XMUT220 Signals and Systems

Demo 5.3. Frequency Domain Analysis

1. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a first-order continuous system with the following transfer function (e.g. effect of a pole):

$$G(j\omega) = \frac{1}{j\omega + 2}$$

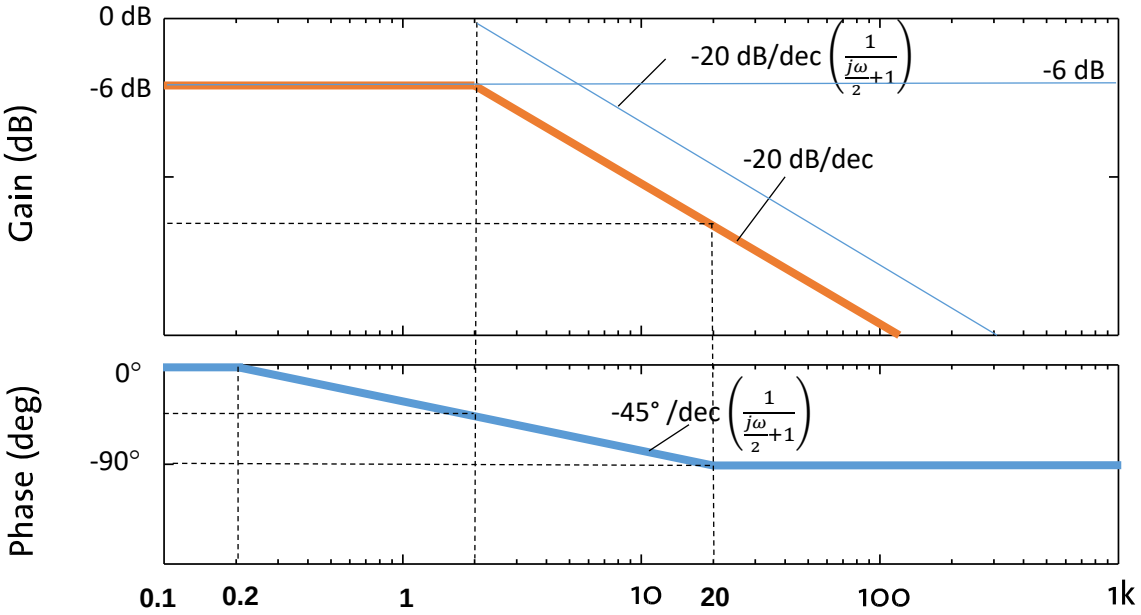
Answer

Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/2)\left[\frac{1}{j\omega/2 + 1}\right]$$

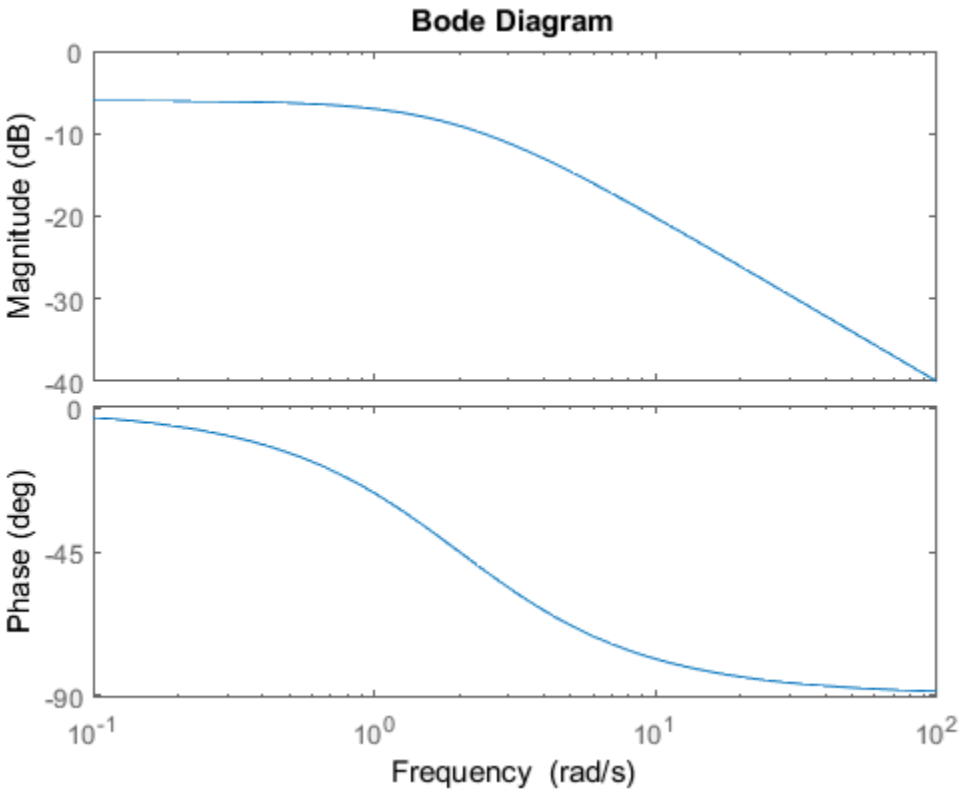
As a result, DC gain is: 20 log (1/2) = -6 dB. Overall phase shift of the system is: -90° (e.g. one pole). Determine the break point on the graph i.e. $j\omega = 2$.



Matlab Code:

```
clear, clc;
% Transfer function
num=[1];
den=[1,2];
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab



For the magnitude plot, at low frequency the gain is -6 dB, a break point at 2 rad/s and after the break point is -20 dB/decade slope.

For the phase plot, at low frequency the phase is 0°, a decade before the break point the phase is at -45°/decade slope and at a decade after the break point the phase settles at -90°.

2. Draw manually, simulate in Matlab and describe the frequency response at critical frequencies of a second-order continuous system with the following transfer function (e.g. effect of double poles):

$$G(j\omega) = \frac{1}{(j\omega + 5)(j\omega + 50)}$$

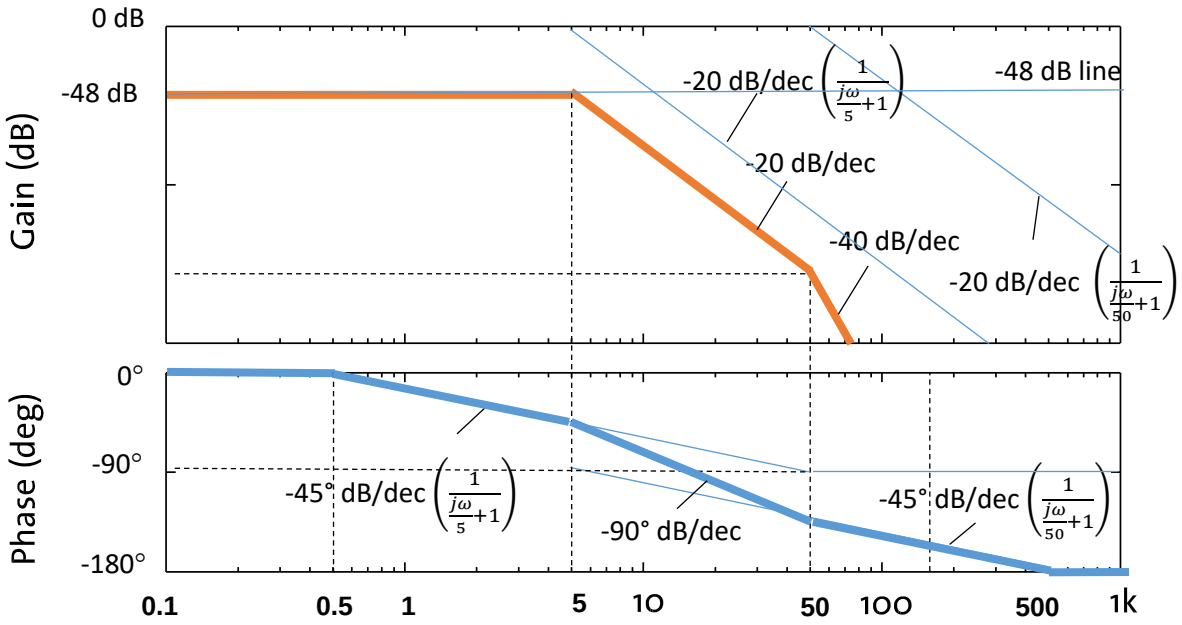
Answer

Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 1/50)\left[\frac{1}{(j\omega/5 + 1)(j\omega/50 + 1)}\right]$$

As a result, DC gain is: 20 log (1/5 x 1/50) = -48 dB. Overall phase shift of the system is: -180° (e.g. two poles). Determine the break points on the graph i.e. $j\omega$ = 5 and $j\omega$ = 50.

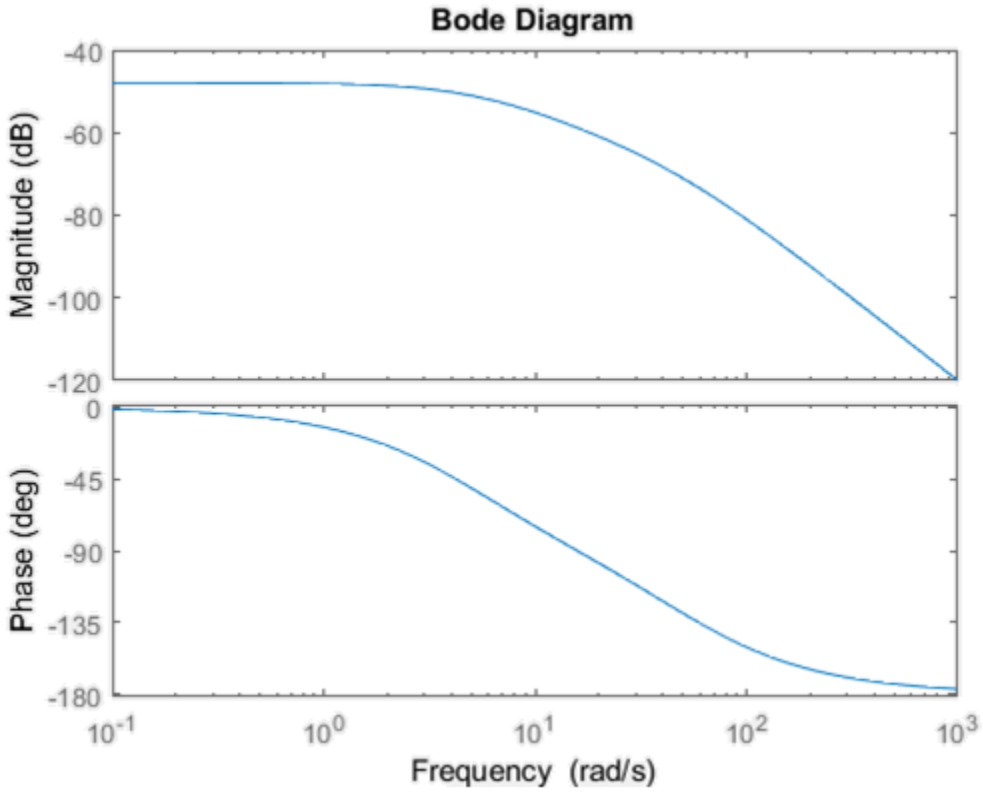


Matlab Code:

```
clear, clc;
% Transfer function
num=[1];
den1=[1,5];
den2=[1,50];
den=conv(den1,den2);
```

```
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab:



For the magnitude plot, at low frequency the gain is -48 dB, the first break point at 5 rad/s and after this break point the gain is -20 dB/decade slope, then the second break point at 50 rad/s and after this break point the gain is -40 dB/decade slope.

For the phase plot, at low frequency the phase is 0°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase settles at -180°. The phase shifts at the first break point is -45° and at the second break point is -135 °.

3. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a system with the following transfer function (e.g. effect of double poles and a zero):

$$G(j\omega) = \frac{j\omega + 50}{(j\omega + 5)(j\omega + 500)}$$

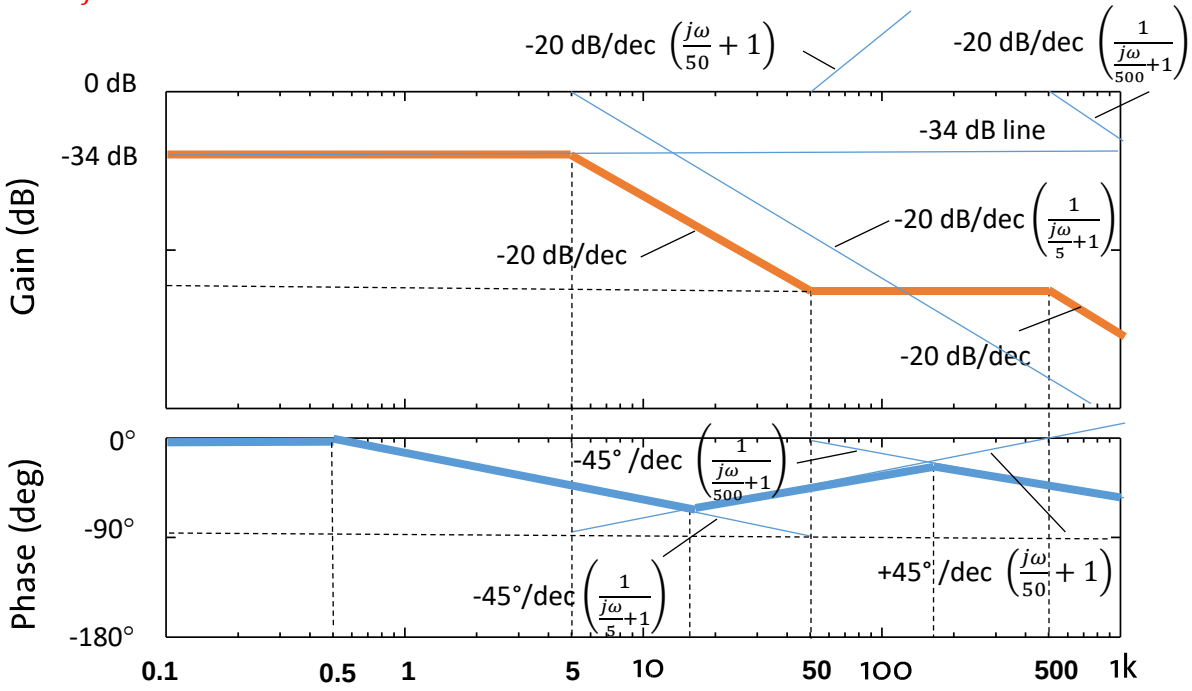
Answer

Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 1/500 \times 50)\left[\frac{(j\omega/50 + 1)}{(j\omega/5 + 1)(j\omega/500 + 1)}\right]$$

As a result, DC gain is: 20 log (1/5 x 1/500 x 50) = -34 dB. Overall phase shift of the system is: -90° (e.g. one pole). Determine the break points on the graph i.e. poles at $j\omega$ = 5 and $j\omega$ = 500 and zero at $j\omega$ = 50.

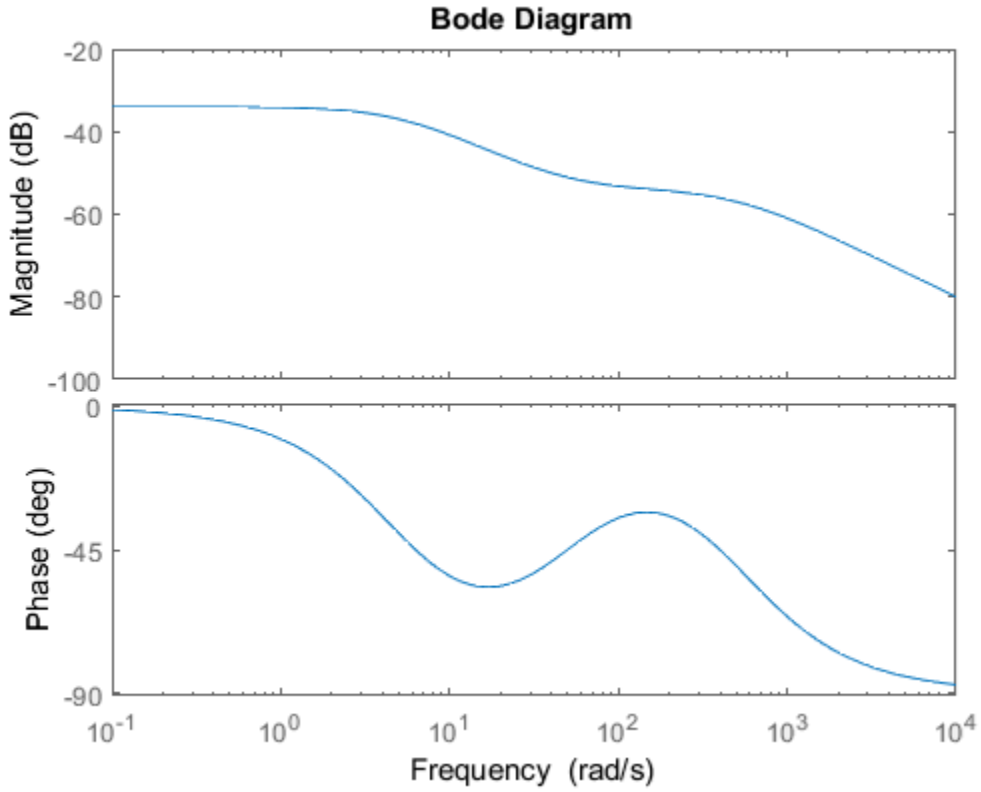


Matlab Code:

```
clear, clc;

% Transfer function

num=[1,50];

den1=[1,5];

den2=[1,500];

den=conv(den1,den2);
```

```
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab



For the magnitude plot, at low frequency the gain is -34 dB, the first break point at 5 rad/s and after this break point the gain is -20 dB/decade slope, and then the second break point at 50 rad/s and after this break point the gain is 0 dB/decade slope, and after the third break point at 500 rad/s the gain is -20 dB/decade slope.

For the phase plot, at low frequency the phase is 0°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and at a decade after the third break point the phase shift is at -45 °/decade again.

4. The following exercises show the effect of a pole at origin, a pole and a zero.

   a. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 5}{j\omega(j\omega + 50)}$$

Answer

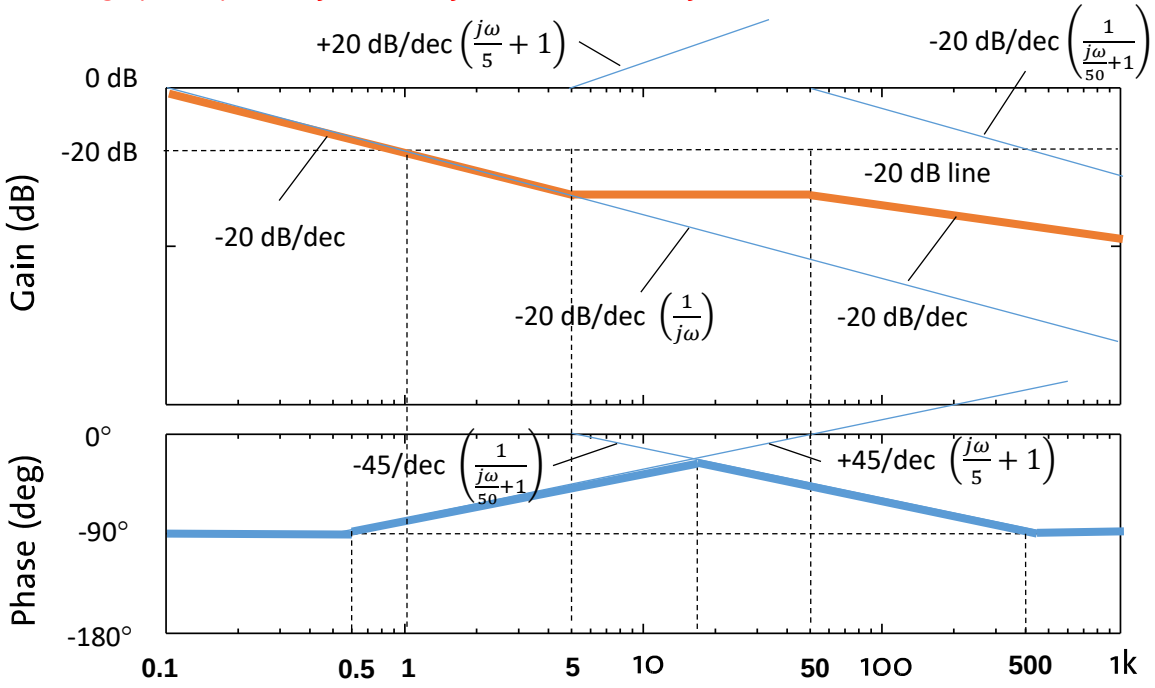
Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/50 \times 5)\left[\frac{(j\omega/5 + 1)}{j\omega(j\omega/50 + 1)}\right]$$

As a result, DC gain at ω = 1 rad/s is: 20 log (1/50 x 5) = -20 dB.

Overall phase shift of the system is: -90° (e.g. one pole). Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 50 and zero at $j\omega$ = 5.

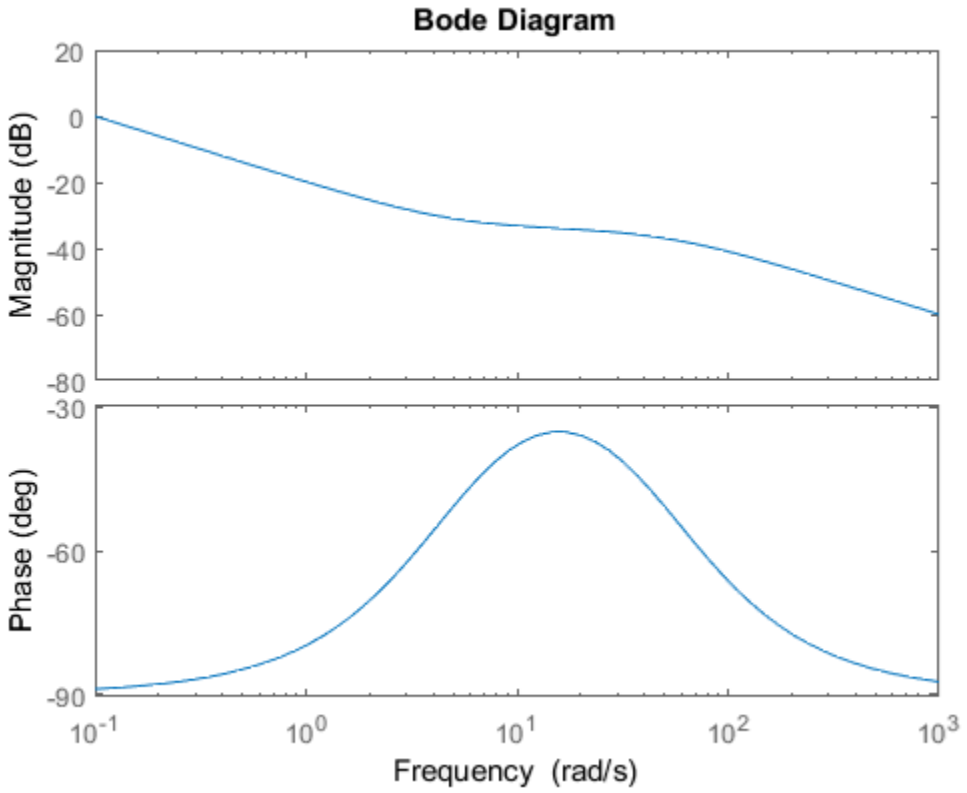


Matlab Code:

```
clear, clc;
% Transfer function
num=[1,5];
```

```
den1=[1,0];
den2=[1,50];
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab:



For the magnitude plot, at low frequency the gain is approximately 0 dB, the first break point at 1 rad/s and at this break point the gain is -20 dB. The first break point is at 5 rad/s and after this break point the gain is 0 dB/decade slope, and after the second break point at 50 rad/s the gain is -20 dB/decade slope.

For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and the phase shift settles to -90° at high frequency.

b. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 5)}$$

Answer

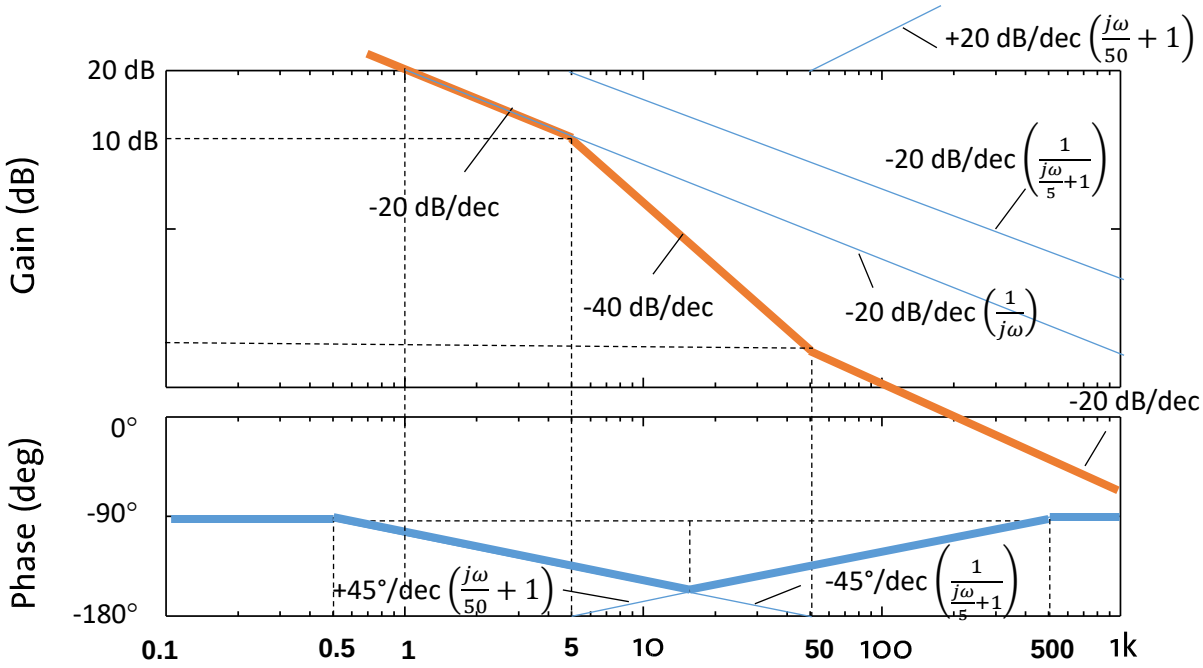
Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/5 \times 50)\left[\frac{(j\omega/50 + 1)}{j\omega(j\omega/5 + 1)}\right]$$

As a result DC gain is at ω = 1 rad/s: 20 log (1/5 x 50) = 20 dB.

Overall phase shift of the system is: -90° (e.g. one pole). Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 5 and zero at $j\omega$ = 50.

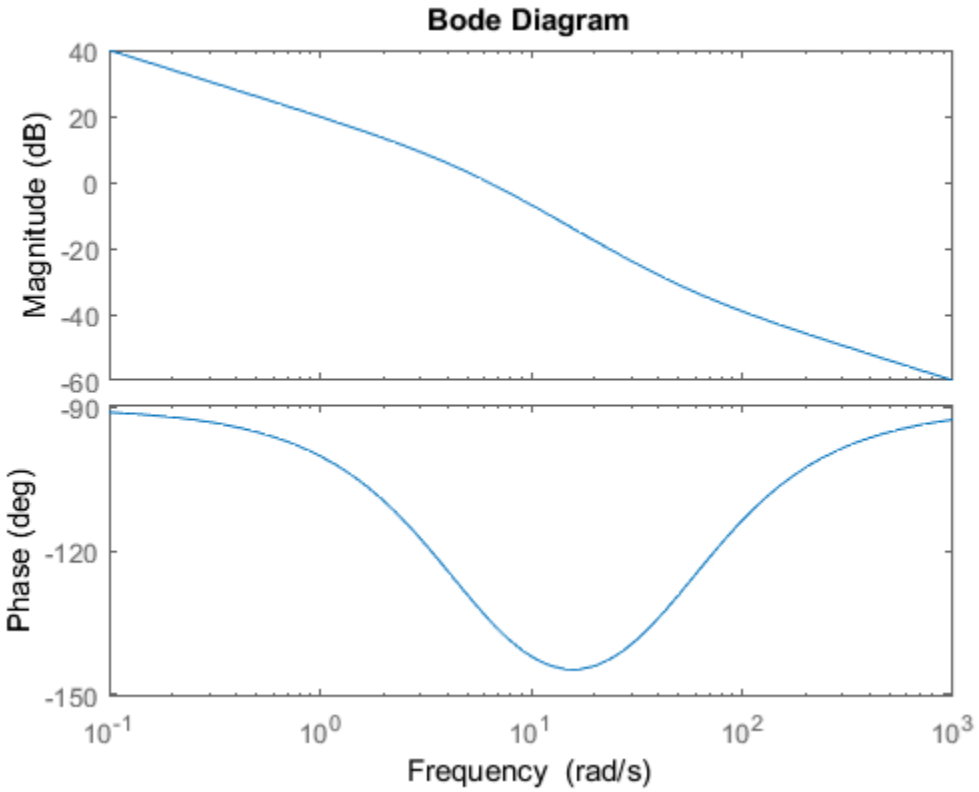


Matlab Code:

```
clear, clc;
% Transfer function
num=[1,50];
den1=[1,0];
den2=[1,5];
```

```
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab



For the magnitude plot, at low frequency the gain is approximately 20 dB at 1 rad/s. The first break point is at 5 rad/s and after this break point the gain is -40 dB/decade slope, and after the second break point at 50 rad/s the gain is -20 dB/decade slope.

For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at +45°/decade and the phase shift settles to -90° at high frequency.

c. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a system with the following transfer function:

$$G(j\omega) = \frac{j\omega + 50}{j\omega(j\omega + 500)}$$

Answer

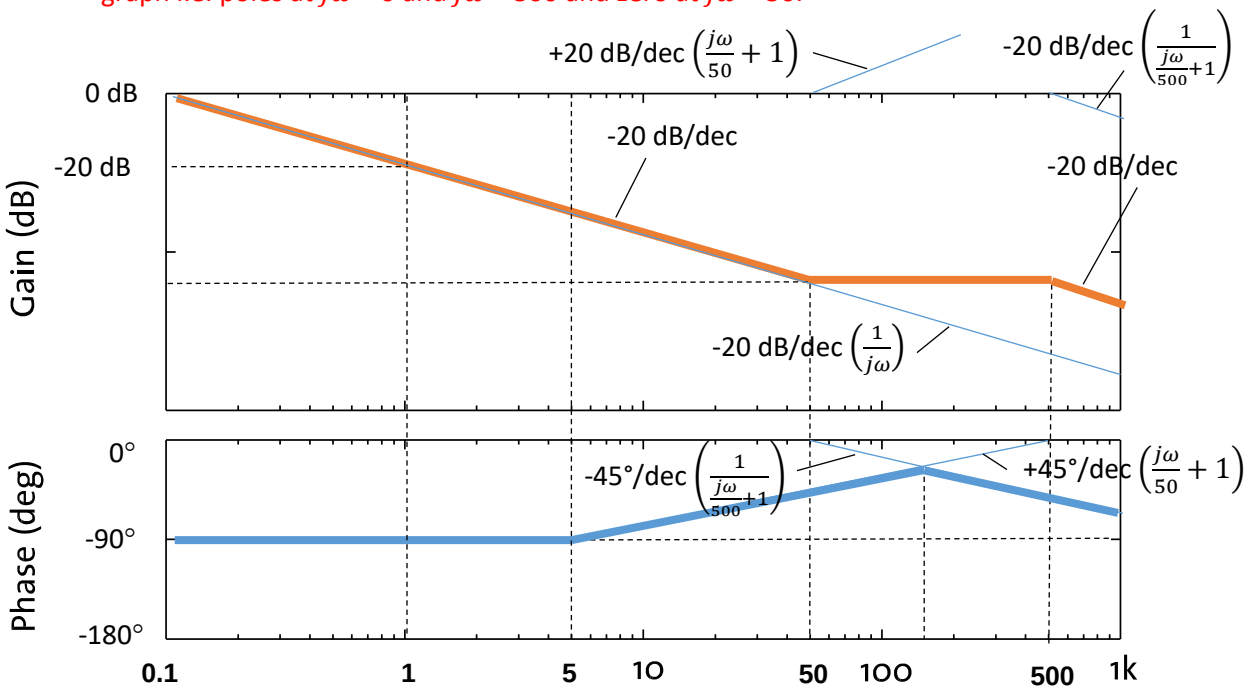
Manual Drawing:

Working out the DC gain of the given transfer function:

$$G(j\omega) = (1/500 \times 50)\left[\frac{(j\omega/50 + 1)}{j\omega(j\omega/500 + 1)}\right]$$

As a result DC gain at ω = 1 rad/s is: 20 log (1/500 x 50) = -20 dB.

Overall phase shift of the system is: -90° (e.g. one pole). Determine the break points on the graph i.e. poles at $j\omega$ = 0 and $j\omega$ = 500 and zero at $j\omega$ = 50.

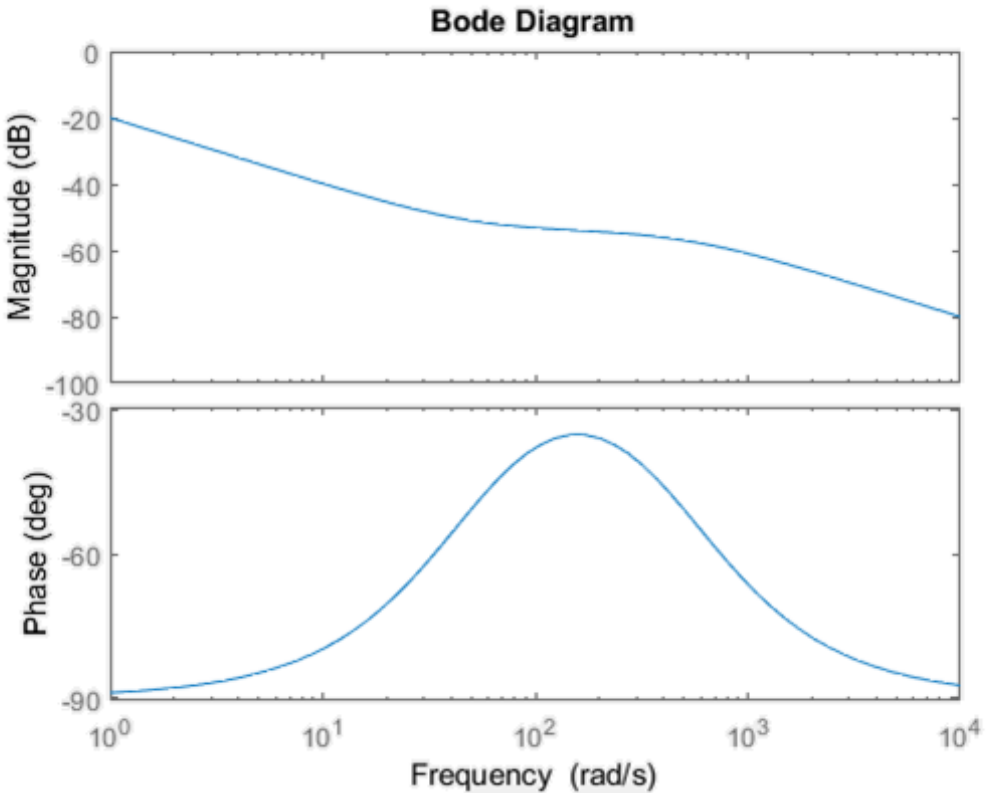


Matlab Code:

```
clear, clc;
% Transfer function
num=[1,50];
den1=[1,0];
den2=[1,500];
```

```
den=conv(den1,den2);
H = tf(num, den);
% Bode Diagram
bode(H);
```

Simulation in Matlab:



For the magnitude plot, at low frequency the gain is approximately -20 dB at 1 rad/s. The first break point is at 50 rad/s and after this break point the gain is 0 dB/decade slope, and after the second break point at 500 rad/s the gain is -20 dB/decade slope.

For the phase plot, at low frequency the phase is -90°, at decade before the first break point the phase is at +45°/decade slope and at a decade after the second break point the phase is at -45°/decade and the phase shift settles to -90° at high frequency.

5. Draw manually, simulate in Matlab, and describe the frequency response at critical frequencies of a complex system with the following transfer function (e.g. effect of a zero at origin, a zero, poles and complex poles):

$$G(j\omega) = \frac{j\omega(j\omega + 10)(j\omega + 25)}{(j\omega + 1)(j\omega + 2 + 2j)(j\omega + 2 - 2j)}$$

Answer

Manual Drawing:

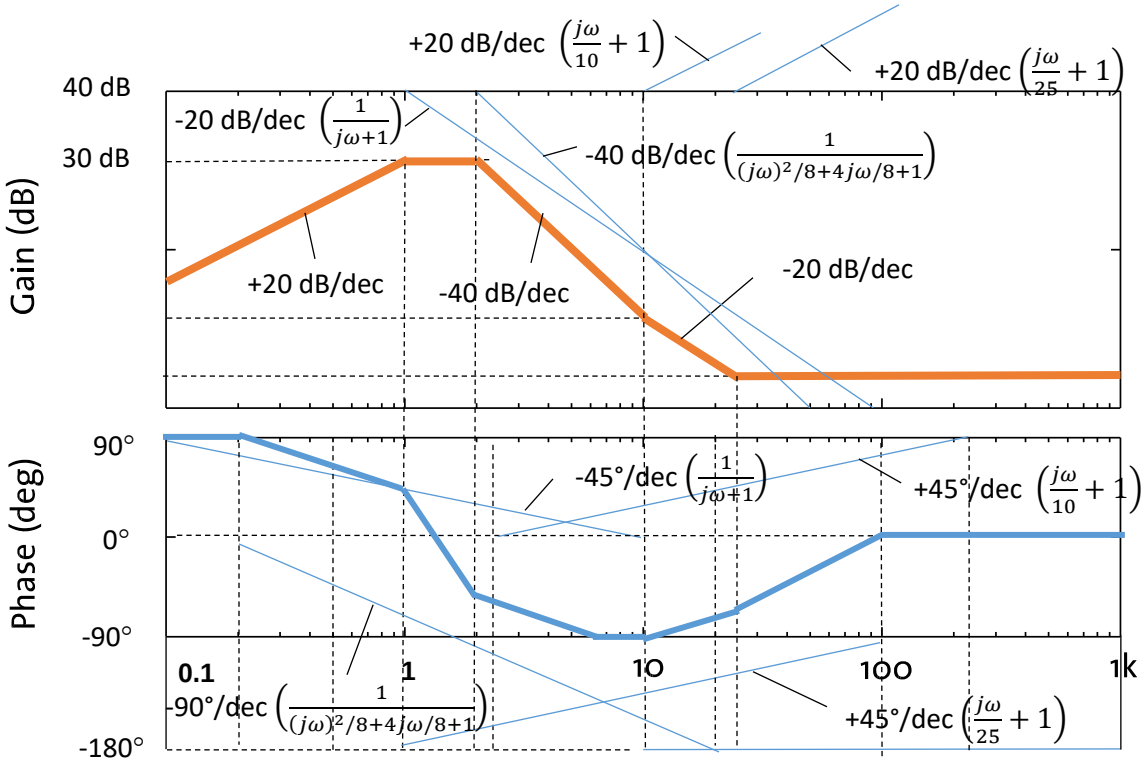
Working out the DC gain of the given transfer function:

$$G(j\omega) = (10 \times 25 \times 1/1 \times 1/8)\left[\frac{j\omega(j\omega/10 + 1)(j\omega/25 + 1)}{(j\omega + 1)((j\omega)^2/8 + 4j\omega/8 + 1)}\right]$$

As a result, DC gain at ω = 1 rad/s is: 20 log (10 x 25 x 1/1 x 1/8) = 30 dB.

Overall phase shift of the system is: 0° (e.g. three zeros are canceled out by a pole and two poles).

Determine the break points on the graph i.e. a pole at $j\omega$ = 1 and two complex poles at $j\omega$ = 2 ± 2j and zeros at $j\omega$ = 0 and $j\omega$ = 10 and $j\omega$ = 25.

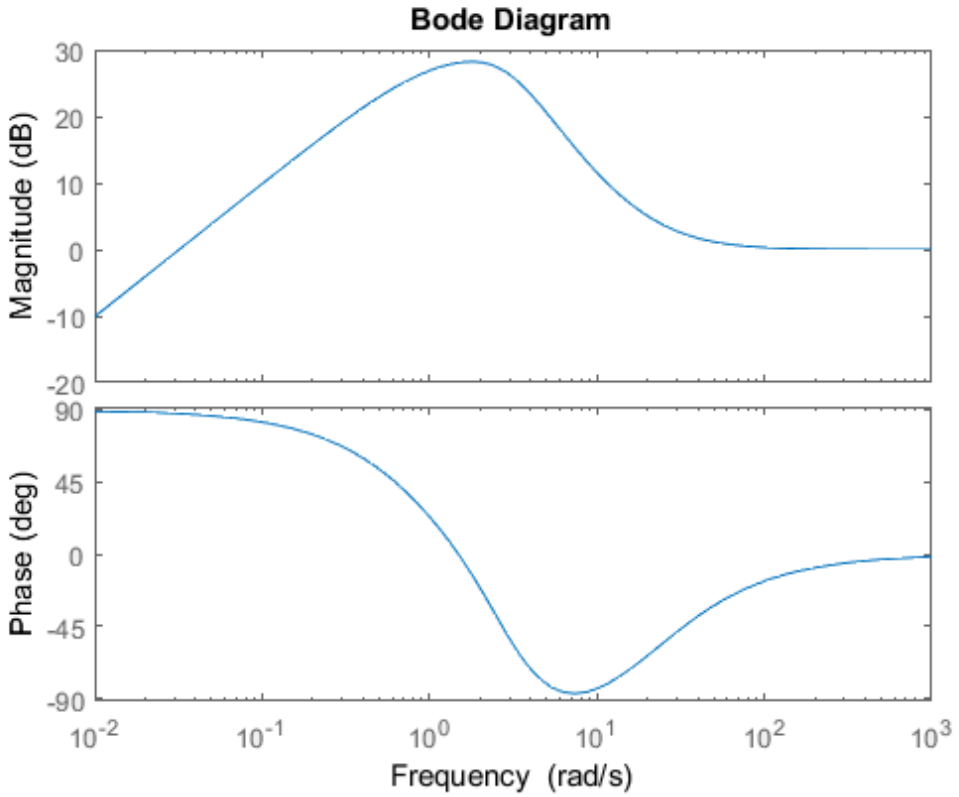


Matlab Code:

```
clear, clc;
```

```
% Transfer function
num1=[1,0];
num2=[1,10];
num3=[1,25];
num=conv(num3,conv(num1,num2));
den1=[1,1];
den2=[1,4,8];
den=conv(den1,den2);
H=tf(num,den);
% Bode Diagram
bode(H);
```

Simulation in Matlab:



For the magnitude plot, at low frequency the gain is approximately 30 dB at 1 rad/s. The first break point is at 1 rad/s and after this break point the gain is 0 dB/decade slope. After the second break

point at $\sqrt{8}$ rad/s the gain is -40 dB/decade slope. After the third break point at 10 rad/s the gain is -20 dB/decade slope and after the fourth break point at 25 rad/s the gain is 0 dB.

For the phase plot, at low frequency the phase is +90°, at decade before the first break point the phase is at -45°/decade slope and at a decade after the second break point the phase is at -90°/decade, after the third and fourth break points the phases are -45° and 0° dB respectively and the phase shift settles to 0° at high frequency.