XMUT220 Signals and Systems

Time and Frequency Characteristics (Introduction)

Topics

1. Fundamentals of the Time and Frequency Characteristics.
2. Time and Frequency Characteristics Analysis.
3. Application of Time and Frequency Analysis.
4. Example of Time and Frequency Domain Analysis.

1. Time and Frequency Domain Characteristics

For the time domain characteristics, we perform analysis and measurement of characteristics and phenomena of the signals and systems in time domain. On other hand, for the frequency domain characteristics, we carry out analysis and measurement of characteristics and phenomena of the signals and systems in frequency domain.

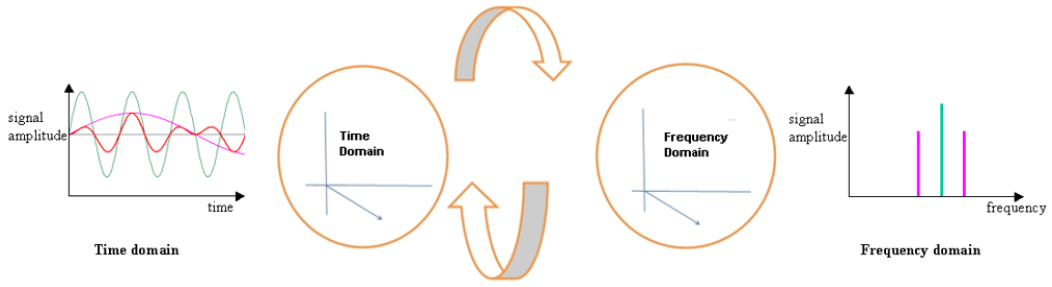


Figure 1: Transformation from time domain to frequency domain

Transformation from the time domain to the frequency domain is often performed through use of transformation techniques: Fourier transform, Laplace transform (continuous time) and z-transform (discrete time).

2. What is Time-Domain Analysis?

Time-domain analysis is the time response of a system to an input that sets the criteria for our systems. Many quantitative criteria have been defined to characterise the time response of a system. The time constant and system gain of a system are useful in its analysis, but other criteria describe the time response more accurately to an engineer i.e. rise time, settling time, etc.

2.1. Things Measured in Time-Domain Analysis

As illustrated in the figure given below, there are the parameters of particular interest in the time-domain analysis e.g. rise time, peak time, overshoot, and settling time.

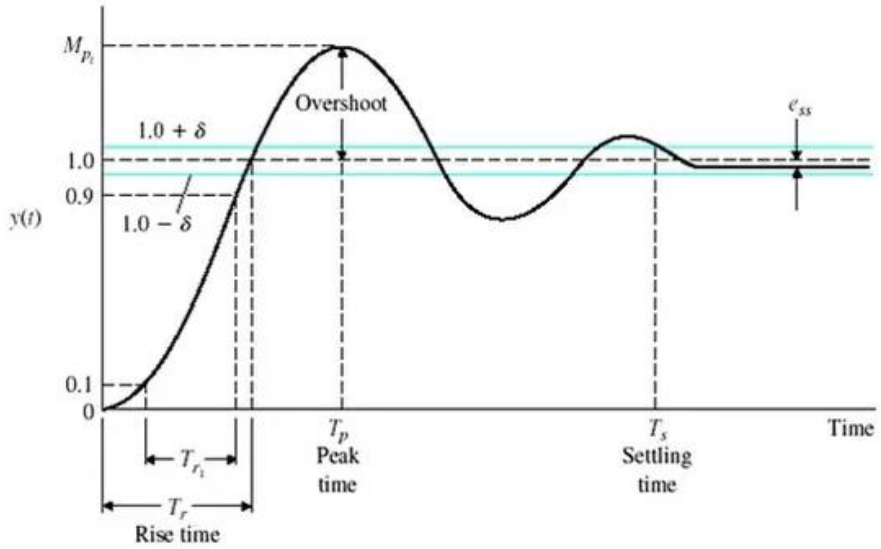


Figure 2: Parameters in the time-domain analysis

Quantitative parameters: *The time-domain analysis quantitative parameters* are the transient response of the system for a given input that can be measured in the time domain.

- Initial transient condition: the response of the system at the start of observation. The most used parameters are time constant, rise time, settling time, damping ratio, time-to-peak, percentage overshoot, etc.
- Steady-state condition: it is the final value or intended outcome of the response of the system. The most used parameters are steady-state amplitude, time and error.

Qualitative phenomena: *Qualitative phenomena* are any observed behaviour of the system in the time domain e.g. oscillations, peaking, wave distortion, etc.

2.2. What Are the Applications of Time-Domain Analysis?

Measuring transient response of the system i.e. how quickly the system reacts to a given input.

- How quick to change state?
- Whether the system is oscillating/ringing or experiencing damped response?

Measuring the steady-state behaviour of the system (i.e. when the time is approaching infinity (∞)).

- When the system settles down?
- Is there any difference between the steady-state value with intended/ final value?

2.3. Application of Time-Domain Analysis 1

Given a mechanical system of a car damper system consisting of a damper, a spring, and a mass of in series and parallel configuration as shown below.

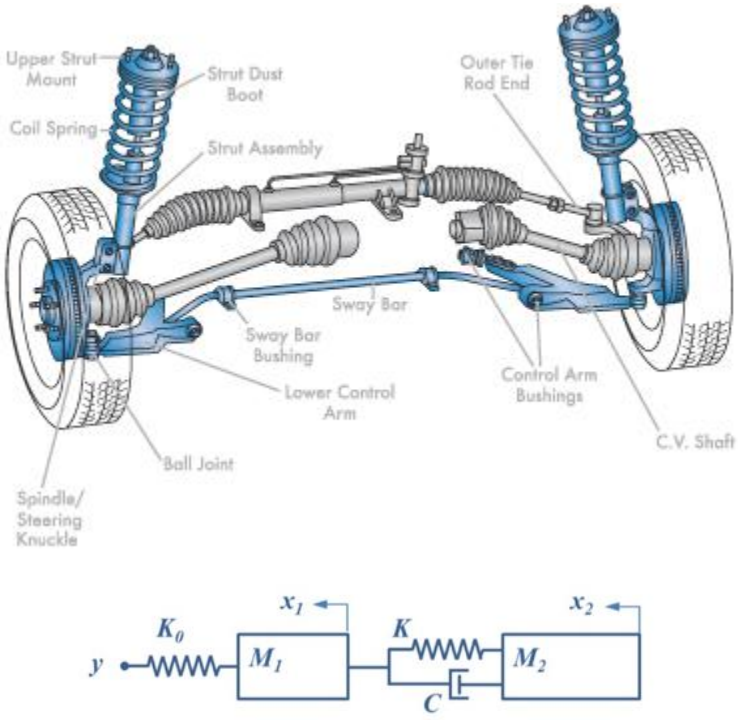


Figure 3: Damper, spring, and mass mechanical system in a car damper system

We can do an analysis of mechanical system performance in terms of transient and steady-state responses.

We can also test the system by obtaining different transient responses of the mechanical system subjected to an input (e.g. unit step) for variety of damping factors.

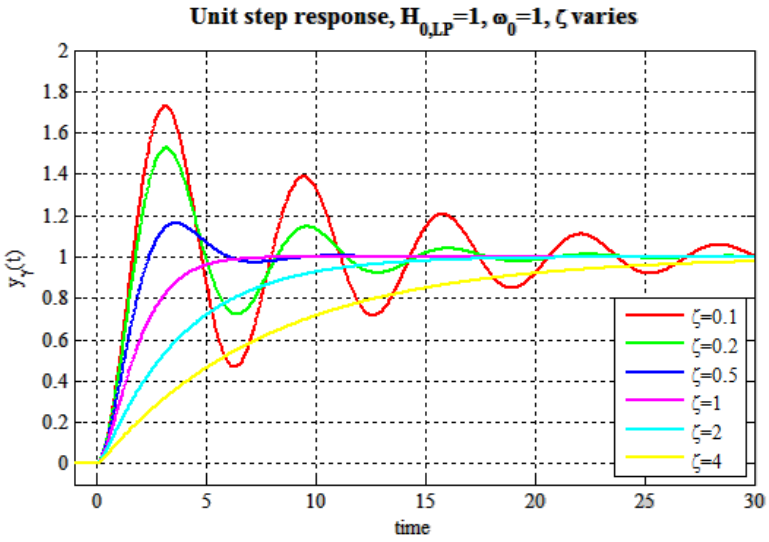


Figure 4: Transient response of the car damper system for variety of damping factors

Note:

$\zeta < 0.707$ - small damping (decaying oscillation).

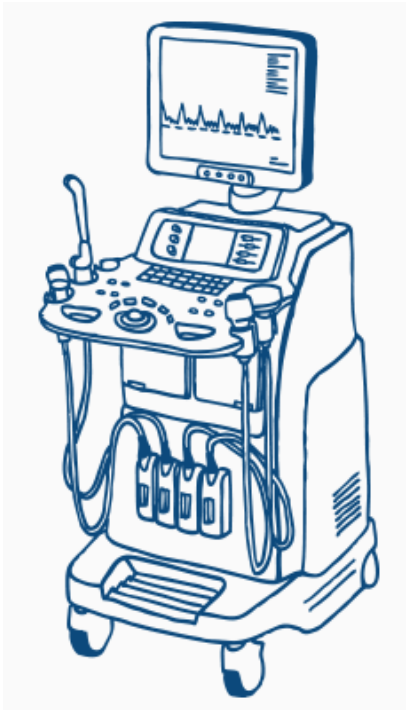
$\zeta = 0.707$ - critical damping (transition).

$\zeta > 0.707$ - large damping (exponential increase).

The figure given above shows an analysis of the mechanical system's performance in terms of transient and steady-state responses i.e. different responses of the system subjected to a unit step for a variety of damping factors.

2.4. Application of Time Domain Analysis 2

Given in the figure below is an ultrasonography (USG) medical equipment used to detect object using ultra high frequency sound wave and shows it on a display. It has a digital signal processing system that works on an analogue input signal and produces specific analogue output signal for image display after the processing.



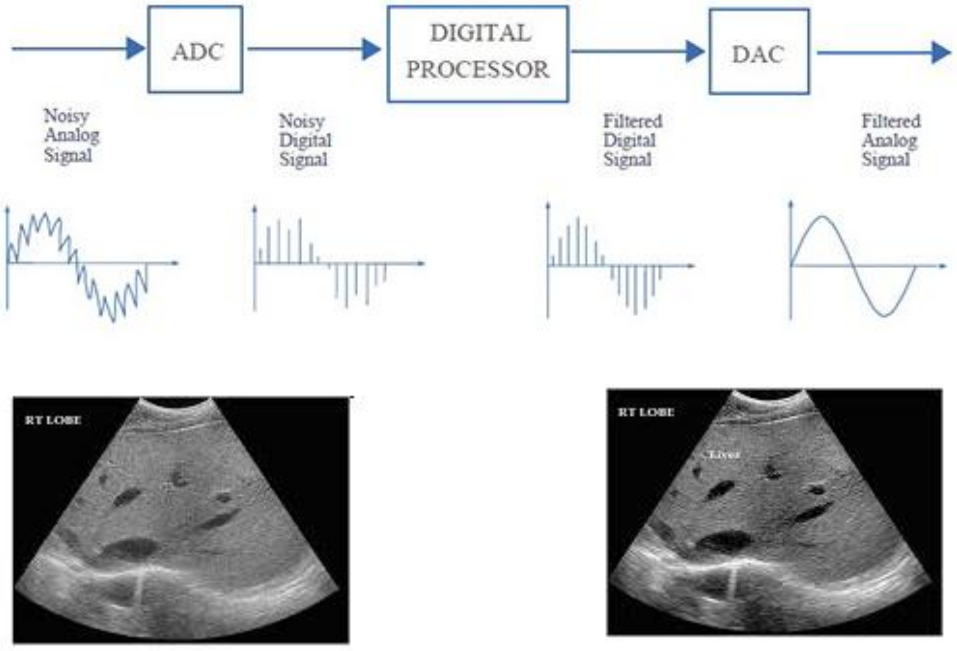
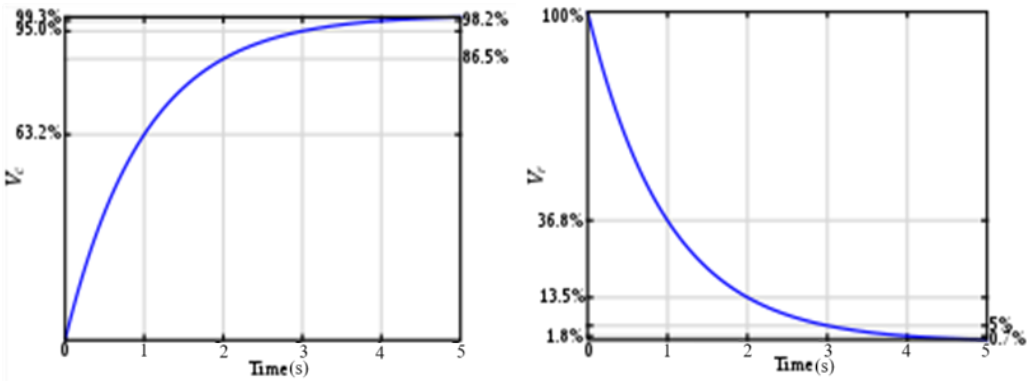


Figure 5: Digital signal processing system in a USG medical equipment

The system outlines reconstruction and filtering of signal contaminated with noise. Using the time domain analysis, we can do e.g. observation and analysis of noises and disturbances and how these might influence the performance of the signal and the overall digital signal processing system.

Example 1 – Parameters in Transient Response of First-Order Systems

For the transient response of a first order system as shown below, determine the following parameters.



a. The time constant of the system. [2 marks]

b. The settling time of the system. [2 marks]

Solution

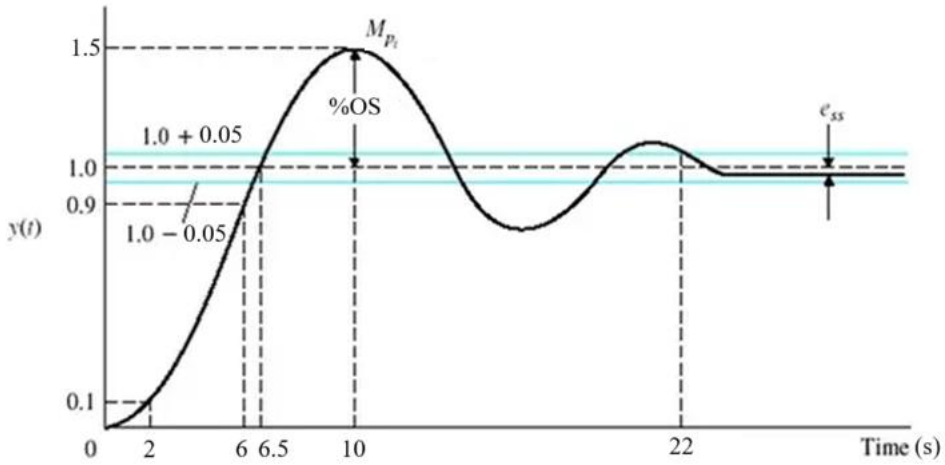
a. The time constant (τ) – For the first-order system, from the example graph:

- For the increasing transient response, it is time taken to reach 63.2% of the final value in the transient response. From the example graph, it is 1 second.
- For the decaying transient response, it is time taken to reach 36.8% of the final value in the transient response. From the example graph, it is also 1 second.

b. The settling time of the system – It is time to reach the final value in the transient response. It is typically about 5 time constant (5τ) of the transient response. From the example graph is it about 5 seconds.

Example 2 – Parameters in Transient Response of Second-Order Systems

Identify the parameters of the transient response of a second-order system shown in the figure below.



a. Rise time. [2 marks]

b. Peak-time. [2 marks]

c. Overshoot. [2 marks]

d. Settling time. [2 marks]

e. Steady-state error. [2 marks]

Solution

a. Rise time – it is the time from 10% to 90% of the transient response of the system. From the example graph, it is T_s = 6 - 2 = 4 seconds.

b. Peak-time – It is the time taken to the n-th (i.e. usually first) peak of the oscillatory transient response. From the example graph, it is 10 seconds.

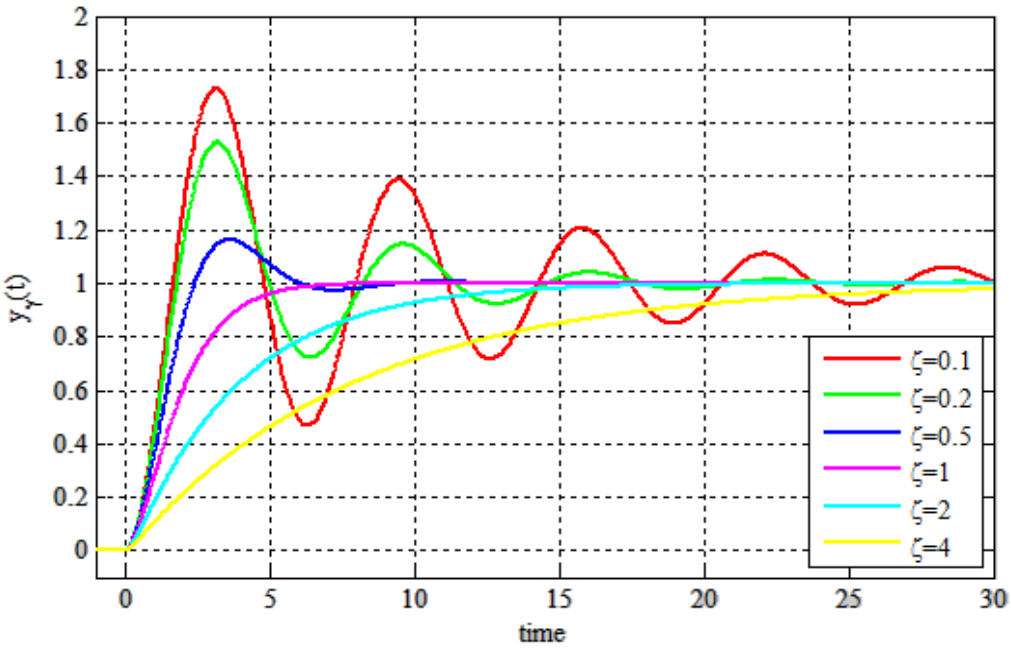
c. Overshoot - It is the largest deviation usually in percentage from the final value of the oscillatory transient response. From the example graph, it is [(1.5 - 1)/1] x 100% = 50%.

d. Settling time – It is the time for the system to settle down to its final value of the transient response. From the example graph, it is 22 seconds.

e. Steady-state error – It is the deviation usually in percentage the final value of the oscillatory transient response. From the example graph if the response settles to 0.98, it is [(1 – 0.98)/1] x 100% = 2%.

Example 3 – Damping in Transient Response of Second-Order Systems

Given in the graph below is a unit step response of a second-order system with $H_0 = 1$, $\omega_0 = 1$, and ζ is varied, which of the following responses belong to the following response categories.



a. Undamped response. [2 marks]

b. Underdamped response. [2 marks]

c. Critical response. [2 marks]

d. Underdamped response. [2 marks]

Solution

a. Undamped response – This is when the damping ratio (ζ) is 0. It is not observed in the example graph.

b. Underdamped response - This is when the damping ratio (ζ) between 0 to 0.707. It is not observed on the red, green, and blue in the example graph.

c. Critical response - This is when the damping ratio (ζ) is exactly 0.707. It is not observed in the example graph.

d. This is when the damping ratio (ζ) between 0.707 or larger. It is not observed on the magenta, light blue and yellow in the example graph.

3. Frequency-Domain Analysis

Frequency-domain analysis of signals and systems are typically about two analytical approaches in systems and signal course:

- Frequency-response (Bode) plot: It is a measure of frequency response of a system e.g. magnitude and phase of the output as a function of frequency, in comparison to the input.
- Frequency-spectrum plot: It is a quantitative measure of the output spectrum of a system or device in response to a stimulus and used to characterize the dynamics of the system.

3.1. Frequency-Response Plot

Frequency-response plot is also informally called Bode plot. It is a graph of the frequency against response of a system. It is usually a combination of a magnitude plot (expressing the magnitude e.g. usually in decibels of the frequency response) and a phase plot (expressing the phase shift). The plot is typically used to analyse and design performance and stability of the system.

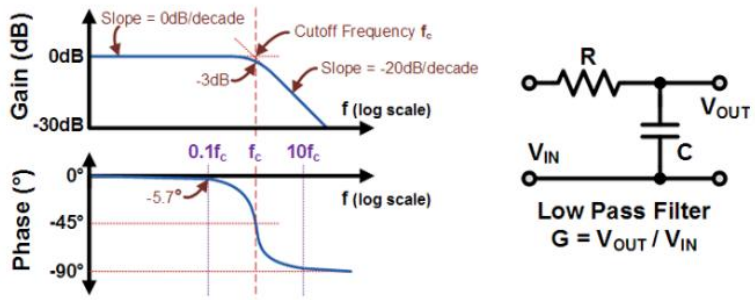


Figure 6: Frequency-response plots (left) of a first-order system (right)

For more complex example of the system, given in the figure below is a second-order RLC circuit that consist of resistor, inductor and capacitor in series and parallel configurations. The system is considered to be a simple second-order system.

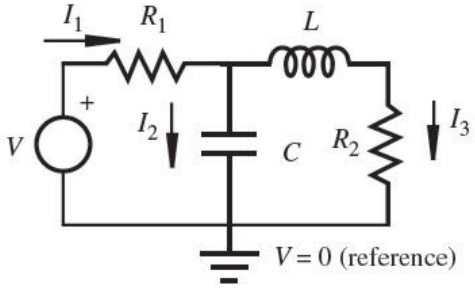


Figure 7: A simple second-order RLC circuit

The following plots shows the frequency response of the circuit e.g. magnitude and phase plots. With these plots, we can determine the characteristics and performance of the system e.g. voltages at output and input, amplification/attenuation of signal, and phase shift.

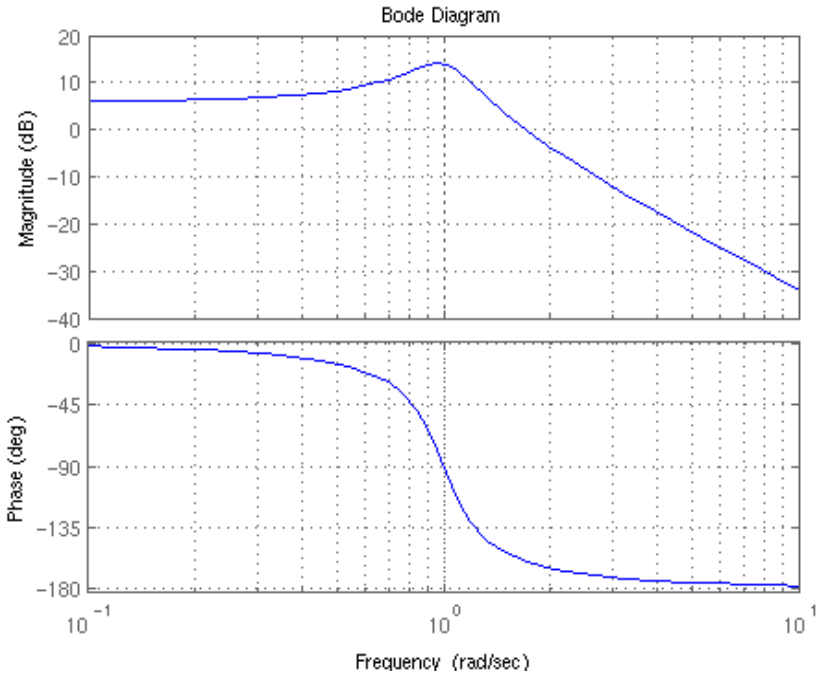


Figure 8: Frequency-response plots of the given second-order RLC circuit

3.2. Frequency Spectrum

Quantitative measure of the frequency components of system or device at the output relative to its input e.g. frequency spectrum:

- Magnitude spectrum: Magnitude spectrum, $|X(\omega)|$ where the power sits in the spectrum. Is it quiet or loud, is it bright or dark?
- Phase spectrum: Phase spectrum, $\measuredangle X(\omega)$ determines where in time the frequency components add or cancel. It essentially describes the waveform, the image.

Frequency-spectrum plots will be extensively used in the sampling and reconstruction chapter of signals and systems.

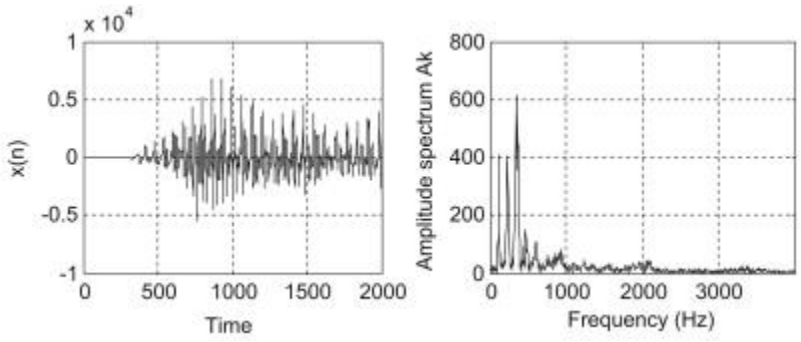


Figure 9: Frequency spectra of a signal burst

Given in the figure below is the frequency spectrum of a signal from a sensor instrument that measure air pressure e.g. spectra of amplitude or power over a range of frequencies plot. With this plot, we can determine the characteristics and behaviour of the parts or components that make up the given signal.

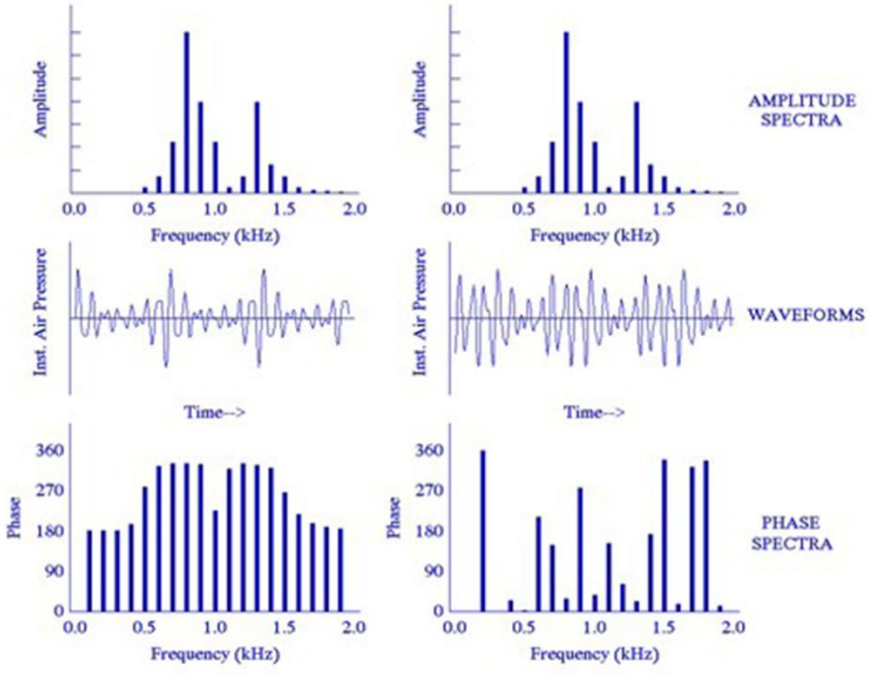


Figure 10: Frequency spectra of two air pressure sensor signals

3.3. Application of Frequency-Domain Analysis 1

Given in the figure below is a public announcement (PA) audio system used for broadcasting information to people in public spaces. For typical audio system, most of the objectives of the system are commonly to reproduce the input signal with no noise and distortion.

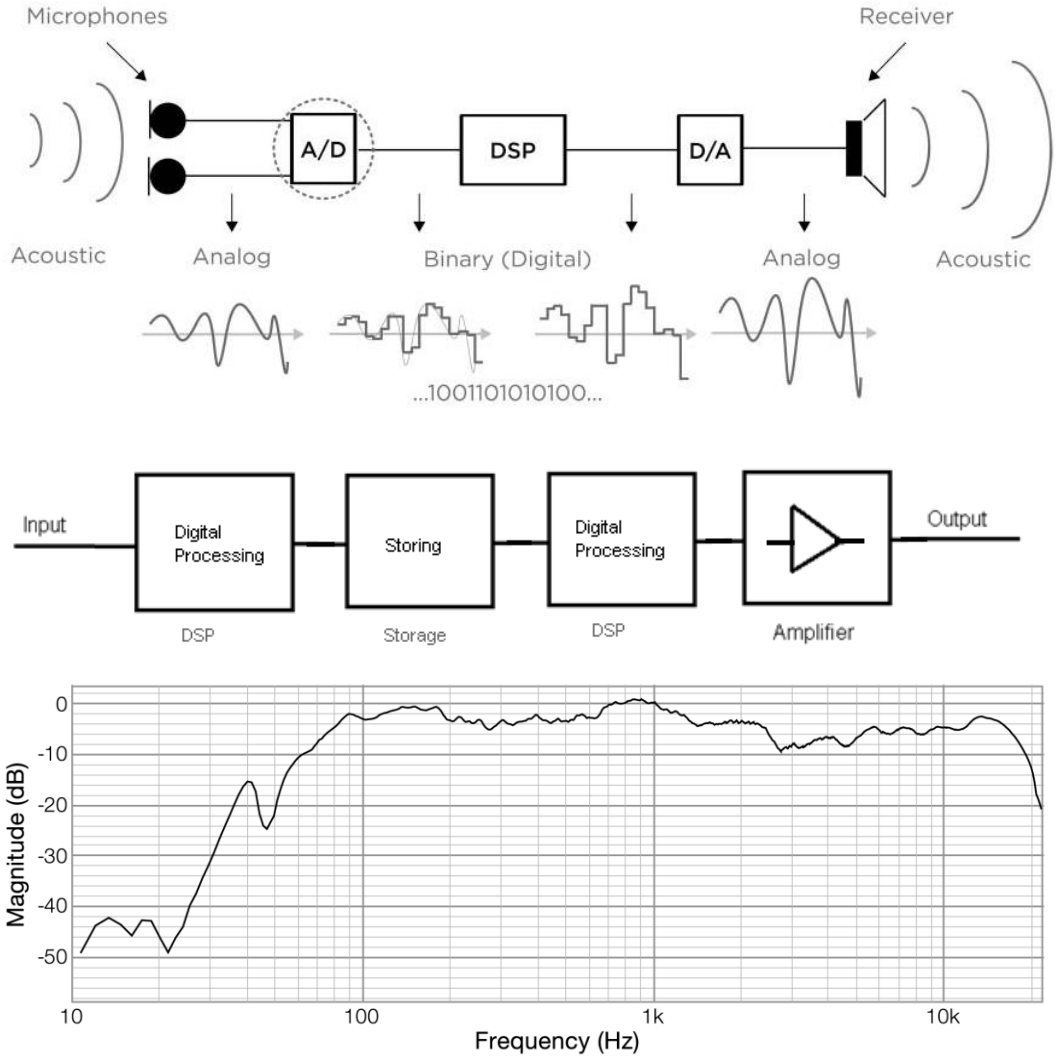


Figure 11: Audio reproduction and amplification system in a PA audio system

As shown in the diagram given above of an audio reproduction and amplification system, to achieve these design goals, we would require a uniform (flat) magnitude of

response up to the bandwidth limitation of the system. Often reproduction is involved with the signal delayed by precisely the same amount of time at all frequencies.

3.4. Application of Frequency-Domain Analysis 2

The following diagram illustrates a filter-amplifier circuit that shows the use of frequency-domain analysis in the signal filtering application. The circuit consists of an amplifier circuit part, a first-order high-pass filter circuit, and a first-order low-pass filter circuit. Together, both of the filter circuits, act as a bandpass filter circuit that pass only a range of signals from certain frequencies.

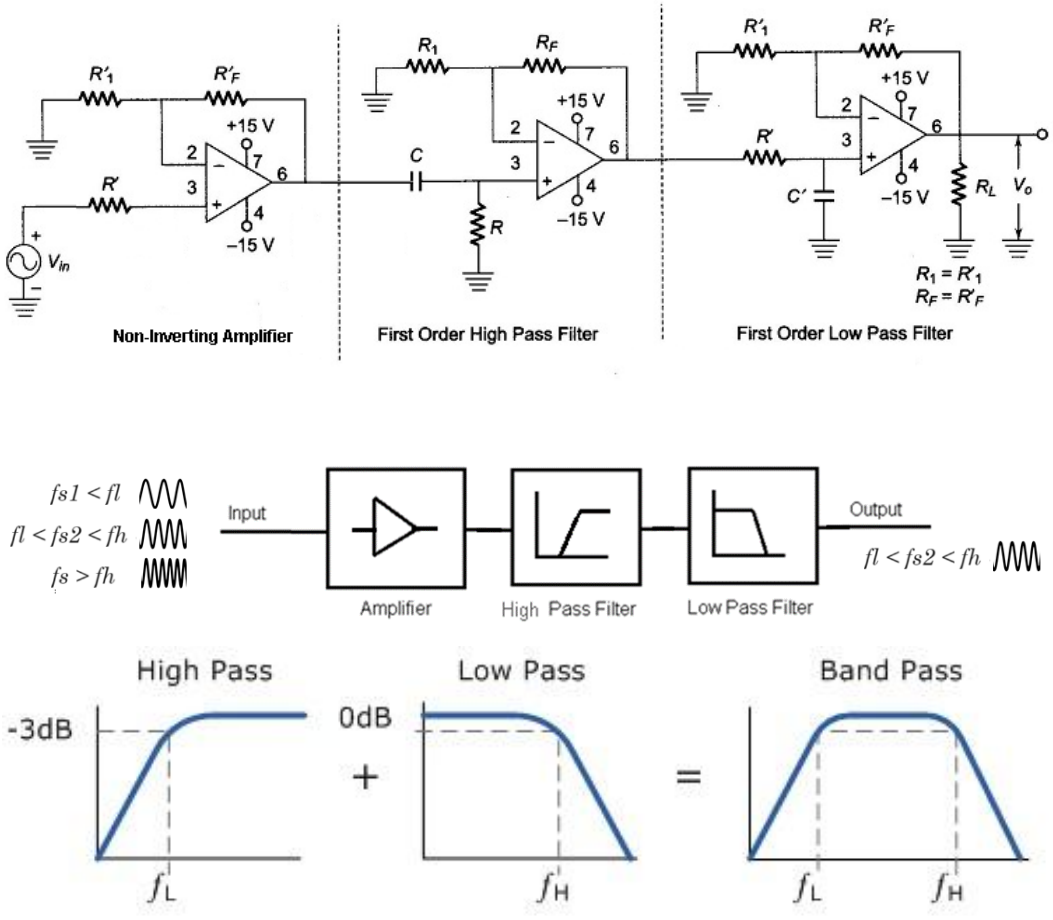


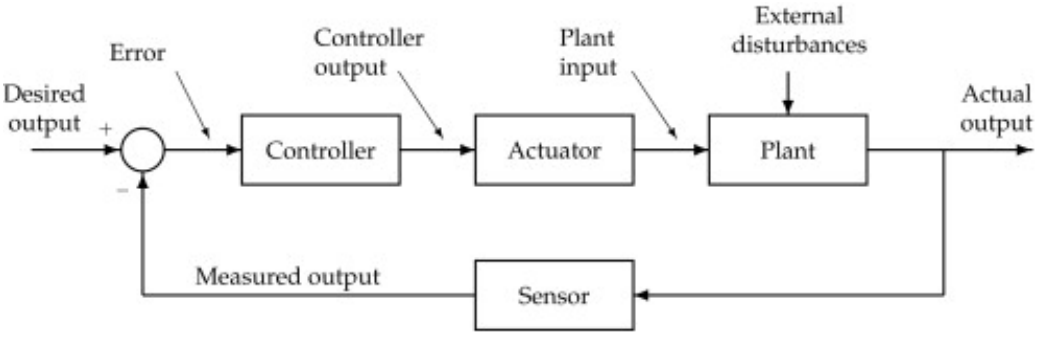
Figure 12: Signal filtering application system

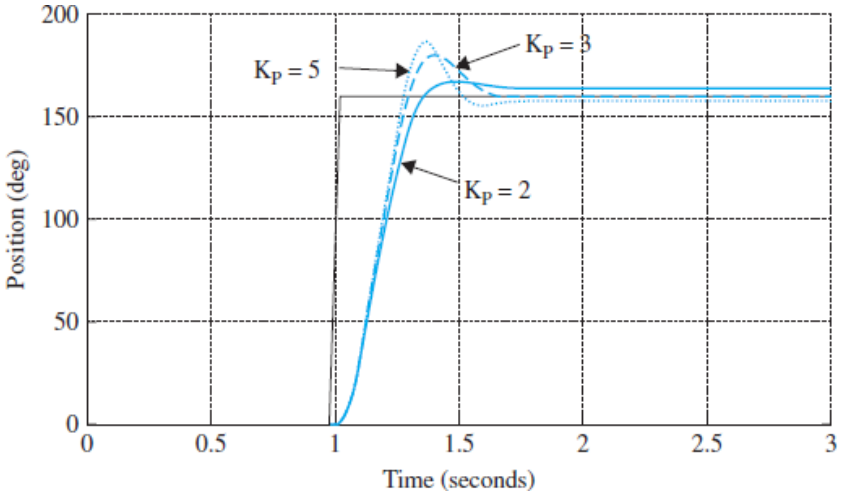
For a filter application as shown above that could be used for both audio systems and feedback control systems. It is used to selectively allow or block specific signal or waveform to go through the system.

In the given system, a high-pass filter e.g. removes low-frequency signals, and a low-pass filter e.g. blocks out high frequency signals. Combining both filters results in bandpass filter that allow only signals with selected range of frequencies $f_L < f_S < f_H$.

3.5. Application of Frequency-Domain Analysis 3

Given a feedback control system in the figure below:





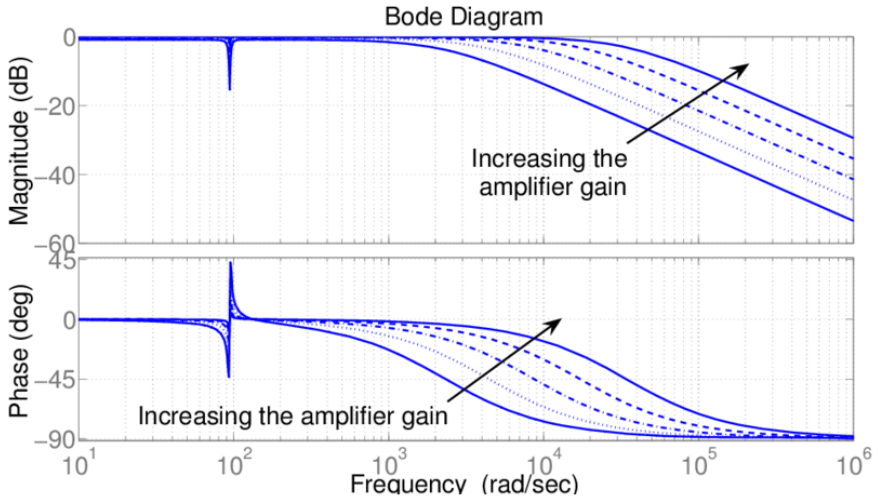


Figure 13: A feedback control system

For a feedback apparatus used to control a dynamic system, the objective is to give the system improved transient response. In this case, we will see more details of this matter in the XMUT315 Control Systems Engineering course.

For improving the transient response of the system, this can be achieved by using feedback system, so we know both the input and output of the system by applying relevant sensor and controller to measure and manage the performance of the system.

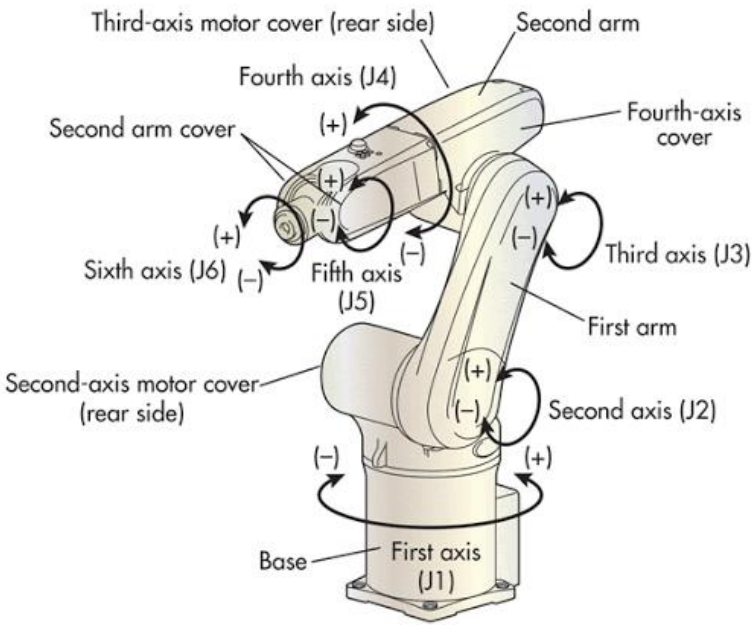
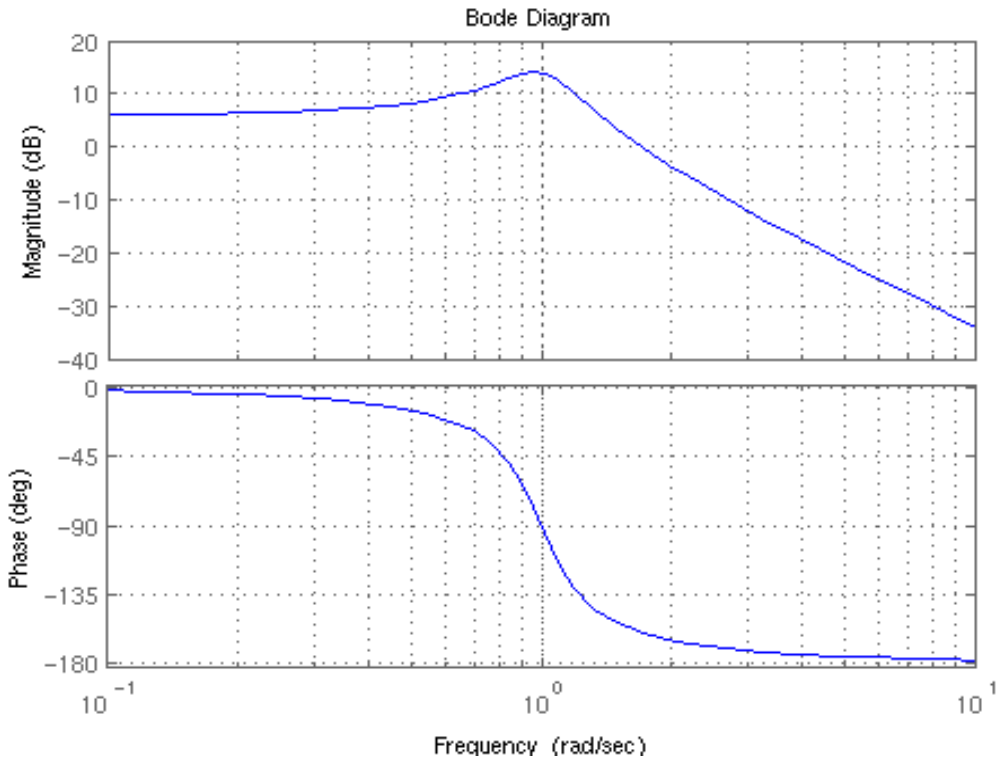


Figure 14: Robotic actuator system realization of the given feedback system

Example 4 – Parameters in Frequency Response of Second-Order System

Given in the figure below is a frequency-response plots of a second-order system.



a. Determine the cut-off frequency of the frequency response. [2 marks]

b. Determine the gain at the following frequencies:

   i. Low frequency. [2 marks]

   ii. Cut-off frequency. [2 marks]

   iii. High frequency. [2 marks]

c. Determine the phase shift at the following frequencies:

   i. Low frequency. [2 marks]

   ii. Cut-off frequency. [2 marks]

   iii. High frequency. [2 marks]

Solution

a. Determine the cut-off frequency of the frequency response. It is the when the magnitude of the gain is at about half power i.e. -3 dB. From the graph, it is around 2 Hz frequency.

b. Determine the gain at the following frequencies:

   i. Low frequency - It is around 6 dB.

   ii. Cut-off frequency – It is about -3 dB.

   iii. High frequency – It is settling at -20 dB/decade.

c. Determine the phase shift at the following frequencies:

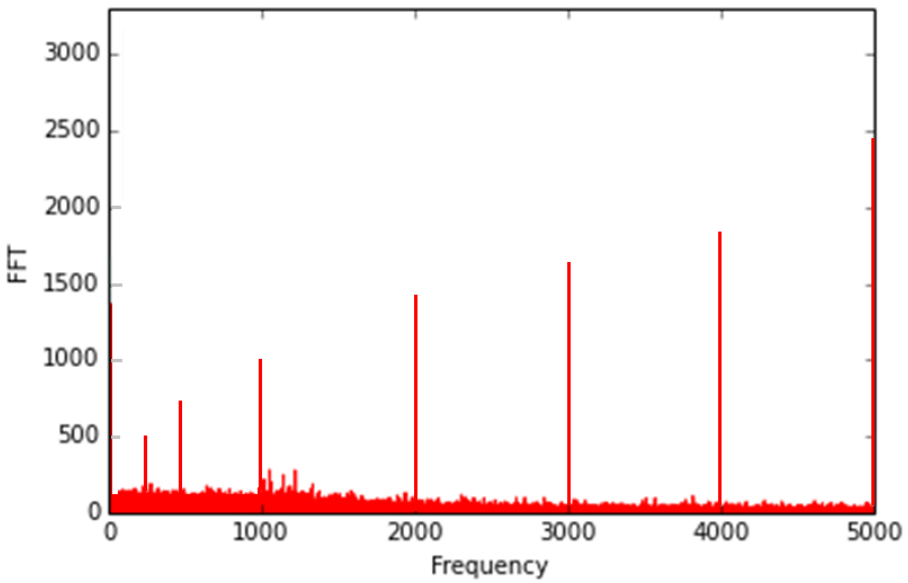
   i. Low frequency – It is about 0 degree.

   ii. Cut-off frequency – it is about -150 degree.

   iii. High frequency – It settles to about -180 degree.

Example 5 – Parameters in Frequency Spectrum of System

For the frequency-spectrum plot of a signal as shown below, perform the following tasks.



a. Determine the frequency component of the signal. [2 marks]

b. Describe the type of the signal being observed. [2 marks]

Solution

a. Determine the frequency component of the signal. From the frequency spectrum graph, the frequency components are 0 Hz (fundamental), 250 Hz, 500 Hz, 1 kHz, 2 kHz, 3 kHz, 4 kHz, and 5 kHz.

b. Describe the type of the signal being observed. From the frequency component observed on the frequency-spectrum graph, it looks like the signal has discrete components of the frequency spectrum and its magnitude of the frequency components are increasing from low frequency to high frequency.

4. Case Study

Suppose a continuous-time system represented as follows:

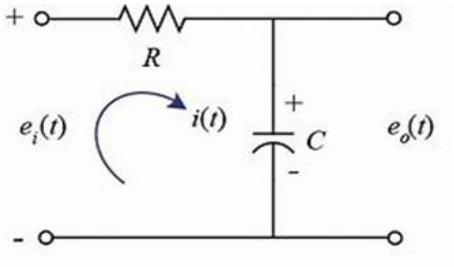


Figure 15: Example of a first-order low pass RC filter circuit

It is an electronic circuit consisting of resistor and capacitor in series. In the circuit, $e_i(t)$ is the input signal and $e_o(t)$ is the output signal. The capacitor charges up when a potential difference is applied across it. The flow of current in the capacitor will be determined by the values of resistor and its capacitance. This system is also known as first-order RC low-pass filter circuit.

4.1. Time Domain Analysis

Diagram below shows the transient response plot of the given first-order RC low-pass filter circuit system.

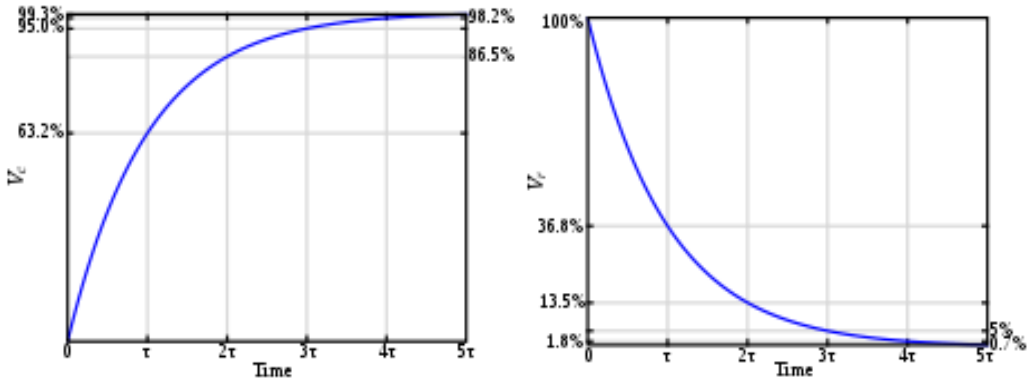


Figure 16: Transient response of the first-order RC low-pass circuit

The response of the system is a first-order system with exponentially surging voltage characteristics of the capacitor ($v_c(t)$) and decaying voltage characteristics of the resistor ($v_r(t)$) and vice versa (i.e. for the system current characteristics).

It takes approximately twice the time constant (τ) for the response of the system to rise and five times of the time constants to settle down.

4.2. Frequency-Domain Analysis (Frequency-Response Analysis)

Frequency-domain analysis is typically performed with frequency-response (Bode) plot. There are two graphs that usually needed to perform the frequency response analysis e.g. gain plot and phase plot.

4.2.1. Gain Plot

The following diagram outlines the first graph of the Bode plot of frequency response of the given first-order RC low-pass filter system e.g. a gain plot.

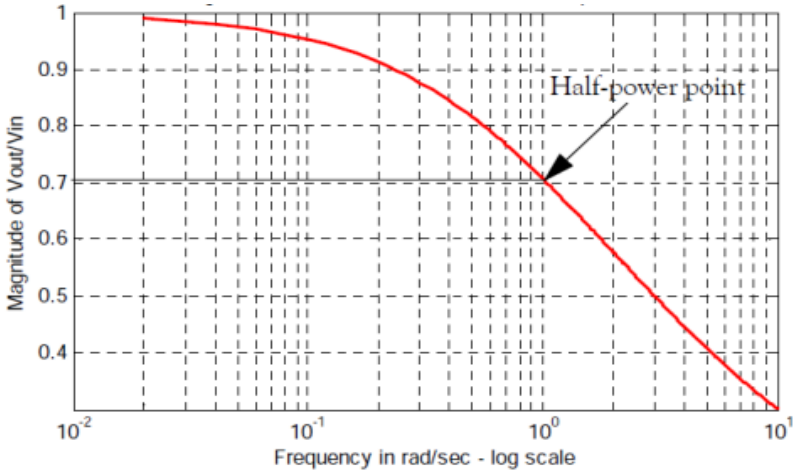


Figure 17: Gain plot of the first-order RC low-pass filter circuit

There are several interesting findings observed in the plot:

First is the half-power point where its value is 0.5 $V_{out}(\omega)/V_{in}(\omega)$. It is also called the cut-off frequency (F_c) e.g. frequency where half-power point is located at F_c.

For the magnitude characteristics of the system, we can observe the following:

- As ω -> 0, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 1 (no change in the output in relation to input).
- For $\omega = 1/RC$, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 0.5 (the output is half of the input).
- As ω -> ∞, magnitude of $V_{out}(\omega)/V_{in}(\omega)$ is equal to 0.

As a result, we could see that at low frequency, the filter circuit lets through low frequency signal and removes high frequency signal from the filter system.

4.2.2. Phase Plot

The diagram given below is the second graph of Bode plots e.g. a phase plot of frequency response of the system.

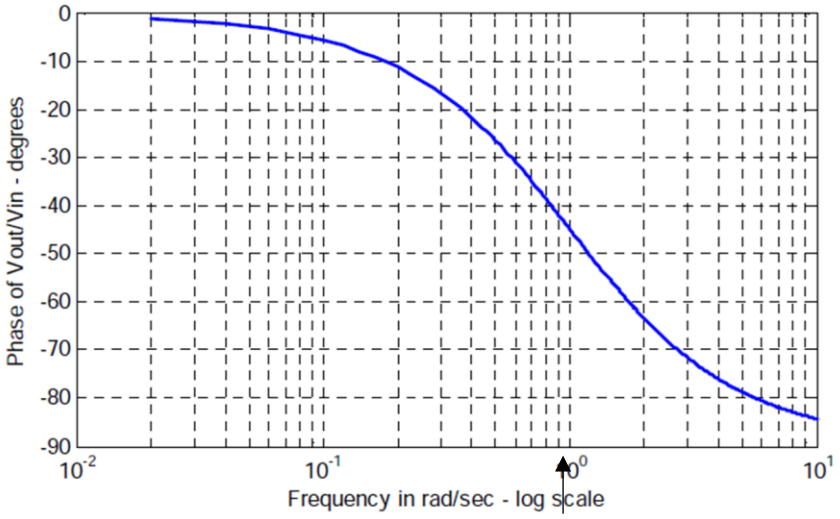


Figure 18: Phase plot of the first-order RC low-pass filter circuit

Just like the magnitude plot, we could observe also a few interesting findings from the phase plot.

For the phase characteristics of the system, we found that:

- As ω -> 0, phase shift is 0 (there is no phase difference between output and input).
- For $\omega = F_C$ (cut off frequency), there is a phase difference of $\pi/4$ between input and output.
- As ω -> $+\omega$, there is a phase difference of $\pi/2$ between input and output.

The phase plot shows that at low frequency, the phase shift of the filter circuit is 0° and at higher frequency, it is approaching 90°.

4.3. Frequency-Domain Analysis (Spectra Analysis)

The graph plot given below is the spectra of a signal of the given first-order RC low-pass filter circuit.

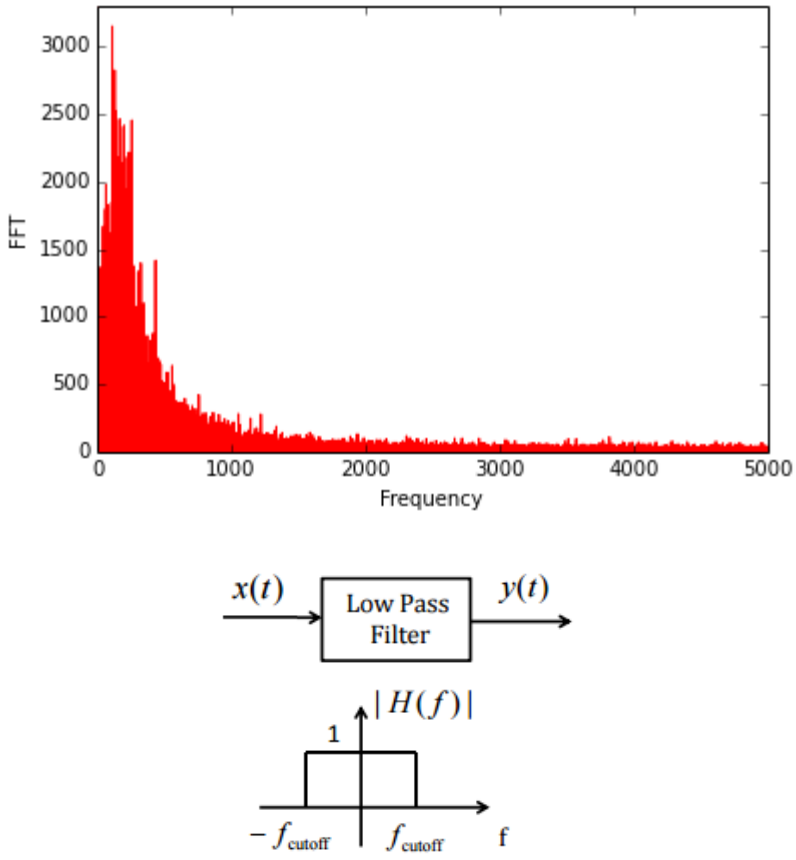


Figure 19: Frequency spectra of the first-order RC low-pass filter circuit

From the frequency-spectrum plot, we can evaluate the frequency components characteristics of the given low-pass filter at the output. We can observe the following frequency-spectrum characteristics of the circuit in the graph:

- At low frequency (100-200 Hz) – large magnitude with value > 2000.
- At cut-off frequency (500 Hz)– magnitude is half of peak (e.g. about 1000).
- At high frequency (1000 Hz) – small magnitude with value < 250.

From the frequency-spectrum graph, we could conclude that the first-order RC low pass filter allows low-frequency components and removes high-frequency components of the signal at the output.