XMUT220 Signals and Systems

Tutorial 6a: Sampling and Reconstruction (Introduction)

A. Introduction to Sampling and Reconstruction

1. Find the Nyquist rate for each of the following signals:
   a. $x(t) = 25\cos(500\pi t)$ [2 marks]
   b. $x(t) = 3\cos(50\pi t) + 10\sin(300\pi t) - \cos(100\pi t)$ [2 marks]
   c. $x(t) = \text{sinc}(200\pi t)$ [2 marks]
   d. $x(t) = \text{sinc}^2(200\pi t)$ [2 marks]
   e. $x(t) = \text{sinc}(200\pi t) + \text{sinc}^2(200\pi t)$ [2 marks]
   f. $x(t) = \sin(1000\pi t)/\pi t * y(t)$ [2 marks]
   with: $X(j\omega) = \begin{cases} 1, & |\omega| < 2000\pi \\ 0, & \text{otherwise} \end{cases}$
   g. $x(t) = 2\cos(5000\pi t)\cos(500k\pi t)$ [2 marks]

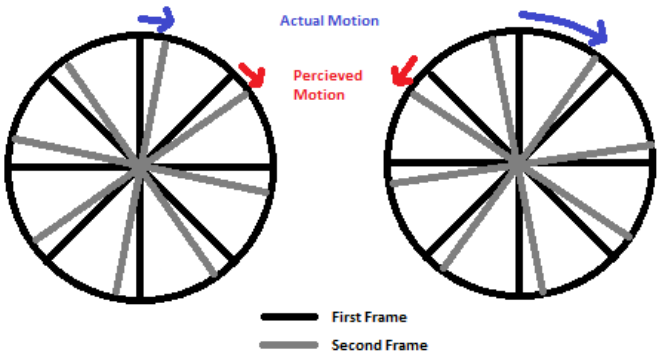
2. A continuous-time signal ($x(t)$) can be represented in its samples ($x_p(t)$) and can be reconstructed back to its original signal when sampling frequency f_s is greater than or equal to the twice the highest frequency component of the signal.
   a. Describe the effect of sampling in the frequency domain. Illustrate the effect in terms of spectrum plots of original signal and sampling function and show also the difference between spectrum plots of sampled signal with $\omega_s > 2\omega_M$ and spectrum of sampled signal with $\omega_s < 2\omega_M$. [8 marks]
   b. Describe the effect of reconstructions of a continuous-time signal from its samples using an ideal low-pass filter: Illustrate your answer with a block diagram of system for sampling and reconstruction, representative spectrum for $x(t)$, corresponding spectrum for $x_p(t)$, ideal low-pass filter to recover $X(j\omega)$ from $X_p(j\omega)$, and spectrum of reconstructed signal, $X_r(t)$.
   [8 marks]

3. Aliasing refers to the incorrect measurement of a signal's frequency due to an inadequate digital sampling rate.

   a. Describe what happens to a signal which is not adequately sampled using enough data points? [2 marks]

   b. What is required to avoid aliasing? [2 marks]

   c. With regard to aliasing, what is the "wagon wheel effect"? [2 marks]



4. Where f_H is highest frequency of the sampled signal and B is the bandwidth of the sampled signal, the minimum sampling rate is given as:

$$f_{s\,(min)} > \frac{2f_H}{\lfloor f_H/B \rfloor}$$

   Let a signal have no nonzero spectral components outside the following range of frequencies, what is the minimum sampling rate (f_s) that avoids aliasing?

   a. $0 < |f| < 600$ kHz. [2 marks]

   b. $15\text{ kHz} < |f| < 45$ kHz. [2 marks]

B. Applications of Sampling and Reconstruction

5. Suppose a signal that is to be acquired by a data acquisition system is known to have an amplitude spectrum that is flat out to 100 kHz and drops suddenly there to zero. The fastest rate at which our data acquisition system can sample the signal is 60 kHz.

   a. Design an anti-aliasing filter that will reduce the signal's amplitude spectrum at 30 kHz to less than 1% of its value at very low frequencies so that aliasing will be minimised. [4 marks]

   b. Determine the CR time constant of the filter. [10 marks]

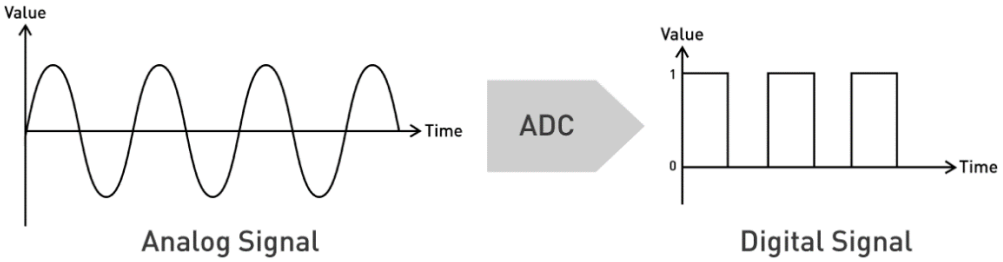
6. Describe the following signal reconstruction methods

a. Zero-order hold. [2 marks]

b. First-order hold. [2 marks]

c. Sinc function. [2 marks]

7. Continuous-time signal sampling to its discrete-time samples can be performed by an analogue-to-digital conversion system.



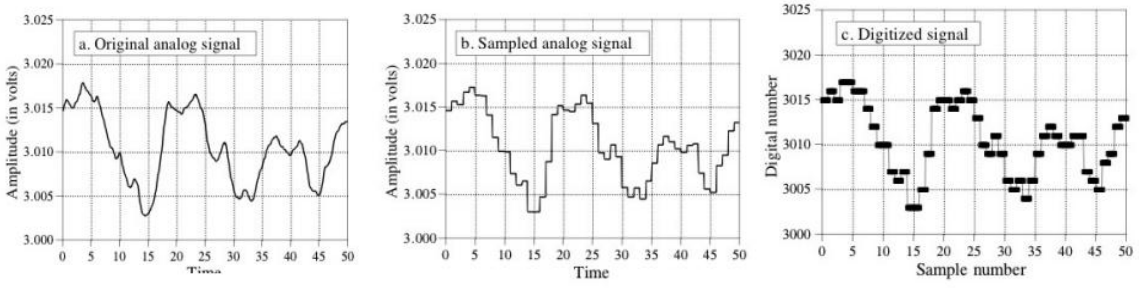
a. Describe sampling with a periodic impulse train followed by conversion to a discrete-time sequence for two different sampling rates (e.g. $T_1 = T$ and $T_2 = 2T$).

Include the following in your description: (a) overall system; (b) discrete-time sampled signals, $x_p(t)$ and its original continuous-time signal, $x_c(t)$; and (c) the output sequence, $x_d(n)$. [6 marks]

b. Describe the relationship between $X_c(j\omega)$, $X_p(j\omega)$, and $X_d(e^{j\Omega})$ for the given different sampling rates. [6 marks]

8. You are given the following analogue-to-digital (ADC) system. The given system employs sample-and-hold (S/H) process and quantisation process inside the ADC system.

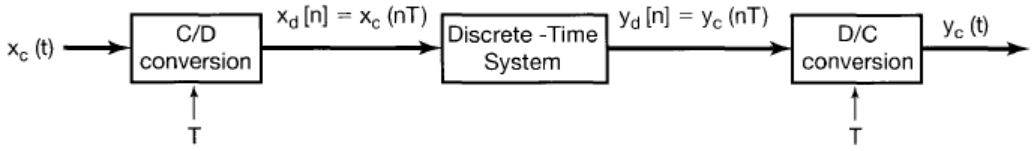




a. Referring to the diagrams, describe the two stages of the conversion process: sample-and-hold and quantization. Describe the effect of quantisation. [2 marks]

b. What happen to the sampled signal after these sample and hold and quantisation process? [2 marks]

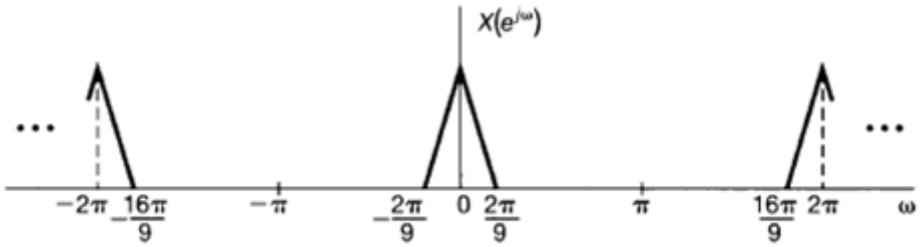
9. Digital signal processing (DSP) is fundamental to digital technology, especially for digital telecommunication and wireless communications. DSP is applicable to both streaming data and static (stored) data.



a. Describe the main reason why digital signal processing is preferred compared with analogue signal processing. [2 marks]

b. Describe the components that make up a typical DSP system. [6 marks]

10. Consider the sequence $x[n]$ whose Fourier transform $X(e^{j\omega})$ is illustrated in the figure below. The lowest rate at which impulse-train sampling may be used on this sequence without incurring aliasing is $2\pi/4$. This corresponds to sampling every 4th value of $x[n]$.



a. Decimate the sampling by a factor of 4. Describe the issue that you see from the reconstruction result. [4 marks]

b. Referring to the results in part (a), can this reconstruction process be improved further? Hint: application of up sampling and down sampling. [6 marks]