Mock Midterm Test: XMUT220 Signals and Systems

Instructions: SOLUTIONS (provided as a reference only)

1. Write your name and student ID on each page of your answer sheets.
2. Write your answers in the order of the questions, with proper labelling of each question number and part.
3. Show your solution steps clearly.
4. Correct answers without solution steps will not get full marks.
5. Calculator is permitted.
6. **Cheating is a serious offence; if you are caught cheating in any forms, you will be reported.**

There are five (5) questions in this examination, answer all questions.

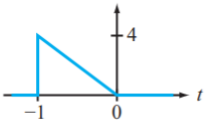
Time allowed: 2 hours.

Q1 Signals

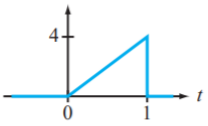
Part a)

Solution: Scale first, then shift, after writing each transformation in the form $x(a(t-b))$.

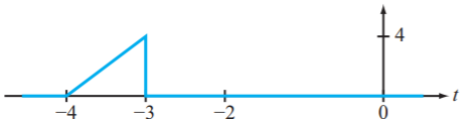
(a) $x(3t+6) = x(3(t+2))$



(b) $x(-3t+6) = x(-3(t-2))$



(c) $x(-3t-6) = x(-3(t+2))$



Part b)

(c) The period of $6\cos(\frac{2\pi}{3}t)$ is $\frac{2\pi}{2\pi/3} = 3$. So $6\cos(\frac{2\pi}{3}t)$ repeats every $3, 6, 9, 12\ldots$

The period of $7\cos(\frac{2}{3}t)$ is $\frac{2\pi}{2/3} = 3\pi$. So $7\cos(\frac{2}{3}t)$ repeats every $3\pi, 6\pi, 9\pi, 12\pi\ldots$

The sum **never** repeats, so it is | not periodic. |

Q2 LTI Systems

Part a) The system is:

$$y(t) = t\,x(t).$$

The response to $[c\,x(t)]$ is the output

$$tc\,x(t) = c\,y(t).$$

The response to $[x_1(t) + x_2(t)]$ is the output

$$t[x_1(t) + x_2(t)] = tx_1(t) + tx_2(t) = y_1(t) + y_2(t).$$

Since the system has both the scaling and additivity properties, it **is linear.**

The response to $x(t - T)$ is the output

$$tx(t - T) \neq (t - T)\,x(t - T) = y(t - T).$$

So the system is **not time-invariant.**

Part b)

Solution: Treat each sinusoidal input and output separately.

$$\cos(\omega t) \rightarrow |\mathbf{H}(\omega)|\cos[\omega t + \angle\underline{\mathbf{H}(\omega)}].$$

$$\mathbf{H}(0) = \frac{0}{1} = 0.$$

$$\mathbf{H}(1) = \frac{6}{2} = 3.$$

$$\mathbf{H}(2) = \frac{6}{3} = 2.$$

The output is

$$y(t) = (0)4 + (3)4\cos(t) + (2)2\cos(2t) = \boxed{12\cos(t) + 4\cos(2t).}$$

Q3 Fourier Series

Solution: $f_2(t)$ appears to be a shifted-down version of $f_1(t)$ with the following differences:

In the case of $f_1(t)$: its period is $T_1 = 4$ s, hence $\omega_{0_1} = \frac{\pi}{2}$ rad/s. Its peak-to-peak value $= 20$.

In the case of $f_2(t)$: its period is $T_2 = 8$ s, hence $\omega_{0_2} = \frac{\pi}{4}$ rad/s. Its peak-to-peak value $= 40$.

$$\therefore \quad f_2(t) = 2[f_1(t) - 10] \quad \text{with } f_1(t) \text{ period modified to be that of } f_2(t)$$

$$\therefore \quad f_2(t) = -\frac{40}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi t}{4}\right).$$

Q4 Fourier Transform

Solution: The waveform can be expressed as

$$f(t) = \begin{cases} 0 & t \le 0 \\ \dfrac{A}{T} t & 0 \le t \le T \\ 0 & t \ge T. \end{cases}$$

Applying the Fourier transform definition to $f(t)$,

$$\begin{aligned} \mathbf{F}(\omega) &= \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt \\ &= \int_0^T \frac{A}{T} t e^{-j\omega t}\, dt \\ &= \frac{A}{T}\left[\frac{jte^{-j\omega t}}{\omega}\bigg|_0^T - \frac{j}{\omega}\int_0^T e^{-j\omega t}\, dt\right] \\ &= \frac{A}{T\omega^2}\left[e^{-j\omega T}(1 + j\omega T) - 1\right]. \end{aligned}$$

Substituting for the values of A and T:

$$\mathbf{F}(\omega) = \frac{5}{3\omega^2}[(1 + j3\omega)e^{-j3\omega} - 1].$$

Q5 DT Convolution and DTFT

Part a)

$$\begin{aligned} \{\underline{1}, 2, -3\} * u[n] &= u[n] + 2u[n-1] - 3u[n-2] \\ &= \boxed{\{\underline{1}, 3\}.} \end{aligned}$$

This is because at $n = 1$, the convolution yields $1 + 2 = 3$, and at $n \ge 2$, it yields $1 + 2 - 3 = 0$.

Part b)

Solution: The DTFT of a signal $x[n]$ is

$$\mathbf{X}(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}.$$

(a) $\mathbf{X}(e^{j\Omega}) = e^{j2\Omega} + e^{j\Omega} + 1 + e^{-j\Omega} + e^{-j2\Omega}$ simplifies to

$$\boxed{\mathbf{X}(e^{j\Omega}) = 1 + 2\cos(\Omega) + 2\cos(2\Omega).}$$

XMUT321 Engineering Statistics Exam Formulae (Page 1 only)

Bayes rule (A_i nonzero, disjoint and exhaustive)

$$p(A_i|B) = \frac{p(B|A_i)p(A_i)}{P(B)} = \frac{p(B|A_i)p(A_i)}{\sum_i P(B|A_i)P(A_i)}$$

Law of total probability (A_i disjoint and exhaustive)

$$p(B) = \sum_i P(B|A_i)P(A_i)$$

Number of ways to choose k from n objects

$$\frac{n!}{(n-k)!}$$

Number of ways to organize n objects into groups of $k_1, \cdots, k_r$ objects:

$$\frac{n!}{k_1! \cdots k_r!}$$

Expectation

$$\text{continuous RV}: \; E[g(X)] = \int_{\infty}^{\infty} g(x) f_X(x) dx$$
$$\text{discrete RV}: \; E[g(X)] = \sum_x g(x) p_X(x)$$

Covariance (continuous case)

$$\text{cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(x-\mu_X)(y-\mu_Y) f_{X,Y}(x,y) dx dy$$

Covariance (discrete case)

$$\text{cov}(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = \sum_x \sum_y (x-\mu_X)(y-\mu_Y) p_{X,Y}(x,y)$$

Binomial

$$p(x) = \begin{cases} \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, & x = 0, 1, \cdots, n \\ 0, & \text{otherwise} \end{cases}$$

$$\sigma_X^2 = np(1-p)$$

$$\mu_X = np$$

Bernoulli: binomial with $n = 1$.

Poisson

$$p(x) = \begin{cases} \mathrm{e}^{-\lambda} \lambda^x / x! & x \text{ nonnegative integer} \\ 0 \text{ otherwise} \end{cases}$$