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WELLINGTON
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z-Transform (Applications)

XMUT220 Signals and Systems

Topics

Applications of z-Transforms:

- Solve DTLTI signals and systems with differential equations.
- Time and frequency responses of DTLTI signals and systems.
- Analysis of DTLTI signals and systems.

Applications of z-Transform

We look into the following applications of z-Transform:

a. Solve DTLTI systems with differential equation:
   - Convert differential equation to sequence.
   - Convert sequence to differential equation.

b. Evaluate responses of DTLTI signals and systems:
   - Time response analysis.
   - Frequency response analysis.

Applications of z-Transform

c. Analysis of DTLTI signals and systems:

- Causality of DTLTI signals and systems.
- Stability of DTLTI signals and systems.

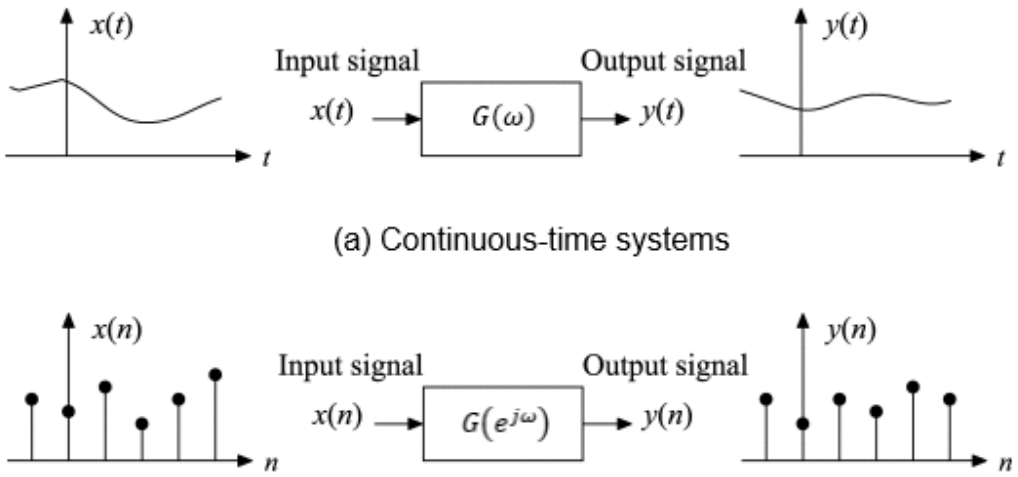
Solve Discrete System with DE

- The output signal of DTLTI system is related to its input signal and its impulse response through the convolution sum.

$$y[n] = x[n] * h[n] = \sum_{k}^{\infty} x[k]h[n - k] \qquad (Eq.1)$$

- The z-transform of the output signal is equal to the product of the z-transforms of the input signal and the impulse response.

$$Y(z) = X(z)H(z) \quad (Eq.2)$$



(a) Continuous-time systems

(b) Discrete-time systems

Solve Discrete System with DE

- Thus, rearranging the above equation, it is now:

$$H(z) = \frac{Y(z)}{X(z)} \qquad (Eq.3)$$

- Discrete-time LTI system can be described by means of a constant-coefficient linear difference equation.

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k] \qquad (Eq.4)$$

- Thus, we can solve the system function and the impulse response from the knowledge of its difference equation.

Solve Discrete System with DE

- Take the z-transform of both sides:

$$Z\left\{\sum_{k=0}^{N} a_k y[n-k]\right\} = Z\left\{\sum_{k=0}^{M} b_k x[n-k]\right\} \qquad (Eq.5)$$

- Using the linearity property of the z-transform:

$$\sum_{k=0}^{N} a_k(Z) y[n-k] = \sum_{k=0}^{M} b_k(Z) x[n-k]$$

- Using the time-shifting property of the z-transform:

$$\sum_{k=0}^{N} a_k z^{-k} Y(z) = \sum_{k=0}^{M} b_k z^{-k} X(z) \qquad (Eq.7)$$

Solve Discrete System with DE

- Factored out of the summations:

$$Y(z)\sum_{k=0}^{N} a_k z^{-k} = X(z)\sum_{k=0}^{M} b_k z^{-k} \qquad (Eq.8)$$

- the system function can be obtained from:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} \qquad (Eq.9)$$

Solve Discrete System with DE

Finding the system function from the difference equation:

1. Separate the terms of the difference equation so that $y[n]$ and its time-shifted versions are on the left of the equal sign, and $x[n]$ and its time-shifted versions are on the right of the equal sign, as in the equation (4).
2. Take the z-transforms of both sides of the difference equation and use the time-shifting property of the z-transform as in the equation (7).
3. Determine the system function as the ratio of $Y(z)$ to $X(z)$ as in the equation (9).
4. If the impulse response is needed, it can now be determined as the inverse *z*-transform of $H(z)$.

Example of DTLTI Systems with DE

Let a discrete-time LTI system be defined by the difference equation:

$$y[n] - 0.4y[n-1] + 0.89y[n-2] = x[n] - x[n-1]$$

Find the system function $H(z)$ for this system. [6 marks]

Example of DTLTI Systems with DE

- Assume zero initial conditions and take the z-transforms of both sides of the difference equation to get:

$$Y(z) - 0.4z^{-1}Y(z) + 0.89z^{-2}Y(z) = X(z) - z^{-1}X(z)$$

- Rearranging the equation above in terms of $H(z) = Y(z)/X(z)$, thus:

$$Y(z)[1 - 0.4z^{-1} + 0.89z^{-2}] = Z(z)[1 - z^{-1}\,]$$

- From which the system function is obtained as:

$$H(z) = \frac{1 - z^{-1}}{1 - 0.4z^{-1} + 0.89z^{-2}}$$

Example of DTLTI Systems with DE

- Or, using non-negative powers of z:

$$H(z) = \frac{z(z-1)}{z^2 - 0.4z + 0.89}$$

Time-Response of DTLTI Signals and Systems

Response to Complex Exponential Signal:

- The response of a discrete-time LTI system to a complex exponential signal in the form:

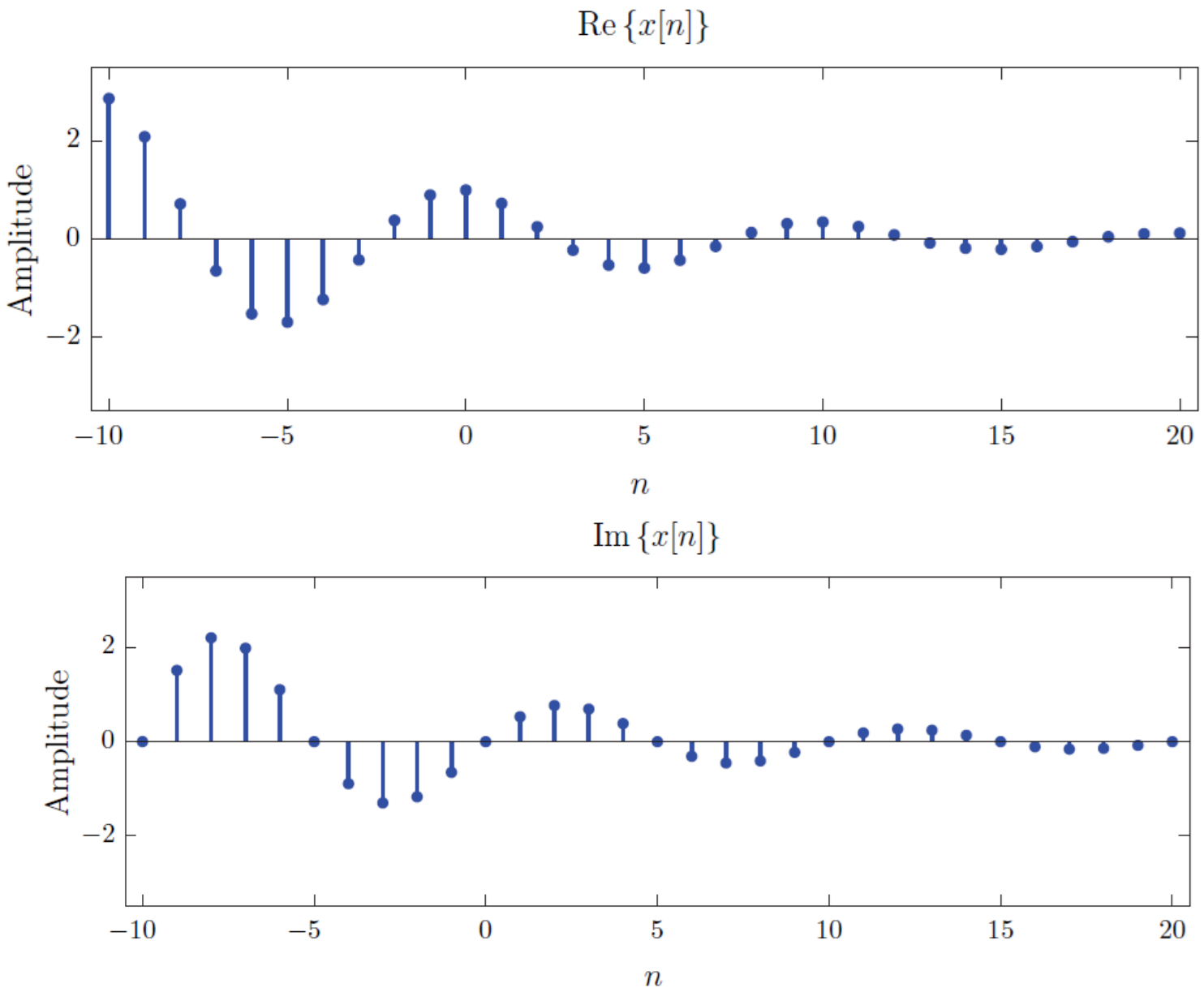
$$x[n] = z_0^n$$

  Where: z_0 represents a point in the z-plane within the ROC of the system function

- The output signal is determined using the convolution sum:

$$y[n] = h[n] * x[n] = \sum_{k=-\infty}^{\infty} h[k]x[n-k] \qquad (Eq.10)$$

Time-Response of DTLTI Signals and Systems



Time-Response of DTLTI Signals and Systems

- Substituting $x[n-k] = z_0^{n-k}$ into the equation:

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] z_0^{n-k} = z_0^n \sum_{k=-\infty}^{\infty} h[k] z_0^{-k} \qquad (Eq.11)$$

- Evaluated at the point $z = z_0$, so the response of the discrete-time LTI system to the input signal $x[n] = z_0^n$ is:

$$y[n] = z_0^n H(z_0) \qquad (Eq.12)$$

Example of Time-Response of DTLTI Systems

Consider a discrete-time LTI system with the system function

$$H(z) = \frac{z+1}{z^2 - \frac{5}{6}z + \frac{1}{6}}$$

a. Write $x[n]$ in Cartesian form. [10 marks]

b. Simulate the real and imaginary parts of $x[n]$ in MATLAB. [4 marks]

c. Find the response of the system to an exponential signal: [6 marks]

$$x[n] = \left(0.9e^{j0.2\pi}\right)^n$$

d. Simulate the real and imaginary parts of $y[n]$ in MATLAB. [4 marks]

Example of Time-Response of DTLTI Systems

a. The input signal is complex-valued.

$$x[n] = \left(0.9e^{j0.2\pi}\right)^n$$

Using Euler's formulae,

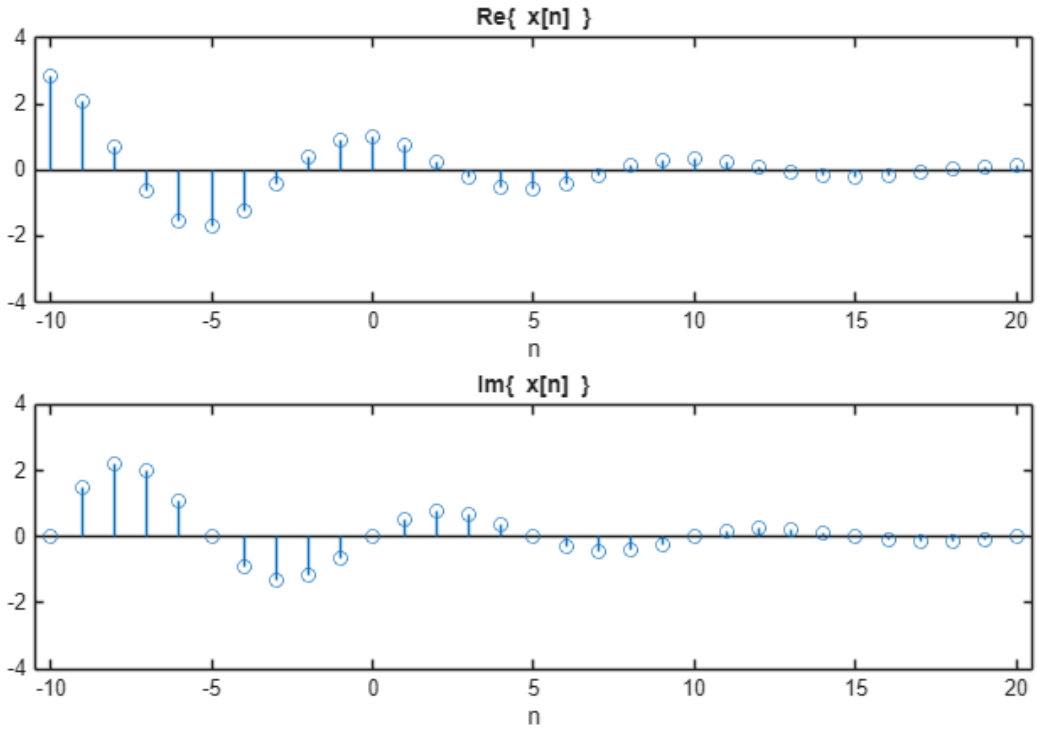
$$r(\cos\theta + j\sin\theta) = r^{j\theta}$$

we can write $x[n]$ as:

$$x[n] = (0.9)^n \cos(0.2\pi n) + j(0.9)^n \sin(0.2\pi n)$$

Example of Time-Response of DTLTI Systems

b. Real and imaginary parts of the input signal, $x[n]$ are shown in figure (a) and (b) below respectively.



Example of Time-Response of DTLTI Systems

c. The value of the system function at $z_0 = 0.9e^{j0.2\pi}$ is:

$$H(0.9e^{j0.2\pi}) = \frac{0.9e^{j0.2\pi} + 1}{(0.9e^{j0.2\pi})^2 - \frac{5}{6}(0.9e^{j0.2\pi}) + \frac{1}{6}}$$

$$= -1.0627 - j4.6323 = 4.7526e^{-j1.7963}$$

- The output signal $y[n]$ is found using equation below as:

$$y[n] = z_0^n H(z_0) = \left(0.9e^{j0.2\pi}\right)^n (4.7526e^{-j1.7963})$$

$$= 4.7526(0.9)^n e^{j(0.2\pi n - 1.7963)}$$

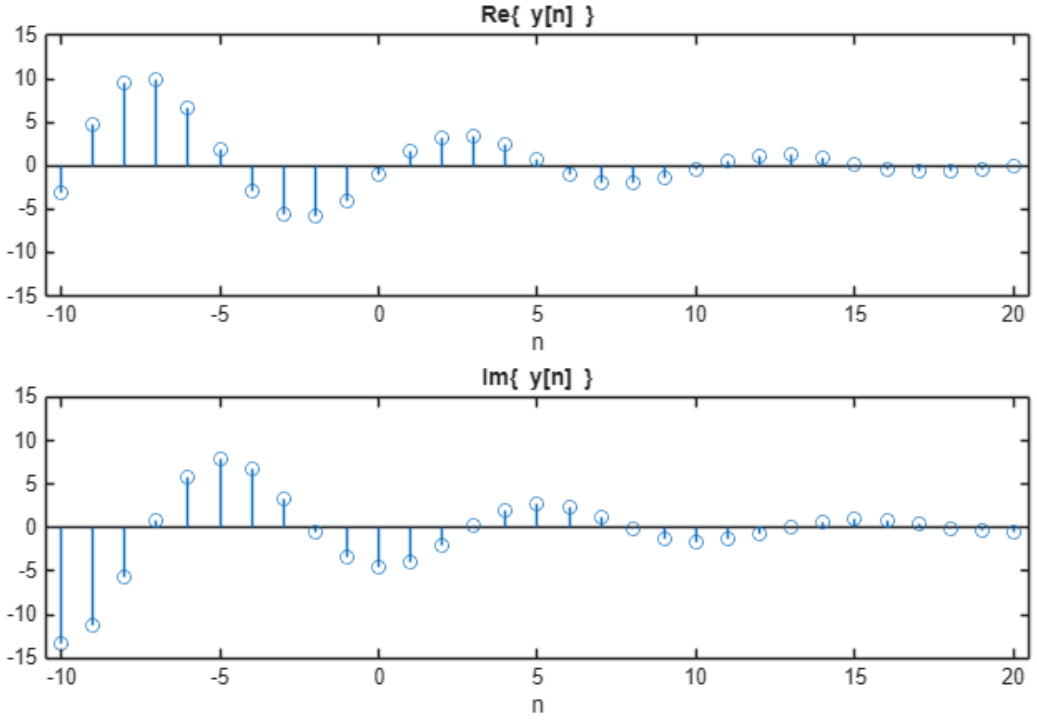
- or, in Cartesian form:

$$y[n] = 4.7526(0.9)^n \cos(0.2\pi n - 1.7963)$$

$$+ j4.7526(0.9)^n \sin(0.2\pi n - 1.7963)$$

Example of Time-Response of DTLTI Systems

d. Real and imaginary parts of the output signal $y[n]$ are shown in the figure (a) and (b) below respectively.

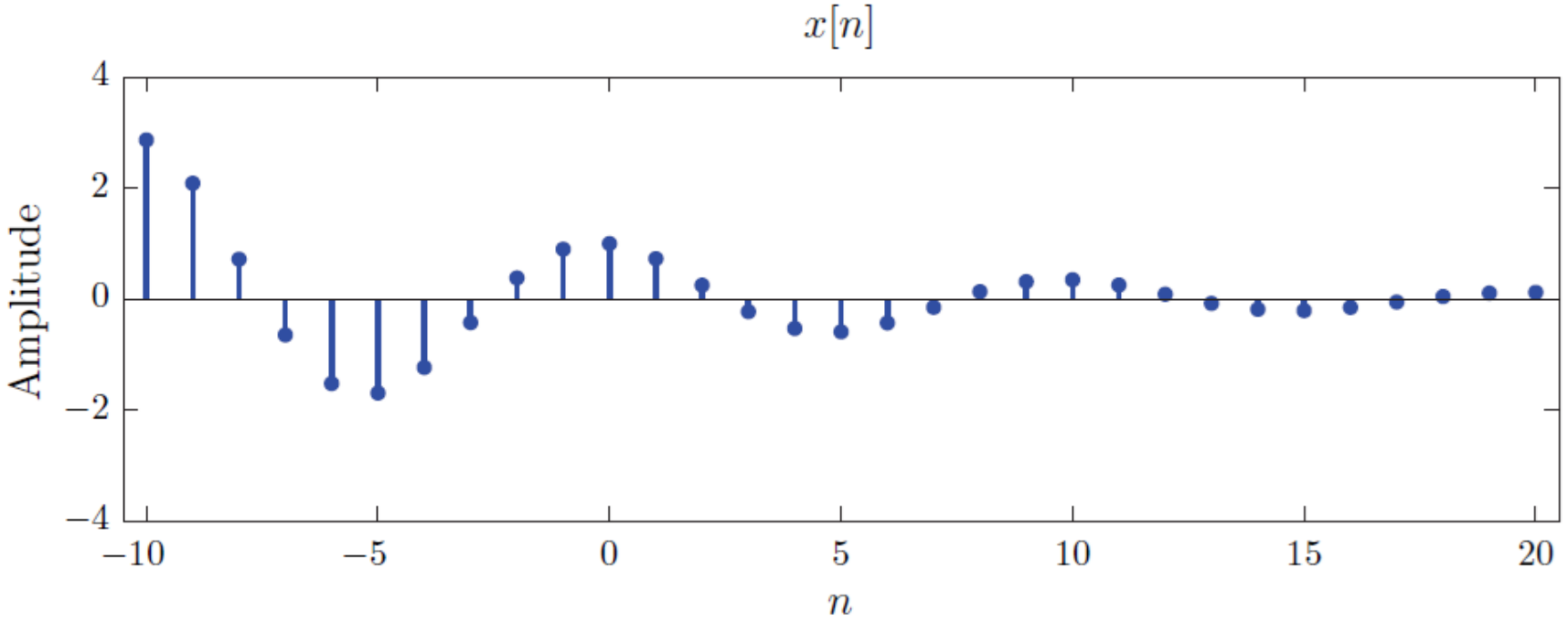


Time-Response of DTLTI Signals and Systems

Response to Exponentially Damped Sinusoid:

- Consider an input signal in the form of an exponentially damped sinusoid such as:

$$x[n] = r_0^n \cos(\Omega_0 n) \qquad (Eq.13)$$



Time-Response of DTLTI Signals and Systems

- Write $x[n]$ using Euler's formula:

$$x[n] = 0.5r_0^n e^{j\Omega_0 n} + 0.5r_0^n e^{-j\Omega_0 n} \qquad (Eq.\,14)$$

- Let z_0 be defined as:

$$z_0 = r_0 e^{j\Omega_0}$$

- It follows that:

$$z_0^* = r_0 e^{-j\Omega_0}$$

- The equation (14) can be written as:

$$x[n] = 0.5Z_0^n + 0.5(Z_0^*)^n \qquad (Eq.\,15)$$

Time-Response of DTLTI Signals and Systems

- The response of the system is determined using the linearity of the system:

$$y[n] = \text{Sys}\{0.5Z_0^n + 0.5(Z_0^*)^n\}$$
$$= 0.5\text{Sys}\{Z_0^n\} + 0.5\text{Sys}\{(Z_0^*)^n\} \qquad (Eq.16)$$

- We already know that the response of the system to the term z_0^n is:

$$\text{Sys}\{z_0^n\} = z_0^n H(z_0) \qquad (Eq.17)$$

- Its response to the term $(z_0^*)^n$ is:

$$\text{Sys}\{(z_0^*)^n\} = (z_0^*)^n H(z_0^*) \qquad (Eq.18)$$

Time-Response of DTLTI Signals and Systems

- So that the output signal $y[n]$ can be written as:

$$y[n] = 0.5z_0^n H(z_0) + 0.5(z_0^*)^n H(z_0^*) \qquad (Eq.19)$$

- The result obtained in the equation (19) can be further simplified.
- Let the value of the system function evaluated at the point $z = z_0$ be written in polar complex form as:

$$H(z_0) = H_0 e^{j\theta_0} \qquad (Eq.20)$$

- Where: H_0 and θ_0 represent the magnitude and the phase of the system function at the point $z = z_0$ respectively:

$$H_0 = |H(z_0)| \qquad (Eq.21)$$

- and

$$\theta_0 = \angle H(z_0) \qquad (Eq.22)$$

Time-Response of DTLTI Signals and Systems

- Shown that for a system with a real-valued impulse response, the value of the system function at the point $z = z_0^*$ is the complex conjugate of its value at the point $z = z_0$, that is,

$$H(z_0^*) = [H(z_0)]^* = H_0 e^{-j\theta_0} \qquad (Eq. 23)$$

- Using the equation (20) and (23) in the equation (19), the output signal $y[n]$ becomes:

$$\begin{aligned} y[n] &= 0.5 z_0^n H_0 e^{j\theta_0} + 0.5 (z_0^*)^n H_0 e^{-j\theta_0} \\ &= 0.5 \left(r_0^n e^{j\Omega_0}\right)^n H_0 e^{j\theta_0} + 0.5 \left(r_0^n e^{-j\Omega_0}\right)^n H_0 e^{-j\theta_0} \\ &= 0.5 r_0^n H_0 \left[e^{j(\Omega_0 n + \theta_0)} - e^{-j(\Omega_0 n + \theta_0)}\right] \\ &= r_0^n H_0 \cos(\Omega_0 n + \theta_0) \qquad (Eq. 24) \end{aligned}$$

Time-Response of DTLTI Signals and Systems

Comparison of the input signal in the equation (13) and the output signal equation (24) reveals the following:

- The amplitude of the signal is multiplied by the magnitude of the system function evaluated at the point $z = r_0 e^{j\Omega}$.
- The phase of the cosine function is incremented by an amount equal to the phase of the system function evaluated at the point $z = r_0 e^{j\Omega}$.

Example of Time-Response of DTLTI Systems

Considering the system function used in the previous example:

$$H(z) = \frac{z+1}{z^2 - \frac{5}{6}z + \frac{1}{6}}$$

Determine the response of the system to an exponentially damped sinusoid input signal:

$$x[n] = (0.9)^n \cos(0.2\pi n)$$

a. Obtain the output signal, $y[n]$ in Cartesian form. [6 marks]

b. Simulate the input signal $x[n]$ and output signal $y[n]$ in MATLAB. [4 marks]

Example of Time-Response of DTLTI Systems

- In the previous example, we have evaluated the system function at the point $z = 0.9e^{j0.2\pi}$ and have obtained:

$$H(0.9e^{j0.2\pi}) = H_0 e^{j\Theta_0} = 4.7526e^{-j1.7963}$$

- Using the equation given below,

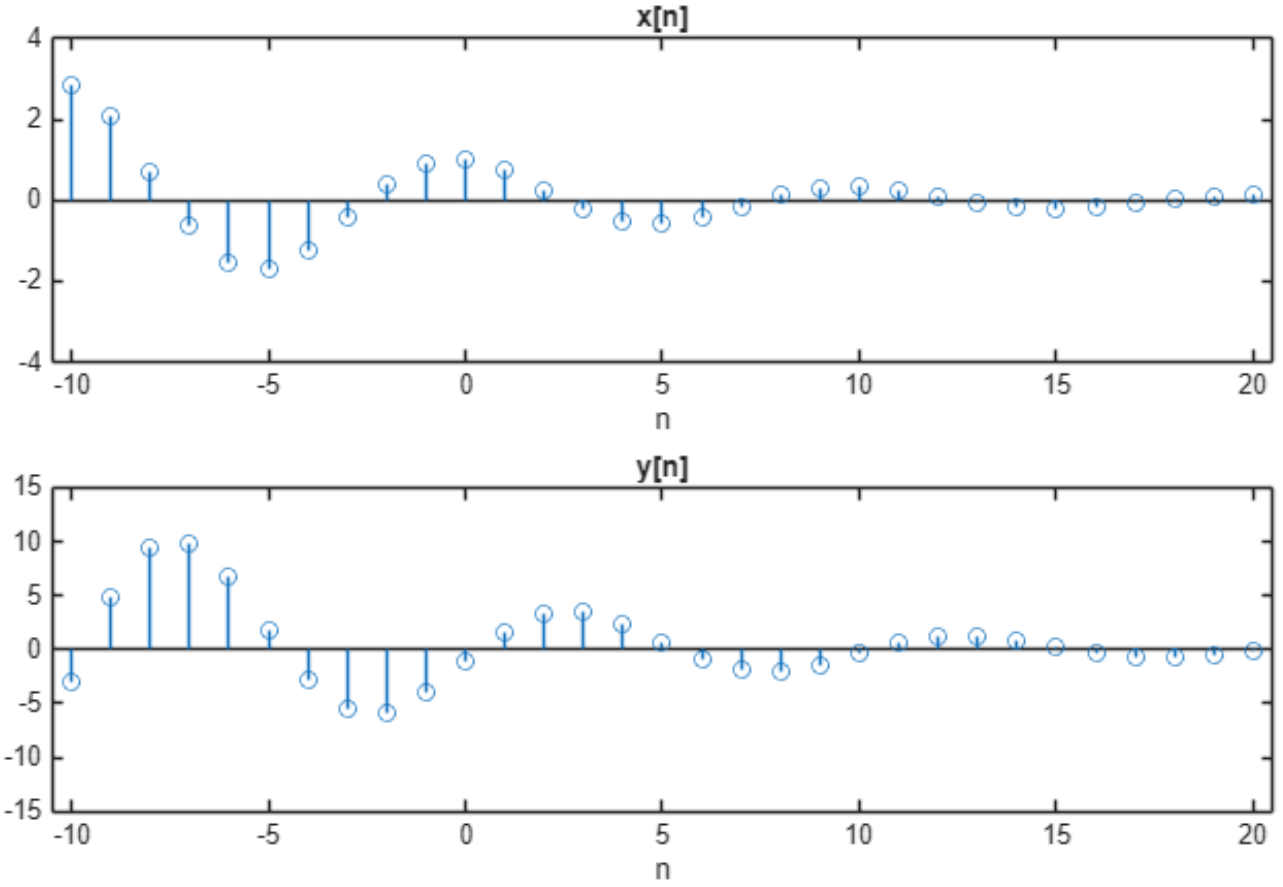
$$y[n] = H_0 r_0^n \cos(\Omega_0 n + \theta_0)$$

- With $H_0 = 4.7526$ and $\theta_0 = -1.7963$ radians, we obtain the output signal, $y[n]$ as:

$$y[n] = 4.7526(0.9)^n \cos(0.2\pi n - 1.7963)$$

Example of Time-Response of DTLTI Systems

b. The following figures show the input signal $x[n]$ and the output signal $y[n]$ of the MATLAB simulation.



Frequency Response of DTLTI Systems

- z-transform to the DTFT is with transformation $z \rightarrow e^{j\Omega}$ with Ω a real variable representing discrete-time radian frequency.
- Determining frequency response, the only values of z that we are now considering are those on the unit circle in the z-plane because $|e^{j\Omega}| = 1$ for any real Ω.
- Suppose the transfer function of a system is:

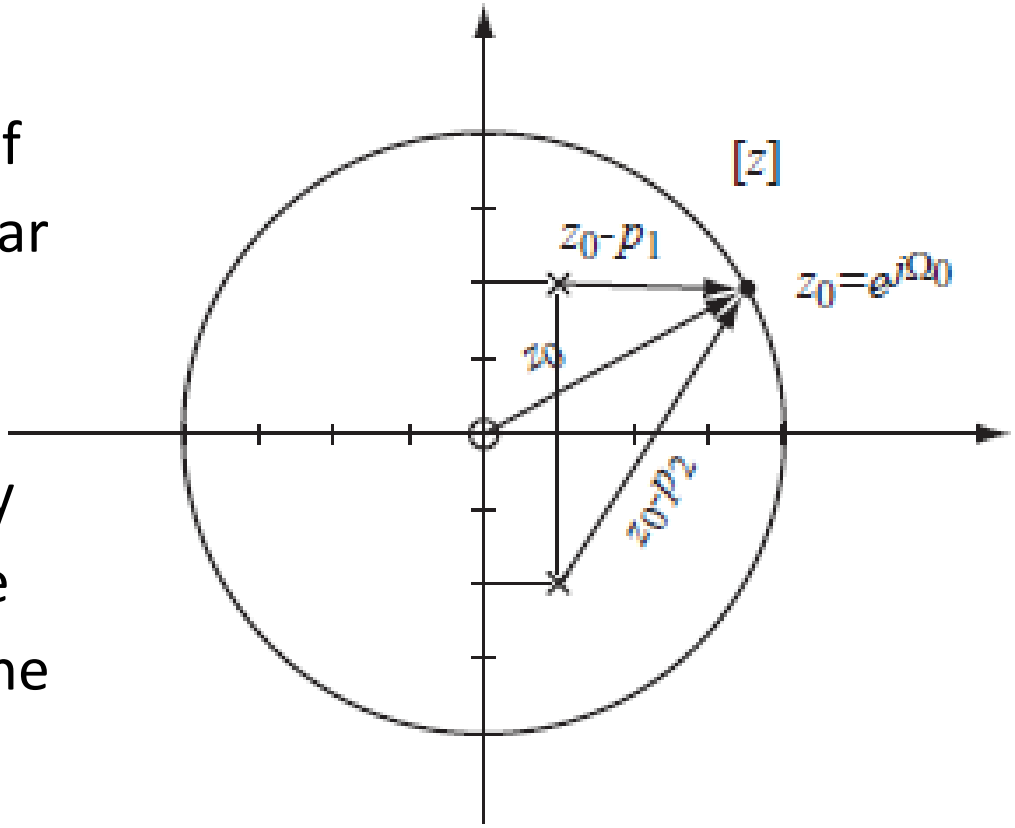
$$H(z) = \frac{z}{(z - p_1)(z - p_2)} = \frac{z}{z^2 - \frac{z}{2} + \frac{5}{16}} \qquad (Eq. 25)$$

- Where:

$$p_1 = \frac{1 + j2}{4} \qquad \text{and} \qquad p_2 = \frac{1 - j2}{4}$$

Frequency Response of DTLTI Systems

- Transfer function has a zero at the origin and two complex-conjugate finite poles.
- The frequency response of the system at any particular radian frequency Ω_0 is determined (to within a multiplicative constant) by the vectors from the finite poles and finite zeros of the transfer function to the *z*-plane point $z_0 = e^{j\Omega_0}$.



Frequency Response of DTLTI Systems

- The magnitude of the frequency response is the product of the magnitudes of the zero vectors divided by the product of the magnitudes of the pole vectors. In this case,

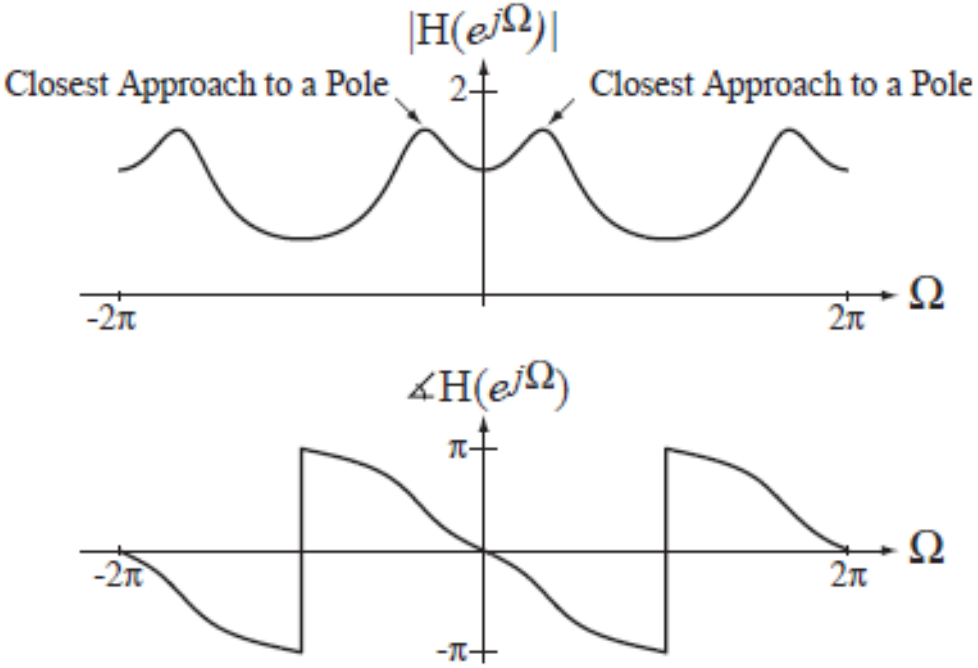
$$\left|H\left(e^{j\Omega}\right)\right| = \frac{\left|e^{j\Omega}\right|}{|e^{j\Omega} - p_1||e^{j\Omega} - p_2|} \qquad (Eq. 26)$$

- It is apparent that as $e^{j\Omega}$ approaches a pole, p_1 for example, the magnitude of the difference $e^{j\Omega} - p_1$ becomes small, making the magnitude of the denominator small and tending to make the magnitude of the transfer function larger.
- The opposite effect occurs when $e^{j\Omega}$ approaches a zero.

Frequency Response of DTLTI Systems

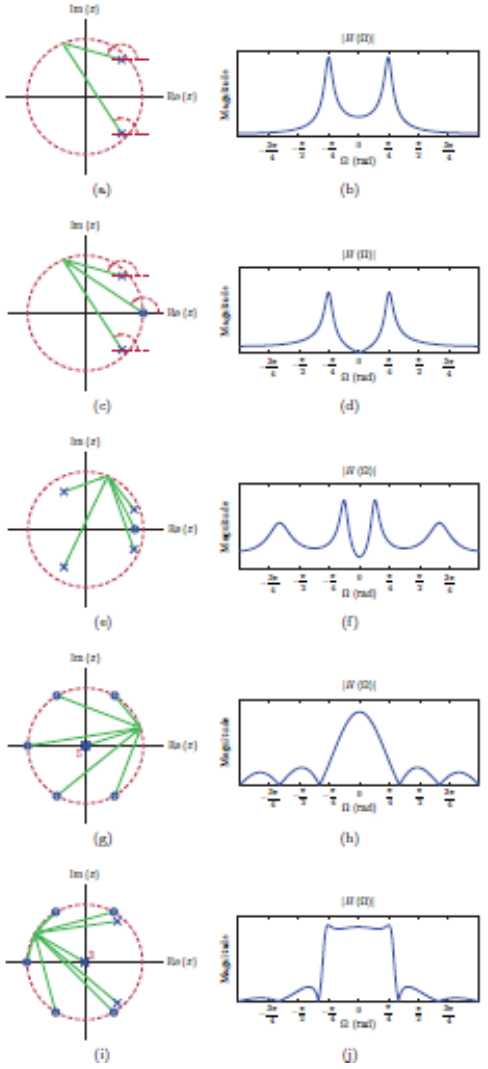
- The phase of the frequency response is the sum of the angles of the zero vectors minus the sum of the angles of the pole vectors.
- In this case, as shown in the figure below,

$$\measuredangle H\left(e^{j\Omega}\right) = \measuredangle e^{j\Omega} - \measuredangle\left(e^{j\Omega} - p_1\right) - \measuredangle\left(e^{j\Omega} - p_2\right) \qquad (Eq.27)$$



Frequency Response of DTLTI Systems

- Maximum magnitude frequency response occurs at approximately $z = e^{\pm j1.11}$.
- These are the points on the unit circle at the same angle as the finite poles of the transfer function.
- Therefore, the points on the unit circle at which the denominator factors $e^{j\Omega_0} - p_1$ and $e^{j\Omega_0} - p_2$ in equation (27) reach their minimum magnitudes.



Example of Frequency Response of DTLTI Systems

A system with transfer function given in the equation below is excited by a unit sequence.

$$H(z) = \frac{z}{(z - 0.3)(z + 0.8)} \qquad |z| > 1$$

Find the total response of the system using the following method:

a. Z-transform. [10 marks]

b. Discrete-time Fourier transform. Comment on the difference between two methods. [32 marks]

Example of Frequency Response of DTLTI Systems

a. The z transform of the response is:

$$Y(z) = H(z).X(z) = \left[\frac{z}{(z-0.3)(z+0.8)}\right]\left(\frac{z}{z-1}\right) \qquad |z| > 1$$

- Expanding in partial fractions,

$$Y(z) = \frac{z^2}{(z-0.3)(z+0.8)(z-1)}$$

$$= \frac{0.1169}{z-0.3} + \frac{0.3232}{z+0.8} + \frac{0.7937}{z-1} \qquad |z| > 1$$

- Therefore, the total response is:

$$y[n]$$
$$= [-0.1169(0.3)^{n-1} + 0.3232(-0.8)^{n-1} + 0.7937]u[n-1]$$

Example of Frequency Response of DTLTI Systems

b. The z transform of the response is:

$$Y(z) = H(z)X(z) = \left[\frac{z}{(z-0.3)(z+0.8)}\right]\left(\frac{z}{z-1}\right) \qquad |z| > 1$$

- Expanding in partial fractions, the equation above becomes:

$$Y(z) = \frac{z^2}{(z-0.3)(z+0.8)(z-1)}$$

$$= \frac{0.1169}{z-0.3} + \frac{0.3232}{z+0.8} + \frac{0.7937}{z-1} \qquad |z| > 1$$

- Therefore, the overall response is:

$$y[n] = [-0.1169(0.3)^{n-1} + 0.3232(-0.8)^{n-1}$$

$$+0.7937]u[n-1]$$

Example of Frequency Response of DTLTI Systems

- Or

$$Y(e^{j\Omega}) = H(e^{j\Omega})X(e^{j\Omega})$$

$$= \frac{e^{j\Omega}}{(e^{j\Omega} - 0.3)(e^{j\Omega} + 0.8)(1 - e^{j\Omega})}$$

$$+\pi \left[\frac{e^{j\Omega}}{(e^{j\Omega} - 0.3)(e^{j\Omega} + 0.8)}\right] \delta_{2\pi}(\Omega)$$

- Expanding in partial fractions, the equation above becomes:

$$Y(e^{j\Omega}) = \frac{-0.1169}{e^{j\Omega} - 0.3} + \frac{0.3232}{e^{j\Omega} + 0.8}$$

$$+ \frac{0.7937}{e^{j\Omega} - 1} + \left(\frac{\pi}{(1 - 0.3)(1 + 0.8)}\right) \delta_{2\pi}(\Omega)$$

Example of Frequency Response of DTLTI Systems

- Using the equivalence property of the impulse and the periodicity of both $\delta_{2\pi}(\Omega)$ and $e^{j\Omega}$, the equation above becomes.

$$Y\left(e^{j\Omega}\right) = \frac{-0.1169e^{-j}}{1-0.3e^{-j\Omega}} + \frac{0.3232e^{-j\Omega}}{1+0.8e^{-j\Omega}} + \frac{0.7937e^{-j\Omega}}{1-e^{-j\Omega}} + 2.4933\delta_{2\pi}(\Omega)$$

- Then, manipulating this expression into a form for which the inverse discrete-time Fourier transform is direct:

$$Y\left(e^{j\Omega}\right) = \frac{0.2269e^{-j\Omega}}{1-0.3e^{-j\Omega}} + \frac{0.3232e^{-j\Omega}}{1+0.8e^{-j\Omega}} + 0.7937\left[\frac{e^{-j\Omega}}{1-e^{-j\Omega}} + \delta_{2\pi}(\Omega)\right] - 0.7937\pi\delta_{2\pi}(\Omega) + 2.4933\delta_{2\pi}(\Omega)$$

Example of Frequency Response of DTLTI Systems

- With $0.7937\pi\delta_{2\pi}(\Omega) + 2.4933\delta_{2\pi}(\Omega) = 0$, thus:

$$Y(e^{j\Omega}) = \frac{0.2269e^{-j\Omega}}{1 - 0.3e^{-j\Omega}} + \frac{0.3232e^{-j\Omega}}{1 + 0.8e^{-j\Omega}}$$

$$+0.7937\left[\frac{e^{-j\Omega}}{1 - e^{-j\Omega}} + \delta_{2\pi}(\Omega)\right]$$

- And, finally, taking the inverse discrete-time Fourier transform of the equation above:

$$y[n] = [-0.1169(0.3)^{n-1} + 0.3232(-0.8)^{n-1} + 0.7937]u[n-1]$$

- The result is the same, but the effort and the probability of error are considerably greater.

Analysis of DTLTI Signals and Systems

- For a discrete-time LTI system to be causal, its impulse response $h[n]$ needs to be equal to zero for $n < 0$.
- Thus, by changing the lower limit of the summation index to n = 0 in the definition of the z-transform, the z-domain system function for a causal discrete-time LTI system can be written as:

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n} = \sum_{n=0}^{\infty} h[n]z^{-n} \qquad (Eq.28)$$

- The ROC for the system function of a causal system is the outside of a circle in the z-plane. Consequently, the system function must also converge at $|z| \to \infty$.

Analysis of DTLTI Signals and Systems

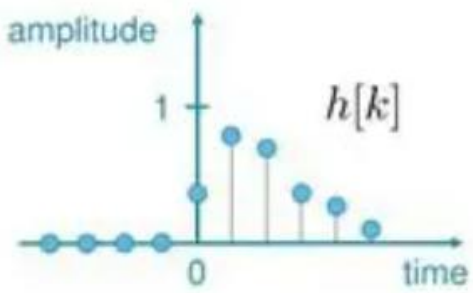
- Consider a system function in the form

$$H(z) = \frac{B(z)}{A(z)} = \frac{b_M z^M + b_{M-1} z^{M-1} + \ldots + b_1 z + b_0}{a_N z^N + a_{N-1} z^{N-1} + \ldots + a_1 z + a_0} \quad (Eq. 29)$$

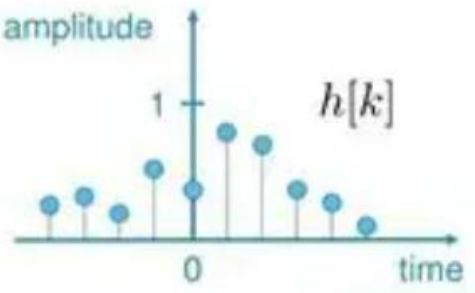
- For the system described by $H(z)$ to be causal, we need:

$$\lim_{z \to \infty} |H(z)| = \lim_{z \to \infty} \left(\frac{b_M}{a_N} \right) z^{M-N} < \infty \quad (Eq. 30)$$

which requires that $M - N \leq 0$ and consequently $M \leq N$.



Causal system



Non-causal system

Analysis of DTLTI Signals and Systems

- A discrete-time LTI system is stable if its impulse response must be absolute summable, that is,

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty \qquad (Eq.31)$$

- The discrete-time Fourier transform of a signal exists if the signal is absolute summable.
- But the discrete-time Fourier transform of the impulse response is equal to the z-domain transfer function evaluated on the unit circle of the z-plane, that is,

$$H(\Omega) = H(z)\Big|_{z=e^{j\Omega}} \qquad (Eq.32)$$

provided that the unit circle of the z-plane is within the ROC.

Analysis of DTLTI Signals and Systems

- The ROC for the system function of a causal system is the outside of a circle with radius r_1, and is expressed in the form:

$$|z| > r_1 \qquad (Eq.33)$$

- For the ROC to include the unit circle, we need $r_1 < 1$.
- Since the ROC cannot have any poles in it, all the poles of the system function must be on or inside the circle with a radius r_1.

Analysis of DTLTI Signals and Systems

- If the system is anti-causal, its impulse response is equal to zero for $n \geq 0$ and the ROC for the system function is the inside of a circle with radius r_2, expressed in the form:

$$|z| < r_2 \qquad (Eq.34)$$

- For the ROC to include the unit circle, we need $r_2 > 1$.
- Also, all the poles of the system function must reside on or outside the circle with radius r_2 since there can be no poles within the ROC.

Analysis of DTLTI Signals and Systems

- In this case the ROC for the system function, if it exists, is the region between two circles with radii r_1 and r_2, and is expressed in the form:

$$r_1 < |z| < r_2 \qquad (Eq. 35)$$

- The poles of the system function may be either:
  - On or inside the circle with radius r_1
  - On or outside the circle with radius r_2
- and the ROC must include the unit circle.

Example of Analysis of DTLTI Systems

A stable system is characterised by the system function:

$$H(z) = \frac{z(z+1)}{(z-0.8)(z+1.2)(z-2)}$$

a. Determine the ROC of the system. [4 marks]

b. Applying partial fraction expansion, determine the impulse response of the system. Based on the result of the partial fraction expansion, comment of the causality of the system. [14 marks]

Example of Analysis of DTLTI Systems

a. The ROC for the system function is not directly stated; however, we are given enough information to deduce it.

- The zeros of the system are $z_1 = 0$ and $z_2 = -1$.
- The poles of the system are at $p_1 = 0.8$, $p_2 = -1.2$ and $p_3 = 2$.

Since the system is known to be stable, its ROC must include the unit circle.

The only possible choice is:

$$0.8 < |z| < 1.2$$

Example of Analysis of DTLTI Systems

b. Applying partial fraction expansion, the equation above becomes:

$$\frac{z(z+1)}{(z-0.8)(z+1.2)(z-2)} = \frac{A}{(z-0.8)} + \frac{B}{(z+1.2)} + \frac{C}{(z-2)}$$

The coefficients in the equation above are:

- Coefficient A:

$$A = \left.\frac{z(z+1)}{(z+1.2)(z-2)}\right|_{z=0.8} = 0.75$$

- Coefficient B:

$$B = \left.\frac{z(z+1)}{(z-0.8)(z-2)}\right|_{z=-1.2} = 0.0312$$

Example of Analysis of DTLTI Systems

- Coefficient C:

$$C = \left.\frac{z(z+1)}{(z-0.8)(z+1.2)}\right|_{z=2} = 0.7813$$

- Partial fraction expansion of $H(z)$ is:

$$H(z) = -\frac{0.75z}{z-0.8} - \frac{0.0312z}{z+1.2} + \frac{0.7813z}{z-2}$$

- Based on the ROC determined above, the first partial fraction corresponds to a causal signal, and the other two correspond to anti-causal signals.

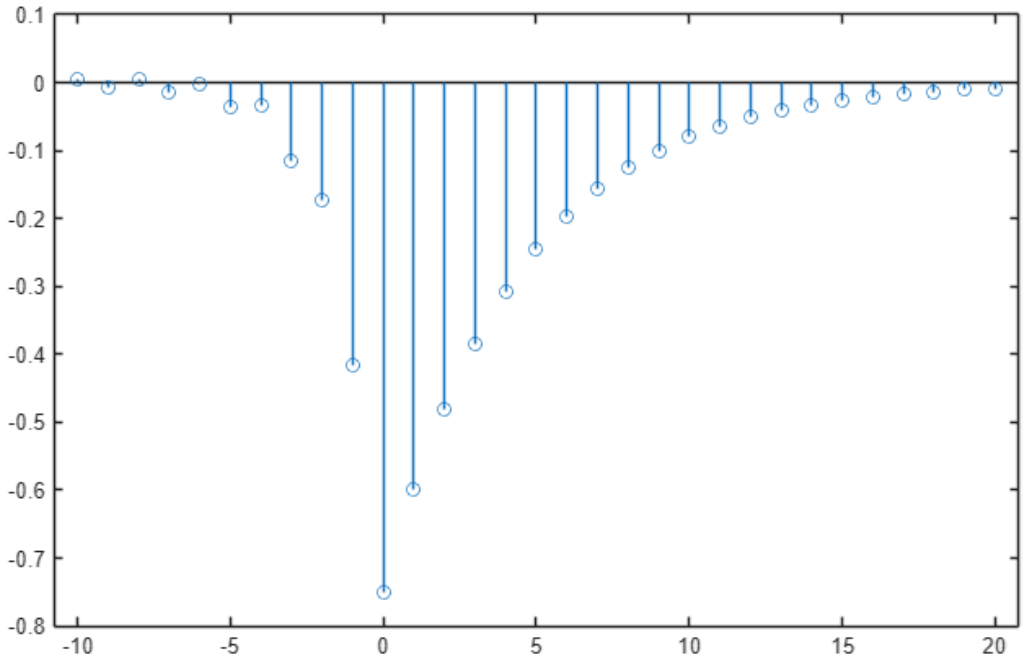
Example of Analysis of DTLTI Systems

- The impulse response of the system is:

$$h[n] = -0.75(0.8)^n u[n] + 0.0312(-1.2)^n u[-n-1]$$
$$-0.7813(2)^n u[-n-1]$$

Example of Analysis of DTLTI Systems

c. The simulated impulse response of the DTLTI system are shown below.



We observe that $h[n]$ tends to zero as the sample index is increased in both directions, consistent with the fact that $h[n]$ must be absolute summable.