

Time and Frequency Characteristics

(Introduction)

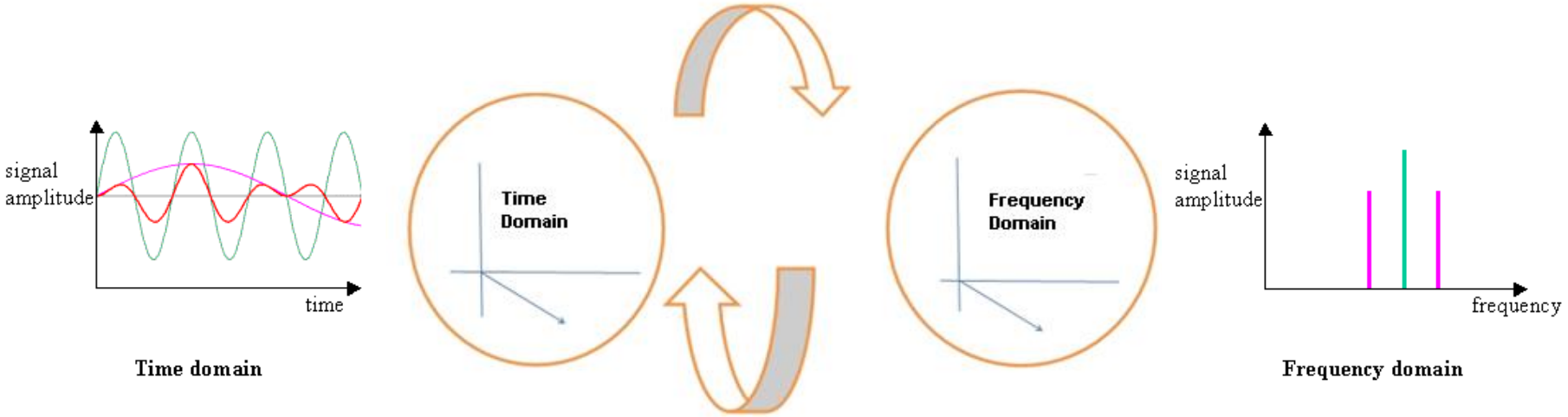
XMUT220 Signals and Systems

Topics

- Fundamentals of the Time and Frequency Characteristics.
- Time and Frequency Characteristics Analysis.
- Application of Time and Frequency Analysis.
- Examples of Time and/or Frequency Domain Analysis.

Time- and Frequency-Domain Characteristics

- *Time-domain characteristics:* analysis and measurement of characteristics and phenomena of the signals and systems in time domain.
- *Frequency-domain characteristics*: analysis and measurement of characteristics and phenomena of the signals and systems in frequency domain.
- Transformation from the time domain to the frequency domain through use of transformation techniques: Fourier transform, Laplace transform (continuous time) and z-Transform (discrete time).



Time-Domain Analysis

Time-domain analysis:

- It is the time response of a system to an input that sets the criteria for our systems.
- Many quantitative criteria have been defined to characterise the time response of a system.
- The time constant and system gain of a system are useful in its analysis.
- But other criteria describe the time response more accurately to an engineer i.e. rise time, settling time, etc.

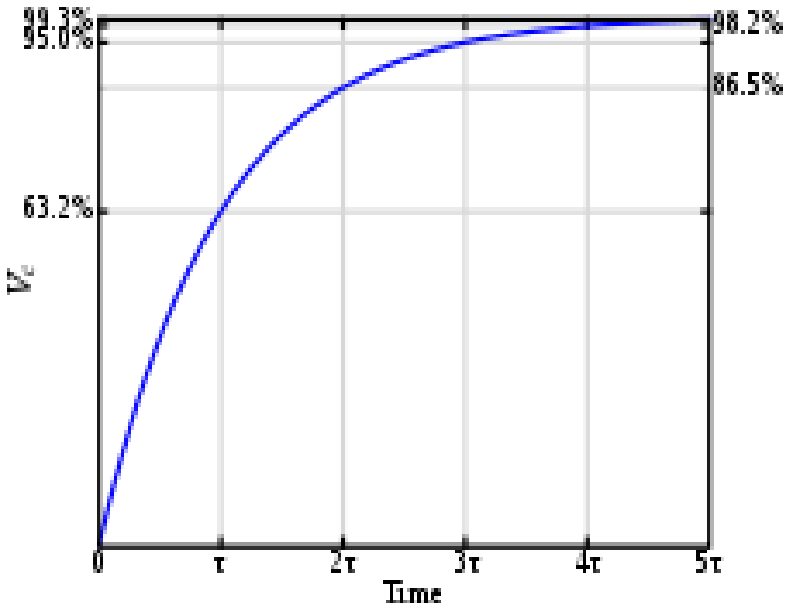
Time-Domain Analysis

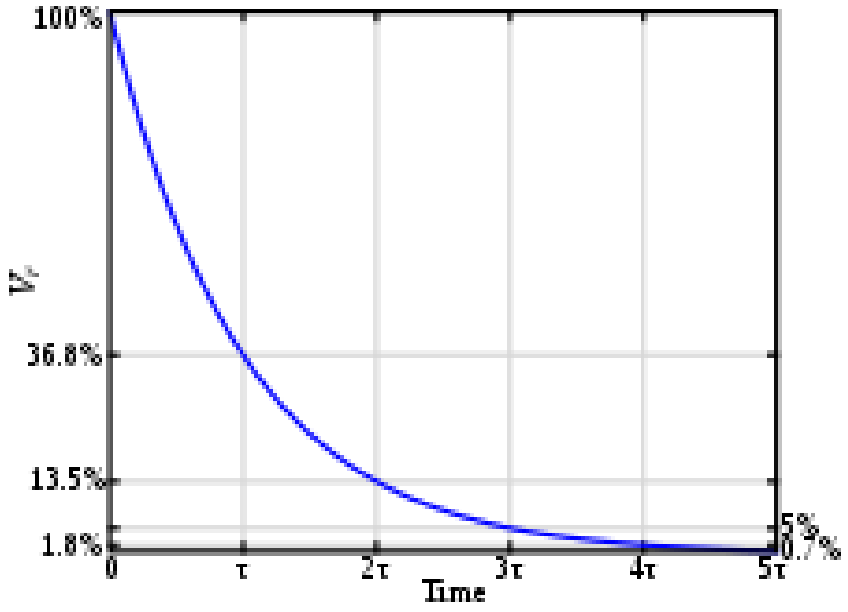
Quantitative Parameters

- *The time-domain analysis' quantitative parameters* are the transient response of the system for a given input measurable in time.

  a. Initial transient response:

  Initial transient response is the response at the start of observation. The most used parameters are time constant, rise time, settling time, damping ratio, time-to-peak, percentage overshoot, etc.



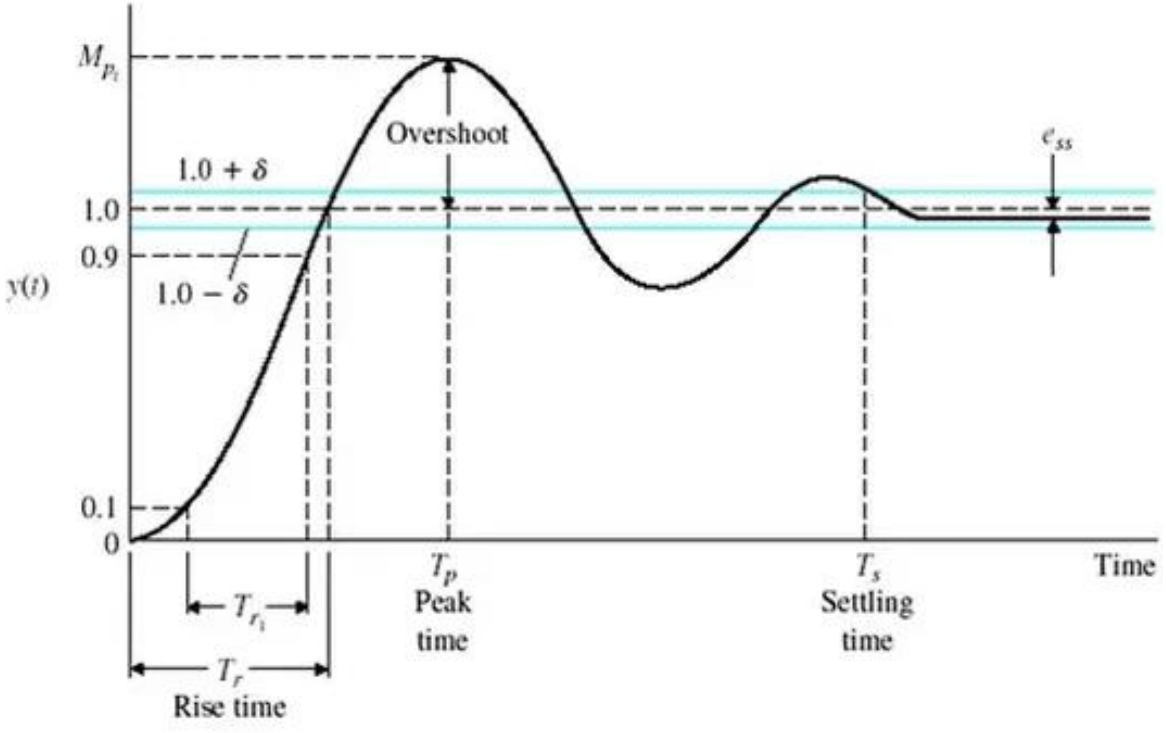


Time-Domain Analysis

b. Steady-state condition:

Steady-state condition is final value or intended outcome of the response of the system.

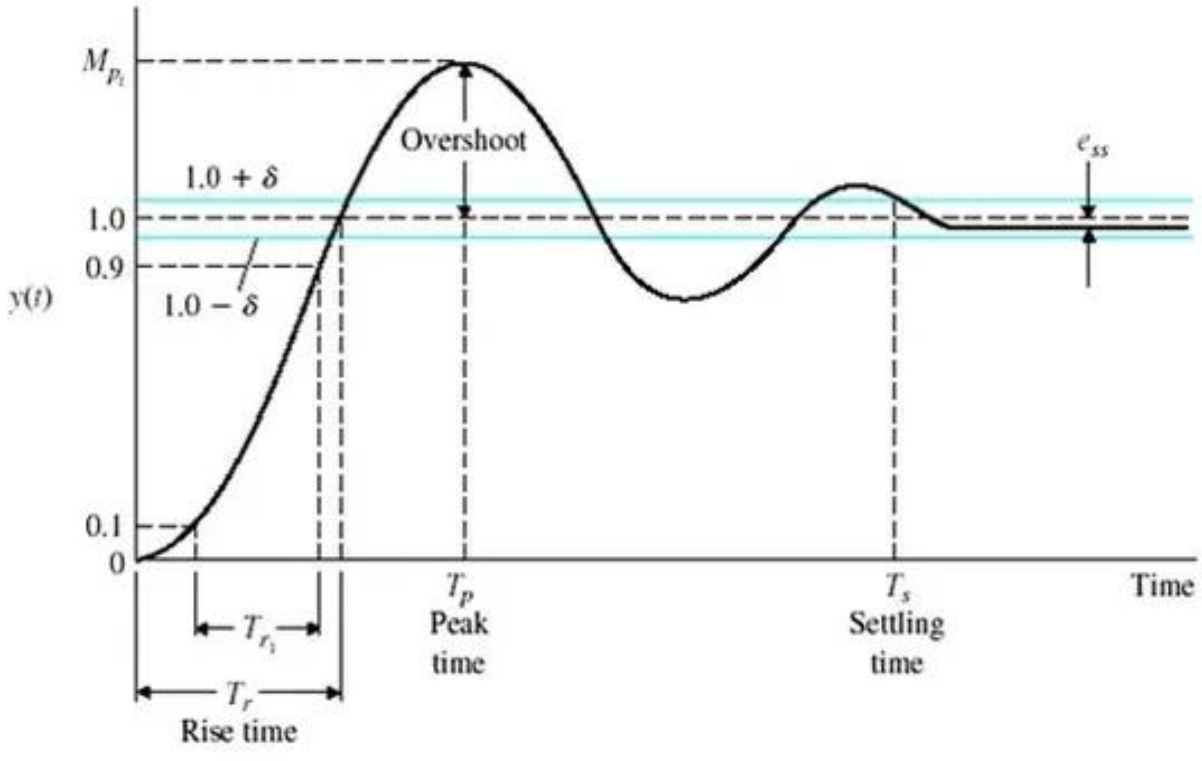
Most used parameters are steady-state amplitude, time and error.



Time-Domain Analysis

Qualitative Phenomena

- *Qualitative phenomena* are any observed behaviour of the system in time domain.
- Most used are oscillations, peaking, wave distortion, noise/disturbance, etc.



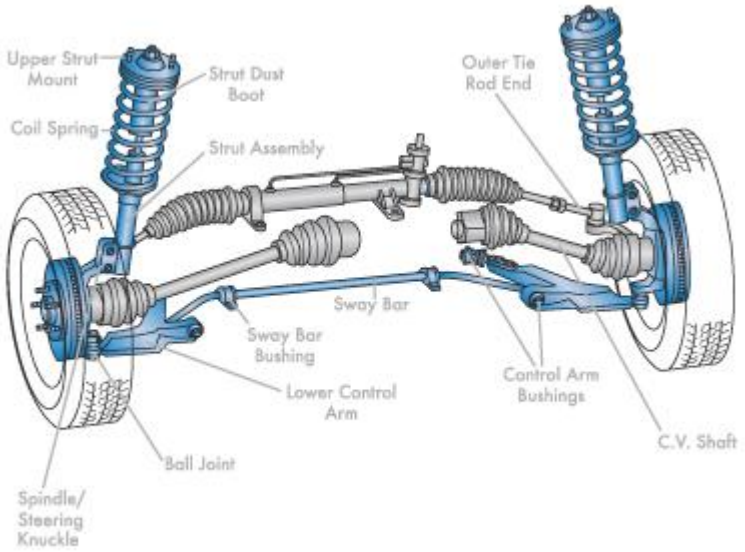
Application of Time-Domain Analysis

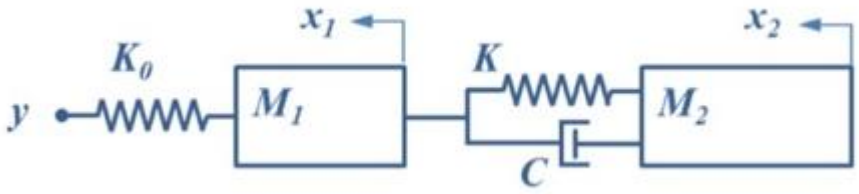
- Measuring transient response of the system e.g. how quickly the system reacts to a given input.
  - How quick to change state?
  - Whether the system is oscillating/ringing or experiencing damped response?

- Measuring the steady-state behaviour of the system e.g. when the time is approaching infinity (∞).
  - When the system settles down?
  - Is there any difference between the steady-state value with intended/ final value?

Application of Time-Domain Analysis 1

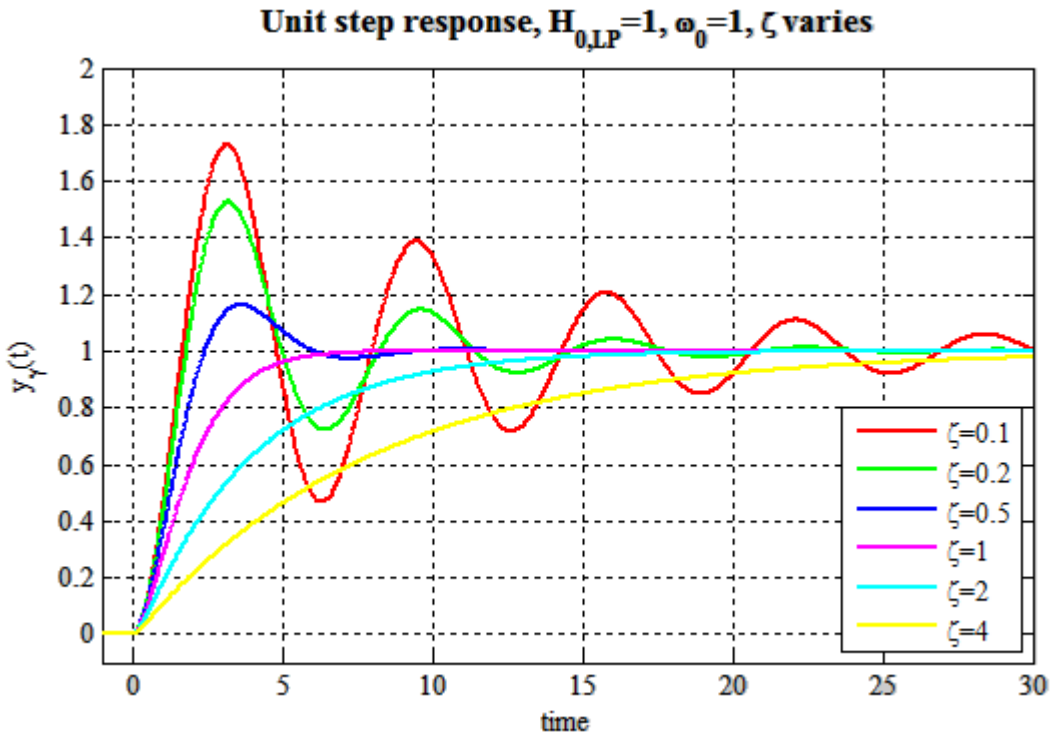
- Damper, spring and mass mechanical system.
- Analysis of system performance in terms of transient and steady-state responses.
- Different transient responses of a system subjected to an input (e.g. unit step) for variety of damping factors.





Application of Time-Domain Analysis 1

- Transient response of the mechanical system

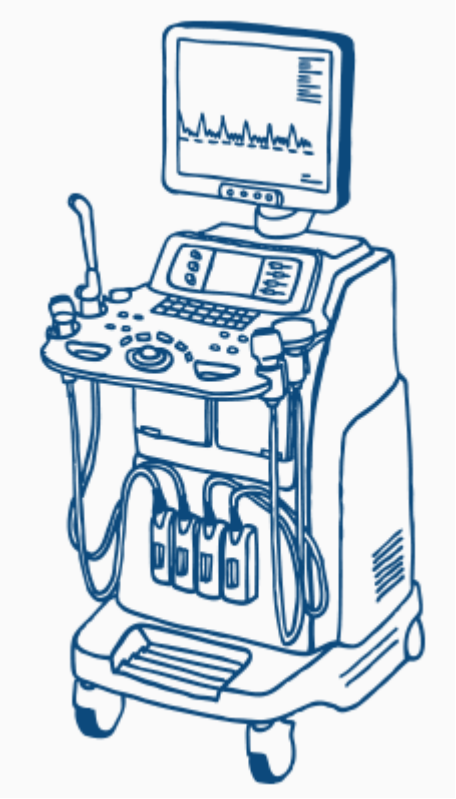


Note:

- $\zeta < 0.707$ – small damping (decaying oscillation).
- $\zeta = 0.707$ - critical damping (transition).
- $\zeta > 0.707$ – large damping (exponential increase).

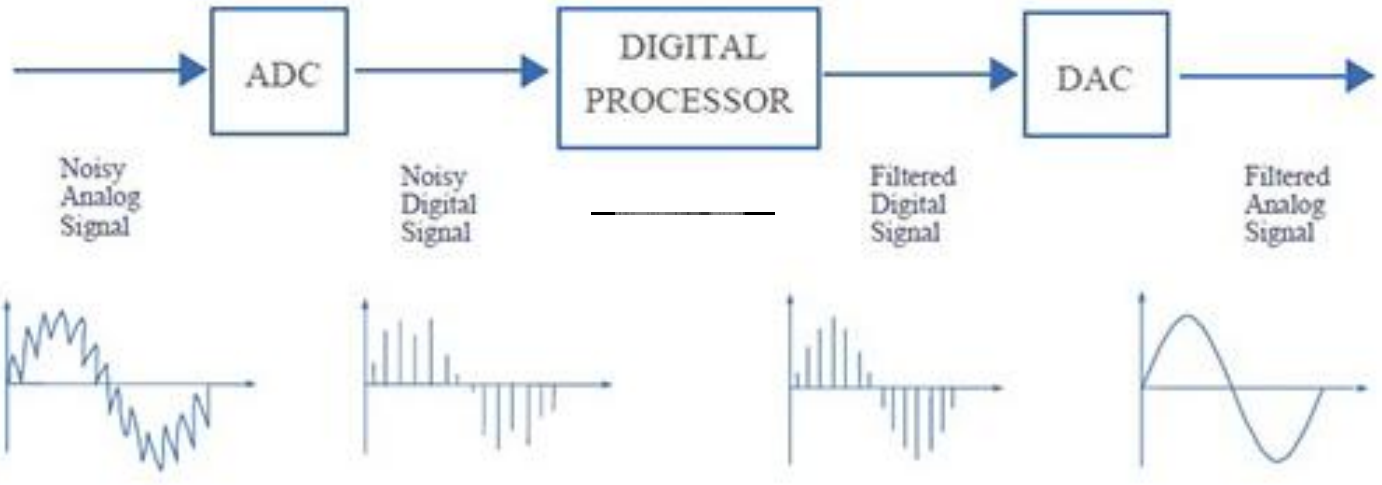
Application of Time-Domain Analysis 2

- Ultrasonography (USG) medical equipment used to detect object using ultra high frequency sound wave and shows it on a display.
  - Digital signal processing system that works on an analogue input signal and produce specific analogue output signal for image display after the processing.
  - Reconstruction and filtering of signal contaminated with noise.
  - Observation and analysis of noises and disturbances.



Application of Time-Domain Analysis 2

- Time-domain plots of the signals in the system.

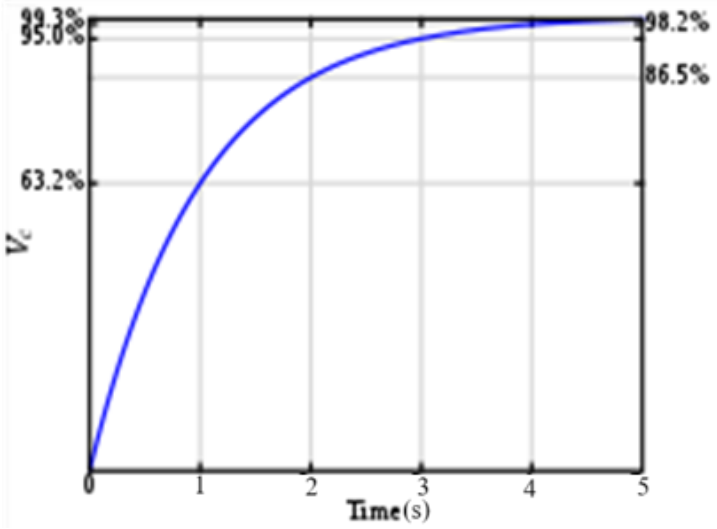


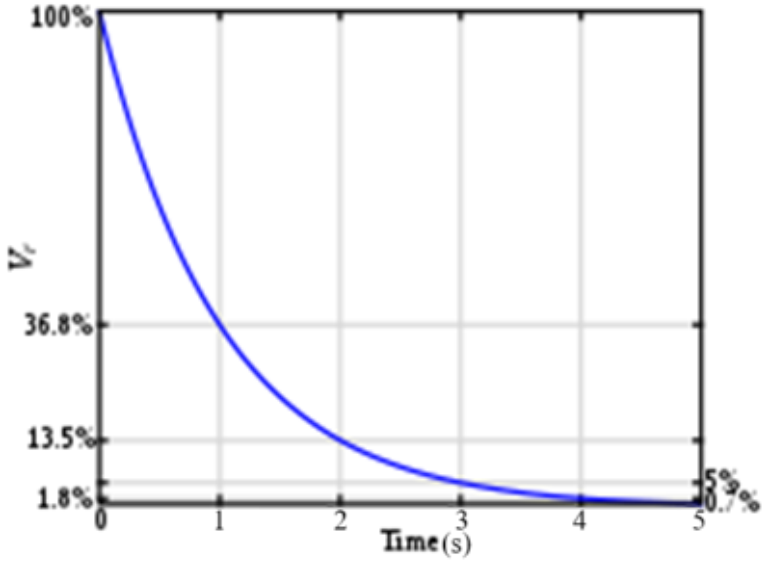




Example of Time-Domain Analysis

1. For the transient response of a first order system as shown below, determine the following parameters.





a. The time constant of the system. [2 marks]

b. The settling time of the system. [2 marks]

Example of Time-Domain Analysis

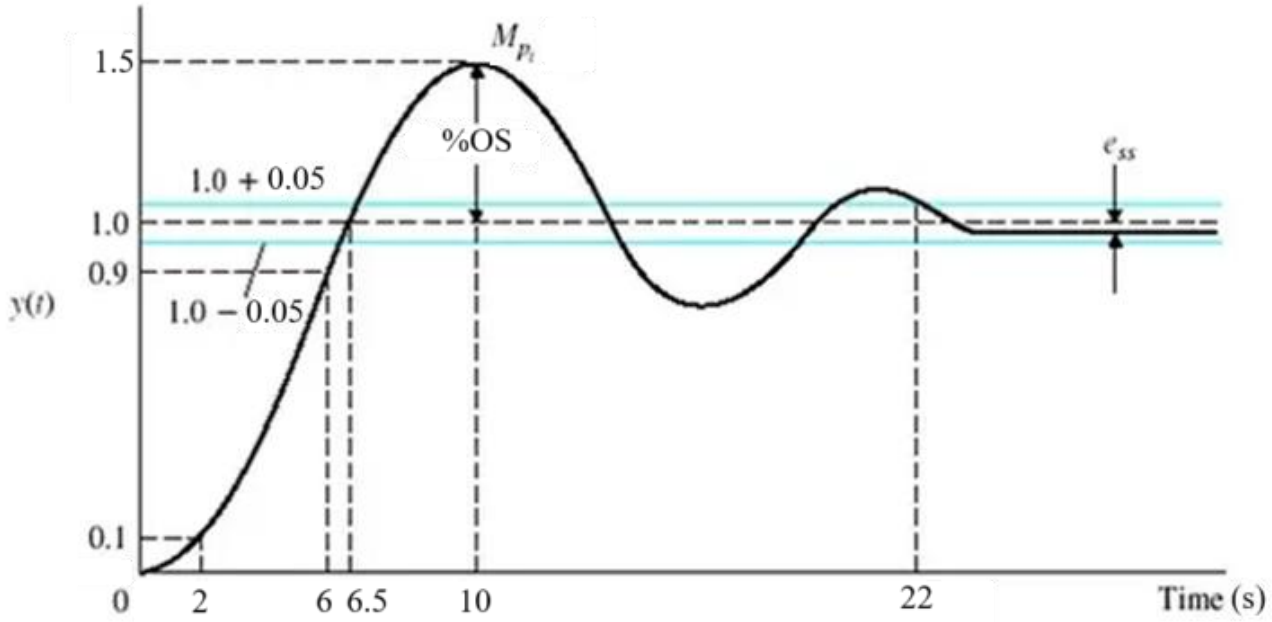
Solution

a. The time constant (τ) – For the first-order system, from the example graph:
   - For the increasing transient response, it is time taken to reach 63.2% of the final value in the transient response. From the example graph, it is 1 second.
   - For the decaying transient response, it is time taken to reach 36.8% of the final value in the transient response. From the example graph, it is also 1 second.

b. The settling time of the system – It is time to reach the final value in the transient response. It is typically about 5 time constant (5τ) of the transient response. From the example graph is it about 5 seconds.

Example of Time-Domain Analysis

2. Identify the parameters of the transient response of a second-order system shown in the figure below.



a. Rise time. [2 marks]

b. Peak-time. [2 marks]

c. Overshoot. [2 marks]

d. Settling time. [2 marks]

e. Steady-state error. [2 marks]

Example of Time-Domain Analysis

Solution

a. Rise time – it is the time from 10% to 90% of the transient response of the system. From the example graph, it is T_s = 6 - 2 = 4 seconds.

b. Peak-time – It is the time taken to the n-th (i.e. usually first) peak of the oscillatory transient response. From the example graph, it is 10 seconds.

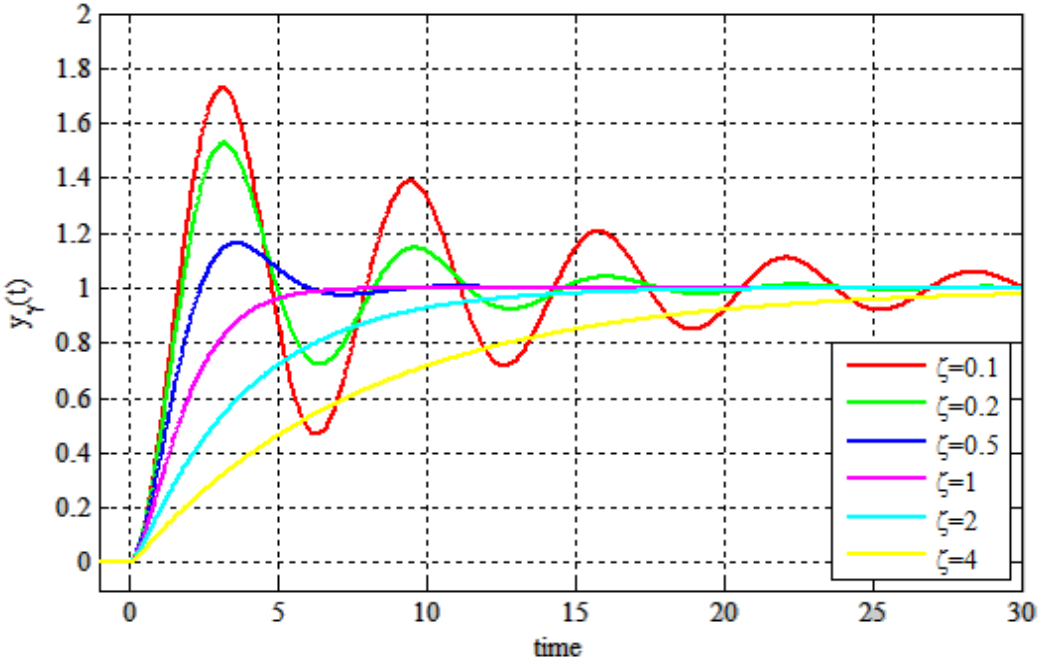
c. Overshoot - It is the largest deviation usually in percentage from the final value of the oscillatory transient response. From the example graph, it is [(1.5 - 1)/1] x 100% = 50%.

d. Settling time – It is the time for the system to settle down to its final value of the transient response. From the example graph, it is 22 seconds.

e. Steady-state error – It is the deviation usually in percentage the final value of the oscillatory transient response. From the example graph, assuming that the response settles to 0.98, it is [(1 – 0.98)/1] x 100% = 2%.

Example of Time-Domain Analysis

3. Given in the graph below is a unit step response of a second-order system with $H_0 = 1$, $\omega_0 = 1$, and ζ is varied, which of the following responses belong to the following response categories.



a. Undamped response. [2 marks]

b. Underdamped response. [2 marks]

c. Critical response. [2 marks]

d. Underdamped response. [2 marks]

Example of Time-Domain Analysis

Solution

a. Undamped response – This is when the damping ratio (ζ) is 0. It is not observed in the example graph.

b. Underdamped response - This is when the damping ratio (ζ) between 0 to 0.707. It is not observed on the red, green, and blue in the example graph.

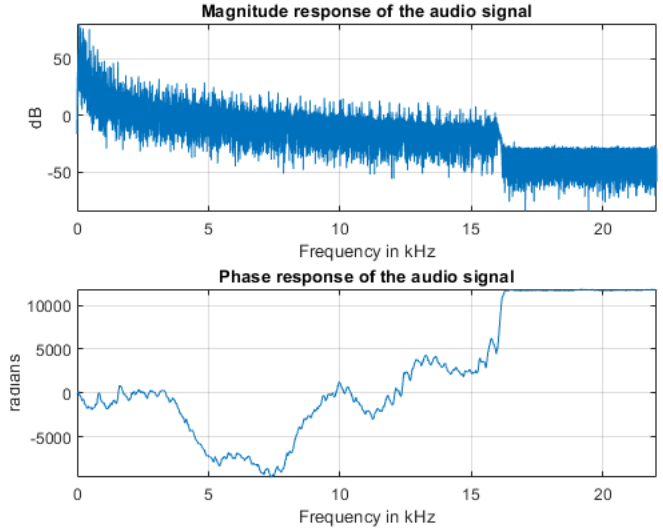
c. Critical response - This is when the damping ratio (ζ) is exactly 0.707. It is not observed in the example graph.

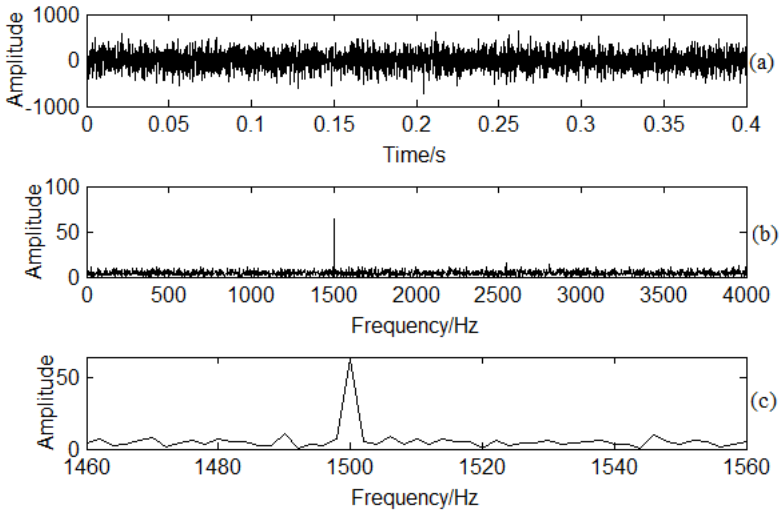
d. This is when the damping ratio (ζ) between 0.707 or larger. It is not observed on the magenta, light blue and yellow in the example graph.

Frequency-Domain Analysis

Frequency-domain analysis of signals and systems:

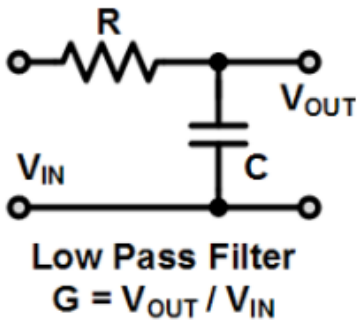
- Frequency response (Bode) Plot: It is a measure of frequency response of a system e.g. magnitude and phase of the output as a function of frequency, in comparison to the input.
- Frequency spectrum plot: It is a quantitative measure of the output spectrum of a system or device in response to a stimulus and used to characterise the dynamics of the system.

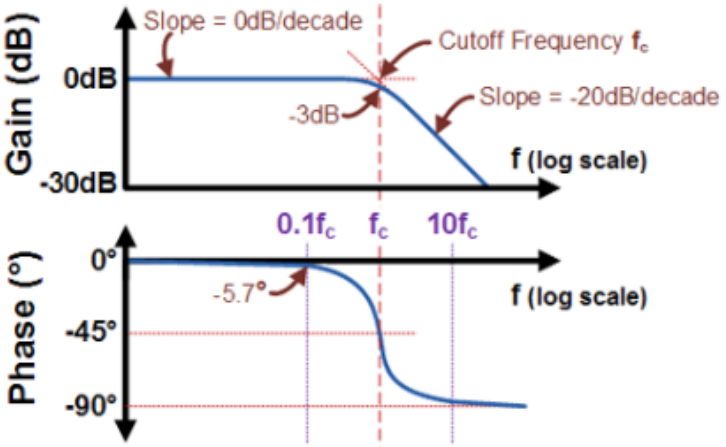




Frequency-Response Plot

- Frequency response plot is also informally called Bode plot.
- It is a graph of the frequency against response of a system.
- It is usually a combination of:
  - a magnitude plot (e.g. expressing the amplitude, usually in decibels of the frequency response) and
  - a phase plot (e.g. expressing the phase shift, in degree or rad/s).

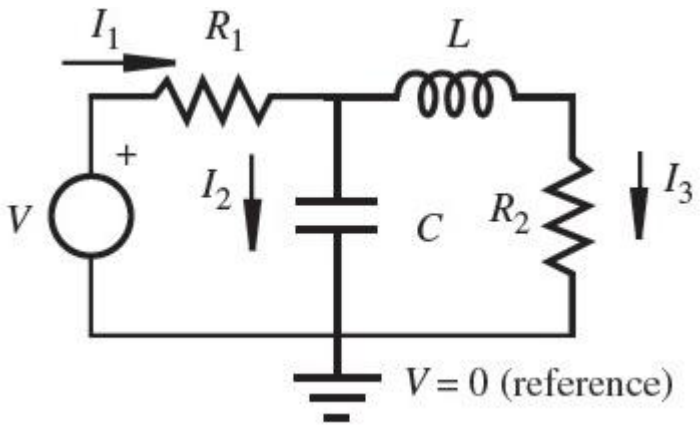


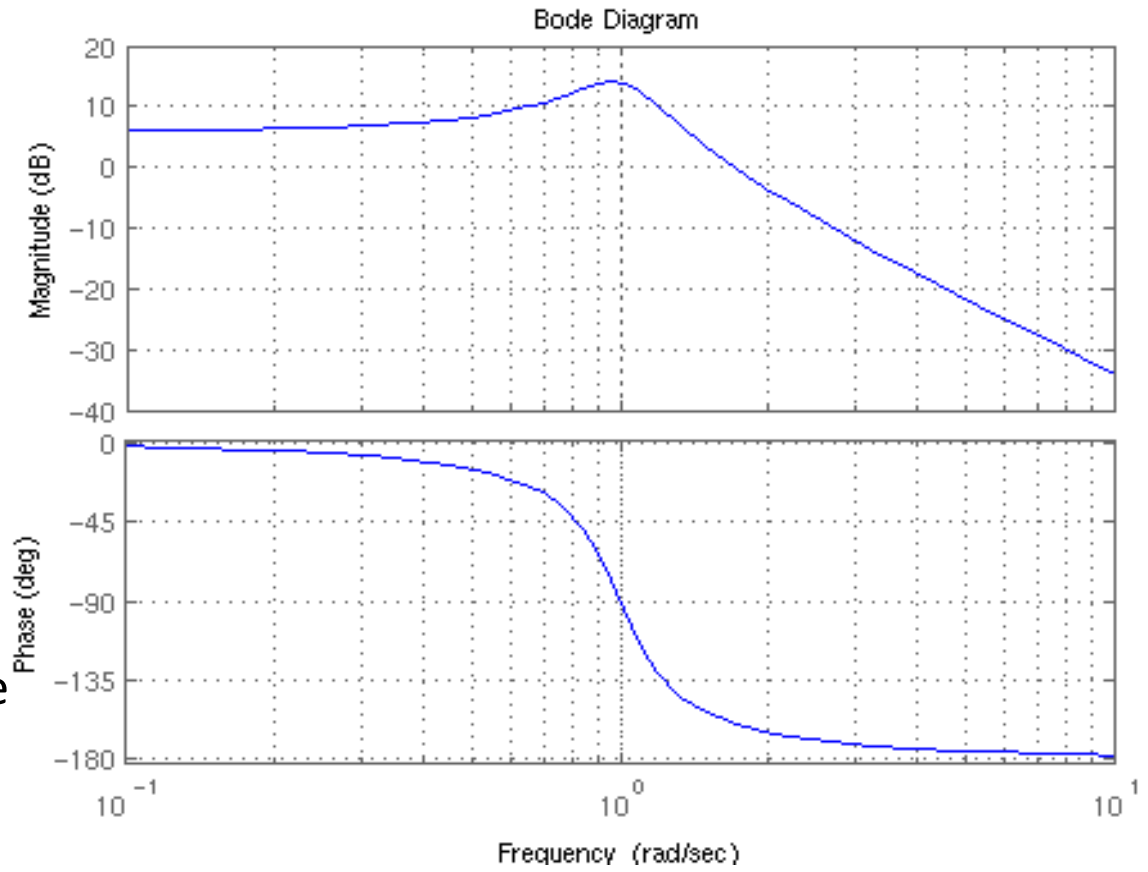


- Bode plot is typically used to analyse and design performance and stability of the system.

Frequency-Response Plot

- A second order RLC circuit.



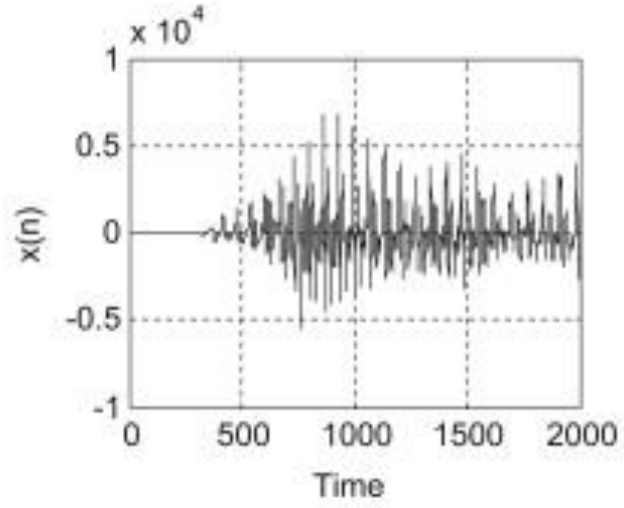


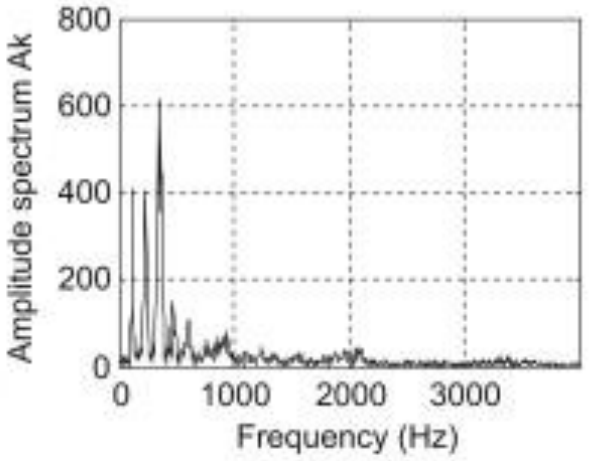
- Frequency response of the circuit e.g. magnitude and phase plots.

- Determine the performance of the system e.g. voltages at output and input, amplification/attenuation of signal, and phase shift.

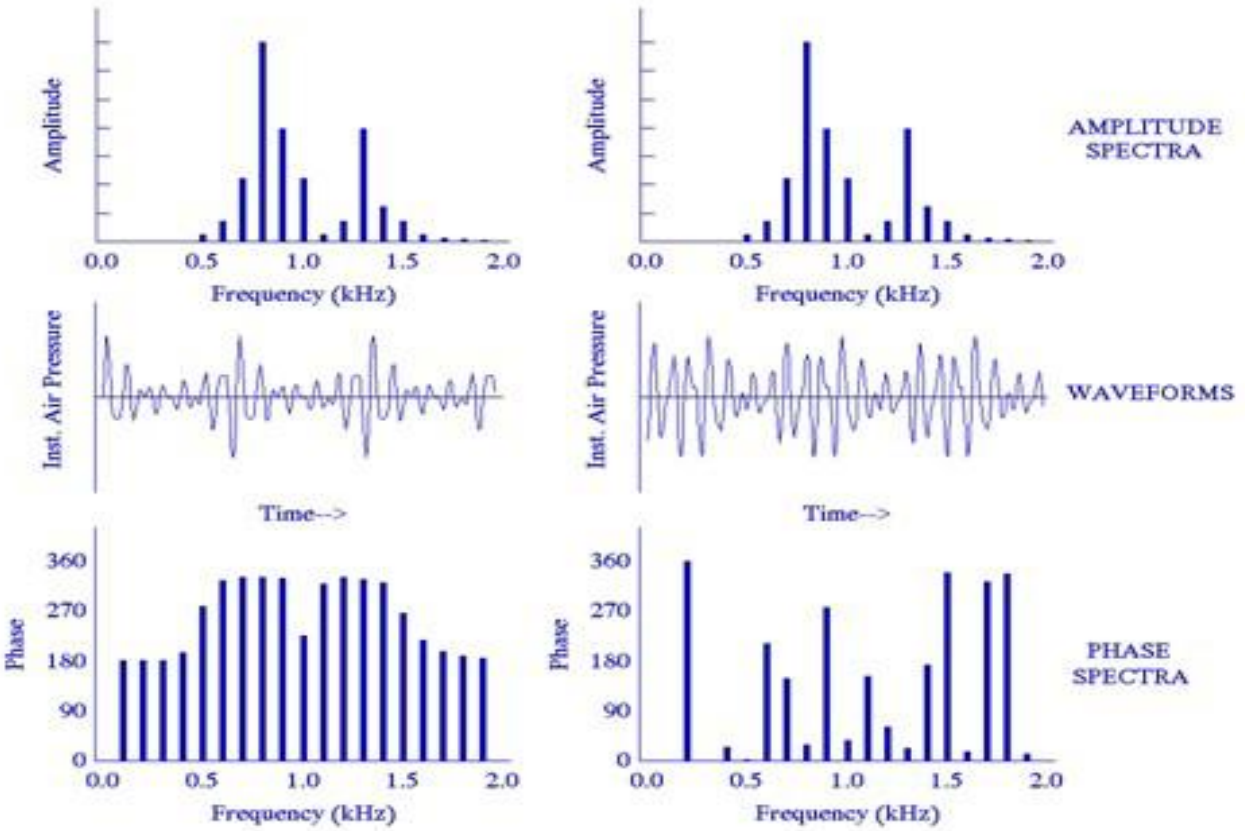
Frequency Spectrum

- Quantitative measure of the parts or components that make up a signal from the output of system or device.
- Frequency spectrum consists of:
  - Magnitude spectrum: Magnitude spectrum, $|X(\omega)|$ where the power sits in the spectrum. Is it quiet or loud, is it bright or dark?
  - Phase spectrum: Phase spectrum, $\measuredangle X(\omega)$ determines where in time the frequency components add or cancel. It essentially describes the waveform, the image.
- Frequency spectrum plots will be extensively used in the sampling and reconstruction of signals and systems.



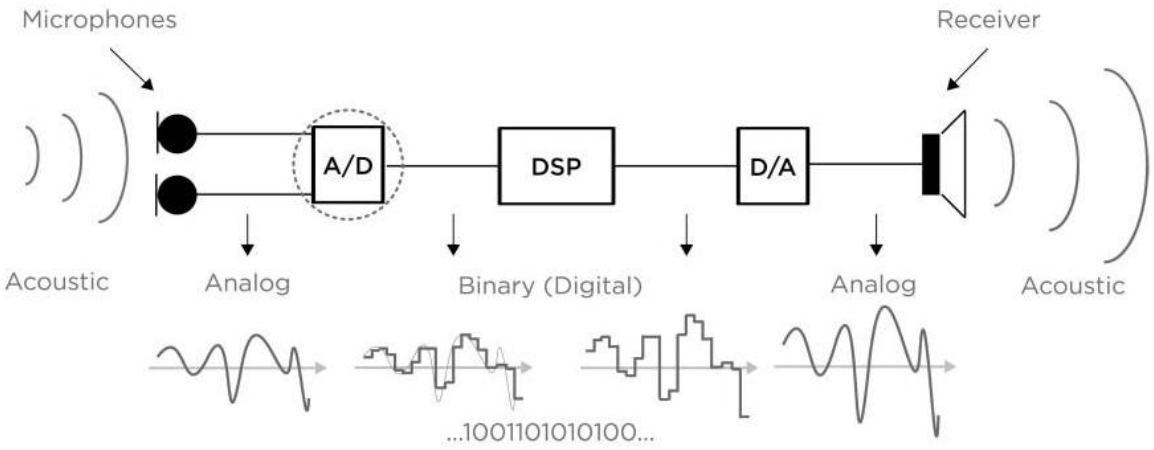


Frequency Spectrum



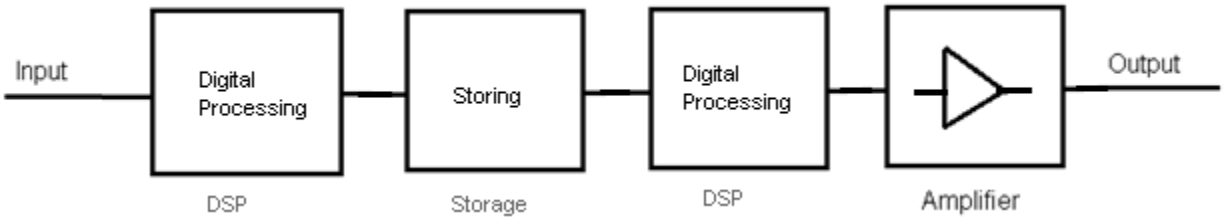
- Frequency spectrum of a signal e.g. spectra of amplitude or power over a range of frequencies plot.
- Determine the characteristics and behavior of the parts or components that make up the given signal.

Application of Frequency-Domain Analysis 1

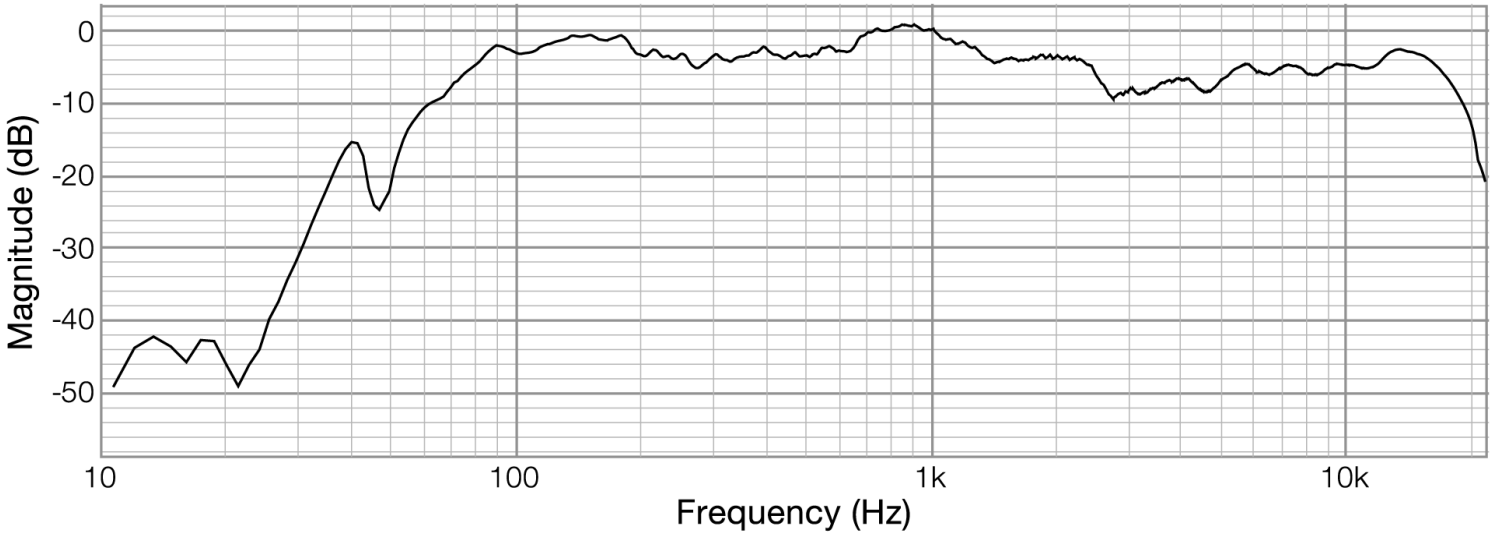


- The figure above is a public announcement (PA) audio system used for broadcasting information to people in public spaces
- For typical audio system, the objectives are to reproduce the input signal with no noise and distortion.
- That would require a uniform (flat) magnitude of frequency response up to the frequency range (bandwidth) limitation of the system.
- Reproduction is with the signal delayed by precisely the same amount of time at all frequencies.

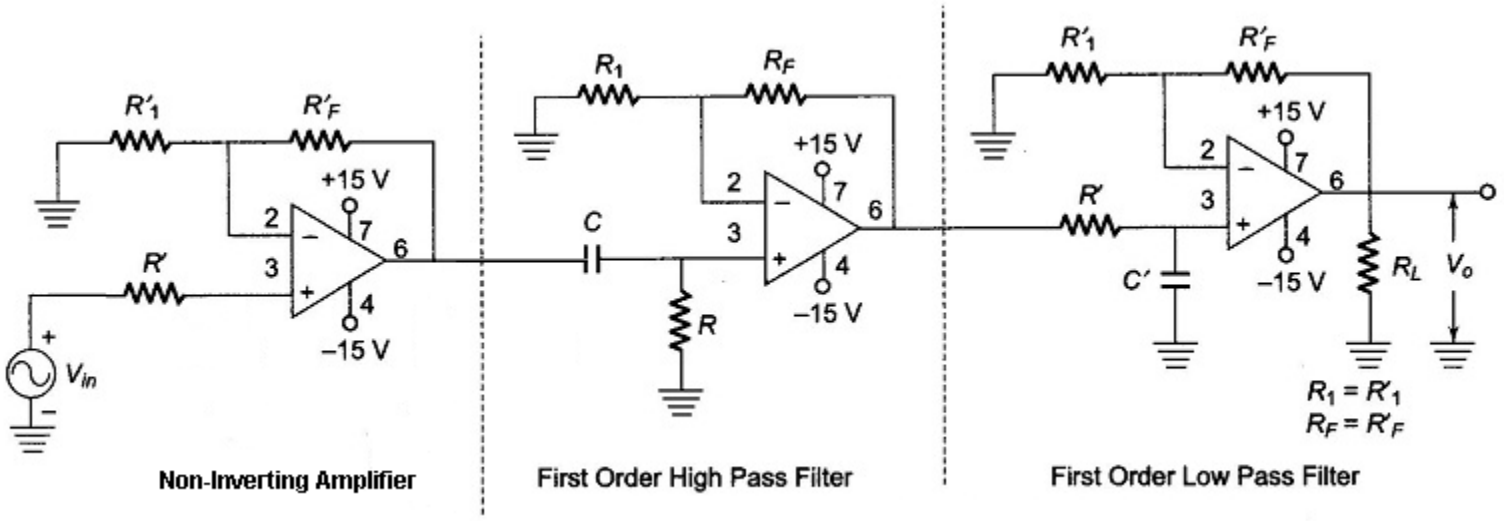
Application of Frequency-Domain Analysis 1



- Frequency response of the audio reproduction system.

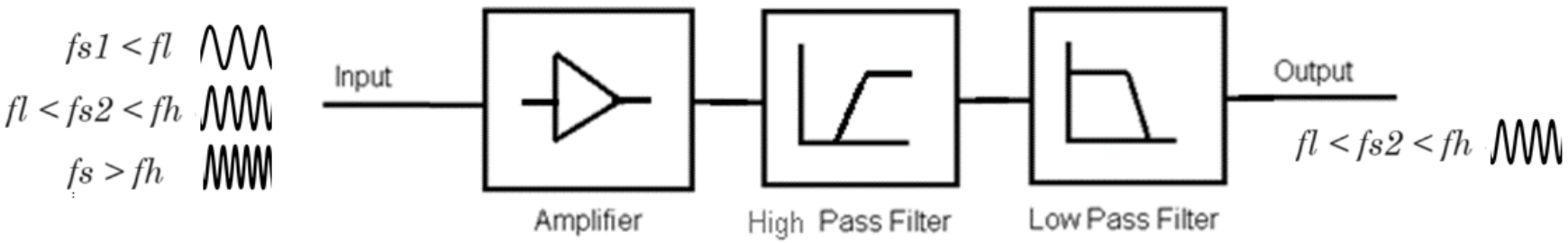


Application of Frequency Domain Analysis 2

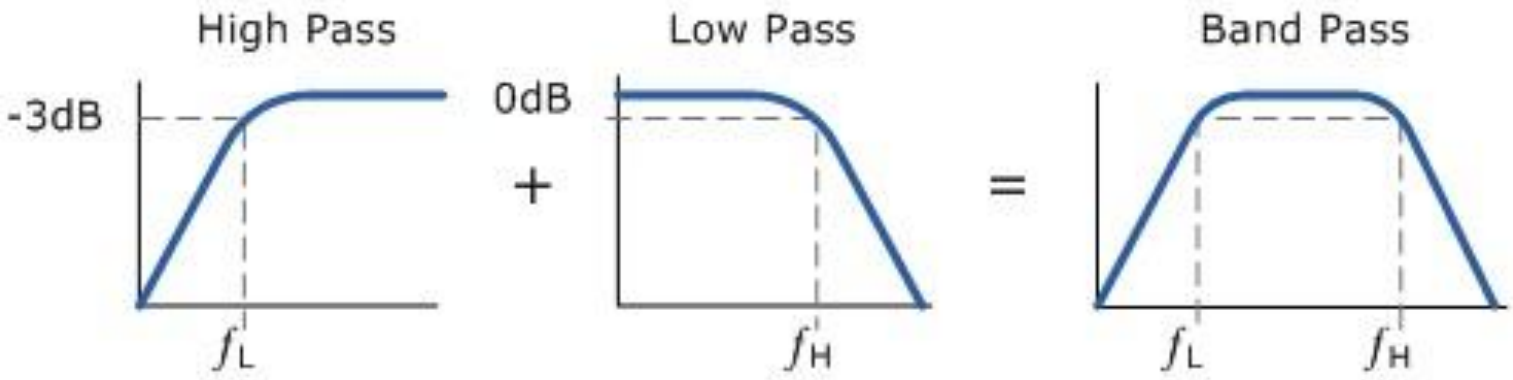


- For a filter-amplifier circuit application as shown above that could be used for the signal and audio systems, it is used to selectively allow or block specific signal or waveform to go through the system.
- High pass filter e.g. takes out low frequency signals, low pass filter e.g. removes out high frequency signals, band pass filter allows selective frequency range, and band reject filter excludes selective frequency range.

Application of Frequency Domain Analysis 2



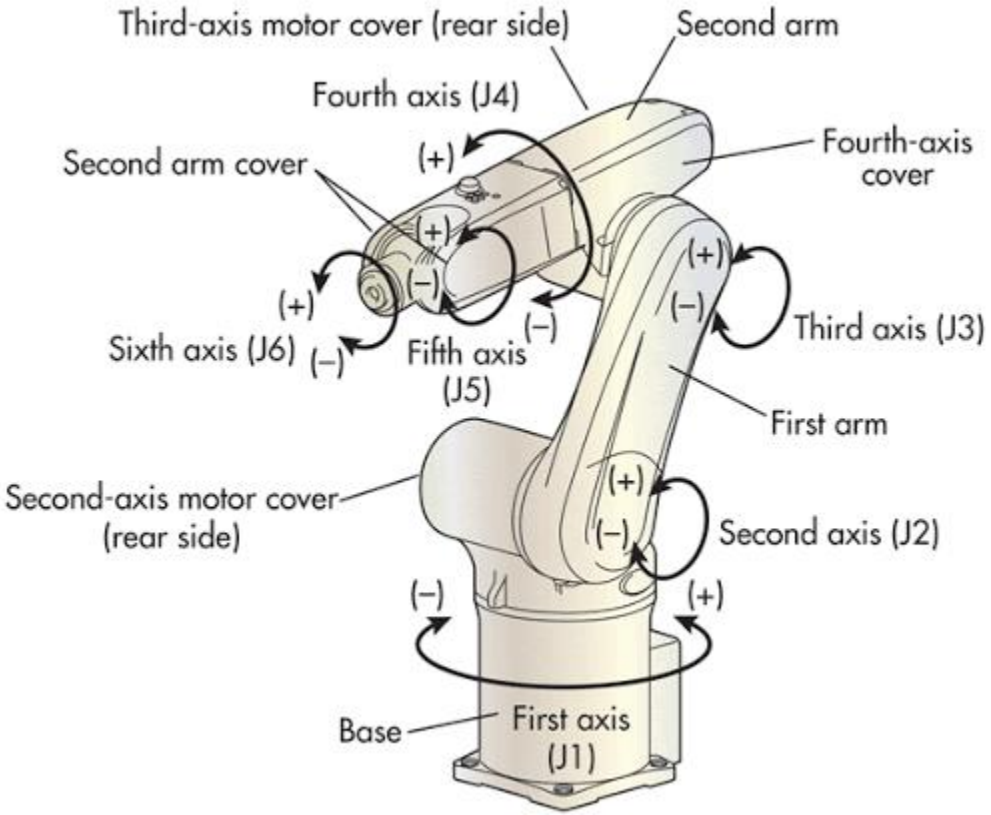
- Frequency responses of signal filtering system. Band pass filter allows through signals of a specific range of frequencies e.g. $f_l < f_s < f_h$.



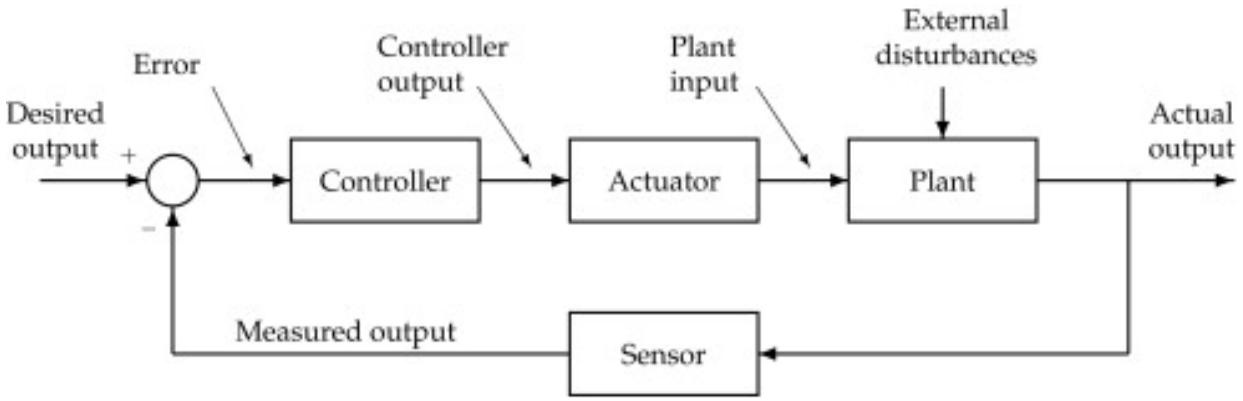
Application of Time & Frequency Domain Analysis

Given a feedback control system of a robotic arm in the figure below:

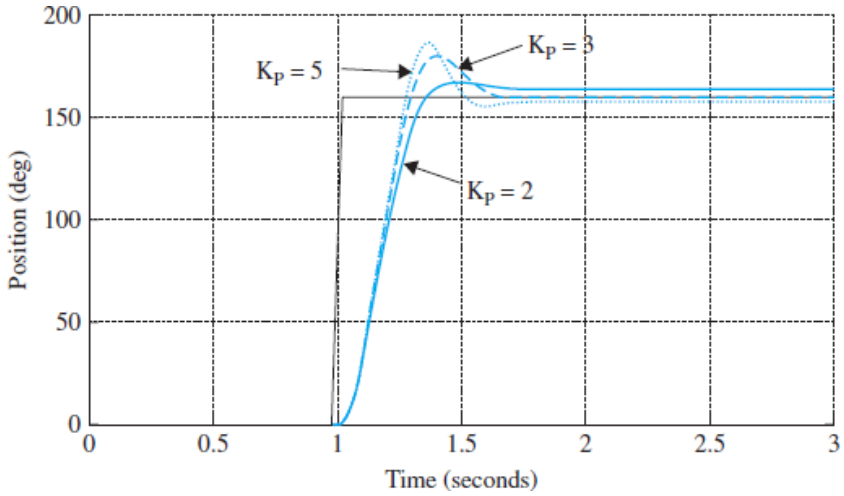
- For a feedback apparatus used to control a dynamic system, the objective is to give the system improved transient response -> XMUT315 Control Systems Engineering course.
- This can be achieved by using feedback system, so we know input and output of the system and by applying sensor and controller to measure and manage the performance of the system.

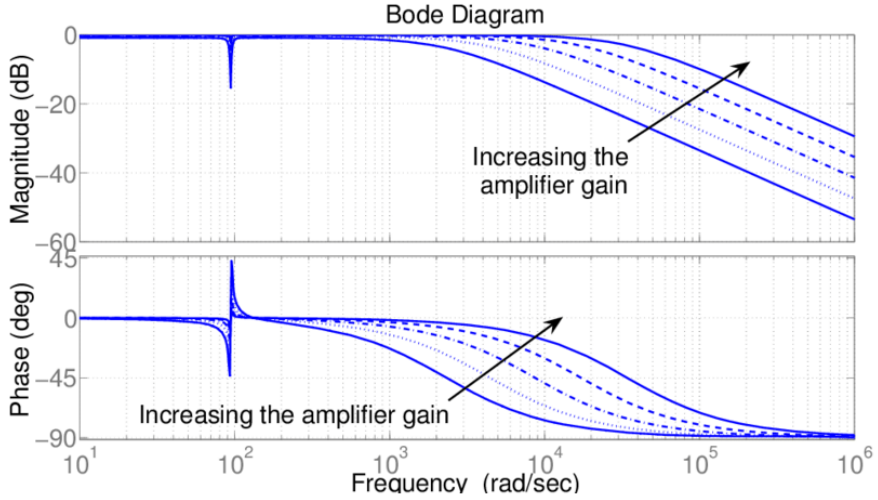


Application of Time & Frequency Domain Analysis



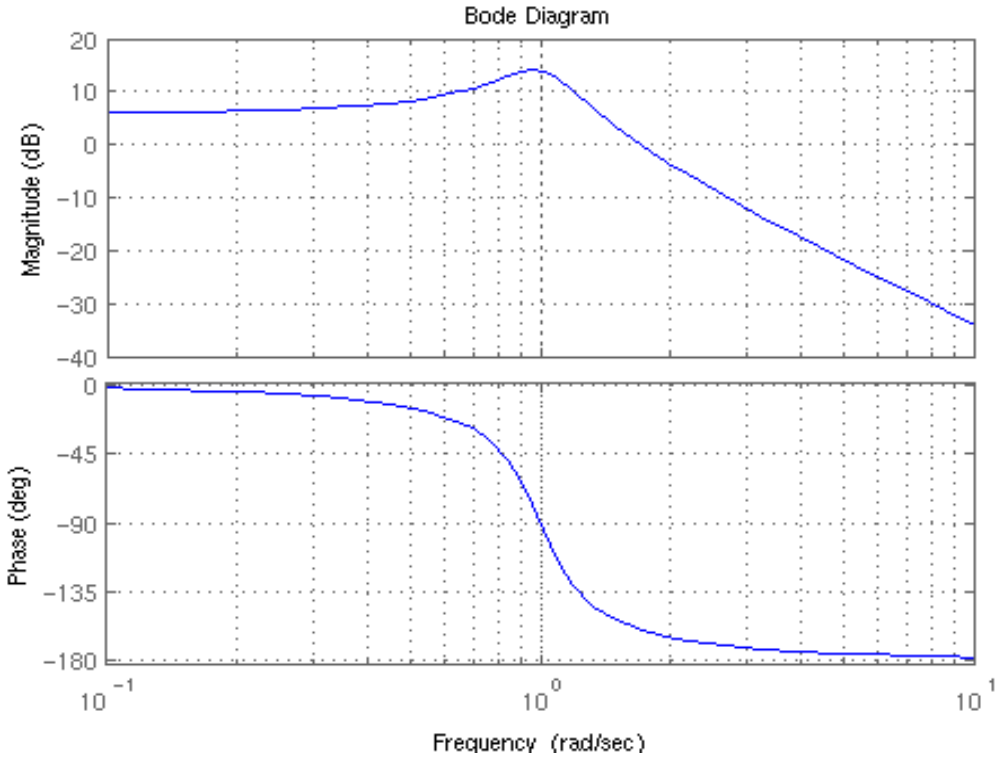
- Time and frequency responses of the robotic system.





Example Frequency-Domain Analysis

4. Given in the figure below is a frequency response plots of a second-order system.



Example of Frequency-Domain Analysis

a. Determine the cut-off frequency of the frequency response. [2 marks]

b. Determine the gain at the following frequencies:

   i. Low frequency. [2 marks]

   ii. Cut-off frequency. [2 marks]

   iii. High frequency. [2 marks]

c. Determine the phase shift at the following frequencies:

   i. Low frequency. [2 marks]

   ii. Cut-off frequency. [2 marks]

   iii. High frequency. [2 marks]

Example of Frequency-Domain Analysis

Solution

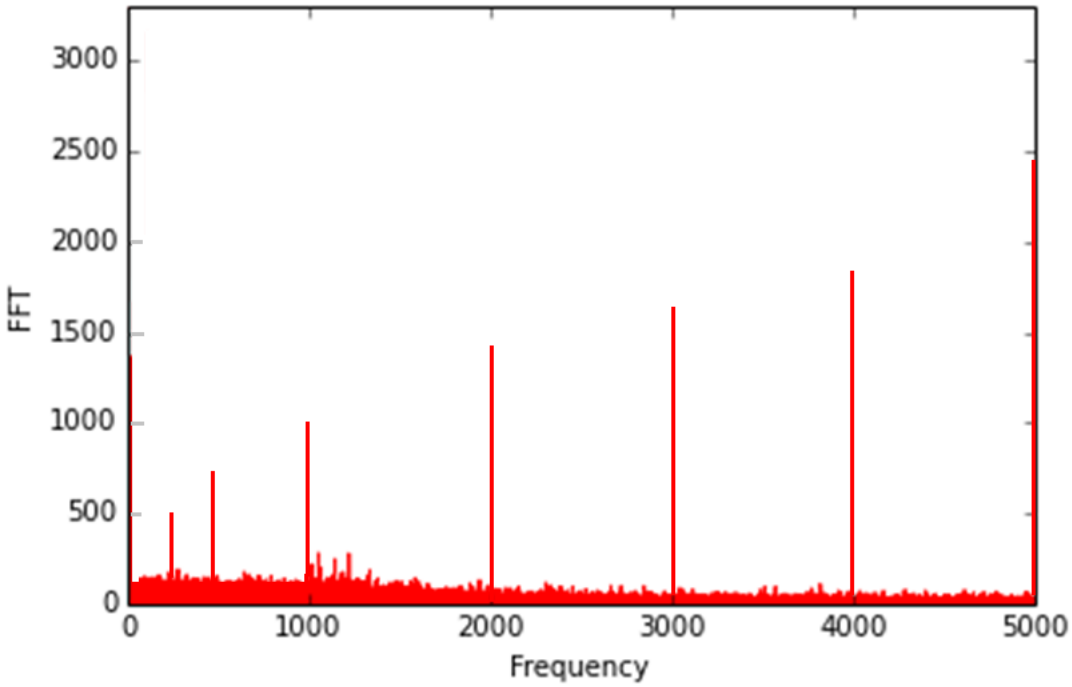
a. Determine the cut-off frequency of the frequency response. It is the when the magnitude of the gain is at about half power i.e. -3 dB. From the graph, it is around 2 Hz frequency.

b. Determine the gain at the following frequencies:
   i. Low frequency - It is around 6 dB.
   ii. Cut-off frequency – It is about -3 dB.
   iii. High frequency – It is settling at -20 dB/decade.

c. Determine the phase shift at the following frequencies:
   i. Low frequency – It is about 0 degree.
   ii. Cut-off frequency – it is about -150 degree.
   iii. High frequency – It settles to about -180 degree.

Example of Frequency-Domain Analysis

5. For the frequency spectrum plot of a signal as shown below, perform the following tasks.



a. Determine the frequency component of the signal. [2 marks]

b. Describe the type of the signal being observed. [2 marks]

Example of Frequency-Domain Analysis

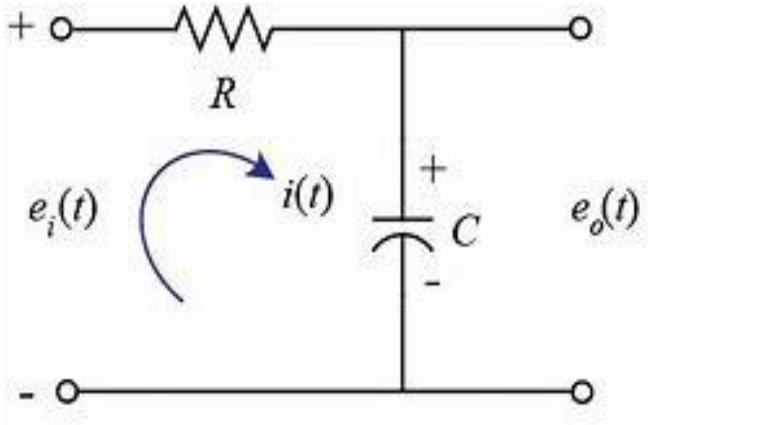
Solution

a. Determine the frequency component of the signal. From the frequency spectrum graph, the frequency components are 0 Hz (fundamental), 250 Hz, 500 Hz, 1 kHz, 2 kHz, 3 kHz, 4 kHz, and 5 kHz.

b. Describe the type of the signal being observed. From the frequency components observed on the frequency spectrum graph, it looks like the signal has a discrete components of the frequency spectrum and its magnitude of the frequency components are increasing from low frequency to high frequency.

Case Study Example

- Suppose a continuous-time system represented as follows:

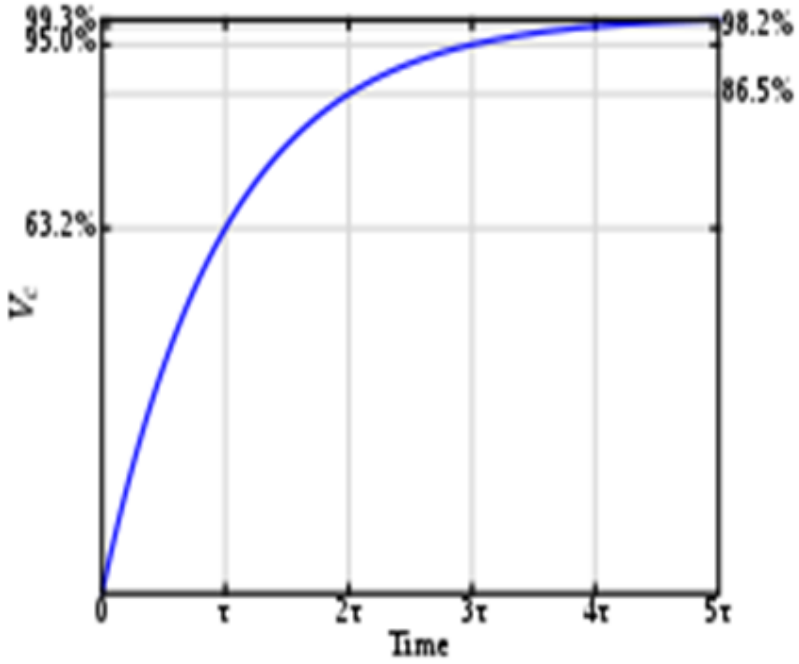


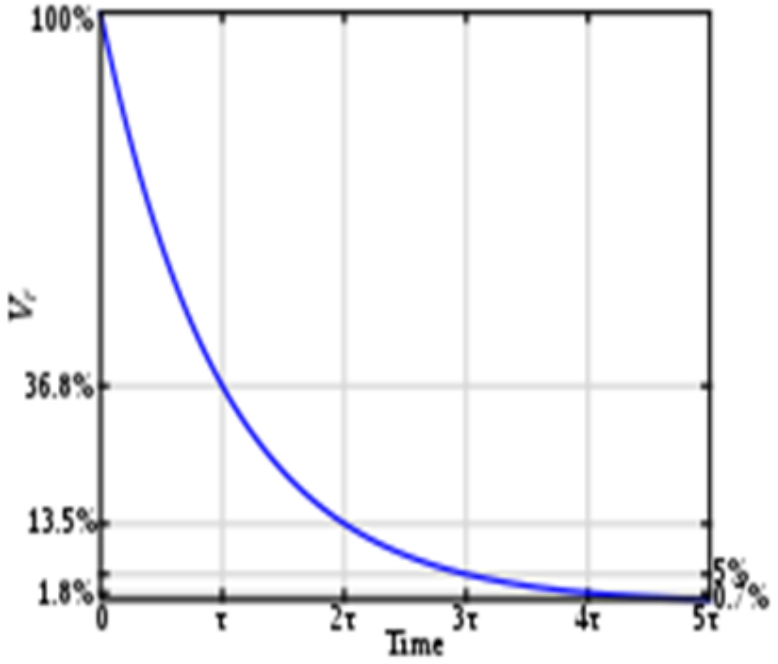
- It is a an electronic circuit of resistor and capacitor in series.
- $e_i(t)$ is the input signal and $e_o(t)$ is the output signal.
- Capacitor charges up when a potential difference is applied across it.
- The flow of current in the capacitor will be determined by the values of resistor and its capacitance.
- It is also known as low-pass first-order RC filter circuit.

Case Study: Time Domain-Analysis

Time domain plot of the system:

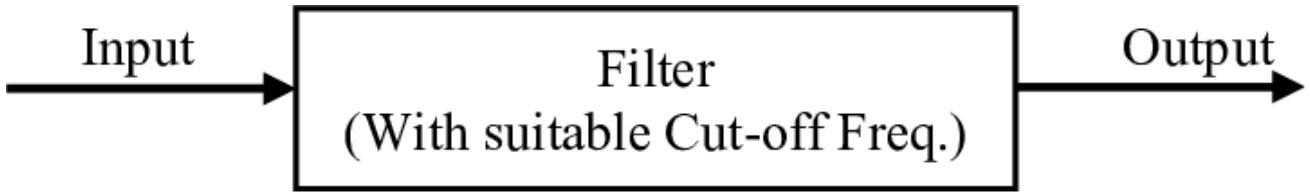
- It is a first order system with exponentially surging voltage characteristics of the capacitor (v_c) and decaying voltage characteristics of the resistor (v_r) and vice versa (i.e. for the system current characteristics).
- It takes approximately 2 x time constant (τ) for the response of the system at the output (v_c) to rise up and 5 x time constant to settle down.

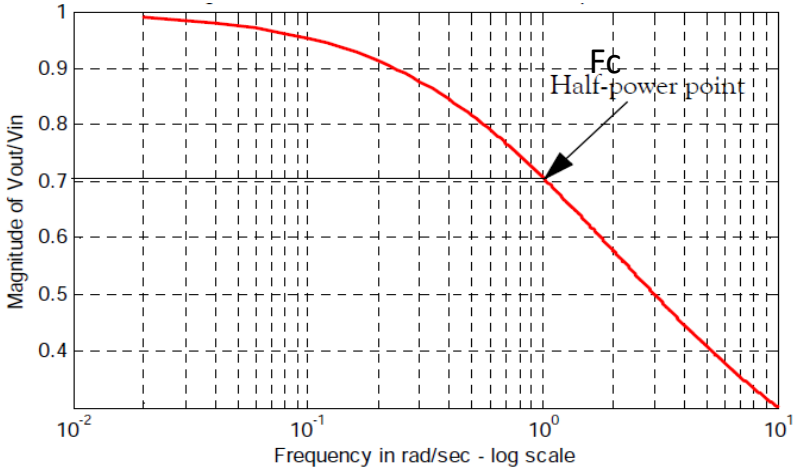


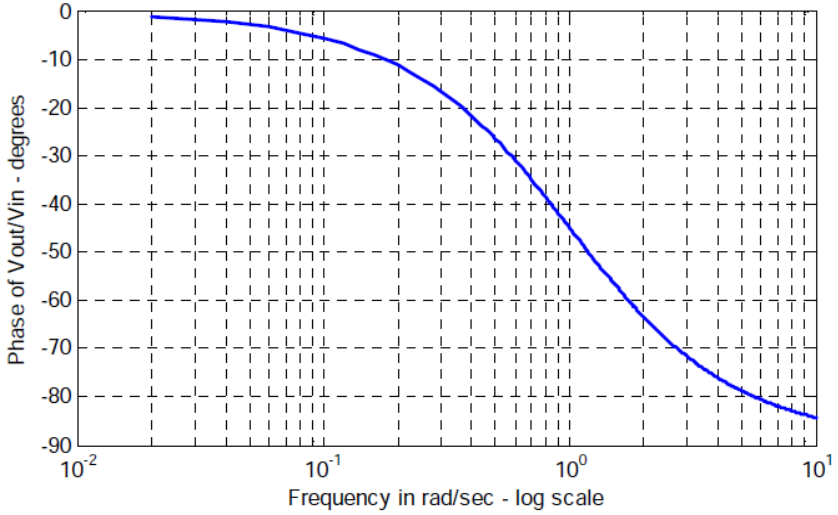


Case Study: Frequency-Domain Analysis

- The frequency response plot of an analogue filter system are given in the figures below. Deduce the type of the filter.

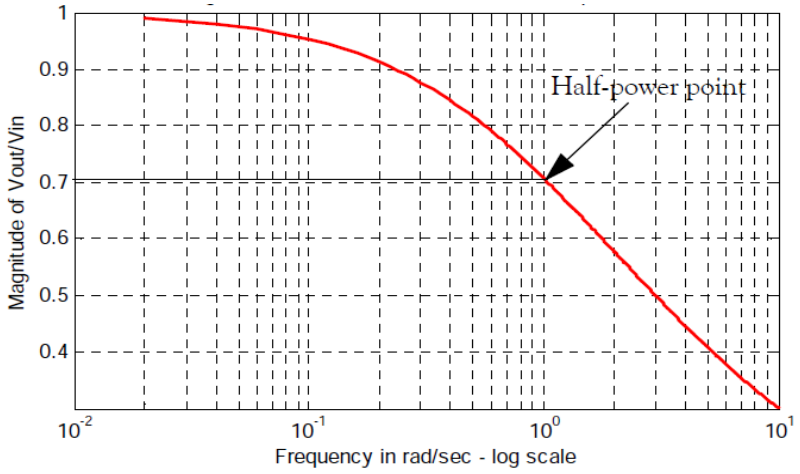






Case Study: Frequency-Domain Analysis

- Bode plot of frequency response of the system (e.g. gain graph).



Fc

- Half-power point:

$$P_{(F_C)} = 0.5\left[\frac{V_{out}(\omega)}{V_{in}(\omega)}\right]$$

- Cut off frequency (F_c): Frequency where half-power point is located.

Magnitude characteristics of the system:

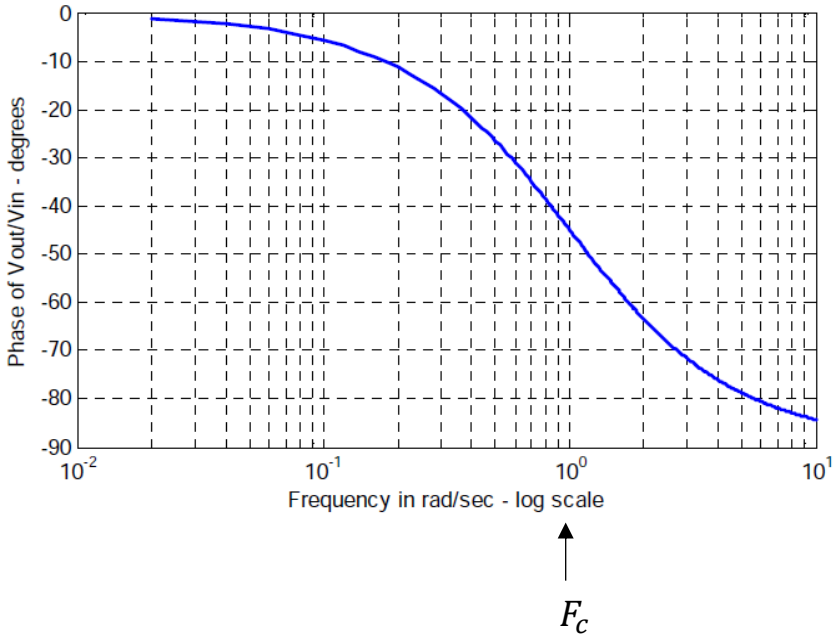
- As ω -> 0, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 1 (no change in the output in relation to input).
- For $\omega = 1/RC$, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 0.5 (e.g. output = 0.5 x input).
- As ω -> ∞, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 0.

Case Study: Frequency-Domain Analysis

- Bode plot of frequency response of the system (e.g. phase graph)

Phase characteristics of the system:

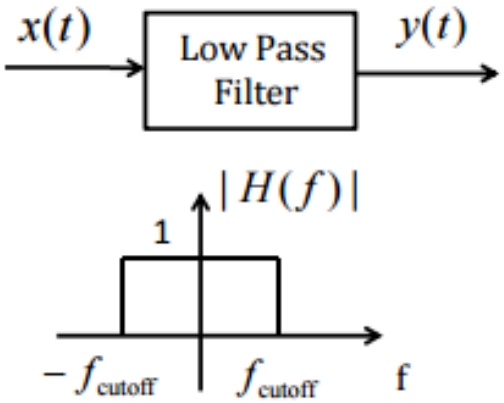
- As ω -> 0, phase shift is 0 (there is no phase difference between output and input.
- For $\omega = F_c$ (cut-off frequency), there is a phase difference of $\pi/4$ between input and output.

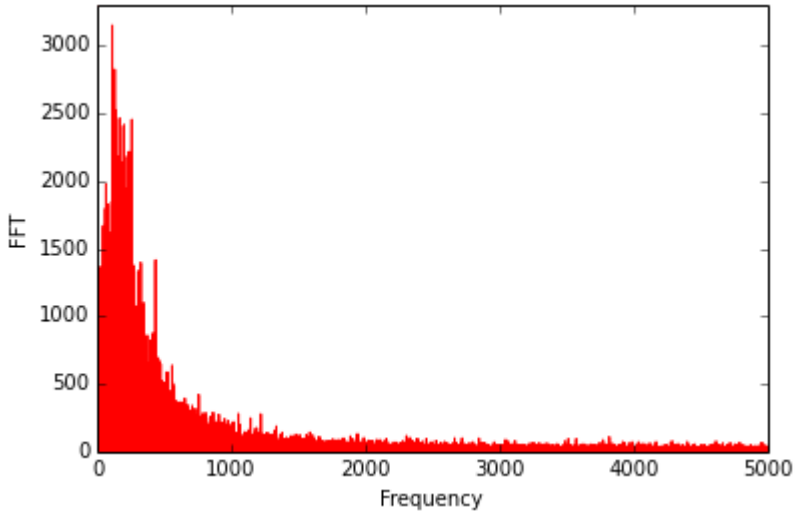


- As ω -> $+\infty$, there is a phase difference of $\pi/2$ between input and output.

Case Study: Frequency-Domain Analysis

- Spectra of a complex signal going through the given low-pass filter.





Frequency spectrum characteristics:

- At low frequency (100 - 200 Hz) – large magnitude with value > 2000.
- At cut-off frequency (500 Hz) – magnitude is half of peak e.g. about 1000.
- At high frequency (1000 Hz) – small magnitude with value < 250.