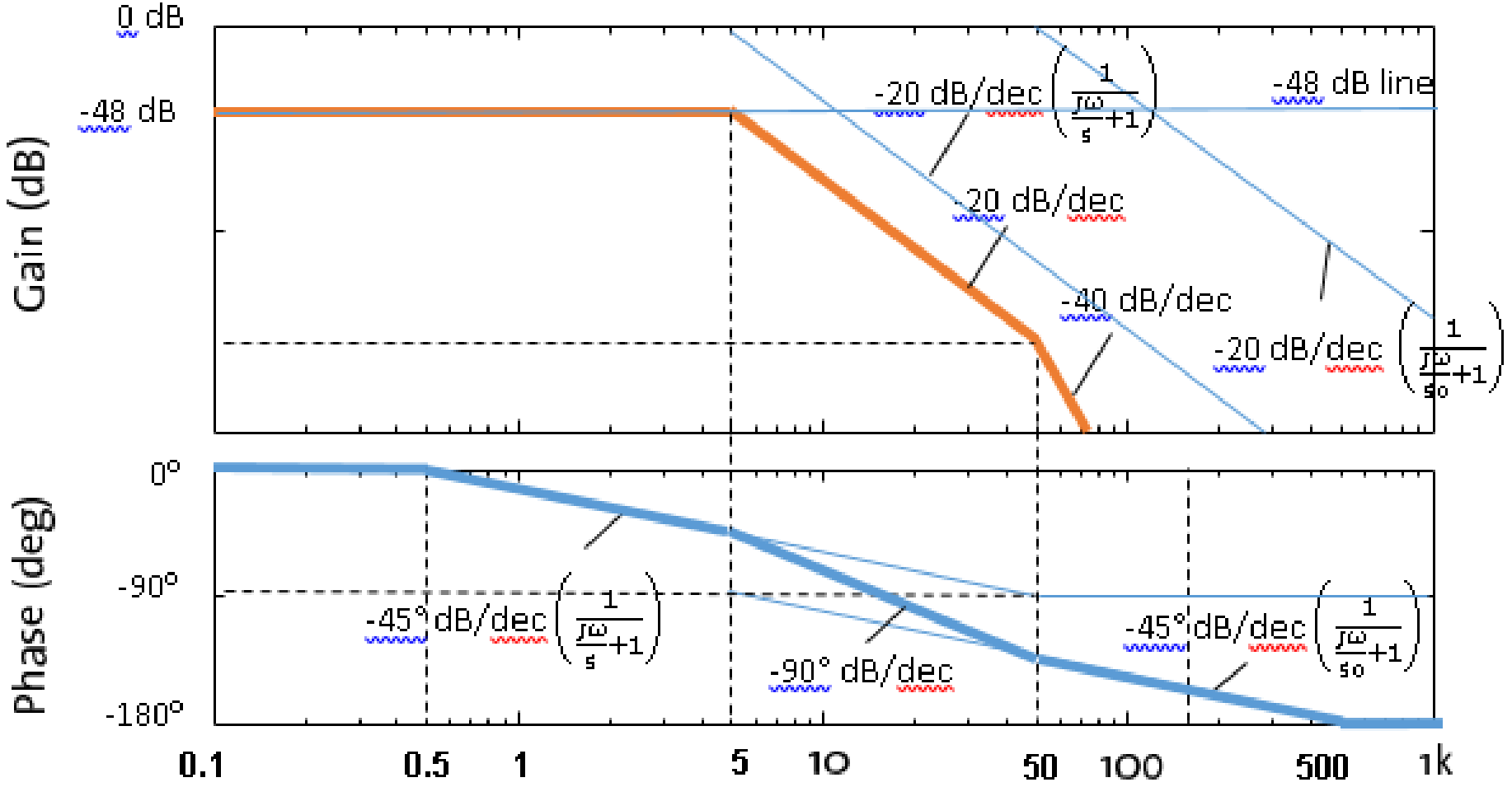
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 1/50)\left[\frac{1}{(j\omega/5 + 1)(j\omega/50 + 1)}\right]$$

As a result, DC gain is: 20 log (1/5 x 1/50) = -48 dB.

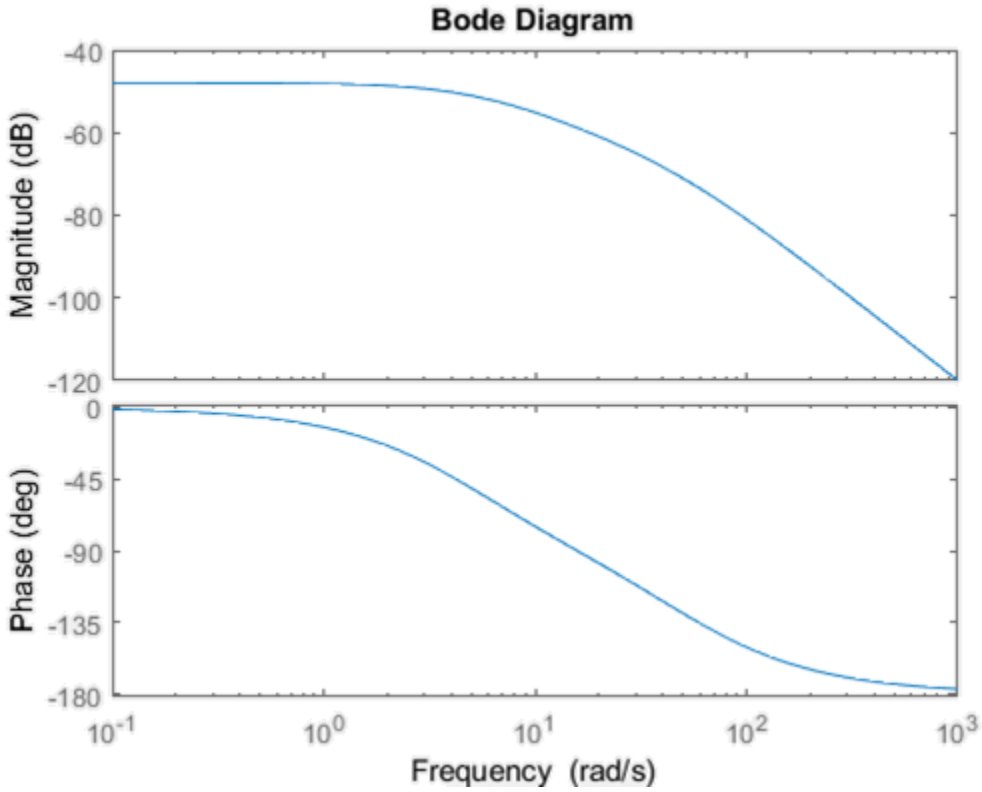
Overall phase shift of the system is: -180° (e.g. two poles).

Determine the break points on the graph i.e. poles at $j\omega$ = 5 and $j\omega$ = 50.


0 dB
-48 dB
Gain (dB)
-20 dB/dec
-48 dB line
-20 dB/dec
-40 dB/dec
-20 dB/dec
0°
-90°
-180°
Phase (deg)
-45° dB/dec
-90° dB/dec
-45° dB/dec
0.1
0.5
1
5
10
50
100
500
1k

MATLAB Code:

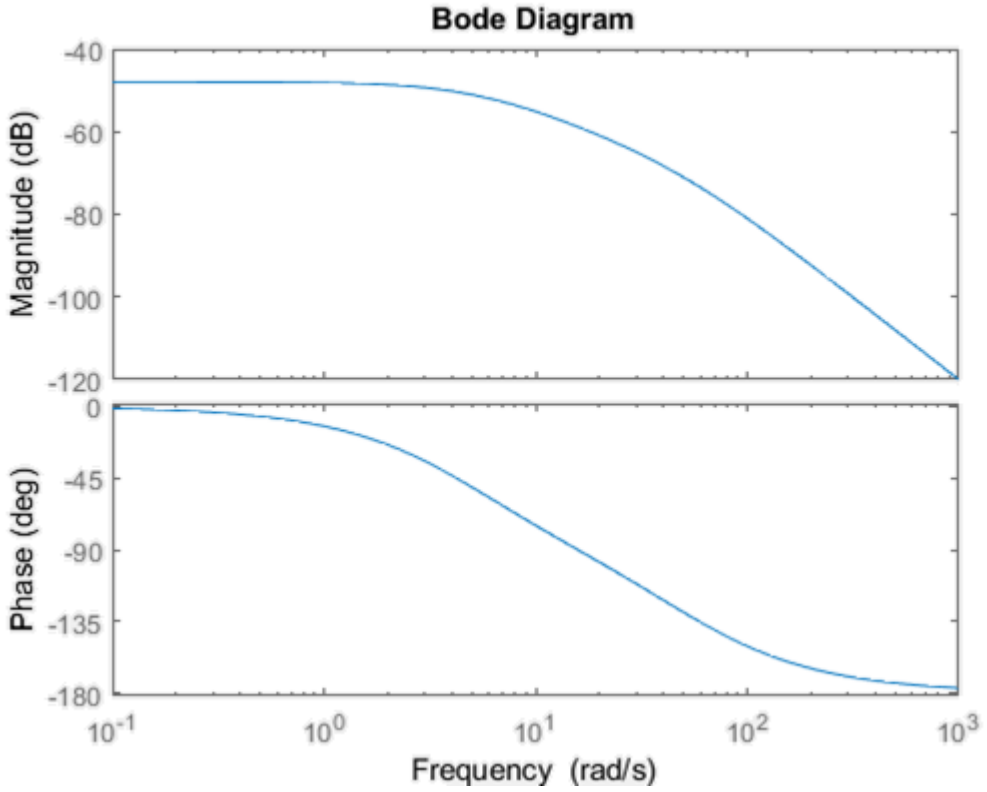
```
clear, clc;
% Transfer function
num=[1];
den1=[1,5];
den2=[1,50];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{1}{(j\omega + 5)(j\omega + 50)}$$

For the magnitude plot, at low frequency the gain is -48 dB, the first break point at 5 rad/s and after this break point the gain is -20 dB/decade slope, then the second break point at 50 rad/s and after this break point the gain is -40 dB/decade slope.



For the phase plot, at low frequency the phase is 0°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase settles at -180°. The phase shifts at the first break point is -45° and at the second break point is -135 °.

3. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a system with the following transfer function (effect of double poles and a zero):

$$G(j\omega) = \frac{j\omega + 50}{(j\omega + 5)(j\omega + 500)}$$

Solution

Manual Drawing:

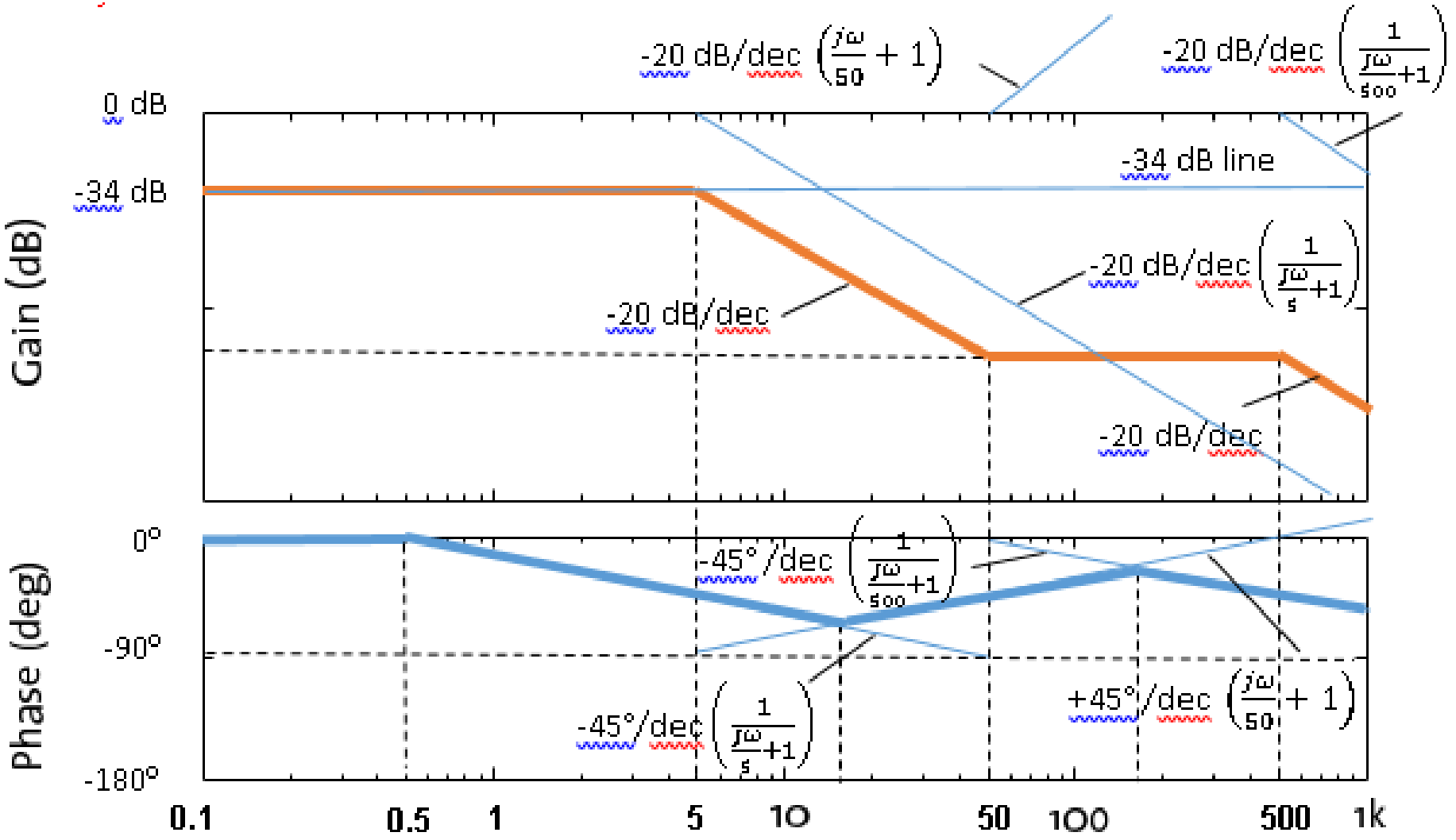
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 1/500 \times 50)\left[\frac{(j\omega/50 + 1)}{(j\omega/5 + 1)(j\omega/500 + 1)}\right]$$

As a result, DC gain is: 20 log (1/5 x 1/500 x 50) = -34 dB.

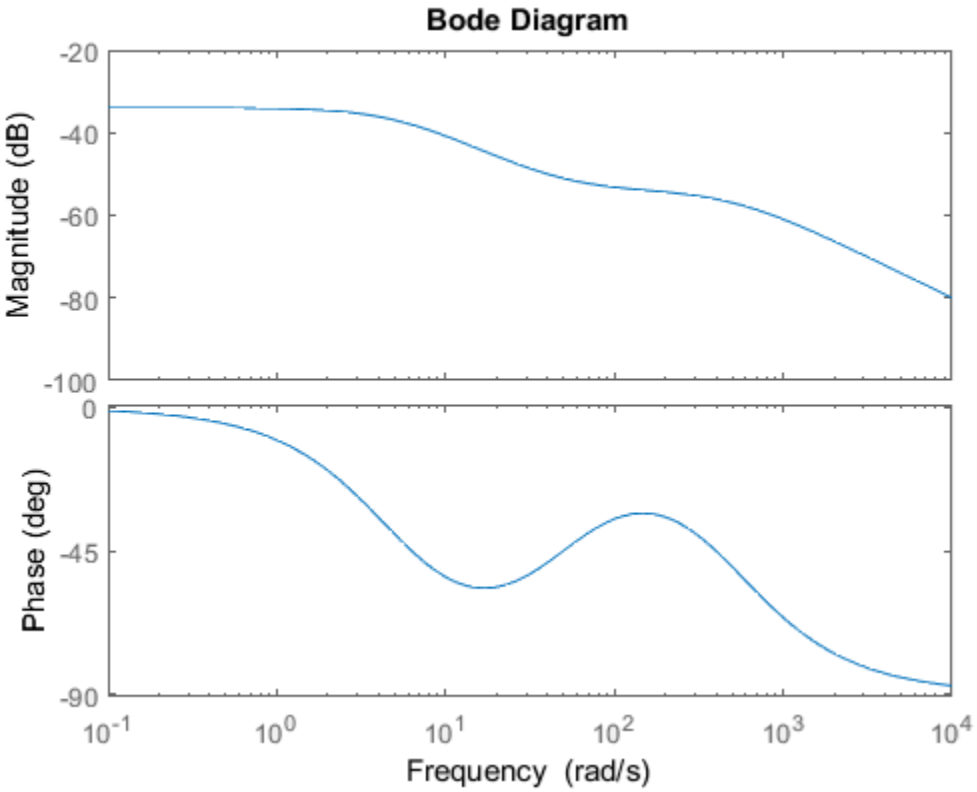
Overall phase shift of the system is: -90° (e.g. one pole).

Determine the break points on the graph i.e. poles at $j\omega$ = 5 and $j\omega$ = 500 and a zero at $j\omega$ = 50.


Gain (dB)
0 dB
-34 dB
-20 dB/dec $\left(\frac{j\omega}{50}+1\right)$
-20 dB/dec $\left(\frac{1}{\frac{j\omega}{500}+1}\right)$
-34 dB line
-20 dB/dec $\left(\frac{1}{\frac{j\omega}{5}+1}\right)$
-20 dB/dec
-20 dB/dec
Phase (deg)
0°
-90°
-180°
-45°/dec $\left(\frac{1}{\frac{j\omega}{500}+1}\right)$
-45°/dec $\left(\frac{1}{\frac{j\omega}{5}+1}\right)$
+45°/dec $\left(\frac{j\omega}{50}+1\right)$
0.1
0.5
1
5
10
50
100
500
1k

MATLAB Code:

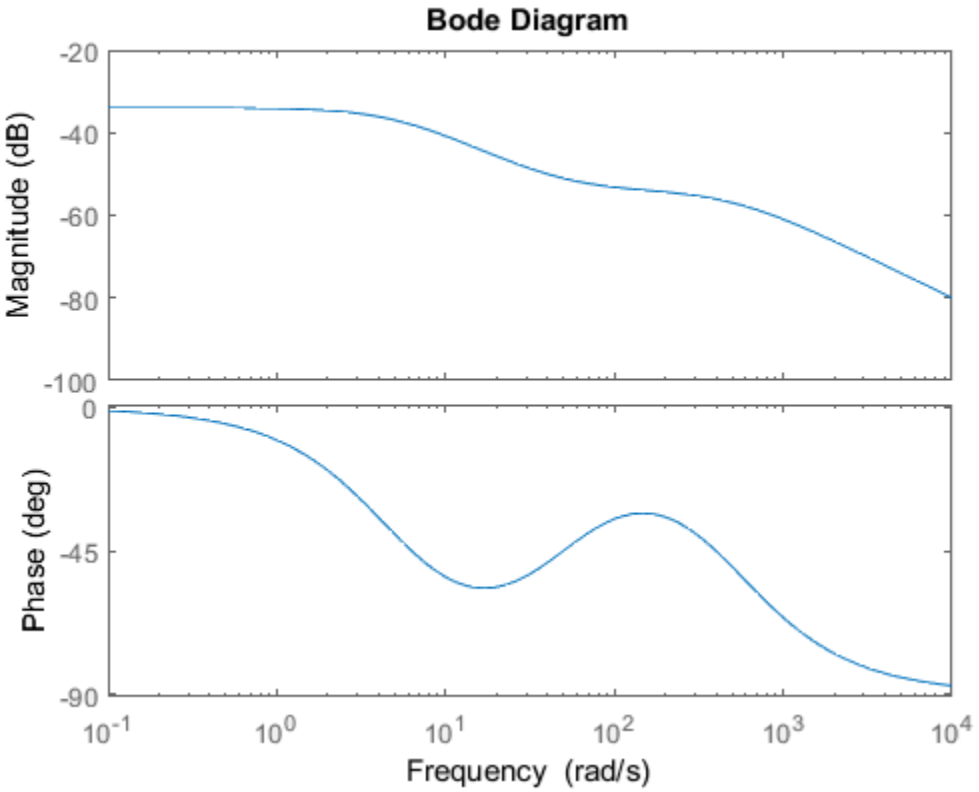
```
clear, clc;
% Transfer function
num=[1,50];
den1=[1,5];
den2=[1,500];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{j\omega + 50}{(j\omega + 5)(j\omega + 500)}$$

For the magnitude plot, at low frequency the gain is -34 dB, the first break point at 5 rad/s and after this break point the gain is -20 dB/decade slope, and then the second break point at 50 rad/s and after this break point the gain is 0 dB/decade slope, and after the third break point at 500 rad/s the gain is -20 dB/decade slope.



For the phase plot, at low frequency the phase is 0°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and at a decade after the third break point the phase shift is at -45 °/decade again.

4. The following exercises show the effect of a pole at origin, a pole and a zero.

a. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 5}{j\omega(j\omega + 50)}$$

Solution

Manual Drawing:

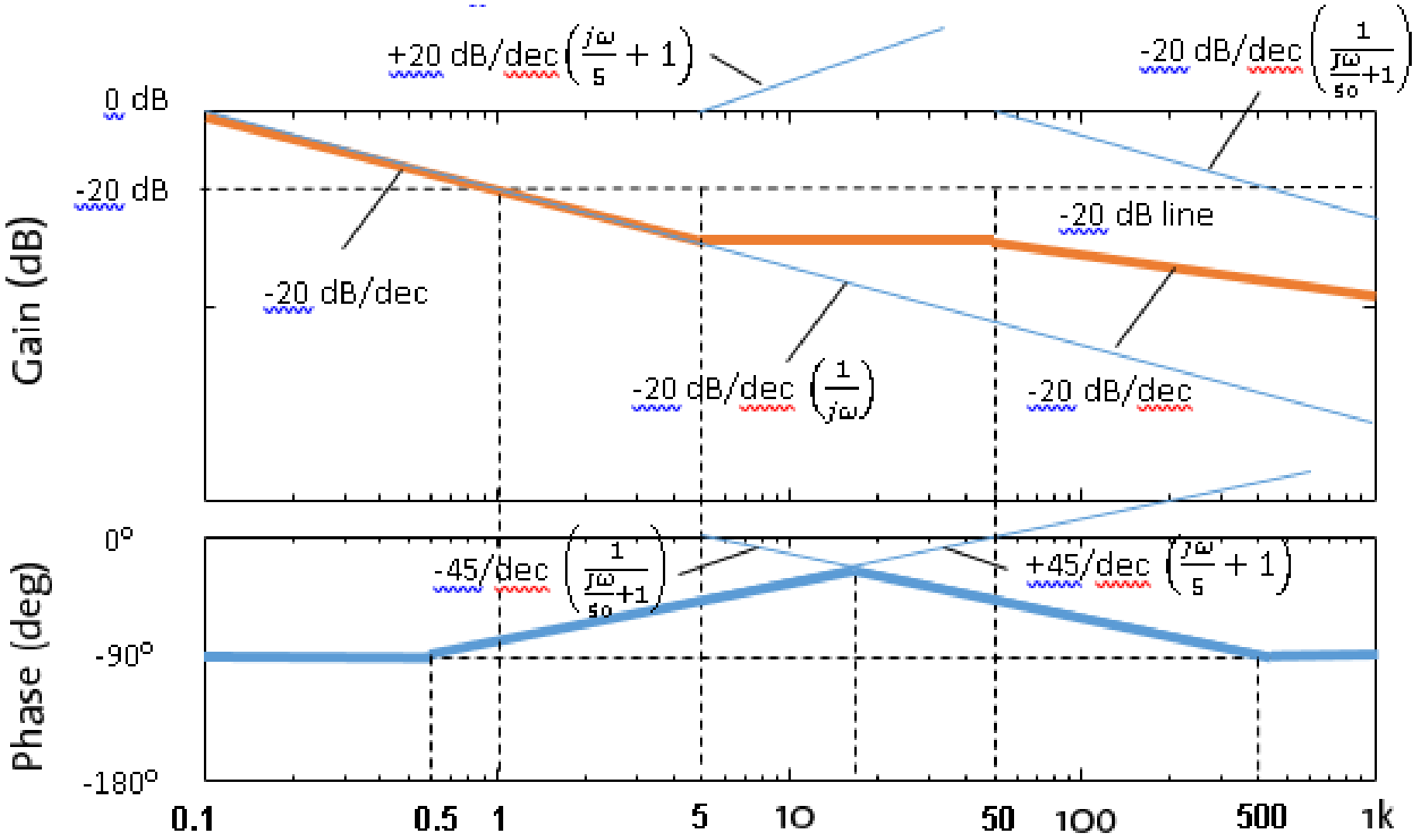
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/50 \times 5)\left[\frac{(j\omega/5 + 1)}{j\omega(j\omega/50 + 1)}\right]$$

As a result, DC gain is: 20 log (1/50 x 5) = -20 dB.

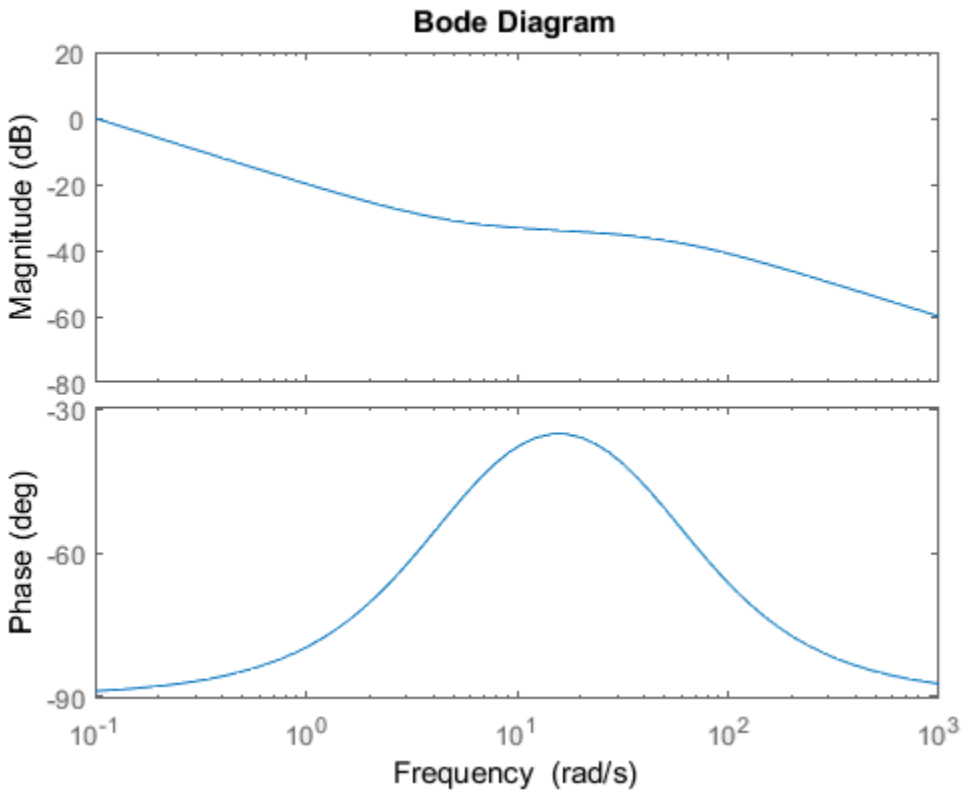
Overall phase shift of the system is: -90° (e.g. one pole).

Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 50 and a zero at $j\omega$ = 5.


+20 dB/dec $\left(\frac{j\omega}{5}+1\right)$
-20 dB/dec $\left(\frac{1}{\frac{j\omega}{50}+1}\right)$
0 dB
-20 dB
-20 dB line
-20 dB/dec
-20 dB/dec $\left(\frac{1}{j\omega}\right)$
-20 dB/dec
Gain (dB)
0°
-45/dec $\left(\frac{1}{\frac{j\omega}{50}+1}\right)$
+45/dec $\left(\frac{j\omega}{5}+1\right)$
-90°
-180°
Phase (deg)
0.1
0.5
1
5
10
50
100
500
1k

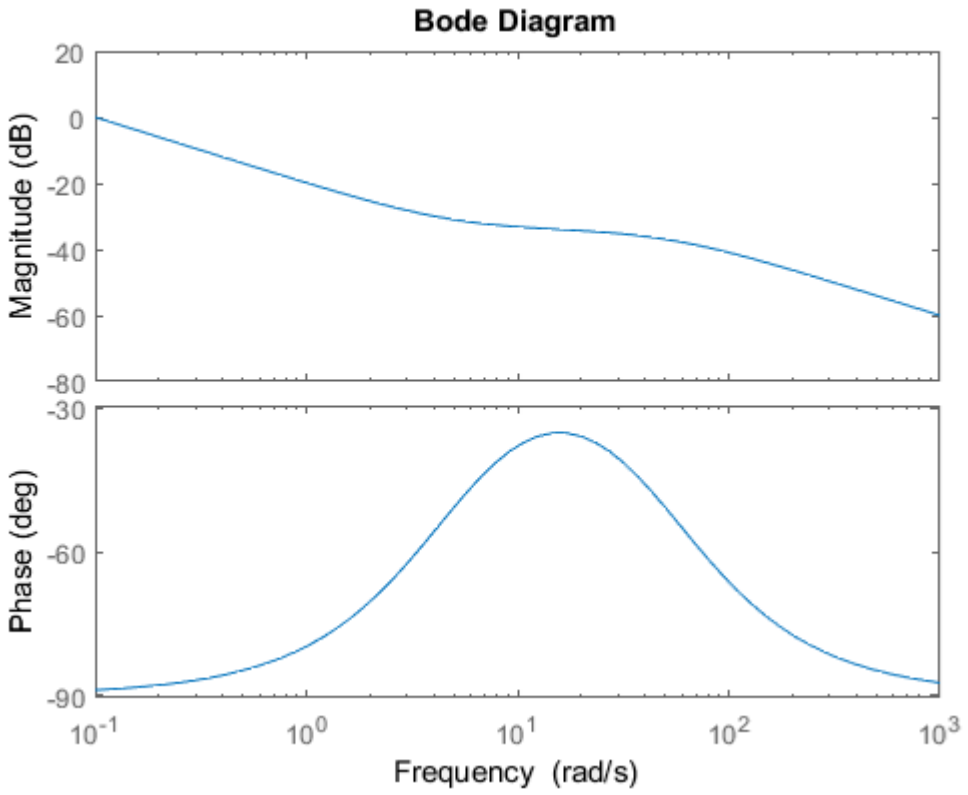
MATLAB Code:

```
clear, clc;
% Transfer function
num=[1,5];
den1=[1,0];
den2=[1,50];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{j\omega + 5}{j\omega(j\omega + 50)}$$



For the magnitude plot, at low frequency the gain is approximately 0 dB, the first break point at 1 rad/s and at this break point the gain is -20 dB. The first break point is at 5 rad/s and after this break point the gain is 0 dB/decade slope, and after the second break point at 50 rad/s the gain is -20 dB/decade slope.

For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and the phase shift settles to -90° at high frequency.

b. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 5)}$$

Solution

Manual Drawing:

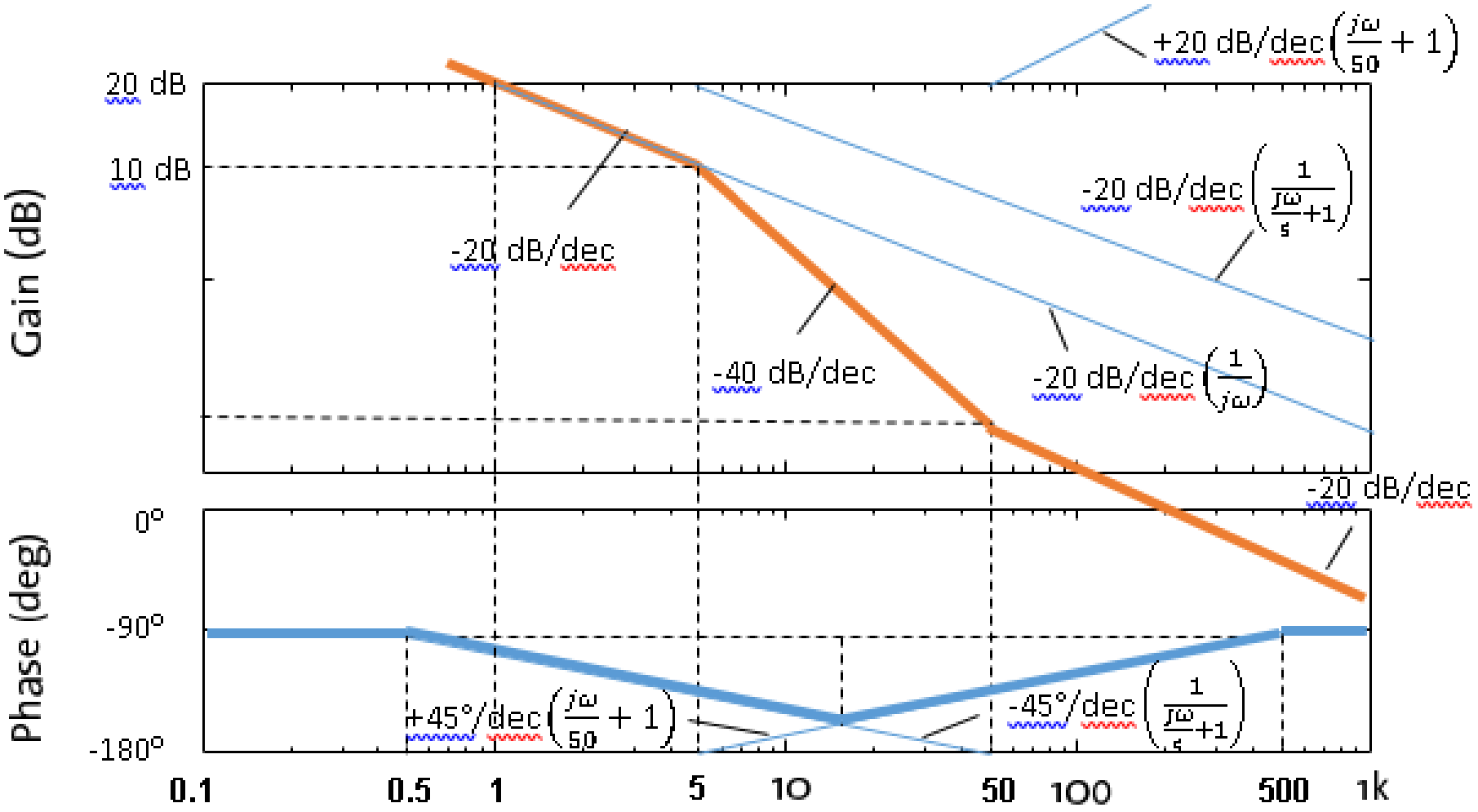
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 50)\left[\frac{(j\omega/50 + 1)}{j\omega(j\omega/5 + 1)}\right]$$

As a result, DC gain is: 20 log (1/5 x 50) = 20 dB.

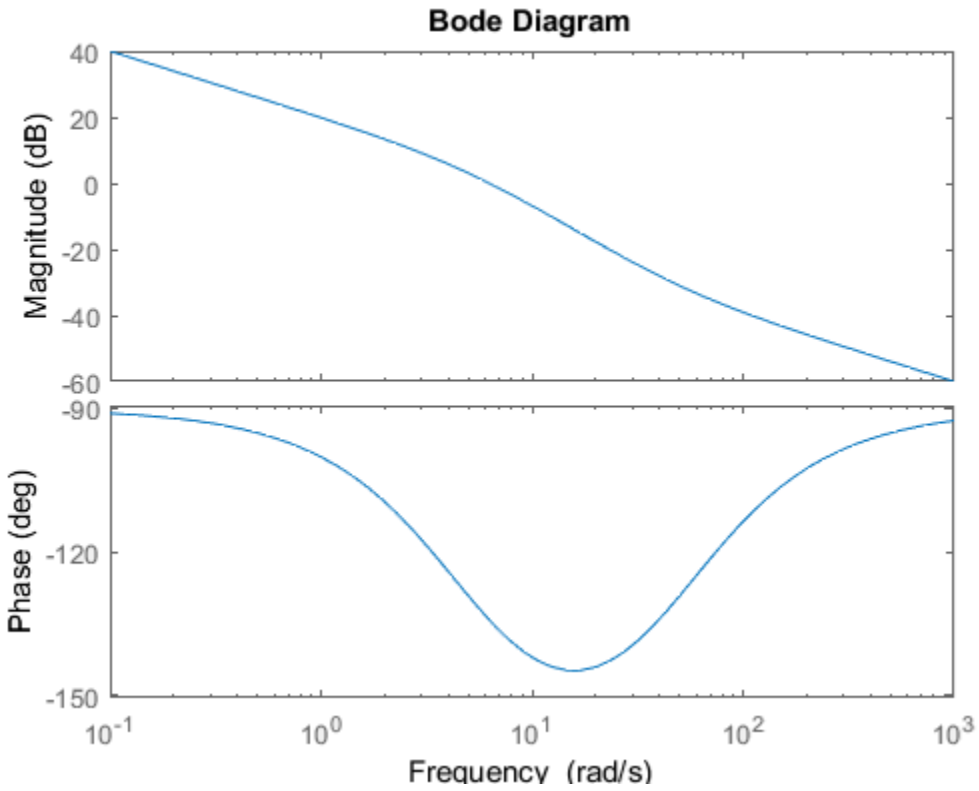
Overall phase shift of the system is: -90° (e.g. one pole).

Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 5 and a zero at $j\omega$ = 50.


Gain (dB)
20 dB
10 dB
-20 dB/dec
-40 dB/dec
-20 dB/dec
+20 dB/dec
-20 dB/dec
-20 dB/dec
Phase (deg)
0°
-90°
-180°
+45°/dec
-45°/dec
0.1
0.5
1
5
10
50
100
500
1k

MATLAB Code:

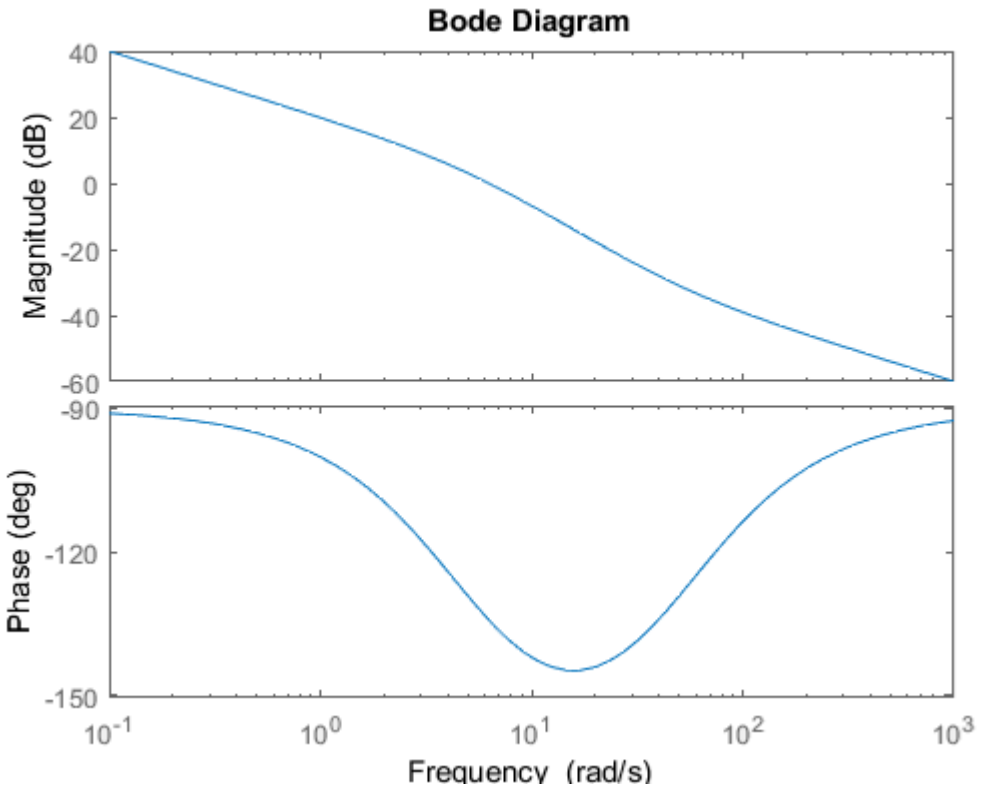
```
clear, clc;
% Transfer function
num=[1,50];
den1=[1,0];
den2=[1,5];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 5)}$$

For the magnitude plot, at low frequency the gain is approximately 20 dB at 1 rad/s. The first break point is at 5 rad/s and after this break point the gain is -40 dB/decade slope, and after the second break point at 50 rad/s the gain is -20 dB/decade slope.



For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and the phase shift settles to -90° at high frequency.

c. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 500)}$$

Solution

Manual Drawing:

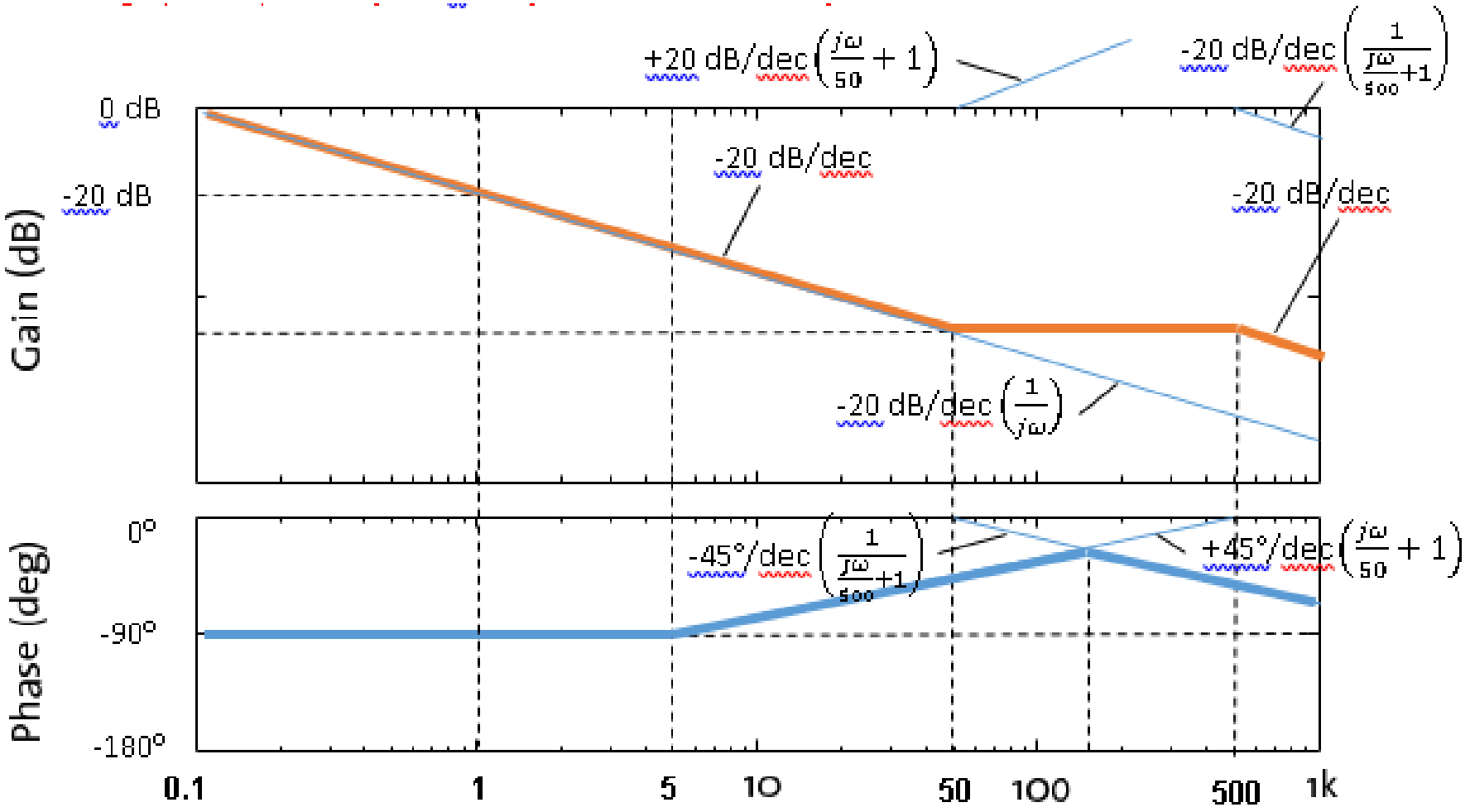
Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/500 \times 50)\left[\frac{(j\omega/50 + 1)}{j\omega(j\omega/500 + 1)}\right]$$

As a result, DC gain is: 20 log (1/500 x 50) = -20 dB.

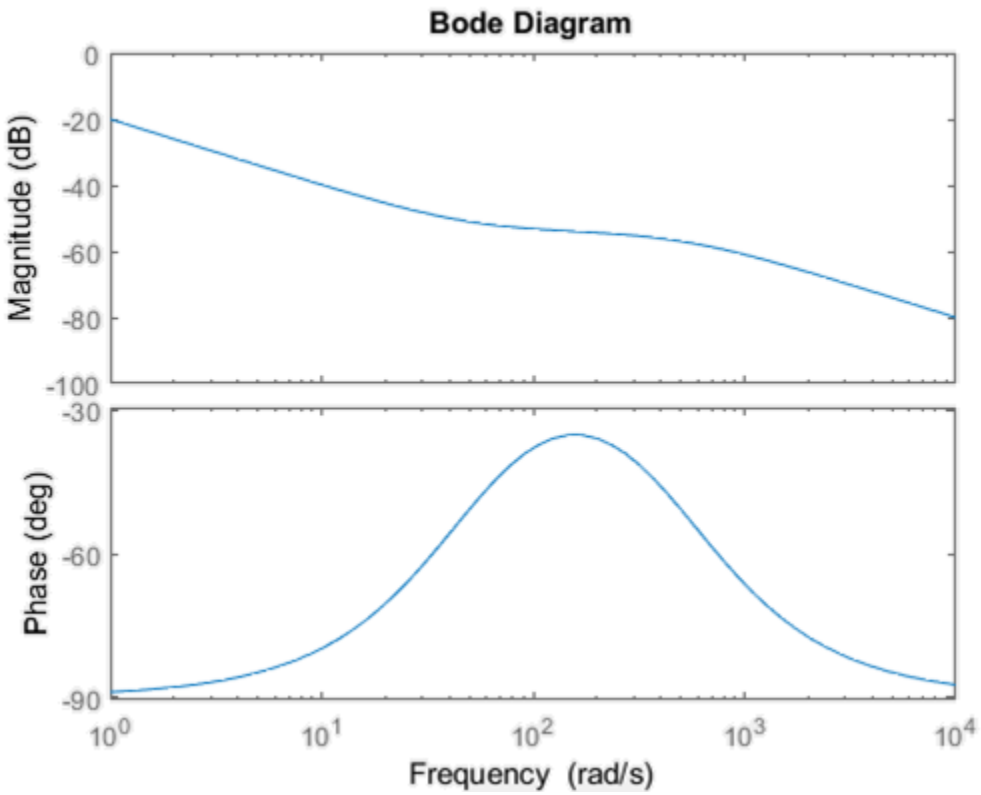
Overall phase shift of the system is: -90° (e.g. one pole).

Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 500 and a zero at $j\omega$ = 50.


+20 dB/dec $\left(\frac{j\omega}{50}+1\right)$
-20 dB/dec $\left(\frac{1}{\frac{j\omega}{500}+1}\right)$
0 dB
-20 dB
-20 dB/dec
-20 dB/dec
-20 dB/dec $\left(\frac{1}{j\omega}\right)$
Gain (dB)
0°
-90°
-180°
-45°/dec $\left(\frac{1}{\frac{j\omega}{500}+1}\right)$
+45°/dec $\left(\frac{j\omega}{50}+1\right)$
Phase (deg)
0.1
1
5
10
50
100
500
1k

MATLAB Code:

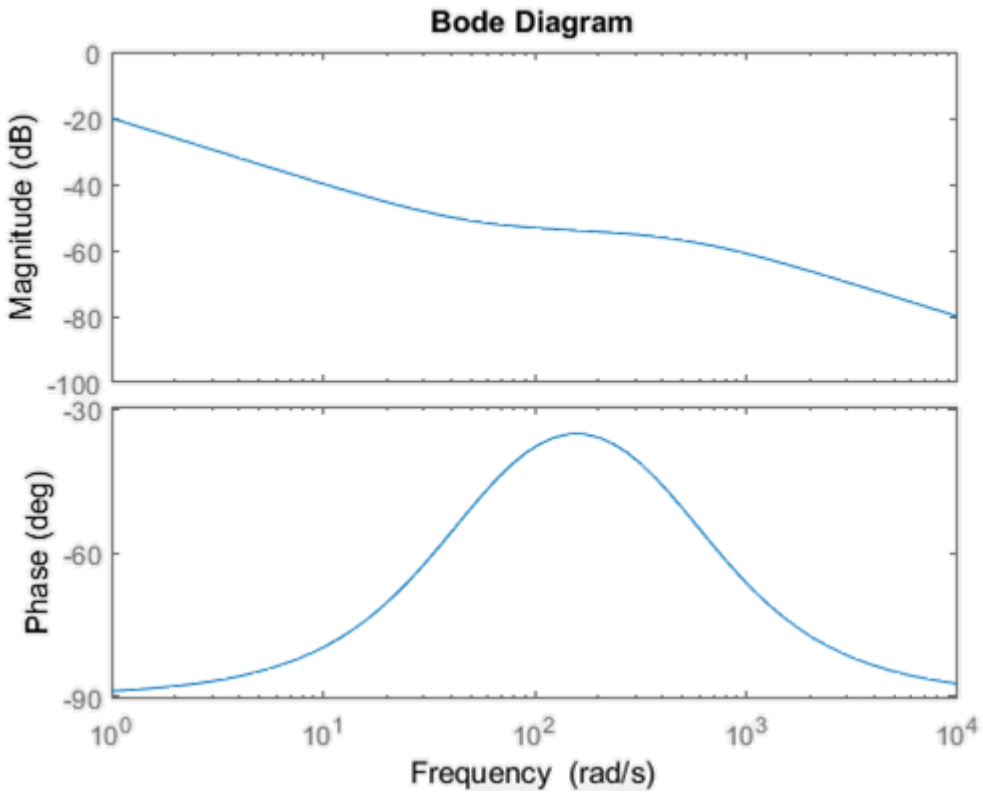
```
clear, clc;
% Transfer function
num=[1,50];
den1=[1,0];
den2=[1,500];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 500)}$$

For the magnitude plot, at low frequency the gain is approximately -20 dB at 1 rad/s. The first break point is at 50 rad/s and after this break point the gain is 0 dB/decade slope, and after the second break point at 500 rad/s the gain is -20 dB/decade slope.



For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at +45°/decade slope and at a decade after the second break point the phase is at -45°/decade and the phase shift settles to -90° at high frequency.

5. Draw manually, simulate in MATLAB, and describe the frequency response at critical frequencies of a complex system with the following transfer function (effect of a zero at origin, a zero, poles and complex poles):

$$G(j\omega) = \frac{j\omega(j\omega + 10)(j\omega + 25)}{(j\omega + 1)(j\omega + 2 + 2j)(j\omega + 2 - 2j)}$$

Solution

Manual Drawing:

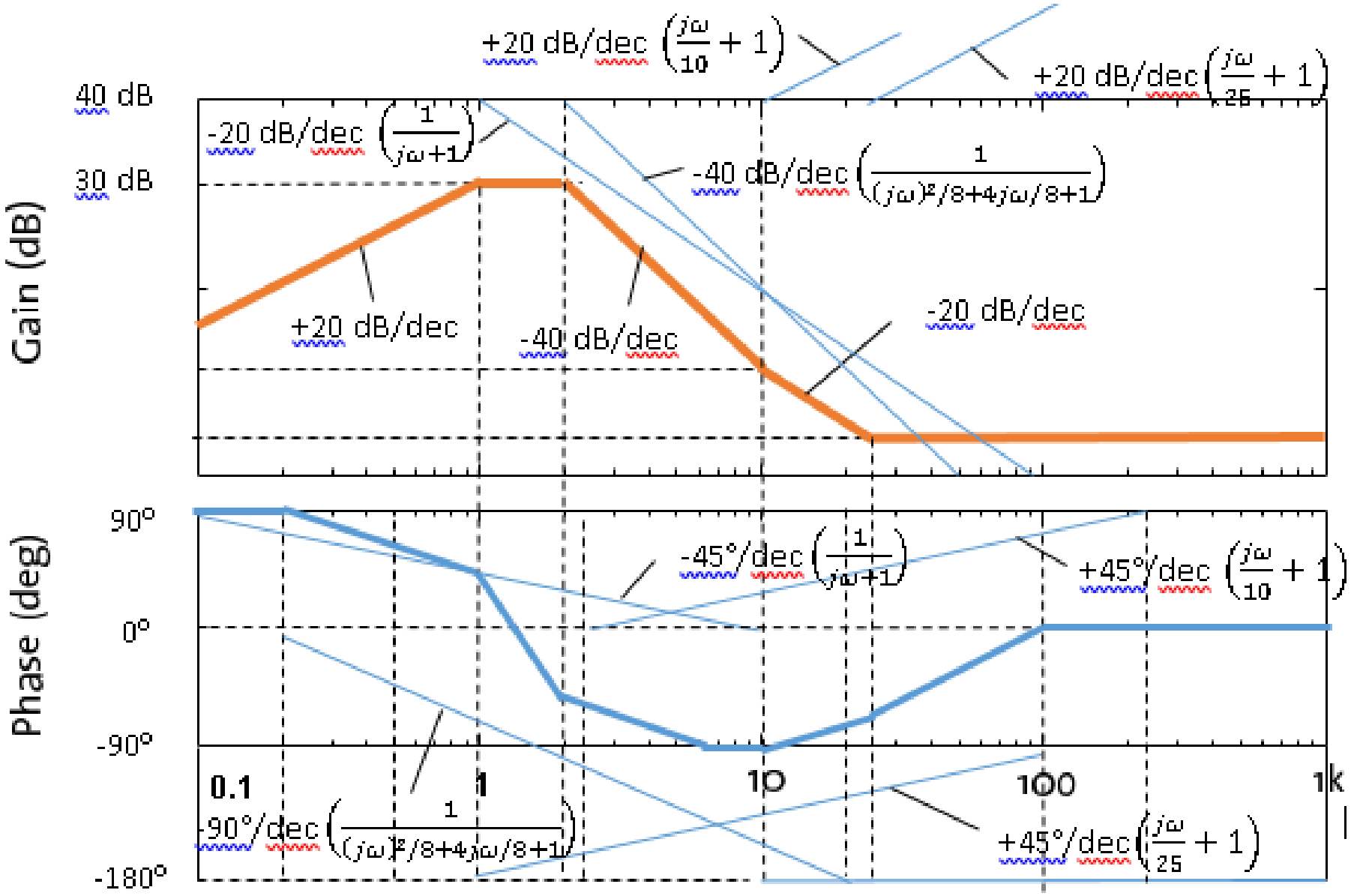
Working out the DC gain of the given transfer function:

$$G(j\omega) = (10 \times 25 \times 1/1 \times 1/8)\left[\frac{j\omega(j\omega/10 + 1)(j\omega/25 + 1)}{(j\omega + 1)\left((j\omega)^2/8 + 4j\omega/8 + 1\right)}\right]$$

As a result, DC gain at ω = 1 rad/s is: 20 log (10 x 25 x 1/1 x 1/8) = 30 dB.

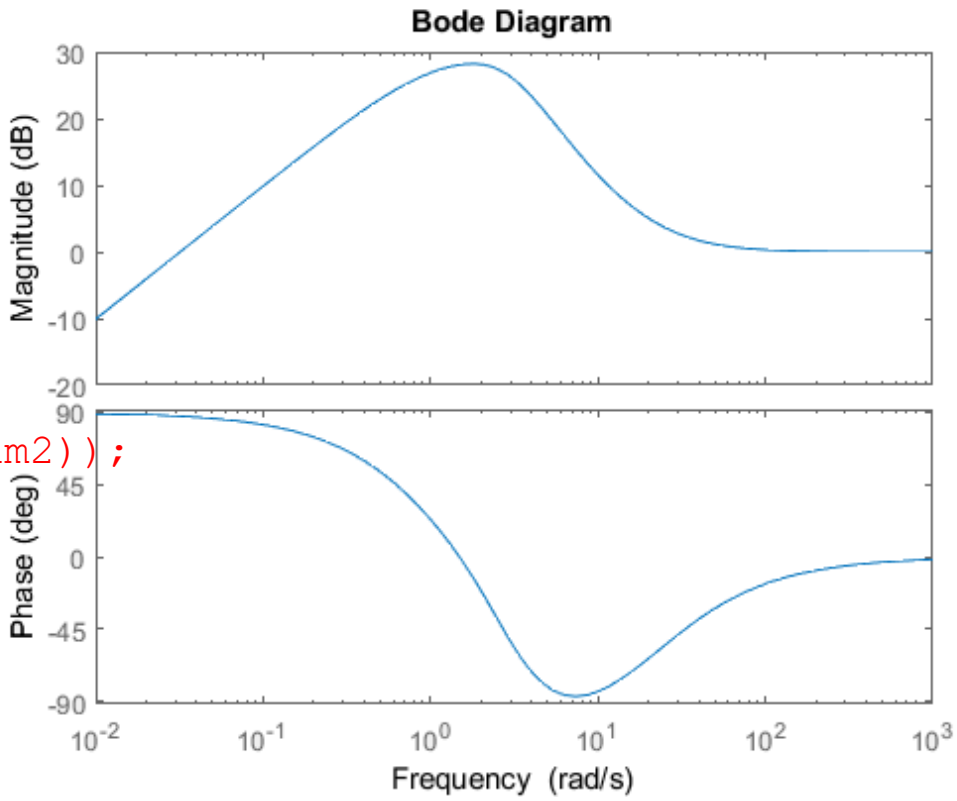
Overall phase shift of the system is: 0° (e.g. three zeros are canceled out by three poles).

Determine the break points on the graph i.e. a pole at $j\omega$ = 1 and two complex poles at $j\omega$ = 2 $\pm$ 2j and zeros at $j\omega$ = 0 and $j\omega$ = 10 and $j\omega$ = 25.


+20 dB/dec $\left(\frac{j\omega}{10}+1\right)$
+20 dB/dec $\left(\frac{j\omega}{25}+1\right)$
40 dB
30 dB
-20 dB/dec $\left(\frac{1}{j\omega+1}\right)$
-40 dB/dec $\left(\frac{1}{(j\omega)^2/8+4j\omega/8+1}\right)$
+20 dB/dec
-40 dB/dec
-20 dB/dec
Gain (dB)
90°
0°
-90°
-180°
-45°/dec $\left(\frac{1}{j\omega+1}\right)$
+45°/dec $\left(\frac{j\omega}{10}+1\right)$
0.1
1
10
100
1k
-90°/dec $\left(\frac{1}{(j\omega)^2/8+4j\omega/8+1}\right)$
+45°/dec $\left(\frac{j\omega}{25}+1\right)$
Phase (deg)

MATLAB Code:

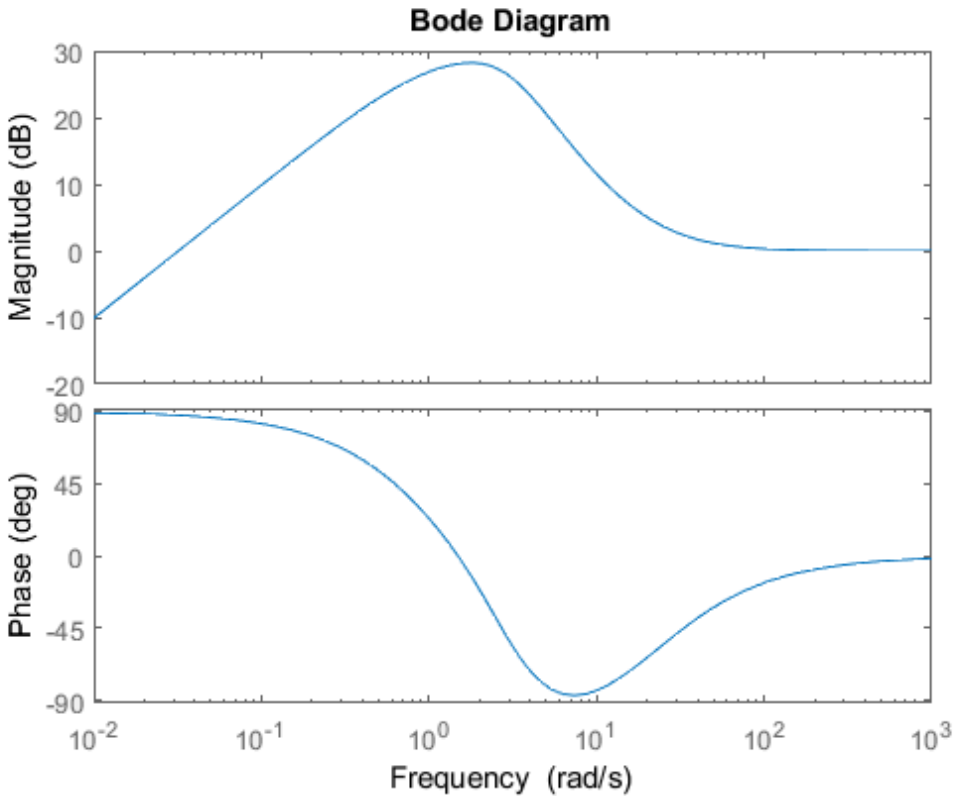
```
clear, clc;
% Transfer function
num1=[1,0];
num2=[1,10];
num3=[1,25];
num=conv(num3,conv(num1,num2));
den1=[1,1];
den2=[1,4,8];
den=conv(den1,den2);
H=tf(num,den);
% Bode Diagram
bode(H);
```



The simulation results confirm the sketches for the magnitude plot and for the phase plot.

$$G(j\omega) = \frac{j\omega(j\omega + 10)(j\omega + 25)}{(j\omega + 1)(j\omega + 2 + 2j)(j\omega + 2 - 2j)}$$

For the magnitude plot, at low frequency the gain is approximately -30 dB at 1 rad/s. The first break point is at 1 rad/s and after this break point the gain is 0 dB/decade slope. After the second break point at $\sqrt{8}$ rad/s the gain is -40 dB/decade slope. After the third break point at 10 rad/s the gain is -20 dB/decade slope and after the fourth break point at 25 rad/s the gain is 0 dB.



For the phase plot, at low frequency the phase is +90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at -90°/decade, after the third and fourth break points the phases are -45° and 0° dB respectively and the phase shift settles to 0° at high frequency.