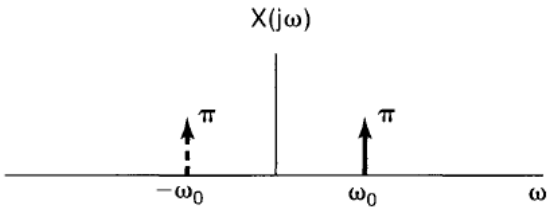
XMUT220 Signals and Systems

Tutorial 6c: Sampling and Reconstruction (Aliasing)

A. Aliasing in Signal Sampling

1. Aliasing influences the reconstruction of the original continuous-time signals from their discrete-time samples.



Given the frequency spectrum plot of a signal as shown above, for the following case studies of a sampling frequency at ω_s:

i. $\omega_o = \omega_s/6$

ii. $\omega_o = 2\omega_s/6$

iii. $\omega_o = 4\omega_s/6$

iv. $\omega_o = 5\omega_s/6$

a. Sketch and describe the effect of aliasing in the frequency domain. [8 marks]

b. If the signal is a sinusoidal signal, sketch and describe the effect of aliasing in time domain. [16 marks]

2. You are given a task to do signal sampling of a sinusoidal signal of frequency f_0 as follows with a sampling frequency $F = 8$ Hz.

$$x(t) = \sin(2\pi f_0 t)$$

Where: $t_0 = 1/F = 1/8$ is the time interval between two consecutive samples.

a. Considering the sampling effects in time domain as well as in frequency domain, determine and sketch original signals, reconstructed signals and spectrum of reconstructed signals for three different signal frequencies f_0= 3, 5 and 9 Hz. [20 marks]

b. Comment further on the marginal case where the signal frequency $f_0 = F/2$ is precisely the Nyquist frequency. [4 marks]

3. Given a signal composed of two frequency components $f_1 = 7$ Hz and $f_2 = 2$ Hz:

$$x(t) = 2\cos(2\pi 7t) + \cos(2\pi 2t)$$

a. Assume the sampling frequency is $F = 1/t_0 = 10$ samples/second, and the sampling period is $t = 0.1$ seconds. Determine and sketch the discrete signal after sampling. [10 marks]

b. If the first frequency component is increased to $f_1 = 8$ Hz, while the sampling frequency is still $F = 1/t_0 = 10$. Determine and sketch the discrete signal after sampling. [10 marks]

c. Consider this signal in (b) at the same sampling rate at $f = 10$. Determine the discrete signal after sampling. [10 marks]

4. Providing a continuous signal as shown below:

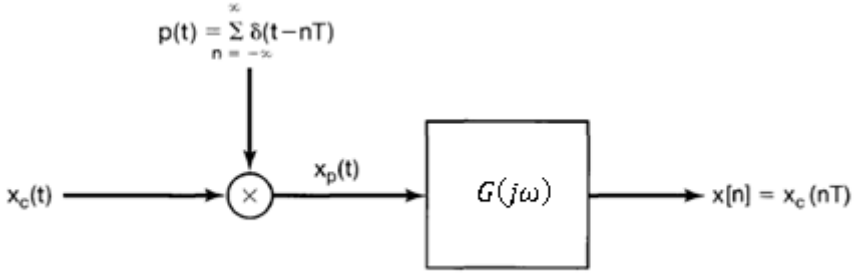
$$x(t) = 26\cos(2\pi(2)t + \pi/2) + 14\cos(2\pi(5)t + \pi/2)$$

a. If the sampling rate is $F = 12$, determine and sketch the original signal, reconstructed signal and spectrum of reconstructed signals. [8 marks]

b. If the sampling rate is $F = 7$, determine and sketch the original signal, reconstructed signal and spectrum of reconstructed signals. [8 marks]

B. Aliasing in Applications of Sampling and Reconstruction

5. Consider a band-limited signal $x_c(t)$ that is sampled at a rate higher than the Nyquist rate. The samples, spaced T seconds apart, are then converted to a sequence $x[n]$ as indicated in the figure given below. Given in the figure $G(j\omega)$ is a function in the system that performs conversion of the impulse trains to the sequence of discrete signals.



Determine the relation between the energy E_d of the sequence, the energy E_c of the original signal, and the sampling interval T. How aliasing of the signal during the sampling process is prevented from happening? [14 marks]

The energy of a sequence $x[n]$ is defined as:

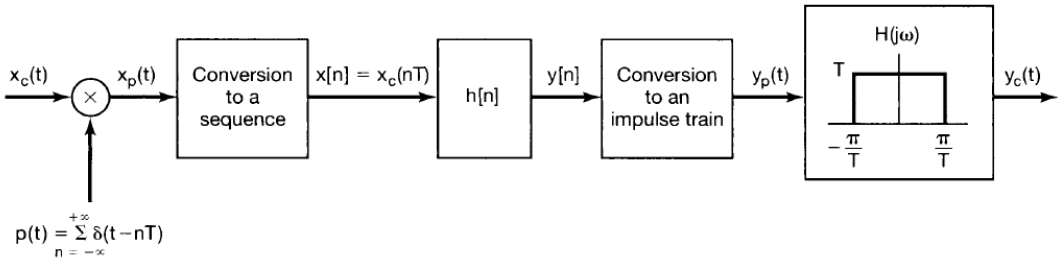
$$E_d = \sum_{n=-\infty}^{+\infty} |x[n]|^2$$

The energy in a continuous-time function $X_c(t)$ is defined as:

$$E_c = \int_{-\infty}^{+\infty} |x_c(t)|^2 dt$$

6. In the system shown in the figure below, the input $x_c(t)$ is band limited with $X_c(j\omega) = 0, |\omega| > 2\pi \times 10^4$. The digital filter $h[n]$ is described by the input-output relation:

$$y[n] = T \sum_{k=-\infty}^{n} x[k]$$



a. What is the maximum value of T allowed if aliasing is to be avoided in the transformation from $x_c(t)$ to $x_p(t)$. [2 marks]

b. With the discrete-time LTI system specified through equation given above, determine its impulse response $h[n]$. [4 marks]

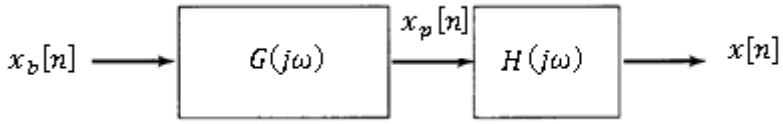
c. Determine whether there is any value of T for which:

$$\lim_{n\to\infty} y[n] = \lim_{t\to\infty} \int_{-\infty}^{t} x_c(\tau) d\tau$$

If so, determine the maximum value. If not, explain and specify how T would be chosen so that the equality in equation given above is best approximated. [16 marks]

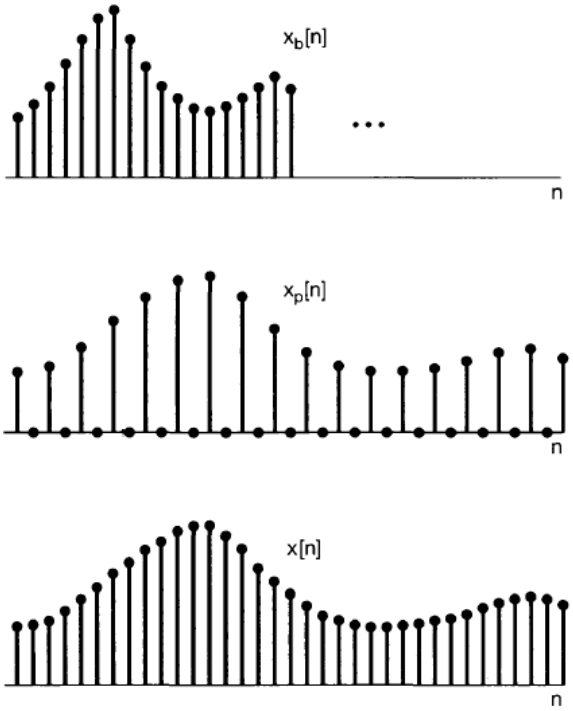
C. Aliasing in Down Sampling and Up Sampling

7. Suppose the impulse response of an ideal discrete-time low-pass filter with cut-off frequency $\pi/2$ is interpolated to obtain an up sampling by a factor of 2.



In accordance with the figure given above, $G(j\omega)$ is a function that performs conversion of decimated sequence to sampled sequence and $H(j\omega)$ is an ideal low-pass filter ($H(e^{j\omega})$).

Note that the illustrated sampling processes given below are associated with sequences and spectra for up sampling by a factor of 2. [8 marks]



What is the frequency response corresponding to this up sampled impulse response? For up sampling and down sampling processes, how do you preserve the sampling of the signal without aliasing?

8. A real-valued discrete-time signal $x[n]$ has a Fourier transform $X(e^{j\omega})$ that is zero for $3\pi/14 \leq |\omega| \leq \pi$.

   By considering aliasing free sampling process, the nonzero portion of the Fourier transform of one period of $X(e^{j\omega})$ can be made to occupy the region $|\omega| < \pi$ by first performing up sampling by a factor of L and then performing down sampling by a factor of M.

   Specify the values of L and M. How do you know that the sampling process is aliasing free? [8 marks]