XMUT220 Signals and Systems

Tutorial 5a: Time and Frequency Characteristics (Introduction) – Solution

A. Fundamentals of Time and Frequency Characteristics

1. In the time-domain, parameters such as time, time constant, rise time, peak time, settling time, overshoot and steady-state error are typically used for its analysis.

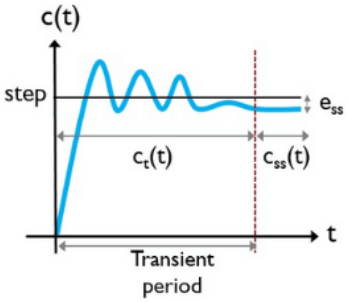
   a. What is time-domain analysis? Which system is typically analysed with the time-domain analysis? What does time domain analysis measure (i.e. say for example an electronic signal)? List several devices used in the time-domain analysis. [10 marks]

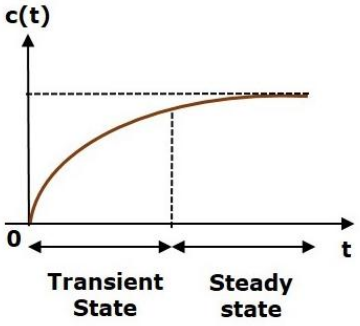
   b. Complete the following table with description and application of the given time-domain parameters. [14 marks]

Time-Domain Parameters	**Description**	**Use/Application**
Time constant		
Rise time		
Peak time		
Settling time		
Overshoot		
Steady-state error		

Solution

a. Vaguely speaking, time domain analysis is analysing the data over a given period-of-time.





Systems that involve electronic signals, biological systems or even business market behaviour are some of the things that can be analysed using time domain analysis.

For an electronic signal, the time domain analysis is mainly based on the voltage – time plot or the current – time plot. We could measure and analyse also other parameters such as power, frequency, phase shift, etc. In a time domain analysis, the variable is always measured against time.

There are several devices used to analyse data on a time domain basis. The analogue or digital oscilloscope is the most common device when analysing electrical signals on a time domain. Other computer instrumentation, graphs and raw numerical data can be used to analyse data on a time domain.

b. The answers for the specified time-domain parameters are listed in the table below.

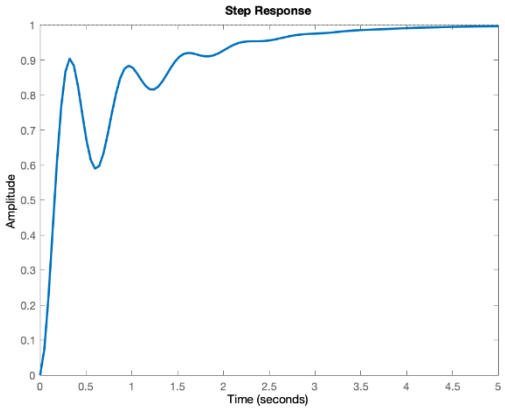
Time-Domain Parameters	**Description**	**Use/Application**
Time constant	The time taken to reach 63% of the final value of the signal.	Measurement of the timing of the rising of decreasing of the signal.
Rise time	The time taken from 10% to 90% of the final value of the signal.	Measurement of the rising time of the signal
Peak time	The time taken to reach the first peak of the oscillation.	Measurement of the timing to reach the first peak.
Settling time	The time taken to reach at least 5 time constant.	Measurement of the timing of the steady-state condition.

Overshoot	The amplitude difference between the peak and the final value of the signal.	Measurement of the amplitude differences of the oscillations.
Steady-state error	The time taken to reach 5 time constant.	Measurement of the steady-state condition of the system.

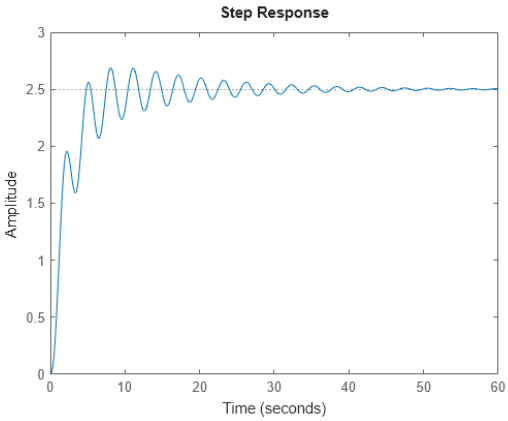
2. The transient response such as the step response is often used in the time-domain analysis for measuring the characteristics and performance of the system.

   a. What variables are extensively used in time-domain analysis? What measuring instrument is used for measuring these variables? What does a typical graph/plot in time domain analysis say about itself? [6 marks]

   b. Analyse the following transient responses of the systems in terms of its response time, settling time, and steady-state error. [6 marks]



System A

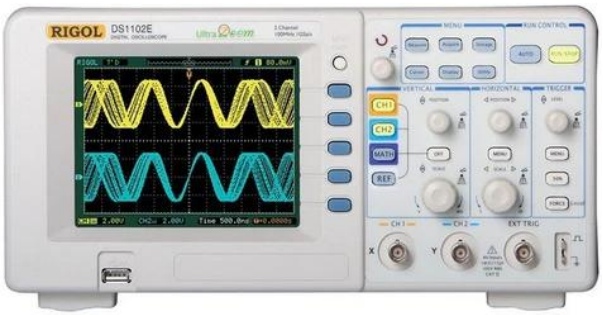


System B

Solution

a. In the time domain, the signal or function's value is known for all real numbers, for the case of continuous time, or at various separate instants in the case of discrete time.

   An oscilloscope is a tool commonly used to visualise real-world signals in the time domain and for measuring the time-domain variables.



A time-domain graph shows how a signal changes with time, whereas a frequency-domain graph shows how much of the signal lying within each given frequency band over a range of frequencies.

b. The there are more oscillations in the step response of System A compared with System B. The settling time of System A is relatively shorter than System B. There is no steady-state error in the step response of System A, but the step response of System B has significant steady-state error.

3. Frequency domain analysis involves parameters such as magnitude, gain, phase shift, frequency, cut-off frequency, bandwidth, and phase/gain margins.

   a. What is frequency-domain analysis? Which field uses this type of analysis? Which system is suitable for its use? Which concept is fundamental in frequency-domain analysis (i.e. related with the concept of transformation)? [8 marks]

   b. Complete the following table with the description and use of the listed frequency-domain parameters. [14 marks]

Frequency-Domain Parameters	**Description**	**Use/Application**
Magnitude		
Gain		
Phase shift		
Frequency		

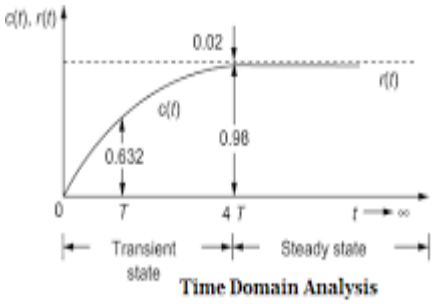
Cut-off frequency		
Bandwidth		
Gain/phase margins		

Solution

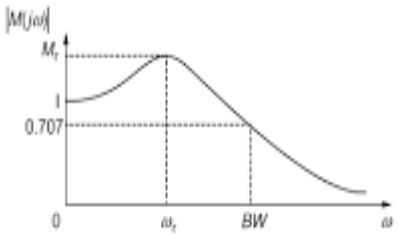
a. Frequency-domain analysis is analysing a mathematical function or a signal with respect to the frequency.

Frequency-domain analysis is widely used in fields such as control systems engineering, electronics and statistics.

Frequency-domain analysis is mostly used to deal with signals or functions that are periodic over time. This does not mean that frequency-domain analysis cannot be used in signals that are not periodic.



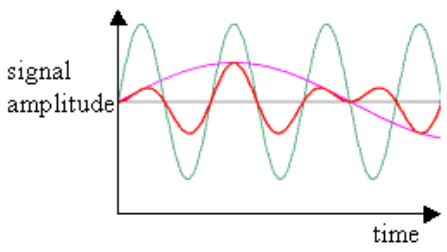
Time Domain Analysis



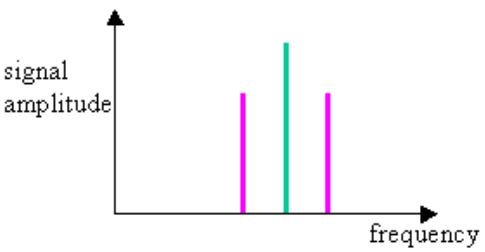
Frequency Domain Analysis

The most important concept in the frequency domain analysis is the transformation. Transformation is used to convert a time domain function to a frequency domain function and vice versa. The most common transformation used in the frequency domain is the Fourier transformations.

Fourier transformation is used to convert a signal of any shape into a sum of infinite number of sinusoidal waves. Since analysing sinusoidal functions is easier than analysing general shaped functions, this method is very useful and widely used.



Time domain



Frequency domain

b. The answers for the description and use/application of the specified frequency domain parameters are listed in the table below.

Frequency-Domain Parameters	**Description**	**Use/Application**
Magnitude	Amplitude of the response of the system over its operating frequency.	Measurement of the strength of the signal.
Gain	The ratio of the amplitudes of the response of the system over its operating frequency.	Measurement of the performance of the system.
Phase shift	Phase of the response of the system over its operating frequency.	Measurement of the delay of the signal.
Frequency	The operating frequency of the system.	Measurement of the frequency response of the system
Cut-off frequency	The frequency when the amplitude of the response of the system is 0.707 of its maximum or minimum.	Measurement of the half-power of the signal. Useful for filter application.
Bandwidth	The width between the cut-off frequencies of the response of the system.	Measurement of the useful frequency range of the system.
Gain/phase margins	The gain and phase difference in the response of the system between 0 dB gain with the gain when the phase shift is 180° and between the 180° phase shift and the phase shift when the gain is 0 dB respectively.	Measurement the stability of the system.

4. Some systems in engineering are better off analysed using the time-domain analysis compared with the frequency-domain analysis, or vice versa.

   a. Compared with time-domain analysis, why people resort to frequency-domain analysis? List important parameters/variables in frequency-domain analysis? What instrument is used for its measurement? Give a common example where frequency-domain analysis is useful. [8 marks]

   b. Fill in the blanks given in the table below with relevant answers. [8 marks]

Context	**Time-domain analysis**	**Frequency-domain analysis**
Transient response of RLC circuit		
Components of a complex signal		
Filter circuit design		
Vibration measurements		

Solution

a. One of the main reasons for using a frequency-domain analysis is to simplify the mathematical analysis.

For mathematical systems governed by linear differential equations, a very important class of systems with many real-world applications, converting the description of the system from the time domain to a frequency domain converts the differential equations to algebraic equations, which are much easier to solve.

In addition, looking at a system from the point of view of frequency can often give an intuitive understanding of the qualitative behaviour of the system.

Variables of particular interest in frequency-domain analysis are bandwidth, frequency response, gain, phase shift, resonant frequencies, time constant, resonance width, damping factor, Q-factor, harmonics, spectrum, power spectral density, eigenvalues, poles, and zeros.

While the time-domain signal can be seen on an oscilloscope, in frequency-domain analysis a spectrum analyser is a device that displays the frequency spectrum, hence its measurements.



An example of a field in which frequency-domain analysis gives a better understanding than time domain is music. The theory of operation of musical instruments and the musical notation used to record and discuss pieces of music is implicitly based on the breaking down of complex sounds into their separate component frequencies (musical notes).

b. The answers for the differences between time-domain analysis and the frequency-domain analysis given are listed in the table below.

Context	**Time domain analysis**	**Frequency domain analysis**
Transient response of RLC circuit	Show the required transient response	Not appropriately used
Components of a complex signal	Difficult to perform	Relatively easier to design using this approach
Filter circuit design	Difficult to perform	Relatively easier to design using this approach
Vibration measurements	Shows the signal related to the vibration problem	Identify the specific component of the signal related to the vibration problem

5. There are some aspects of time domain analysis that make it different from the frequency domain analysis, or vice versa.

   a. What is the difference between time and frequency domain analysis? [2 marks]

   b. Fill in the blanks given in the table below with relevant answers. [6 marks]

Aspect	Time Domain Analysis	Frequency Domain Analysis
Data usage		
Methods		
Applications		

Solution

a. Among the differences between time-and frequency-domain analysis are:
   - Frequency domain analysis is used in conditions where processes such as filtering, amplifying and mixing are required.
   - Time domain analysis gives the behaviour of the signal over time. This allows predictions and regression models for the signal.
   - Frequency domain analysis is very useful in creating desired wave patterns such as binary bit patterns of a computer.
   - Time domain analysis is used to understand data sent in such bit patterns over time.

b. The answers for the differences between the time-domain analysis and frequency domain analysis are listed in the table below.

Aspect	Time Domain Analysis	Frequency Domain Analysis
Data usage	Magnitude of time-domain parameters: voltage, current, power, energy, etc.	Magnitude of frequency-domain parameters: gain, phase shift, spectrum power
Methods	Time-domain methods: transient response, step response, etc.	Frequency-domain methods: frequency response, frequency spectrum, etc.
Applications	Performance analysis, fault analysis, stability analysis, etc.	Filter design, signal processing, design of control system, etc.

6. Signal transformation is often utilised in the time and frequency domain analysis for conversion from one domain to another.

a. What signal transformation techniques are useful in time and frequency domain analysis? Describe also its applications. [4 marks]

b. Fill in the blanks given in the table below with relevant answers. [5 marks]

From	**To**	**Transform Method**
Discrete signal	Frequency component	
Analogue signal	Frequency component	
Complex calculus equation	Simpler algebra equation	
Recurrent signal	Frequency component	
Image/raw data	Processed data	

Solution

a. There are a few different (mathematical) transforms which are used to analyse the systems in both the time-and frequency domain. These are the most common transforms, and the fields in which they are used:

- Fourier series – repetitive signals, oscillating systems.
- Fourier transform – non-repetitive signals, transients.
- Laplace transform – electronic circuits and control systems.
- Z transform – discrete-time signals, digital signal processing.
- Wavelet transform — image analysis, data compression.

b. The answers for the specified transform methods are listed in the table below.

From	**To**	**Transform Method**
Discrete signal	Frequency component	Discrete Fourier transform

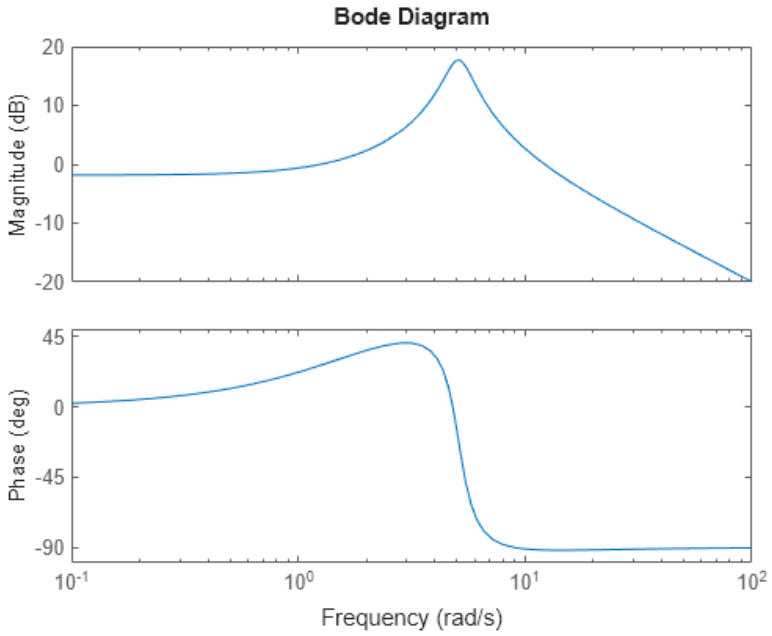
Analogue signal	Frequency component	Continuous Fourier transform
Complex calculus equation	Simpler algebra equation	Laplace transform
Recurrent signal	Frequency component	Generic transform method
Image/raw data	Processed data	Signal processing method

7. Frequency domain analysis is commonly performed using magnitude and phase shift as measured parameters against the operating frequency of the system.

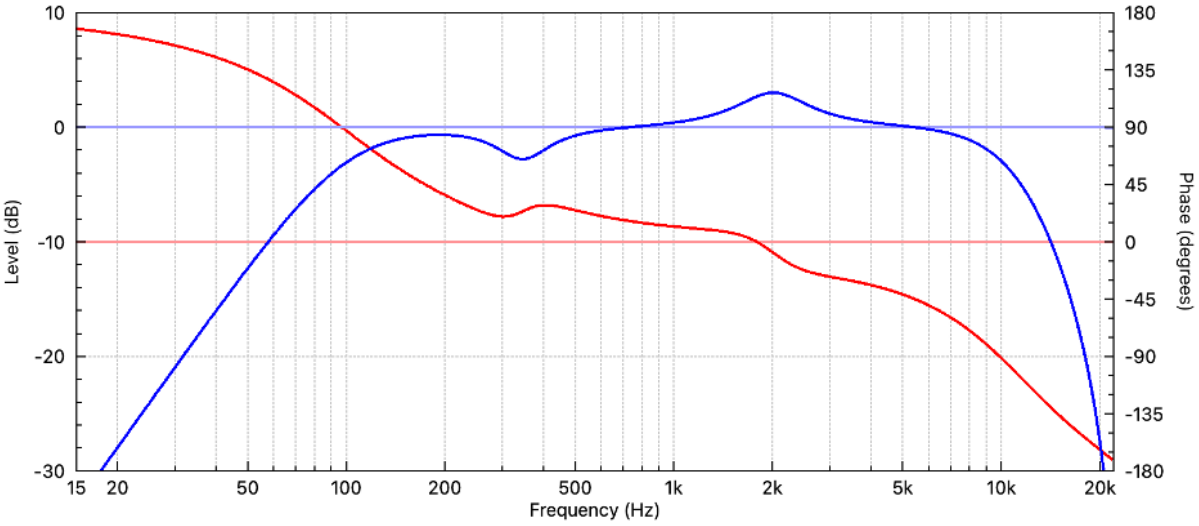
   a. Describe the significant of magnitude and phase in frequency domain analysis and its use for understanding the behaviour of the system? [4 marks]

   b. Analyse the following frequency response plots in terms of its magnitude and phase shift. [4 marks]

System 1: Signal Filter

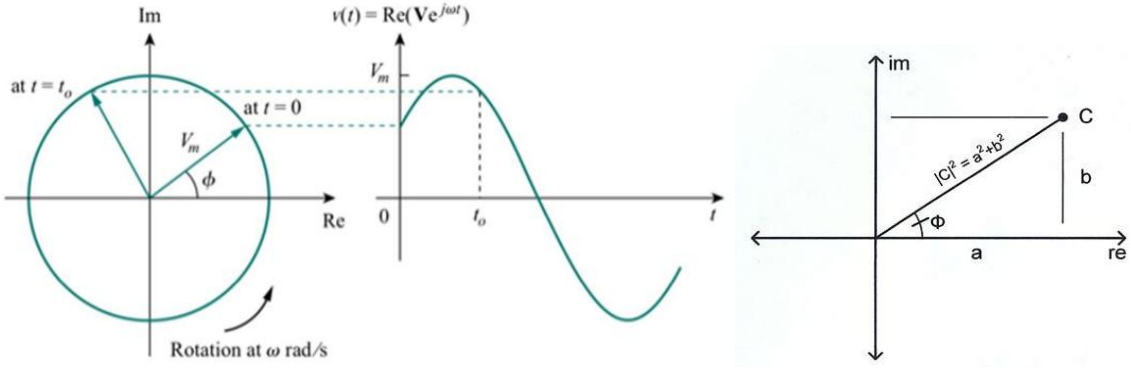


System 2: Audio Amplifier

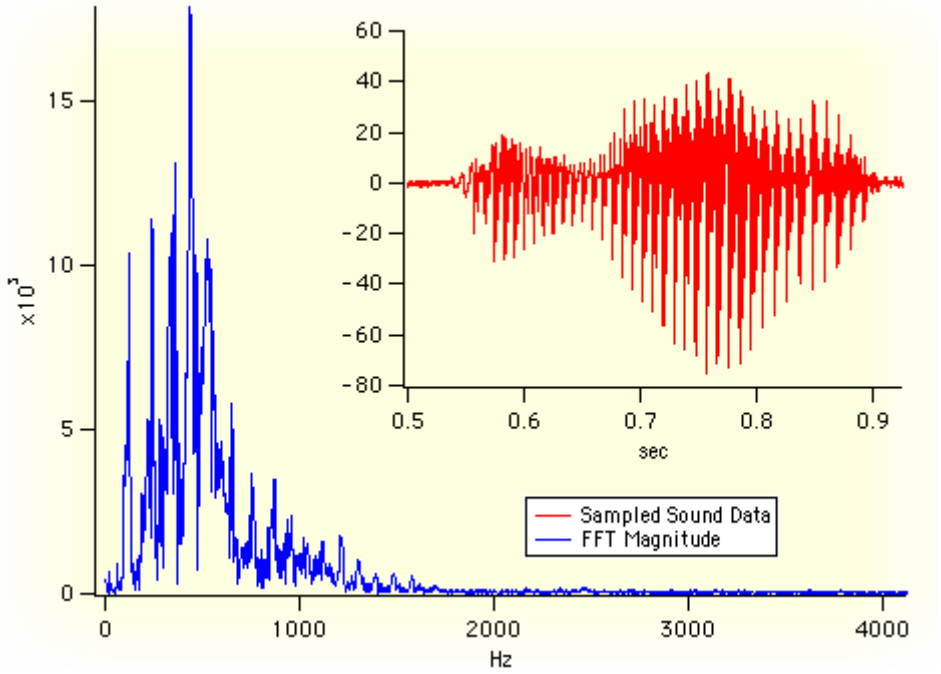


Solution

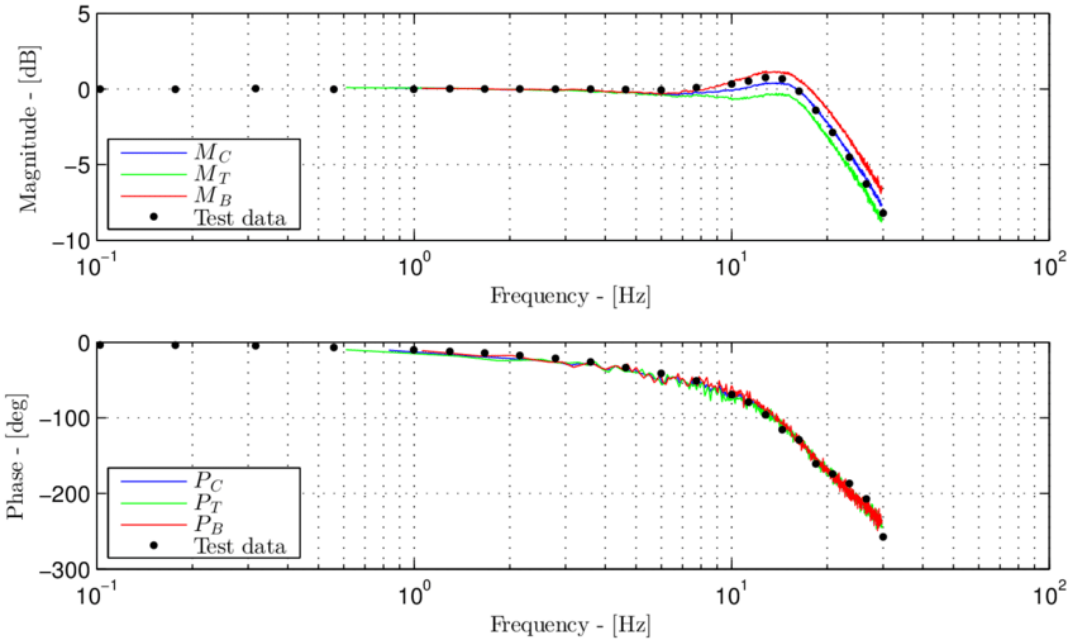
a. In frequency domain analysis, a signal is described by a complex function of frequency. The component of the signal at any given frequency is given by a complex number.
The modulus of the number is the amplitude of that component, and the argument is the relative phase of the wave.



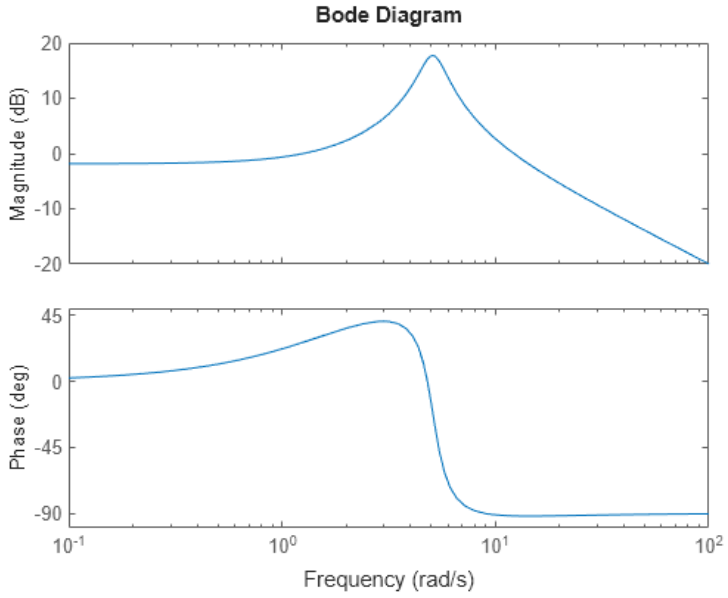
For example, using the Fourier transform, a sound wave, such as human speech, can be broken down into its component tones of different frequencies, each represented by a sine wave of a different amplitude and phase.



The response of a system, as a function of frequency, can also be described by a complex function. In many applications, phase information is not important. By discarding the phase information, it is possible to simplify the information in a frequency-domain representation to generate a frequency spectrum or spectral density.

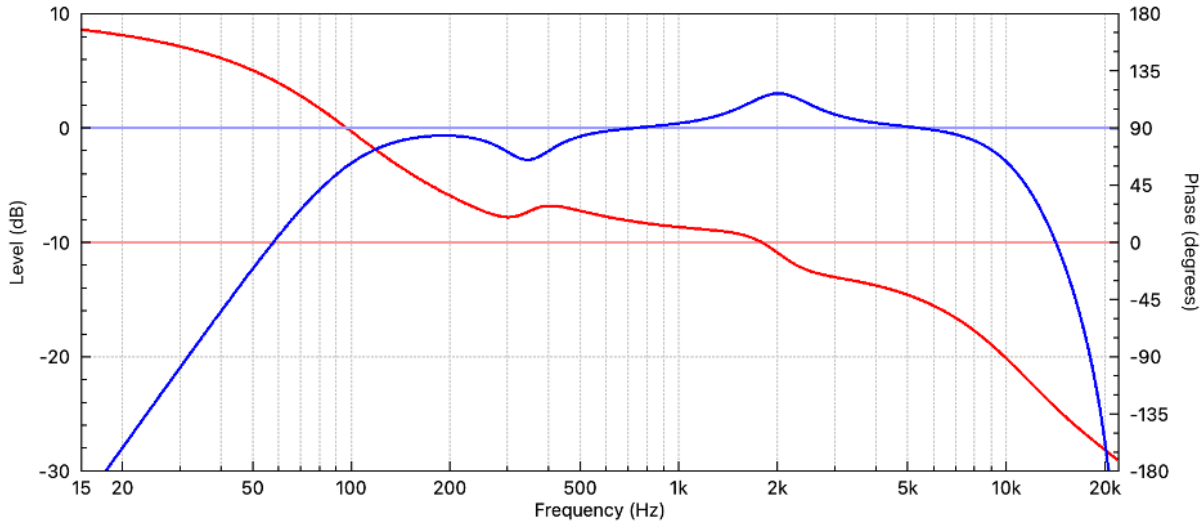


b. In terms of magnitude and phase shift, the differences between System A and System B are:



System A:

The magnitude plot has an 18 dB peak at 5 rad/s. The magnitude is -3 dB at low frequencies, and it has a slope of -40 dB/decade at high frequencies. The phase plot is 0° at low frequencies and -90° at high frequencies. At the peak, it is approx. -45°.



System B:

The magnitude of the frequency response is 0 dB and 0° between 100 rad/s and 10,000 rad/s. At low frequencies, the magnitude plot has a slope of -20 dB/decade and a phase plot of 180°. At high frequencies, the magnitude plot has a slope of -40 dB/decade and a phase plot of -180°.

B. Applications of Time and Frequency Characteristics

8. The following time and frequency domain graphs/plots are referring to the bass guitar signals plucked as open string for the A note (55 Hz). What can you tell about its significant?

[4 marks]

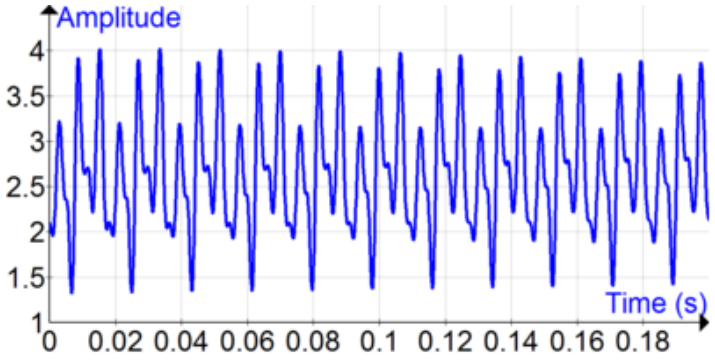


Figure a: Time-domain plot of bass-guitar's open-string A note (55 Hz)

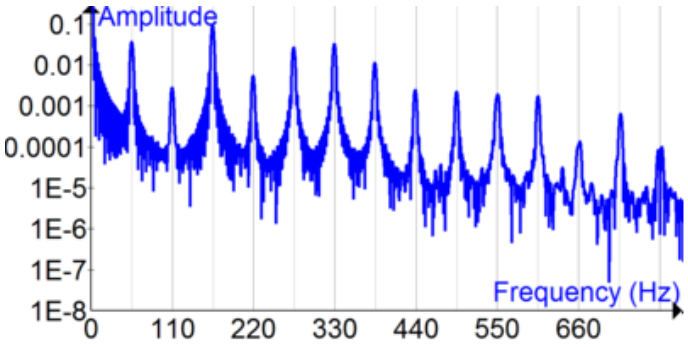
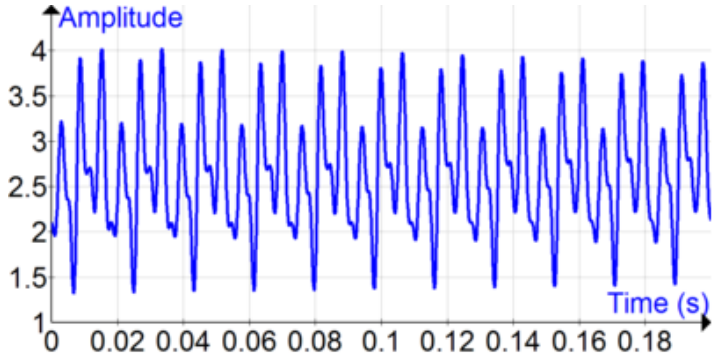


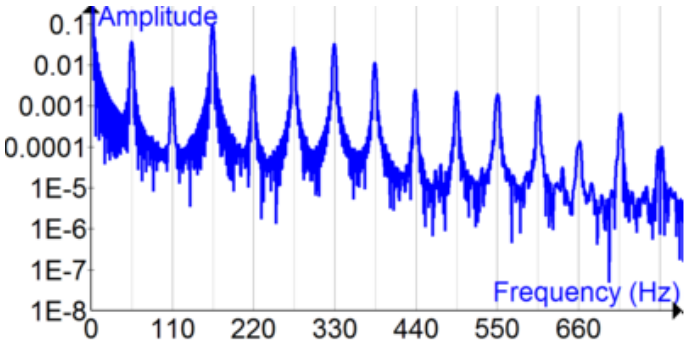
Figure b: Frequency-domain plot of bass-guitar's open-string A note (55 Hz)

Solution

In this example, the top signal at the right is a sound waveform of a bass guitar playing an open string corresponding to an A note with a fundamental frequency of 55 Hz.

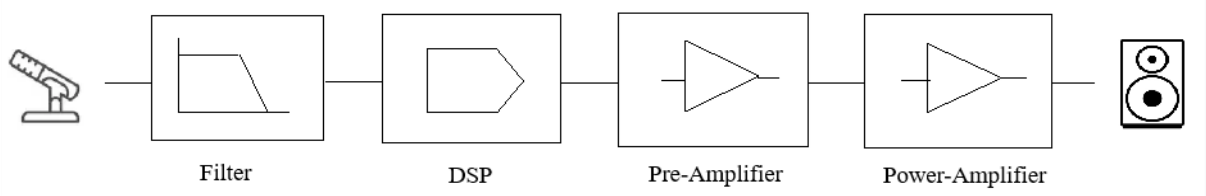


The waveform appears oscillatory, but it is more complex than a simple sine wave, indicating the presence of additional waves. The different wave components contributing to the sound can be revealed by applying a mathematical analysis based on the Fourier transform, the result of which is shown in the lower figure.



Note that there is a prominent peak at 55 Hz, but that there are other peaks at 110 Hz, 165 Hz, and at other frequencies corresponding to integer multiples of 55 Hz. In this case, 55 Hz is identified as the fundamental frequency of the string vibration, and the integer multiples are known as harmonics.

9. The following audio signal processing system works on capturing audio sounds through the microphone, filtering the signal by removing unwanted noise, performing sound reproduction process in the digital signal processing (DSP) unit of the system and amplify the signal before delivering to the speaker.



By applying time and frequency domain analysis through plots of representation of the signals in the time and frequency domain, describe how it works? [4 marks]

Solution

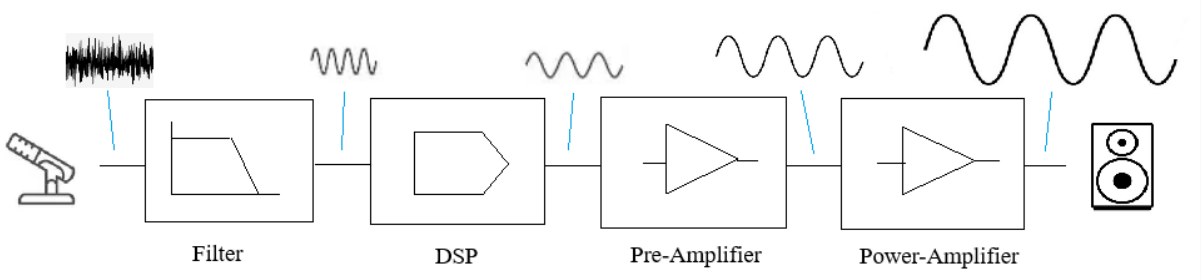
The working principles of the audio signal processing system in time and frequency domain are:

Components of the system:

- Microphone – a sensor that is used to capture the audio signal.
- (Noise) filter – used to remove noises that might exist in the audio signal captured by the microphone.
- DSP – perform signal processing using program and/or algorithms for specific purposes i.e. signal conversion, signal conditioning, storing, etc.
- Pre-amplifier – To increase the quality of the signals after the signal processing.
- Amplifier – To amplify the signals so they can be driven to the speaker.
- Speaker – To emit audio signal.

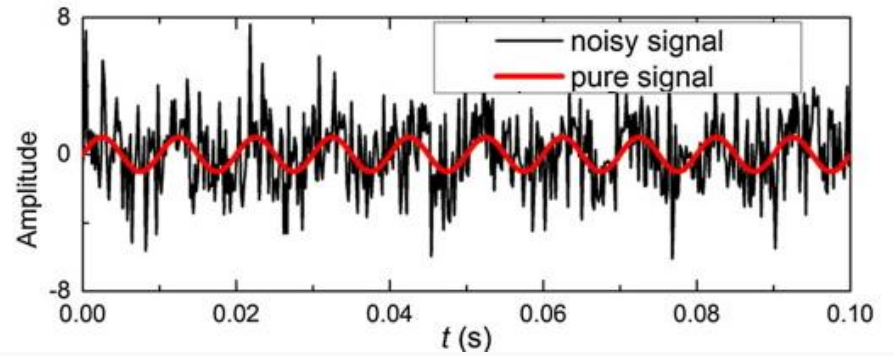
System description:

When the system captures audio signal, the signal undergoes filtering process to remove the noise before being treated for further processing in the DSP i.e. shifting the signal phase (lower frequency signal at the output of the DSP). After the DSP part the signal experiences signal conditioning in the pre-amplifier and finally to be delivered to the speakers.



Time domain analysis:

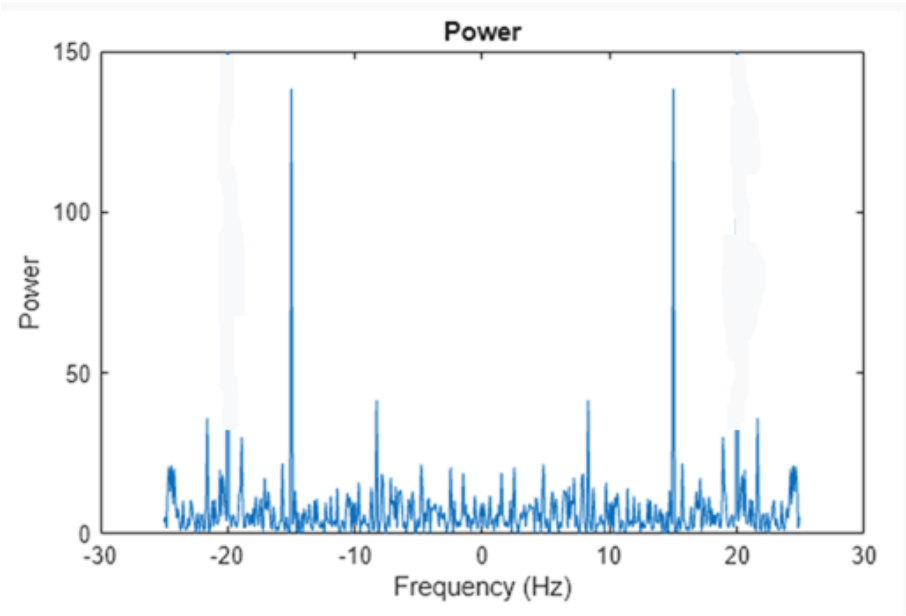
In the time domain plot of the input signal against the time, the input signal could be observed as an audio signal that contains noise.



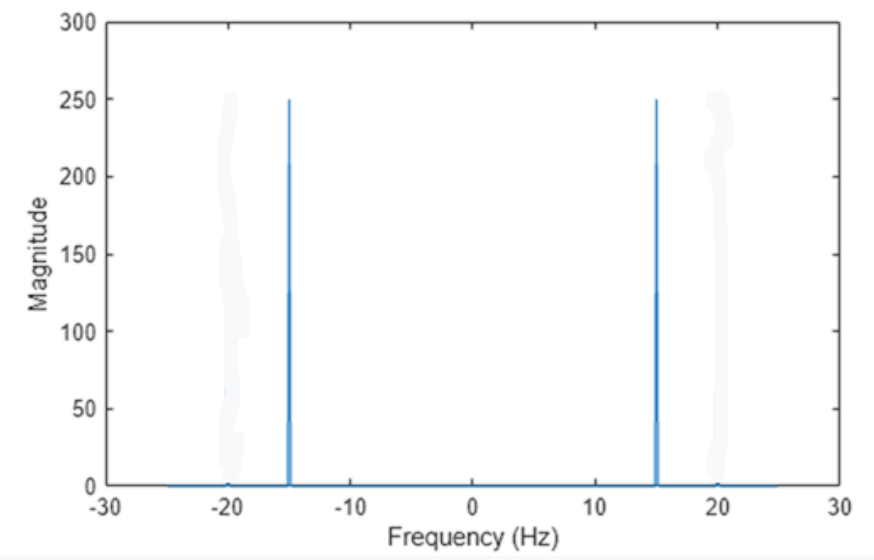
On the other hand, the output signal could be seen as the amplified version of the input signal, and it doesn't have the noise as this has been removed from the system by the noise filter.

Frequency domain analysis:

In the frequency spectrum plot of the input signal over the frequency, the input signal (drawn as plots with noise signals present) could be represented as having a few frequency spectrum components (i.e. fundamental and relevant harmonics). The noise (indicated as spectra with lower magnitudes) is also shown on the plot at the bottom over the frequency.

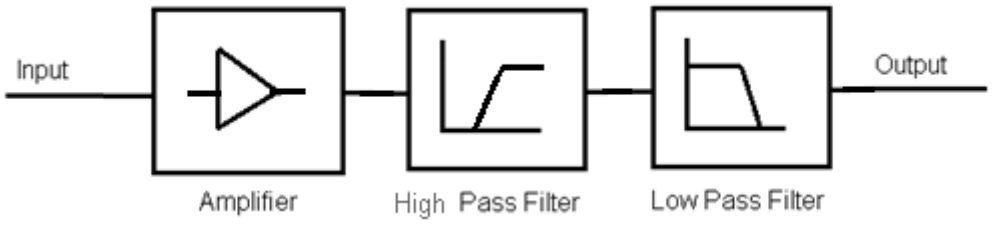


This is quite a contrast on the observed frequency spectrum of the output signal where the noise spectra are not visible as these have been removed by the filter.



It looks like also that the amplitude of the frequency spectrum components of the output signal has been amplified somehow compared with the original input signal.

10. The following block diagram describes a receiver system used in the communication system. The receiver has an operational frequency of 10 Hz (ω_n). The cut-off frequency of the low-pass filter is $0.1\omega_n$ and the cut-off frequency of the high-pass filter is $10\omega_n$.



By using the plots of time and frequency representation of signals in time and frequency domain, describe the working principle of the system. [4 marks]

Solution

The working principles of the receiver system in time and frequency domain are:

Components of the system:

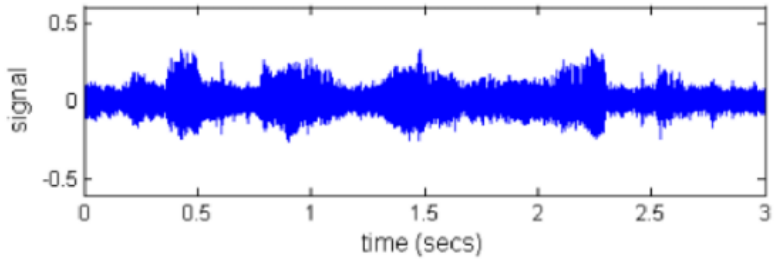
- High-pass Filter – Pass high frequency components of the signal (> cut-off frequency of filter).
- Low-pass Filter – Pass low frequency components of the signal (< cut-off frequency of filter).
- Amplifier - To amplify the signal.

System description:

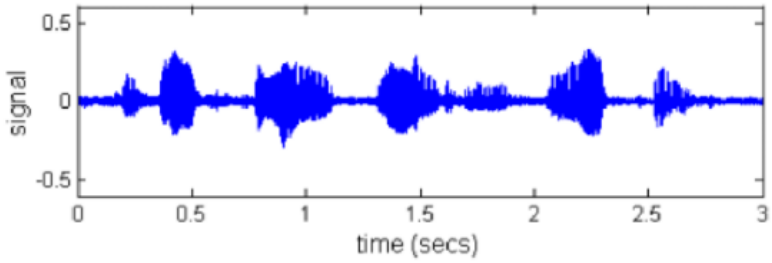
The low-pass filter removes the high-frequency components of the signal and passes any low frequency components below the cut-off frequency of the filter. The high-pass filter performs the opposite, before the signal being amplified in the amplifier.

Time domain analysis:

From the time-domain plot, the input signal is seen with some noise present.



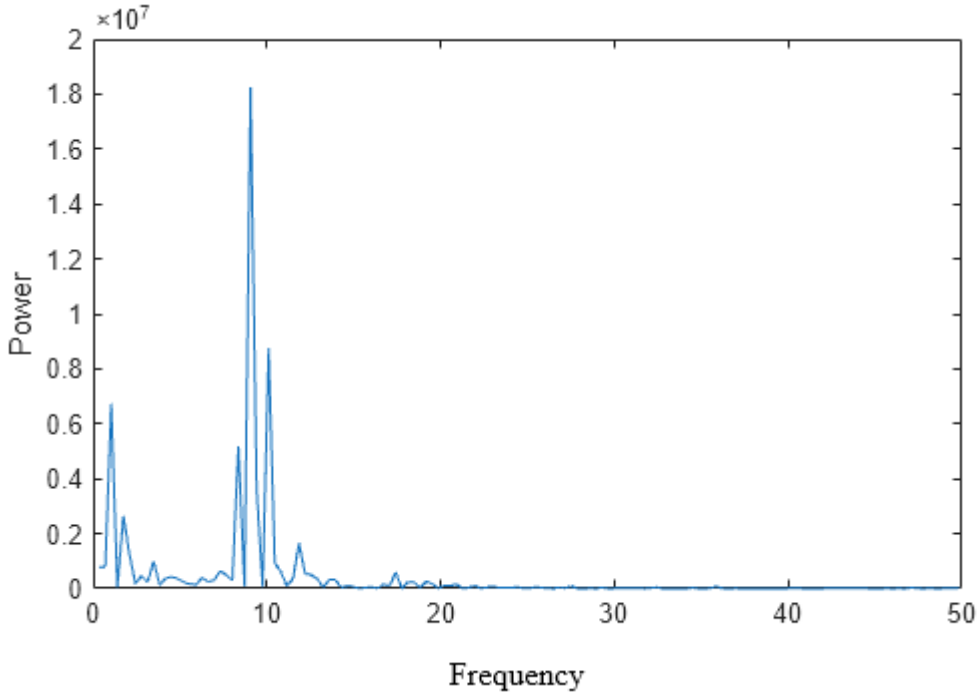
It is very hard to predict the waveforms of the output signal of the system with the system specifications provided.



By default, some frequency spectra of the output signal are also removed from the system by the filters, so the output waveforms can't be determined precisely from the given specifications.

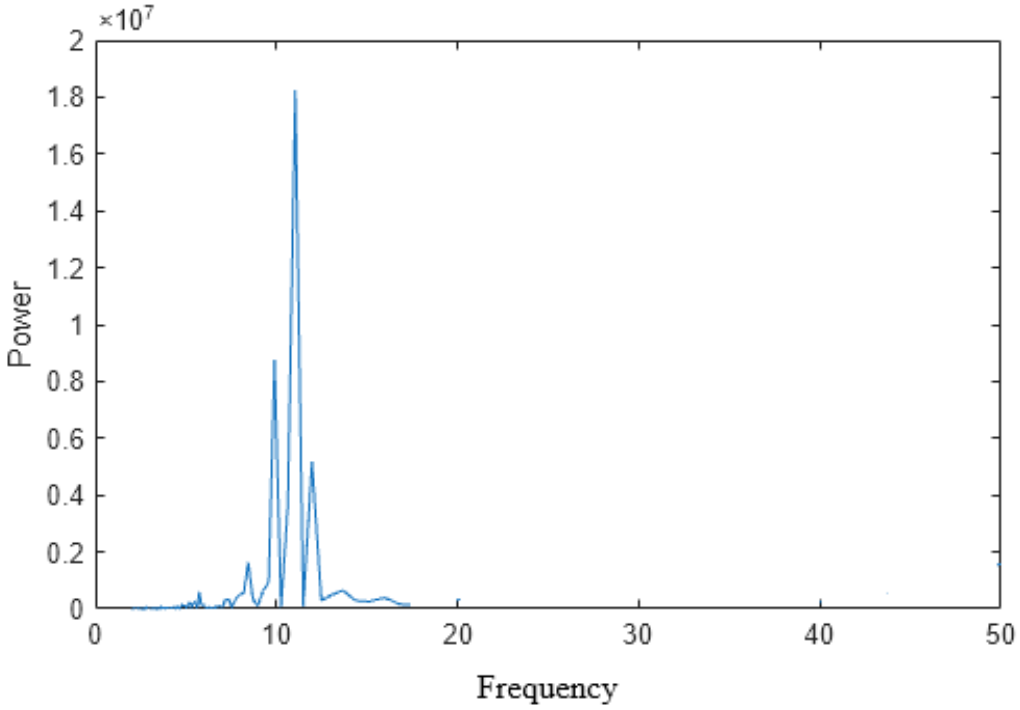
Frequency domain analysis:

In the frequency spectra plot of the input signal, fundamental and relevant harmonics of the signal are shown as discrete impulses with higher amplitudes in the graph and continuous spectra of noise at the bottom are also present in the plot.



In the frequency spectra plot of the output signal, we observe the frequency components of the signal that lie between $0.1\omega_n$ and $10\omega_n$ are not removed from the system and these components also experience slight amplification as well.

Note also that some frequency spectra of the noise (that also exist in the frequency range between $0.1\omega_n$ and $10\omega_n$) pass through the filter and being slightly amplified as well.


×10^7
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
Power
0
10
20
30
40
50
Frequency