

ECEN220 Tutorial 1 Questions (**SOLUTIONS**)

Block 1: Signals

Q1. Solutions:

A continuous-time signal $x(t)$ is a function of continuous time t , where t is a real number which usually represents time, but may represent space. A discrete-time signal $x[n]$ is a function of discrete time n , where n is an integer, which is an index that represents discrete time. The values of both $x(t)$ and $x[n]$ are continuous, i.e., they are real or complex and may assume any value. A digital signal may be defined in either discrete time or continuous time, but its values are discrete, i.e., they are integers or integer multiples of some real number.

Q2. Solutions:

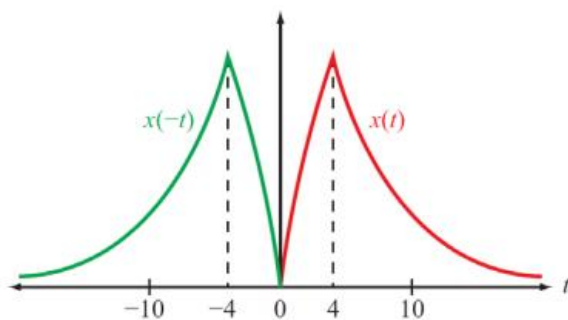
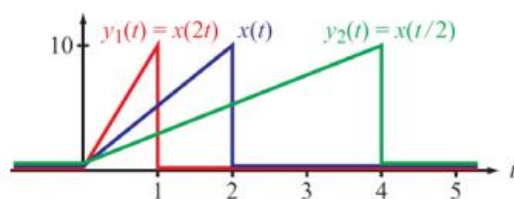
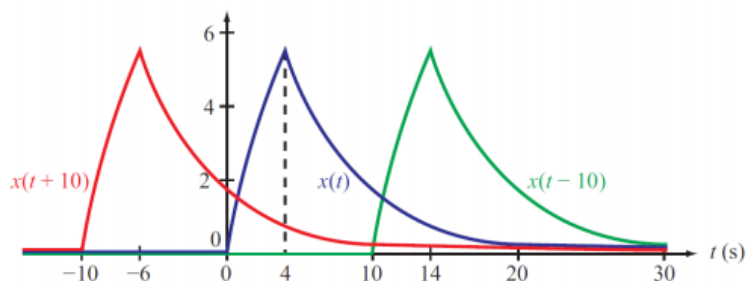
A continuous-time causal signal $x(t)$ is zero for $t < 0$. A discrete-time causal signal $x[n]$ is zero for $n < 0$. At time zero, $x(0)$ may or may not be 0, but it may also be undefined; the step function $u(t)$ is undefined at $t = 0$, while $x[0]$ is (usually) nonzero.

A signal $x(t)$ (or $x[n]$) is noncausal if $x(t) \neq 0$ (or $x[n] \neq 0$) for $t < 0$ (or $n < 0$).

A continuous-time anticausal signal $x(t)$ is zero for $t > 0$. A discrete-time anticausal signal $x[n]$ is zero for $n > 0$, and $x[0] = 0$.

Q3. Solutions:

Time-shift transformation does not alter the shape of a waveform—it moves it back or forth along the t axis. Time-scaling transformation does alter the shape—it compresses or expands it along the t axis. Time-reversal transformation, which is a special case of time-scaling transformation, does alter the shape—it flips the waveform about the vertical axis.



Q4.

$$y(t) = x(at - b) = x(a(t - \frac{b}{a})) = x(a(t - T)) \text{ where } T = \frac{b}{a}.$$

Here, $y(t) = x(-4t - 8) = x(-4(t + 2))$, so $a = -4$ and $T = -2$.

Q5.

Yes, since $x(2t) = (2t)^3 = 8t^3 = y(t)$.

Q6.

Solution: Let $y(t) = x(a(t - T))$ for a time-scaling transformation with factor a and a time-shift transformation with a time delay of T .

Since $x(t) = 4t$, we have $y(t) = 4[a(t - T)] = 4at - 4aT$. We want $y(t) = 2(t + 4) = 2t + 8$.

So $4a = 2$ and $-4aT = 8$. This yields $a = \frac{1}{2}$ and $T = -4$.

Then

$$y(t) = x(a(t - T)) = x\left(\frac{1}{2}(t + 4)\right) = x\left(\frac{t}{2} + 2\right).$$

Q7.

An even-symmetrical waveform has the property that it is invariant to flipping around the vertical axis. If $x(t)$ is the waveform, then $x(-t) = x(t)$, as shown in Fig. 1-11(b).

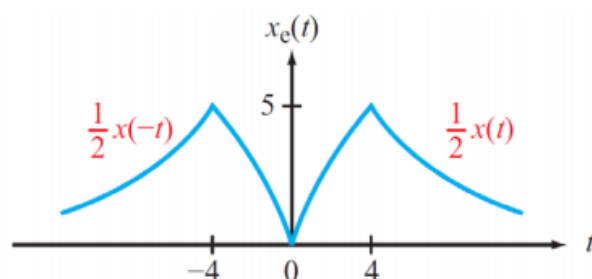


Fig. 1-11(b)

An odd-symmetrical waveform has the property that flipping it around the vertical axis has the same effect as flipping it around the horizontal axis. If $x(t)$ is the waveform, then $x(-t) = -x(t)$, as shown in Fig. 1-11(c).

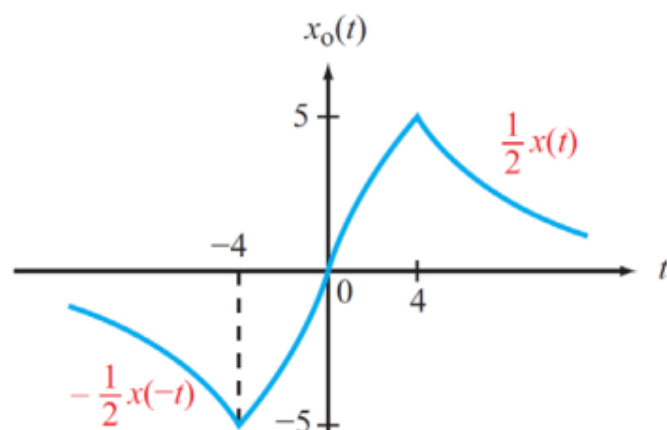


Fig. 1-11(c)

Q8.

A continuous-time signal $x(t)$ is periodic with period T if $x(t) = x(t + kT)$ for all times t and integers k . A sinusoid with nonzero frequency f is periodic with period $T = 1/f$.

A discrete-time signal $x[n]$ is periodic with period N if $x[n] = x[n + kN]$ for all times n and integers k . Unlike in continuous time, most discrete-time sinusoids are not periodic.

Q9.

Solution: A function $x(t)$ has an even-symmetrical waveform if $x(-t) = x(t)$. It is symmetric about the vertical axis.

A function $x(t)$ has an odd-symmetrical waveform if $x(-t) = -x(t)$. Reflecting an odd-symmetric function about the vertical axis, then about the horizontal axis (or vice-versa), leaves it unaltered.

(a) $x_1(-t) = 3(-t)^2 = 3t^2 = x_1(t)$, so $x_1(t)$ has an even-symmetrical waveform.

(b) $x_2(-t) = \sin(-2t) = -\sin(2t) = -x_2(t)$,

so $x_2(t)$ has an odd-symmetrical waveform.

(c) $x_3(-t) = \sin^2(-2t) = (-\sin(2t))^2 = \sin^2(2t) = x_3(t)$,

so $x_3(t)$ has an even-symmetrical waveform.

(d) $x_4(-t) = 4e^t \neq \pm x_4(t)$, so the waveform of $x_4(t)$ has no symmetry.

(e) $x_5(-t) = |\cos(-2t)| = |\cos(2t)| = x_5(t)$,

so $x_5(t)$ has an even-symmetrical waveform.

Note that $\cos(2t)$ also has an even-symmetrical waveform.

Q10.

Solution: $u(-t)$ is simply $u(t)$ reflected about the vertical axis. So

$u(t)$ and $u(-t)$ are mirror images of one another.

This is true for *any* function $x(t)$, not just $u(t)$.

Q11.

$$\delta(t) = du/dt.$$

Q12.

$$x(t) = 2u(t-3) - 2u(t-4).$$

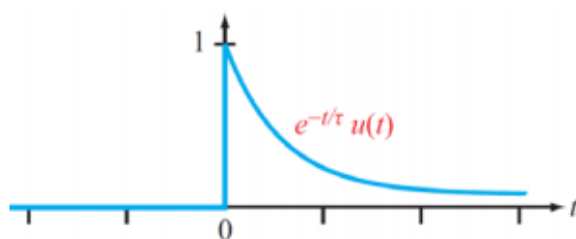
$$\frac{d}{dt} [u(t-T)] = \delta(t-T).$$

So

$$x'(t) = \frac{dx}{dt} = \frac{d}{dt} [2u(t-3) - 2u(t-4)] = 2\delta(t-3) - 2\delta(t-4).$$

Q13.

Slower. The time constant of a decaying exponential function $x(t)$ represents how long it takes the function to decay from $x(0)$ to $x(0)/e = 0.37x(0)$.



Q14.

The energy and average power of a signal $x(t)$, defined in Eqs. (1.33b) and (1.34), are given by:

$$E = \lim_{T \rightarrow \infty} \int_{-T}^T |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt \quad \text{Eq. (1.33b)}$$

$$P_{\text{av}} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t) dt = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt \quad \text{Eq. (1.34)}$$

An **energy signal** has finite energy E , so its average power P (which is averaged over an infinite duration T) must be zero.

A **power signal** has finite average power P , so its energy E must be infinite. Periodic signals are power signals. For example, a sinusoid with amplitude A and nonzero frequency has infinite energy and average power $A^2/2$.

A signal with infinite power, and therefore infinite energy, is neither an energy signal nor a power signal.

Q15.

Solution: $x(t) = \begin{cases} 5 & \text{for } 1 < t < 5, \\ 0 & \text{otherwise.} \end{cases}$

So

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_1^5 |5|^2 dt = 100.$$

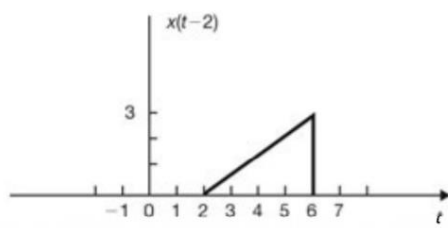
Since E is finite, $P_{\text{av}} = 0$.

Note that E is invariant to time shifts, so we could have used

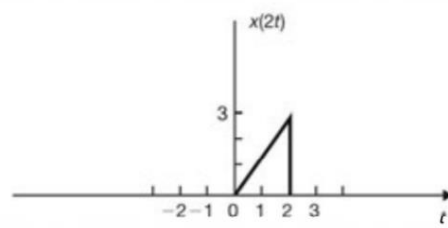
$$E = \int_{-2}^2 |5|^2 dt = 100.$$

Q16. (not provided) 😊

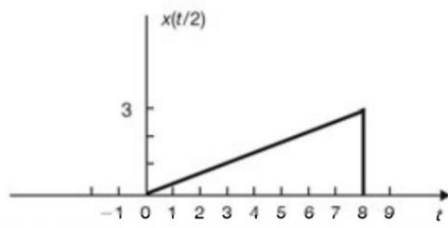
Q17.



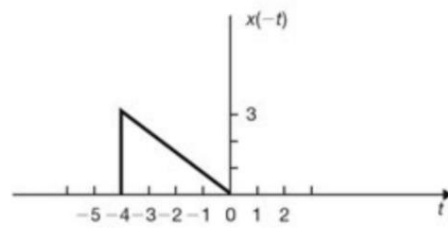
(a)



(b)

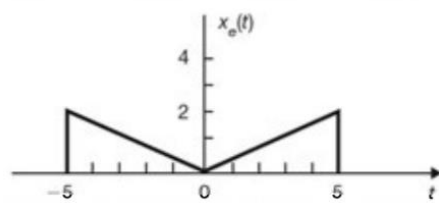


(c)

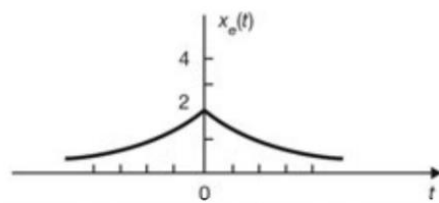
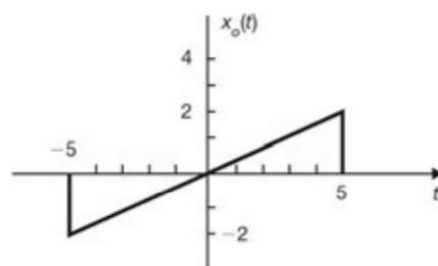


(d)

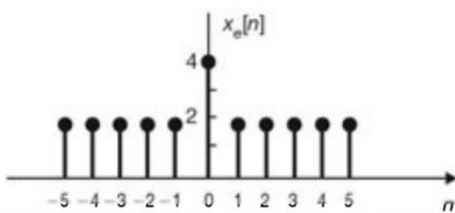
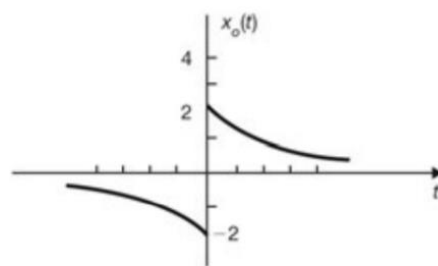
Q18.



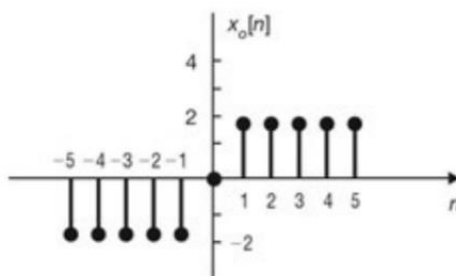
(a)

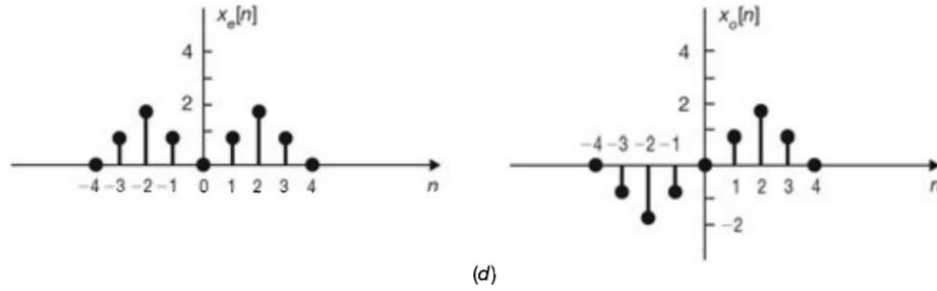


(b)



(c)





Q19.

Let $x(t) = x_1(t)x_2(t)$. If $x_1(t)$ and $x_2(t)$ are both even, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ and $x_2(t)$ are both odd, then

$$x(-t) = x_1(-t)x_2(-t) = -x_1(t)[-x_2(t)] = x_1(t)x_2(t) = x(t)$$

and $x(t)$ is even. If $x_1(t)$ is even and $x_2(t)$ is odd, then

$$x(-t) = x_1(-t)x_2(-t) = x_1(t)[-x_2(t)] = -x_1(t)x_2(t) = -x(t)$$

and $x(t)$ is odd. Note that in the above proof, variable t represents either a continuous or a discrete variable.

Q20.

$$x(t) = \cos\left(t + \frac{\pi}{4}\right) = \cos\left(\omega_0 t + \frac{\pi}{4}\right) \rightarrow \omega_0 = 1$$

$x(t)$ is periodic with fundamental period $T_0 = 2\pi / \omega_0 = 2\pi$.

Q21.

$$x[n] = \cos \frac{\pi}{3} n + \sin \frac{\pi}{4} n = x_1[n] + x_2[n]$$

where

$$x_1[n] = \cos \frac{\pi}{3} n = \cos \Omega_1 n \rightarrow \Omega_1 = \frac{\pi}{3}$$

$$x_2[n] = \sin \frac{\pi}{4} n = \cos \Omega_2 n \rightarrow \Omega_2 = \frac{\pi}{4}$$

$X_1[n]$ is periodic with fundamental period $N_1 = 6$. $X_2[n]$ is periodic with fundamental period $N_2 = 8$. So, $x[n]$ is periodic with the LCM of 6 and 8 = 24.

Q22. The sinusoidal signal $x(t)$ is periodic with $T_0 = 2\pi/\omega_0$. The average power of $x(t)$ is

$$\begin{aligned} P &= \frac{1}{T_0} \int_0^{T_0} [x(t)]^2 dt = \frac{\omega_0}{2\pi} \int_0^{2\pi/\omega_0} A^2 \cos^2(\omega_0 t + \theta) dt \\ &= \frac{A^2 \omega_0}{2\pi} \int_0^{2\pi/\omega_0} \frac{1}{2} [1 + \cos(2\omega_0 t + 2\theta)] dt = \frac{A^2}{2} < \infty \end{aligned}$$

Thus, $x(t)$ is a power signal. Note that periodic signals are, in general, power signals.

Q23.

Solution: In discrete time, the duration of a signal that is zero outside the interval $a \leq n \leq b$ is $b - a + 1$.

Here, $a = -1$, $b = 2$, so the duration is $b - a + 1 =$ 4 .

Q24.

Solution: The mean of the sum of two signals is the sum of the means.

$$y[n] = x[n] - 3.$$

Q25.

$$N_0 = \frac{2\pi k}{\Omega} = \frac{2\pi k}{0.56\pi} = \frac{25k}{7}.$$

Hence, select $k = 7$ and $N_0 = 25$ samples.

Also,

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{25} \text{ rad/sample.}$$

Q26.

$$N_0 = \frac{2\pi k}{\Omega} = \frac{2\pi k}{5.1\pi} = \frac{20k}{51}.$$

Hence, select $k = 51$ and $N_0 = 20$ samples.

$$\Omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{20} = \text{ 0.1π rad/sample.}$$