

Lab 6 – z-Transforms

XMUT220 Signals and Systems

Overview

1. z-Transform characteristics and behaviours.
2. Transform and inverse transform in the z-transform.
3. z-plane and system responses in z-transform.
4. Partial fraction in z-transform

1. The z-transform of a discrete-time signal $x(n)$ is defined as the power series.

$$X(z) = \sum_{n=-\infty}^{+\infty} x(n) z^{-n}$$

Where:

- z is a complex variable.
- n is the order of function.

The relation sometimes called the direct z-transform because it transforms the time-domain signal $x(n)$ into its complex-plane representation $X(z)$.

$$z = re^{j\omega}$$

Hence

$$z^{-n} = r^{-n} e^{-j\omega n} = r^{-n}[\cos(\omega n) - j \sin(\omega n)]$$

Where:

- 'r' is a real number.
- $e^{j\omega}$ is Euler's Number.
- ω is the angular frequency in radians per sample.

a. For a given z-transform relationship, perform the following simulation tasks in MATLAB:

   i. Plot of $r^{-n}\cos(\omega n)$ when $r < 1$ and $\omega = \pi/4$ per samples.
   [10 marks]

   ii. Plot of $r^{-n}\cos(\omega n)$ when $r = 1$ and $\omega = \pi/4$ per samples.
   [10 marks]

   iii. Plot of $r^{-n}\cos(\omega n)$ when $r > 1$ and $\omega = \pi/4$ per samples.
   [10 marks]

b. Describe the results of the simulations as in part (a) i - (a) iii.
   [5 marks]

2. Find the relevant transforms of the following functions using simulation in MATLAB.

   a. z-transform of the functions given below.

      i. $x_1(n) = a^n$ [5 marks]

      ii. $x_2(n) = (1/4)^n$ [5 marks]

      iii. $x_3(n) = \sin(\omega * n)$ [5 marks]

   b. Inverse z-transform of the discrete sequence given below. [5 marks]

$$ix(n) = 2\left[\frac{z}{(2z-1)}\right]$$

3. We can analyse the characteristics and behaviour of discrete system from the z-plane and impulse response. Plot the z-plane and impulse response of the following discrete systems in MATLAB.

   a. System with complex conjugate poles. [10 marks]

$$H_1(z) = \frac{z}{z^2 - 0.707z + 0.2499}$$

   Or

$$H_1(z) = \frac{z}{[z - (0.3535 + j0.3535)][z - (0.3535 - j0.3535)]}$$

   b. System with complex conjugate poles in unit circle. [10 marks]

$$H_2(z) = \frac{z}{z^2 - 1.4144z + 1}$$

   c. System with complex conjugate poles outside of unit circle. [10 marks]

$$H_3(z) = \frac{z^2}{z^2 - 2.02z + 1.14}$$

d. Describe the characteristics and behaviours of the given discrete systems from the z-plane and impulse response simulations in parts (a)-(c). Formulate stability criteria in z-transform.

[10 marks]

4. For finding partial fraction in z-transform, we can use the following function in MATLAB:

```
[R,P,K]=residueZ(num,den)
```

Where: `R`, `P` and `K` forms the partial fraction in the following forms:

$$X(z) = \frac{R_1 z}{(z - p_1)} + \frac{R_2 z}{(z - p_2)} + \cdots + K$$

Find the partial fraction of following discrete sequence expression:

[5 marks]

$$X(z) = \frac{4 - (7/4)z^{-1} + (1/4)z^{-2}}{1 - (3/4)z^{-1} + (1/8)z^{-2}}$$