ECEN 220 - Signals and Systems
Lab 3: Fourier Transform

1) First, let us explore some practical examples of the Fourier transform that we might find in practice.
   a) Start by plotting $x(t) = \cos(\omega_0 t)$ over one period for $\omega_0 = 2\pi/10$.
   b) In the same figure[1] plot the Fourier transform of $x(t)$, $X(\omega)$, which can be found with the Matlab command `fft()`. Note that as $X(\omega)$ is potentially complex for arbitrary $x(t)$ you will need two plots: one showing $|X(\omega)|$ vs. ω, the other showing $\angle X(\omega)$ vs. ω.
   c) Comment on how your plots of $X(\omega)$ differ from your expectations, given your theoretical knowledge of $\mathcal{F}\{x(t) = \cos(\omega_0 t)\}$.
   d) Investigate what happens to the difference between the $X(\omega)$ found by Matlab and the $X(\omega)$ that we expect from theory when the step size of t is reduced.[2]

2) Now that you have confirmed the Fourier transform of $x(t) = \cos(\omega_0 t)$, create figures that show the Fourier transforms of the following functions:
   - $x(t) = \sin(\omega_0 t)$
   - $x(t) = e^{j\omega_0 t}$
   - $x(t) = \delta(t)$
   - $x(t) = u(t)$
   - $x(t) = \delta(t - t_0)$
   - $x(t) = e^{-at}u(t), \quad \mathcal{R}\rceil\{a\} > 0$
   - $x(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T}{2} \end{cases}$

   a) As for part 1, each figure should show three plots: $x(t)$, $|X(\omega)|$, and $\angle X(\omega)$.
   b) Annotate your plots with the idealised mathematical expressions for $X(\omega)$ in each case. Note that you may need to scale your plots or inspect their data in the plot editor to determine some of the expressions, particularly for $\angle X(\omega)$.
   c) Comment on any anomalies that you find.
   Hint: See table 4.2 from Oppenheim et al., reproduced in the Lecture Block 4 notes, if you need help determining expressions for $X(\omega)$.

3) EXTENSION — Complete only if time permits.
   In part 1 we found that the Fourier transform of $x(t) = \cos(\omega_0 t)$ is a pair of impulse functions at $\pm\omega_0$, with amplitude given by $N/2$, where N is the number of time steps in t.
   a) Use Matlab to find the Fourier transform of $X(\omega) = 5(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$ for $\omega_0 = \frac{2\pi}{10}$ and $\omega = 0 : \frac{2\pi}{10} : 2\pi$.[3] Create a figure to show your answer.[4]

[1] Use the `subplot()` command to create multiple plots in the same figure.
[2] We can control step size when we define t: `t = min_value:step_size:max_value`.
[3] You will have to think carefully about how to display $(\omega - \omega_0)$, which is potentially negative, in the range $0 : 2\pi$.
[4] So far we have taken the Fourier transforms of functions of t using `fft()`, but we could just as easily take transforms of functions of ω or any other variable using the same command.

b) Matlab has a built in function, `ifft()`, for calculating inverse Fourier transforms, denoted $\mathcal{F}^{-1}\{\}$. The same way `fft()` implements the analysis equation for the Fourier transform, `ifft()` implements the synthesis equation. On the same figure showing $\mathcal{F}\{X(\omega)\}$, plot the inverse Fourier transform of $X(\omega)$.

c) Comment on the difference between $\mathcal{F}\{X(\omega)\}$, $\mathcal{F}^{-1}\{X(\omega)\}$, and your original function, $x(t) = \cos(\omega_0 t)$.

d) Based on your knowledge of the synthesis and analysis equations of the Fourier transform, what other difference would you expect between the outputs of $\mathcal{F}\{X(\omega)\}$ and $\mathcal{F}^{-1}\{X(\omega)\}$ if those outputs were complex?