XMUT220 Signals and Systems
Signals Sampling and Reconstruction (Introduction)

Topics

1. Sampling of signals
2. Nyquist theorem
3. Aliasing
4. Reconstruction of signals
5. Discrete-time processing of continuous-time signals
6. Analogue-to-digital converter and digital-to-analogue converter (ADC and DAC).
7. Digital signal processing (DSP).
8. Sampling of discrete-time signals.

1. Sampling of Signals

Sampling of signals is the capture of a continuous- time signal into a discrete-time signal.

A common example of sampling is the conversion of a sound wave (continuous signal) to a sequence of samples (a discrete- time signal).

A sample is value or set of values at a point in time and/ or space. The number of samples taken during one second is called the sample rate.

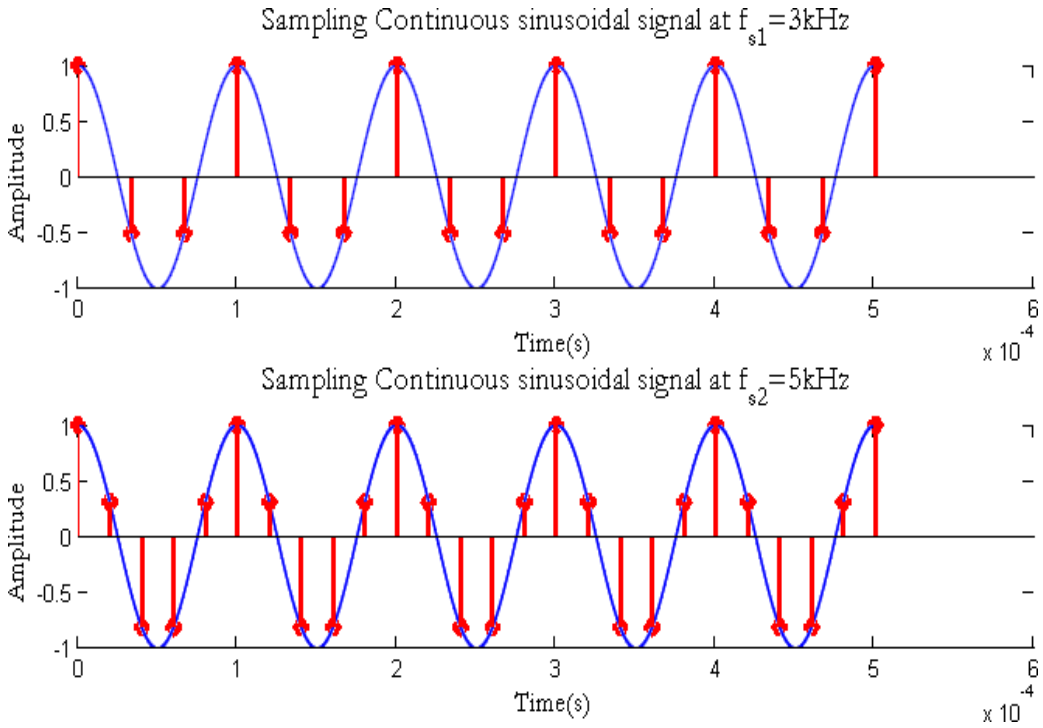


Figure 1: Sampling of signal at various sampling frequencies

In order to perform sampling process on a given signal, there are several components required for performing signal sampling:

- Sampled signals – signals to undergo sampling process.
- Sampling signal – a signal used as a reference for sampling of a given signal.
- Multiplier system – a system that enables sampling of the signals to be sampled with sampling signal.
- Algorithm for conversion – a type of process for converting from a given signal to its sample(s).

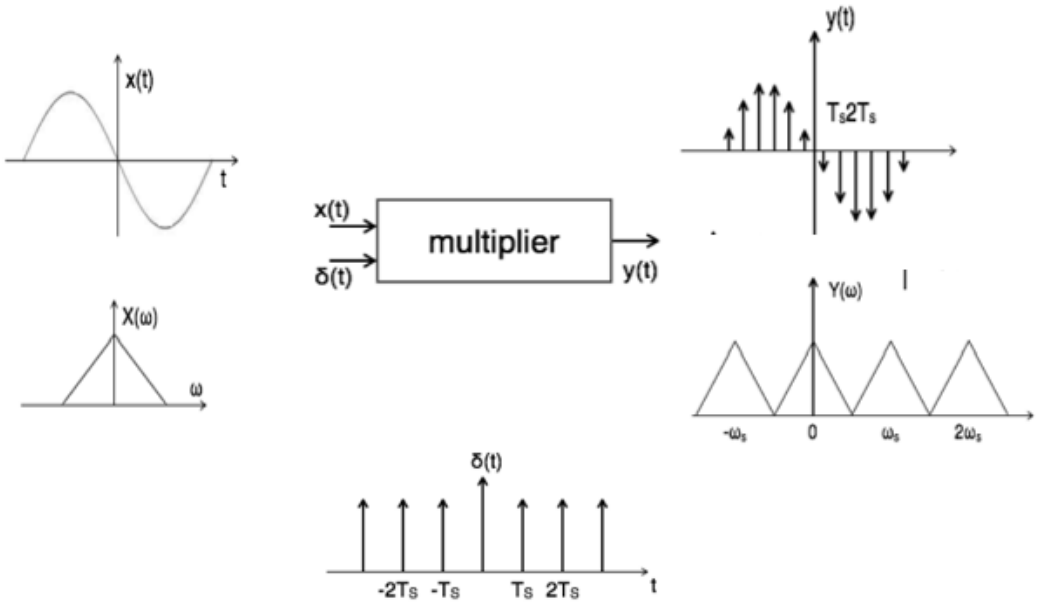


Figure 2: Required components in the sampling process

2. Nyquist Theorem

The highest frequency component in an analogue signal determines the bandwidth of that signal. According to the Nyquist Theorem, the sampling rate must be at least $2f_{max}$, or twice the highest analogue frequency component.

$$\text{sampling rate} = 2f_{max}$$

If the sampling rate is less than $2f_{max}$, some of the highest frequency components in the analogue input signal will not be correctly represented in the digitized output.

When such a digital signal is converted back to analogue form by a digital-to-analogue converter, false frequency components appear that were not in the original analogue signal.

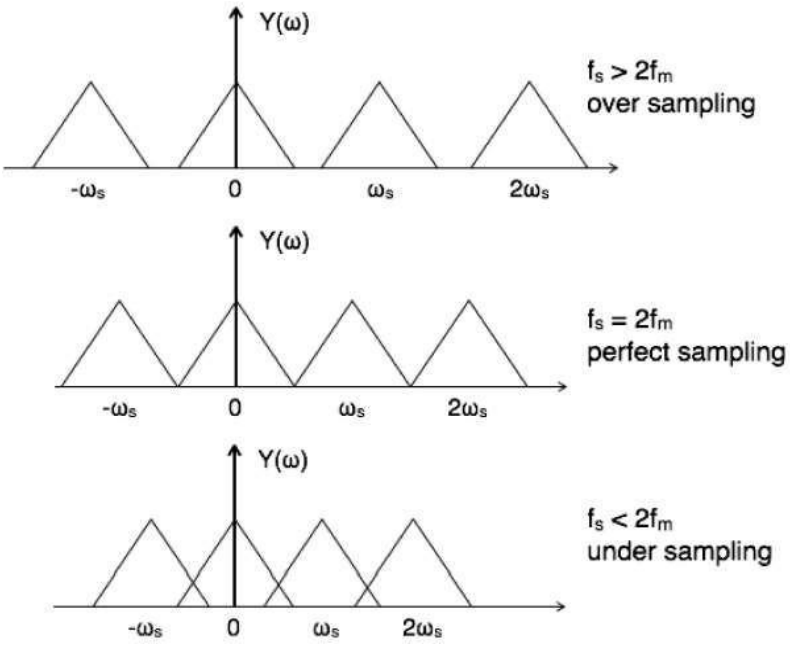


Figure 3: Types of sampling based on Nyquist sampling rate

Sampling rate for an analogue signal must be at least two times as high as the highest frequency in the analogue signal in order to avoid aliasing.

Conversely, for a fixed sampling rate, the highest frequency in the analogue signal can be no higher than a half of the sampling rate.

Analogue signal is usually filtered by a low pass filter prior to being sampled, this filter is called an anti-aliasing filter.

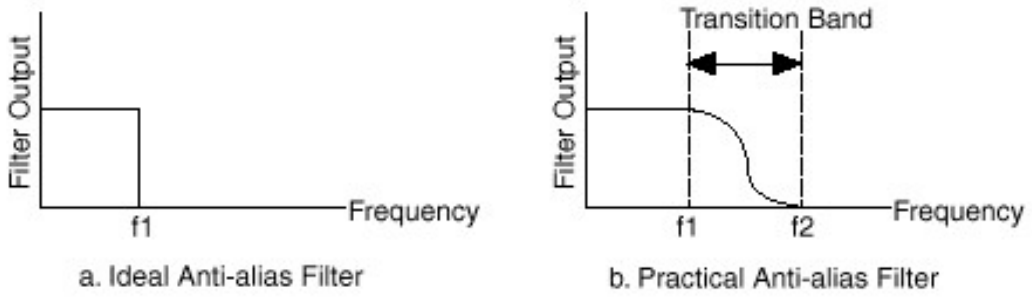


Figure 4: Ideal and practical anti-aliasing filter

Related to the aliasing, specific area in signal and system typically has a specific bandwidth and sampling frequency requirements.

Application	**Bandwidth**	**Sampling Frequency**
Biomedical	< 500 Hz	1 kHz
Telephone speech	< 4 kHz	8 kHz
Music	< 20 kHz	44.1 kHz
Ultrasonic	< 100 kHz	250 kHz
Radar	< 100 MHz	200 MHz

Table 1: Range of frequencies of common applications in signals and systems

3. Aliasing

Aliasing is an effect that causes different signals to become indistinguishable (or aliases of one another) when sampled. It also refers to the distortion when the signal reconstructed from samples is different from the original continuous signal.

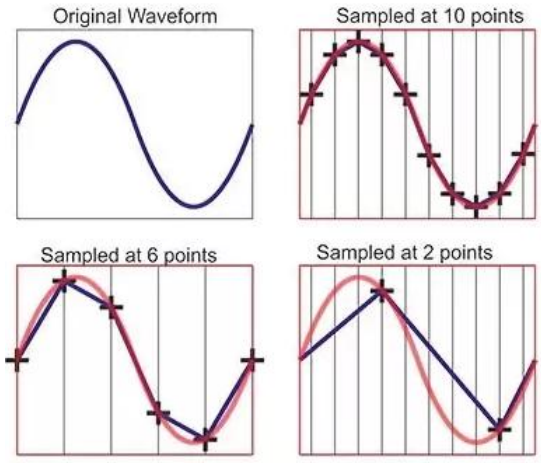


Figure 5: Sampling of a signal at various sampling points

For signals sampled in time, for instance digital audio, it is referred to as temporal aliasing.

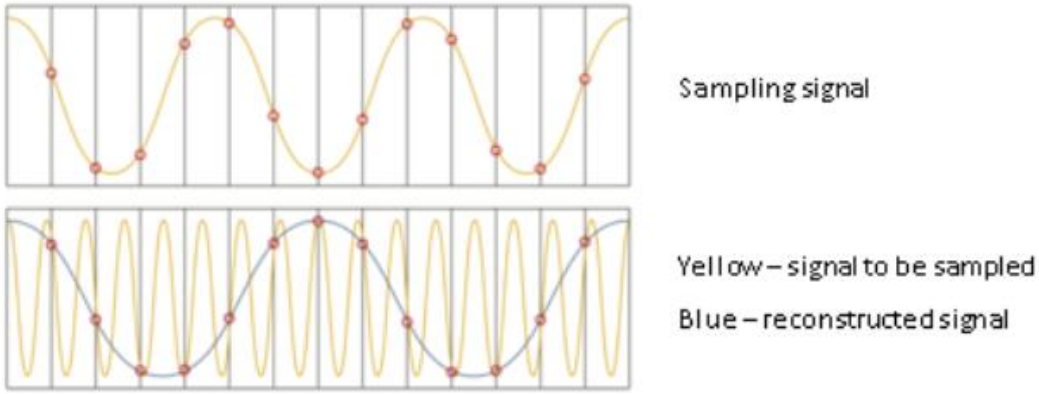


Figure 6: Temporal aliasing in digital audio sampling application

Aliasing can also occur in spatially sampled signals, for instance digital images, and it is called spatial aliasing.

- **Moiré effect** is a visual perception that occurs when viewing a set of lines or dots that is superimposed on another set of lines or dots, where the sets differ in relative size, angle, or spacing.

- **Raster images** (or jaggies/pixelated) are stair-like lines that appear where there should be "smooth" straight lines or curves.





Figure 7: Moire Effect

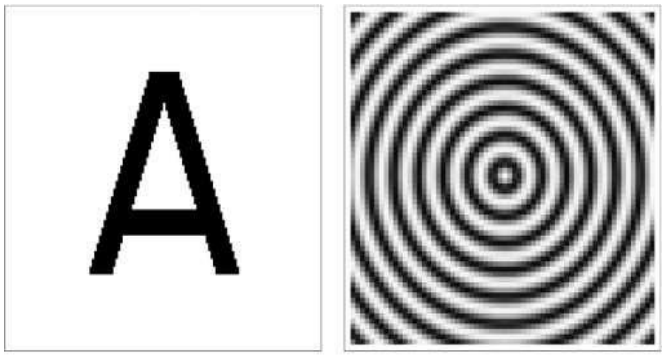


Figure 8: Raster Image/Jaggies/Pixelated

4. Reconstruction Methods

Reconstruction usually means the determination of an original continuous signal from a sequence of equally spaced samples.

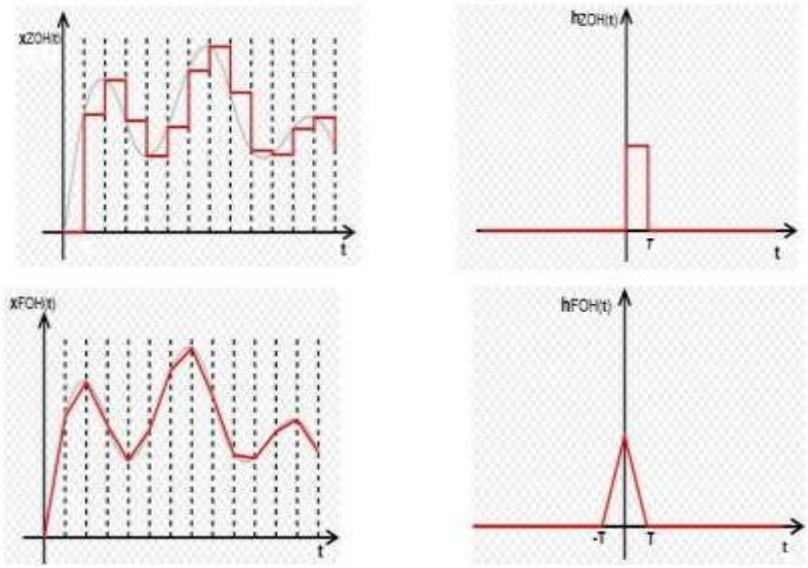


Figure 9: Zero-order hold (top) and first order hold (bottom) methods

Techniques used for reconstruction of signals:

- Zero-Order Hold - stair step shape that follows the original signal.
- First-Order Hold - interpolate between samples with straight lines

4.1. Zero-Order Hold

In a zero order hold reconstruction method, it converts a discrete-time signal to a continuous-time signal by holding each sample value for one sample interval.

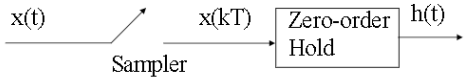


Figure 10: Components of zero-order hold reconstruction method

The zero-order hold is realised with a conventional digital-to-analog converter (DAC). The following diagram shows a zero-order hold as impulse-train sampling followed by an LTI system with a rectangular impulse response. The diagram also show the result of application of zero-order hold on the sampled signals.

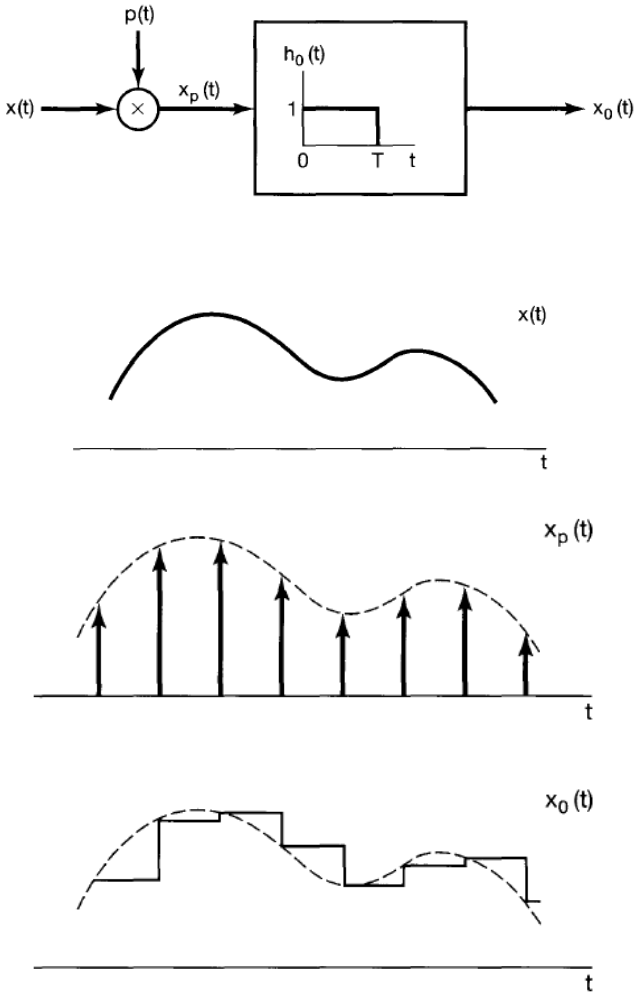


Figure 11: Zero-order hold as impulse-train sampling followed by an LTI system with a rectangular impulse response.

4.2. First-Order Hold

In the first-order hold method, the signal is reconstructed as a piecewise linear approximation to the original signal that was sampled.

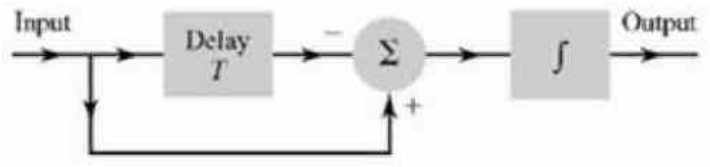
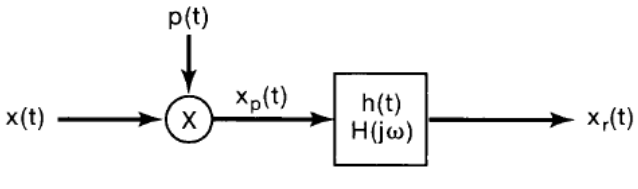


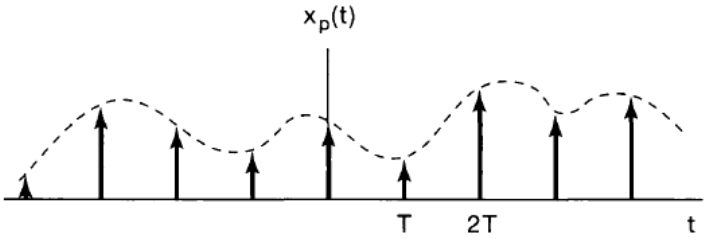
Figure 12: Components of first-order hold reconstruction method

First-order hold is constructed from a conventional digital-to-analog converter (DAC) and an analog circuit called an integrator.

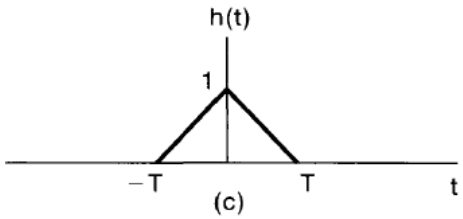
The following diagram shows a linear interpolation (first-order hold) as impulse-train sampling followed by convolution with a triangular impulse response. The diagram shows a system for sampling and reconstruction. It shows impulse train of samples and the impulse response representing a first-order hold. The outcome of first-order hold applied to the sampled signal is shown in the diagram.



(a)



(b)



(c)

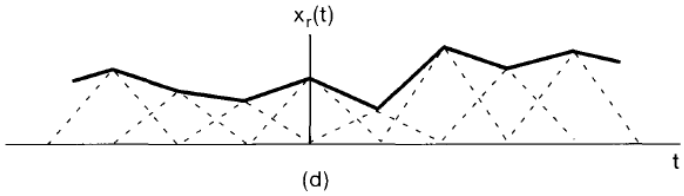


Figure 13: Linear interpolation (first-order hold) as impulse-train sampling followed by convolution with a triangular impulse response: (a) system for sampling and reconstruction; (b) impulse train of samples; (c) impulse response representing a first-order hold; (d) first-order hold applied to the sampled signal.

5. Discrete Processing of Continuous Signals

Application of digital computation to signal processing has many advantages over analog processing in many applications, e.g. complex algorithms, error detection and correction, data compression, etc.

Digital signal processing (DSP) is also fundamental to digital technology, especially for digital telecommunication and wireless communications. DSP is applicable to both streaming data and static (stored) data.

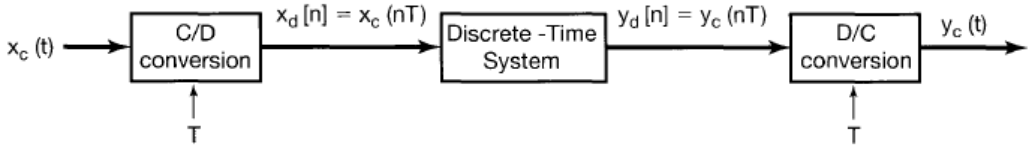


Figure 14: Notation for continuous-to-discrete-time conversion and discrete-to-continuous-time conversion. T represents the sampling period.

5.1. Analogue-to-Digital and Digital-to-Analogue Conversions

Analogue-to-Digital (A2D) conversions change analogue electrical data into digital signals (sampling).



Figure 15: Analogue to digital conversion

Digital-to-Analogue (D2A) conversions attempt to create an analogue output waveform from a digital signal (conversion/ reconstruction).



Figure 16: Digital to analogue conversion

Sampling is usually carried out in two stages e.g. discretization and quantization.

- Discretization - signal is divided into equal intervals of time, and each interval is represented by a single measurement of amplitude.
- Quantization - each amplitude measurement is approximated by a value from a finite set.

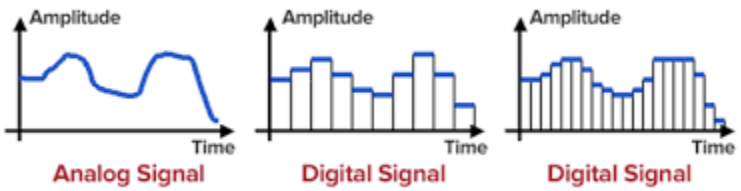


Figure 17: Conversion of digital signals to analogue signals

For converting analogue signals to digital (digitised) signals, it typically involves an application analogue filter. On the other hand, for converting digital signals to analogue signals, it usually involves some sort of sampled and hold mechanism.

5.2. Digital Signal Processing

For a typical digital signal processing system, you might expect to see the following components of the system:

- Analogue Filter (Input) – Remove unwanted signals and noises.
- Digital Processing – Main signal processing of the system e.g. wave shaping, algorithm based processing, signal modulation, etc.
- Analogue Filter (Output) – Increase the quality and performance of the processed signal for the targeted system.

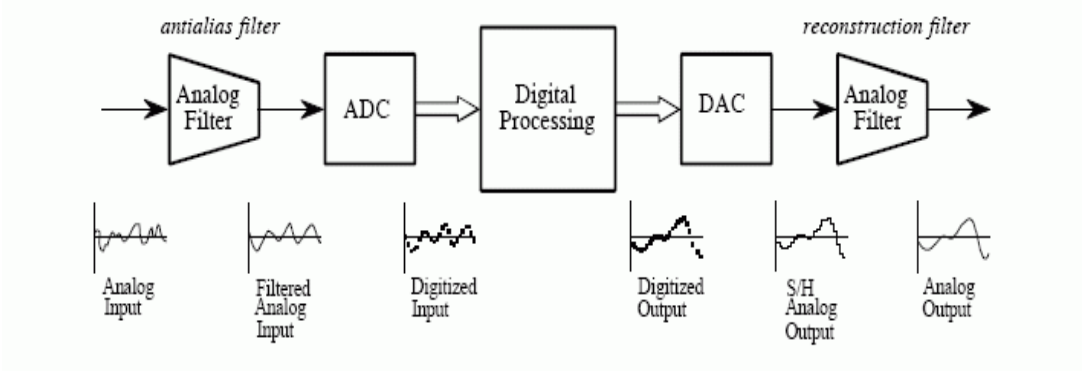


Figure 18: Digital signal processing (DSP) system

The digital processing unit of the DSP system is usually constructed as follow:

- Hardware unit e.g. wave shaping, digital filtering, advanced signal conversions.
- Software unit e.g. algorithm of digital signal processing for manipulating the characteristics and properties of the signals.

DSP applications include audio and speech processing, sensor processing, spectral density estimation, statistical signal processing, digital image processing, data compression, video and audio coding, image compression, etc.

Domains in digital signal processing: time domain (one-dimensional signals), spatial domain (multidimensional signals), frequency domain, and wavelet domains.

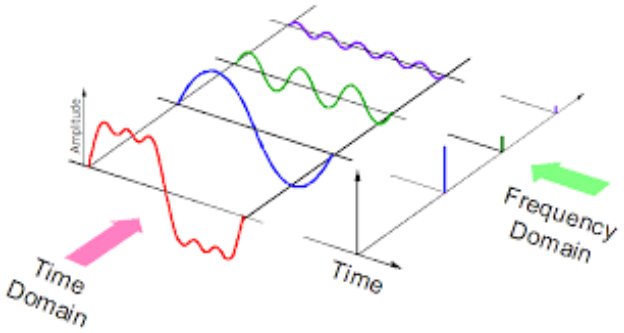


Figure 19: Domains in digital signal processing

The domain is chosen in which to process a signal by making an informed assumption (or by trying different possibilities) as to which domain best represents the essential characteristics of the signal and the processing to be applied to it.

5.3. Application of Digital Signal Processing

Two applications of digital signal processing e.g. digital filter system and digital radio system are described in this section.

5.3.1. Digital Filter System

Digital filter system e.g. a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. Digital filter is typically used for removing unwanted part of the signals, commonly these are noise, disturbance, unwanted signals, etc.

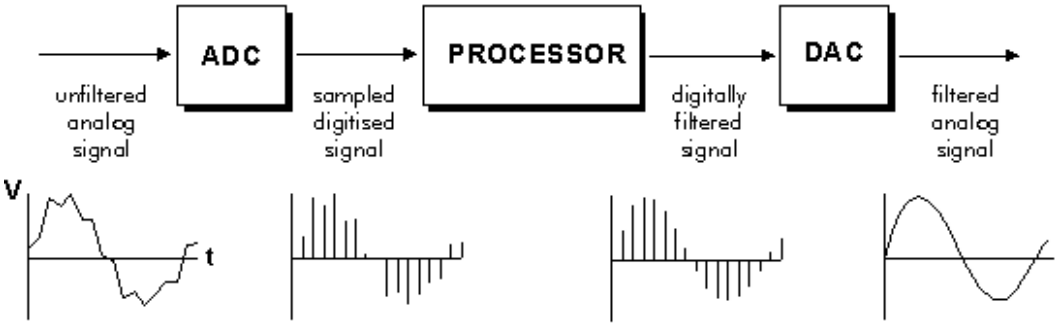


Figure 20: Digital signal processing in the digital filter system

5.3.2. Digital Radio System

Digital radio system e.g. part of digital processing unit for broadcasting the sound signals. DSP in this system is about coding or encoding, filter, modulator or demodulator and amplification processes of the signals.

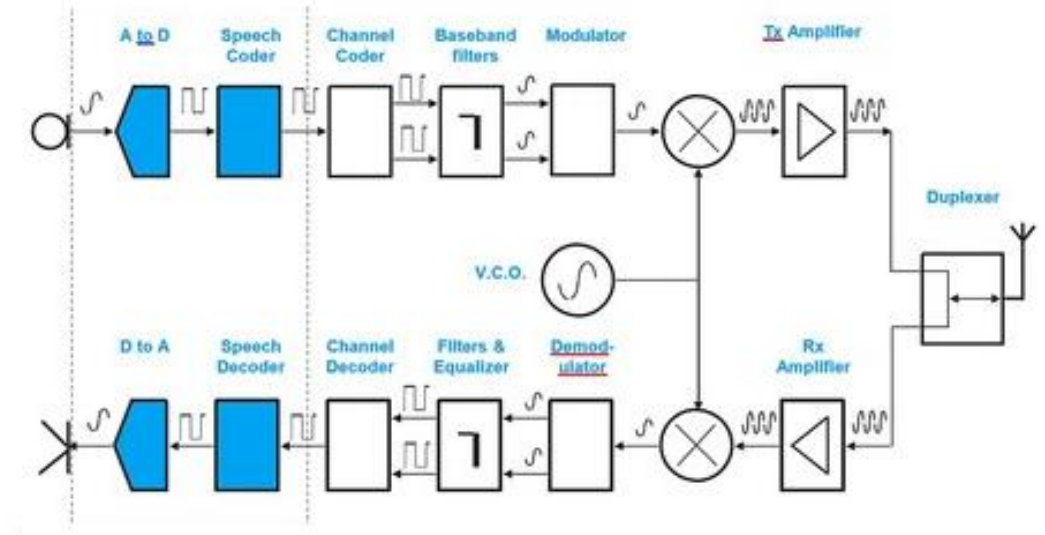


Figure 21: Digital signal processing in the Internet digital radio system

6. Sampling of Discrete-Time Signals

Sampling can also be performed in the discrete-time signals. The principle behind its

application is very similar to its continuous –time signal counterpart.

6.1. Introduction to Sampling of Discrete-Time Signals

Sampling of discrete-time signals implements the basic concept of discrete-time sampling is similar to that of continuous-time sampling. It involves multiplying a discrete-time sequence by a periodic impulse train. It retains every Nth sample and setting the remaining ones to zero (where: N = period of the sampling impulse train).

The following diagram illustrates a discrete –time sampling process.

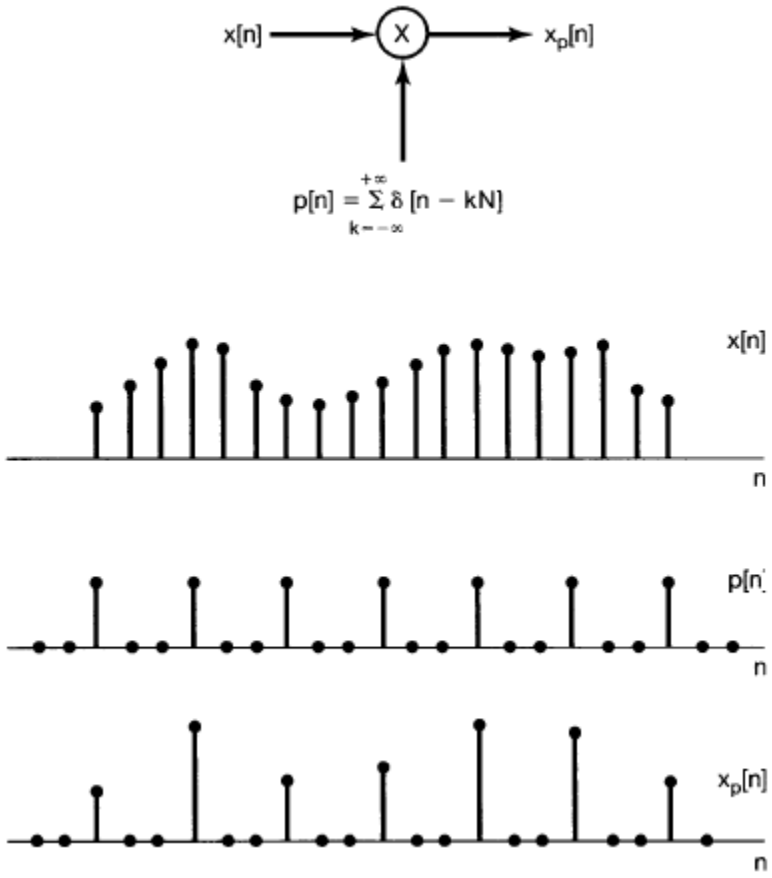


Figure 22: Discrete-time sampling process

Just like in the continuous sampling, we could observe the aliasing effect in the frequency domain of impulse-train sampling of a discrete-time signal.

As illustrated in the following diagram, given a spectrum of original signal and its corresponding spectrum of sampling sequence, we can observe the spectrum of sampled signal with $\omega_s > 2\omega_M$ and the spectrum of sampled signal with $\omega_s < 2\omega_M$.

Note that aliasing occurs whenever the sampling signal is less than twice of maximum signal to be sampled.

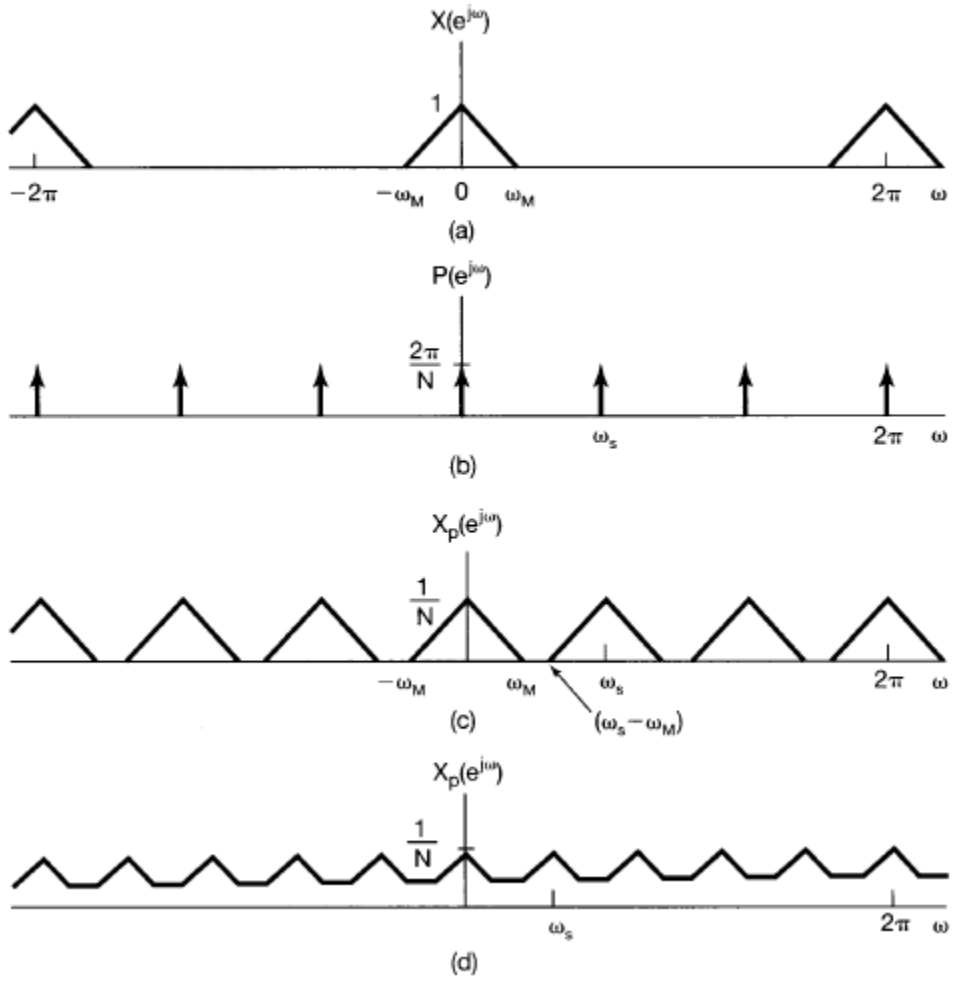
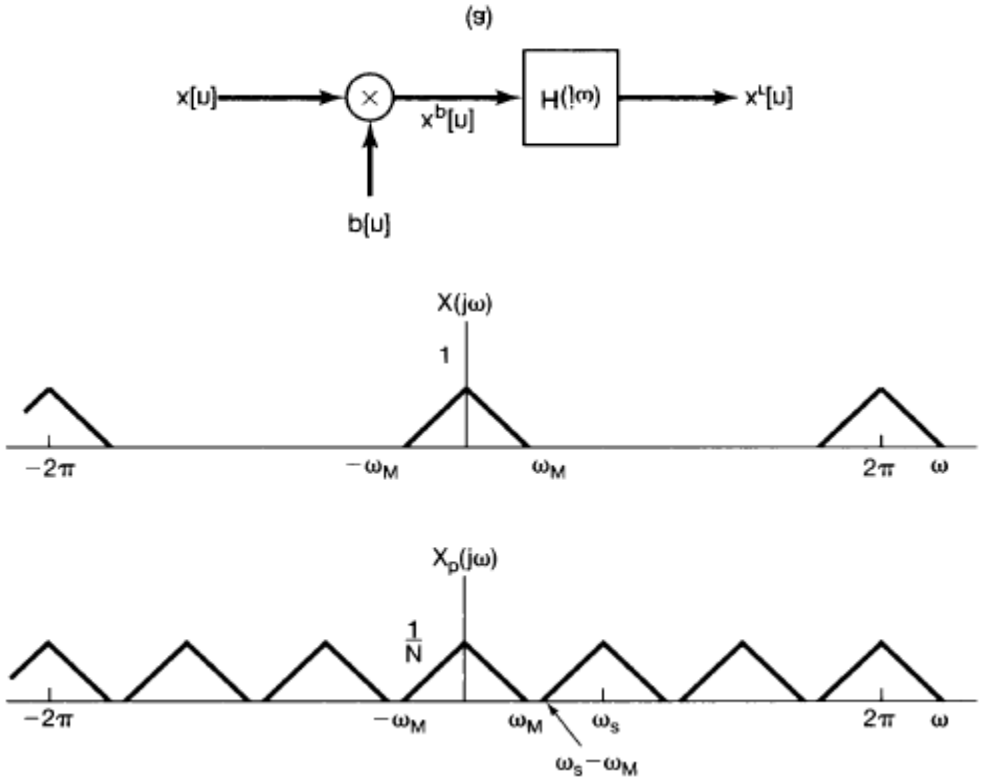


Figure 23: Effect in the frequency domain of impulse-train sampling of a discrete-time signal: (a) spectrum of original signal; (b) spectrum of sampling sequence; (c) spectrum of sampled signal with $\omega_s > 2\omega_M$; (d) spectrum of sampled signal with $\omega_s < 2\omega_M$. Note that aliasing occurs.

For the exact recovery of a discrete-time signal from its samples using an ideal low-pass filter, reconstruction of discrete-time signal is very similar to reconstruction of continuous-time signal.

The following diagram illustrates a block diagram for sampling and reconstruction of a band-limited signal from its samples. In the diagram, we can see the spectrum of the signal $x[n]$ and the spectrum of $x_p[n]$.

Providing the frequency response of an ideal low-pass filter with cut-off frequency $\omega_s/2$, we can observe the spectrum of the reconstructed signal $x_r[n]$. For the example depicted here $\omega_s > 2\omega_M$, so that no aliasing occurs and consequently $x_r[n] = x[n]$.



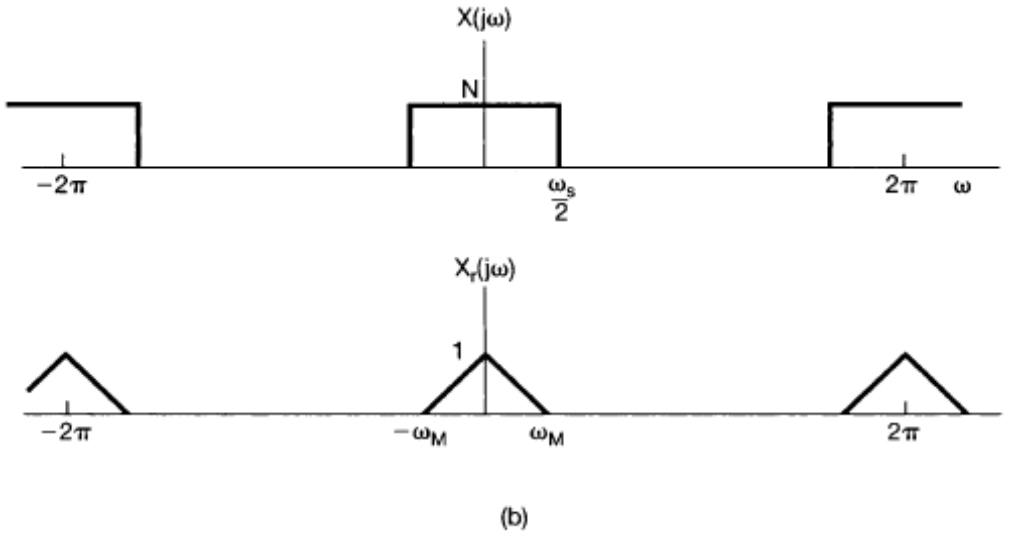


Figure 24: Exact recovery of a discrete-time signal from its samples using an ideal low-pass filter: (a) block diagram for sampling and reconstruction of a band-limited signal from its samples; (b) spectrum of the signal $x[n]$; (c) spectrum of $x_p[n]$; (d) frequency response of an ideal lowpass filter with cut-off frequency $\omega_s/2$; (e) spectrum of the reconstructed signal $x_r[n]$. For the example depicted here $\omega_s > 2\omega_M$ so that no aliasing occurs and consequently $x_r[n] = x[n]$.

6.2. Discrete-Time Down sampling and Up sampling

Two topics which are related closely to discrete –time sampling process are the down sampling (decimation) and up sampling (interpolation) processes.

6.2.1. Down sampling

Down-sampling (decimation) is process of reducing the sampling rate. In practice, this usually implies low-pass-filtering a signal, then throwing away some of its samples.

Down-sampling factor is simply the ratio of the input rate to the output rate (e.g. M = input rate /output rate).

The purpose of down sampling is to operate at a lower sampling rate and to reduce the cost of process.

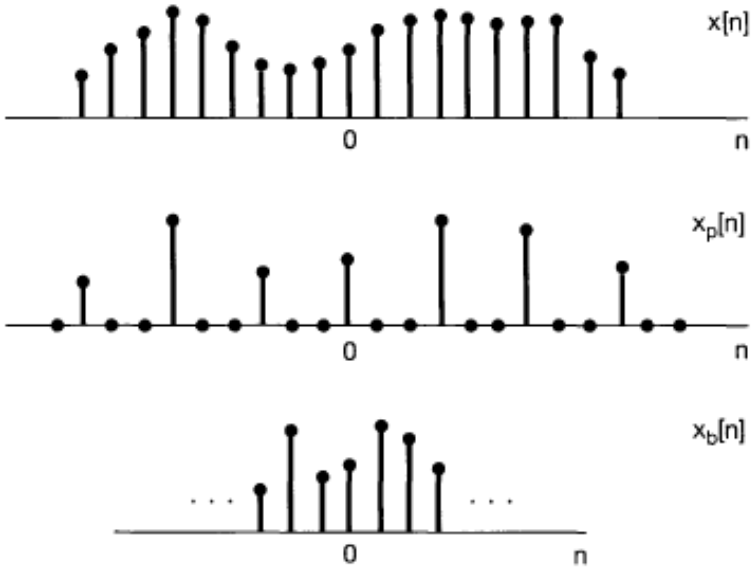


Figure 25: Relationship between $x_p[n]$ corresponding to sampling and $x_b[n]$ corresponding to decimation.

The following diagram describes the frequency-domain illustration of the relationship between sampling and down sampling by a factor of M = 2.

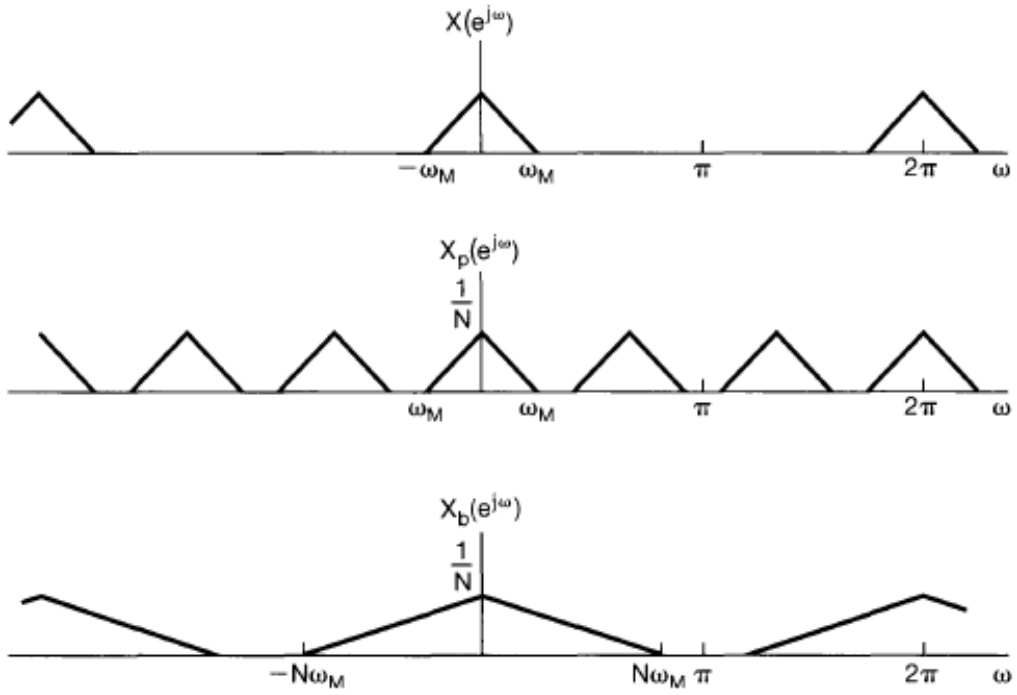


Figure 26: Frequency-domain illustration of the relationship between sampling and decimation.

6.2.2. Up sampling

Up-sampling (interpolation) is the process of inserting zero-valued samples between original samples to increase the sampling rate (this is called “zero-stuffing”).

The interpolation factor is simply the ratio of the output rate to the input rate (L = output rate /input rate).

The usual purpose of applying up sampling is to increase the sampling rate at the output of one system so that another system operating at a higher sampling rate can input the signal

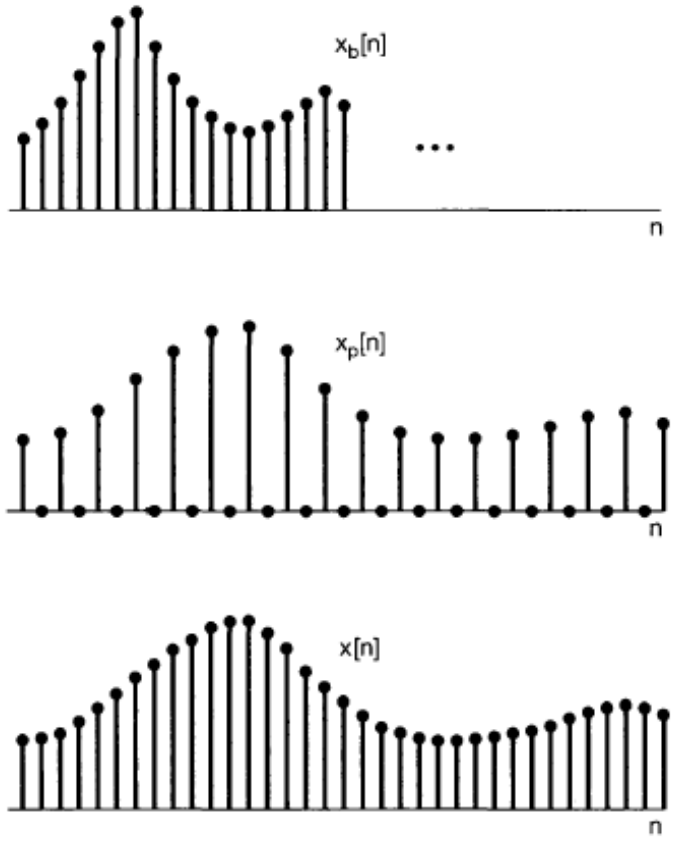


Figure 27: Overall system of an up sampling.

The following diagram shows the frequency domain illustrations showing sequences and spectra for up sampling by a factor of L = 2.

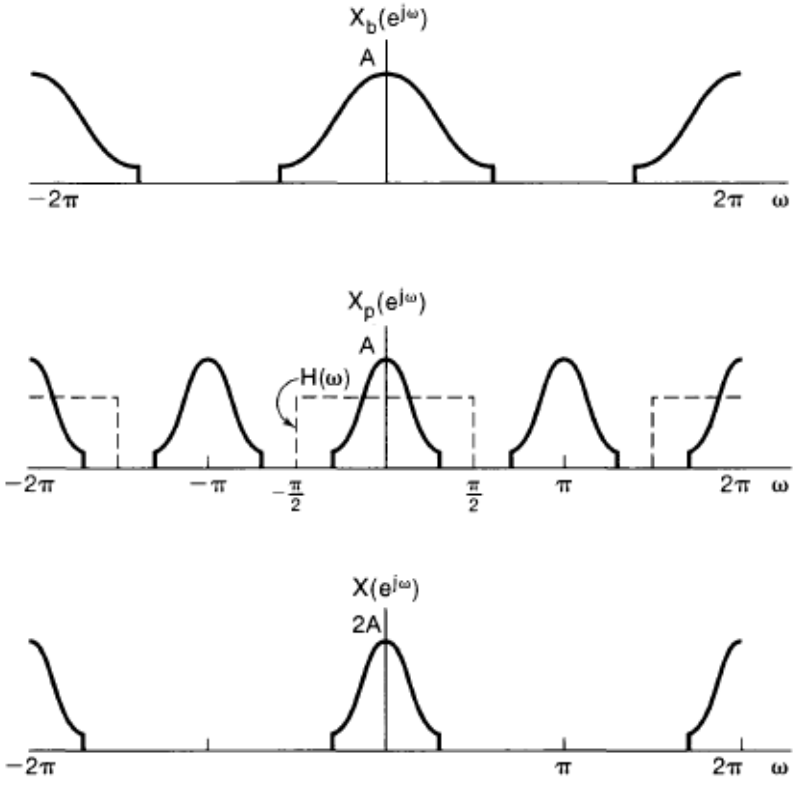


Figure 28: Associated sequences and spectra for up sampling by a factor of 2.