XMUT220 Signals and Systems

Tutorial 5b: Time and Frequency Characteristics (Time-Domain Analysis)

A. Introduction to Time-Domain Analysis

1. Performing time-domain analysis for a given first-order system is different from performing the analysis for a second-order system.
   a. Describe the parameters in the time-response analysis which are common for first- and second-order systems. [2 marks]
   b. List the parameters in the time-response analysis which are specific for second-order system. [2 marks]
   c. Why is it not accurate to use the rise time equation for first-order system for calculating the rise time of second-order system? [2 marks]

2. The transient response of a second-order system can be determined from its transfer function equation. Depending on the type of roots in the equation, the response can be categorised as underdamped, critically damped and overdamped.
   a. Prove the roots of the equation for second-order system are: [6 marks]

$$j\omega = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} \quad \text{or} \quad j\omega = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

   b. Prove the time-domain equation of the underdamped response of a second-order system when it is given a step input is as shown below. [12 marks]

$$c(t) = 1 - \frac{e^{-\zeta\omega_t}}{\sqrt{1-\zeta^2}}\sin[\omega_d t + \phi]$$

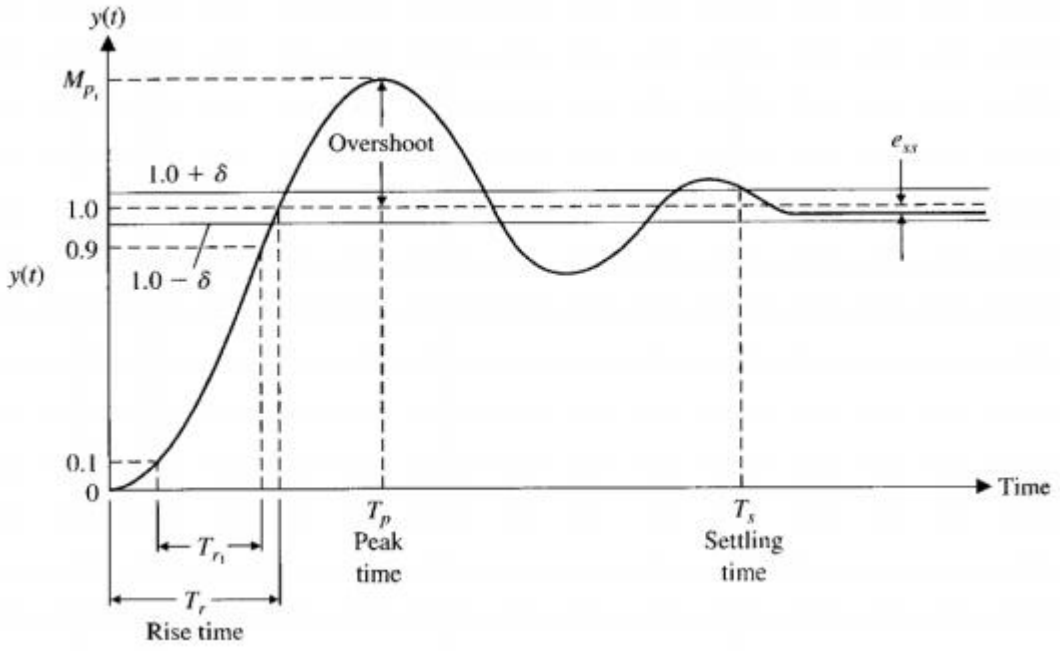
   Where: $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and $\phi = \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right)$

3. The rise time of the transient response of a given second-order system can't be determined very accurately from the rise time equation for a first-order system.

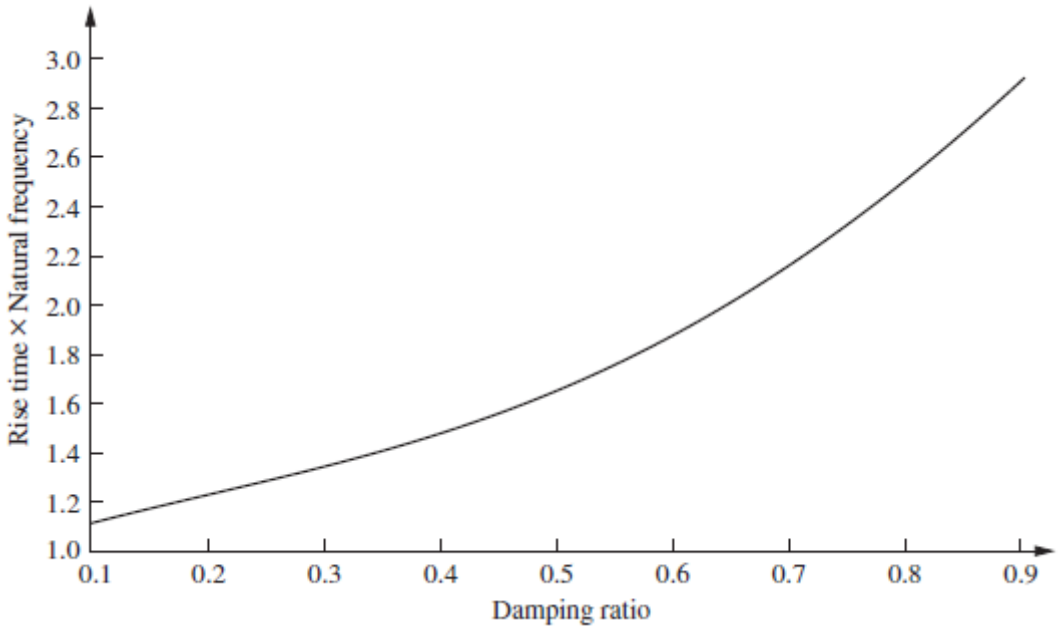
a. Derive the rise time equation for a second-order system as shown below. [8 marks]

$$T_r = \frac{\pi - \phi}{\omega_n \sqrt{1-\zeta^2}}$$

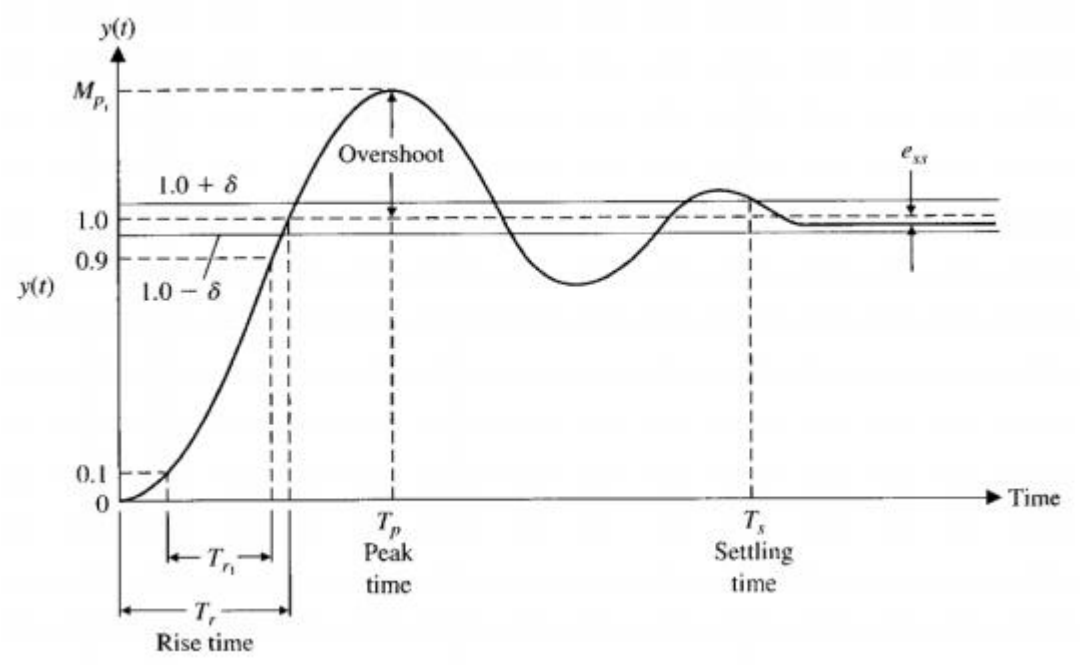
Where: $\phi = \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right)$



b. Derive the graph of normalised rise time versus damping ratio for a second-order underdamped response. [8 marks]



4. The transient-response behaviour of second order systems is typically determined from the time-domain analysis parameters which are specific for second-order system.



a. Prove that the time-to-peak (T_p) of the second-order system is determined from the following equation. [12 marks]

$$T_p = \frac{n\pi}{\omega_n\sqrt{1-\zeta^2}} \qquad \text{where } n = \text{nth peak}$$

b. Derive the percentage overshoot ($\%OS$) of the second-order system which is as shown in the equation below. [14 marks]

$$\%OS = e^{-\left(\zeta\pi/\sqrt{1-\zeta^2}\right)} \times 100\%$$

B. Time-Domain Analysis of Systems

5. You are given the following first-order systems:

   i. System 1:

$$G_1(\omega) = \frac{5}{j\omega + 5}$$

   ii. System 2:

$$G_2(\omega) = \frac{20}{j\omega + 20}$$

a. For both systems, calculate the time constant, rise time, and settling time for each system. [6 marks]

b. Based on the results in part (d), comment on the differences in transient and steady-state responses of the first system with the second system. [4 marks]

c. Simulate and describe the transient responses of the systems when each of the systems is subjected to a step input. [6 marks]

d. Simulate and describe the transient responses of the systems when each of the systems is subjected to an impulse input. [6 marks]

6. You are given the following second-order systems:

i. System 1:

$$G_1(\omega) = \frac{16}{(j\omega)^2 + 3j\omega + 16}$$

ii. System 2:

$$G_2(\omega) = \frac{0.04}{(j\omega)^2 + 0.02j\omega + 0.04}$$

iii. System 3:

$$G_2(\omega) = \frac{1.05 \times 10^7}{(j\omega)^2 + 1.6 \times 10^3 j\omega + 1.05 \times 10^7}$$

a. Determine the transient responses of the systems based on the roots of equations. [12 marks]

b. For each of the second-order systems given above, simulate and find the time-response characteristics of the systems in terms of: [36 marks]

- Damping Ratio (ζ),
- Natural Frequency (ω_n),
- Settling Time (T_s),
- Peak Time (T_p),
- Rise Time (T_r), and
- Percentage Overshoot ($\%OS$).

c. Comment on the transient-response results of the three systems given in (a) and (b). [6 marks]

7. The steady-state error of a particular system depends on the type of input given to that system.

a. Derive the steady-state response of the given system for an underdamped second-order system with step input is as shown below. [10 marks]

$$C(\infty) = \lim_{\omega \to 0} j\omega R(\omega) G(\omega)$$

b. Determine and simulate in MATLAB the steady-state error of the systems given below when given a step input: [20 marks]

i. System 1:

$$G_1(\omega) = \frac{1}{j\omega + 0.3}$$

ii. System 2:

$$G_2(\omega) = \frac{1}{[0.2(j\omega)^2 + 5j\omega + 5]}$$

EXTRA

Prove that the time-domain equations of the transient responses of the given second-order system when it is given a step input are as shown below.

a. For the critically damped response second-order system, time-domain equation is. [12 marks]

$$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \quad \text{for} \quad t \geq 0$$

b. For the overdamped response second-order system, time-domain equation is. [14 marks]

$$c(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{e^{-j\omega_1 t}}{j\omega_1} - \frac{e^{-j\omega_2 t}}{j\omega_2}\right) \quad \text{for} \quad t \geq 0$$

Where: $\omega_1 = \left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n$ and $\omega_2 = \left(\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n$

Appendix 1 - Selective Fourier Transforms Table

$f(t)$	$\hat{F}(\omega)$	**Notes**
$f(t)$	$\int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$	Definition of Fourier transform
$\delta(t)$	1	Impulse function
$\delta(t-t_0)$	$e^{-j\omega t_0}$	Impulse function with delay
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$	Step function
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	Exponential function
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	a constant, $\Re e(a)>0$
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	a constant, $\Re e(a)>0$
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$	$\frac{1}{(a+j\omega)^n}$	a constant, $\Re e(a)>0$
$\cos(\omega_0 t)$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	Cosine function
$\sin(\omega_0 t)$	$\frac{\pi}{j}[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	Sine function
$f'(t)=\frac{df(t)}{dt}$	$j\omega\hat{f}(\omega)$	First derivative in time
$f''(t)=\frac{d^2f(t)}{dt^2}$	$(j\omega)^2\hat{f}(\omega)$	Second derivative in time
$tf(t)$	$j\frac{d}{d\omega}\hat{f}(\omega)$	First derivative in frequency
$t^2f(t)$	$j^2\frac{d^2}{d\omega^2}\hat{f}(\omega)$	Second derivative in frequency