XMUT303 Analogue Electronics

Note 12: Inductors and Transformers

Topics

- Ideal transformer
- Equivalent circuit of transformer.
- Impedance matching using transformer.
- Design of a simple line power supply transformer.
- Design of inductor for power converter application.

12.1. Introduction to Transformer

Before embarking on the analysis and design of transformer and inductor, it is best to start with fundamental concepts and parameters related to these devices.

12.1.1. Ideal Transformer

We have so far considered the impedance of resistors, capacitors and inductors. One other component we are likely to meet in practice is the transformer. Ideal transformers are simple enough to model.

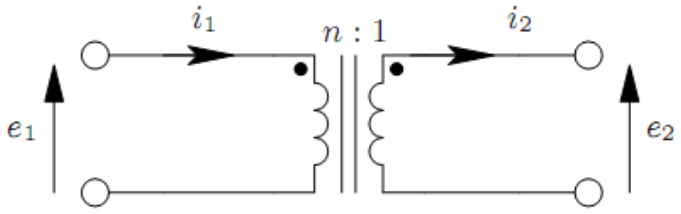


Figure 12.1: Ideal transformer

We have two windings on a common core of high permeability material (μ). An electromotive force (EMF) is induced across each winding. In accordance with Faraday's law of induction, the EMFs are proportional to the rate of change of flux ($d\Phi/dt$).

$$e_1 = N_1\left(\frac{d\Phi_1}{dt}\right)$$

And

$$e_2 = N_2\left(\frac{d\Phi_2}{dt}\right)$$

Where:

- e_1 and e_2 are the induced EMFs across the primary and secondary windings.
- N_1 and N_2 are the numbers of turns in the primary and secondary windings.
- Φ_1 and Φ_2 are the flux linking the primary and secondary windings respectively.

In the ideal transformer, all flux produced by the primary winding also links the secondary, and so $\Phi_1 = \Phi_2$. The ratio of primary to secondary voltage is therefore the same as the ratio of the number of turns.

Note that the flux density (B) depends amount of magnetic flux (Φ) on the permittivity of the medium (μ) and field density (H).

$$B = \frac{\Phi}{A} \qquad \text{and} \qquad B = \mu H$$

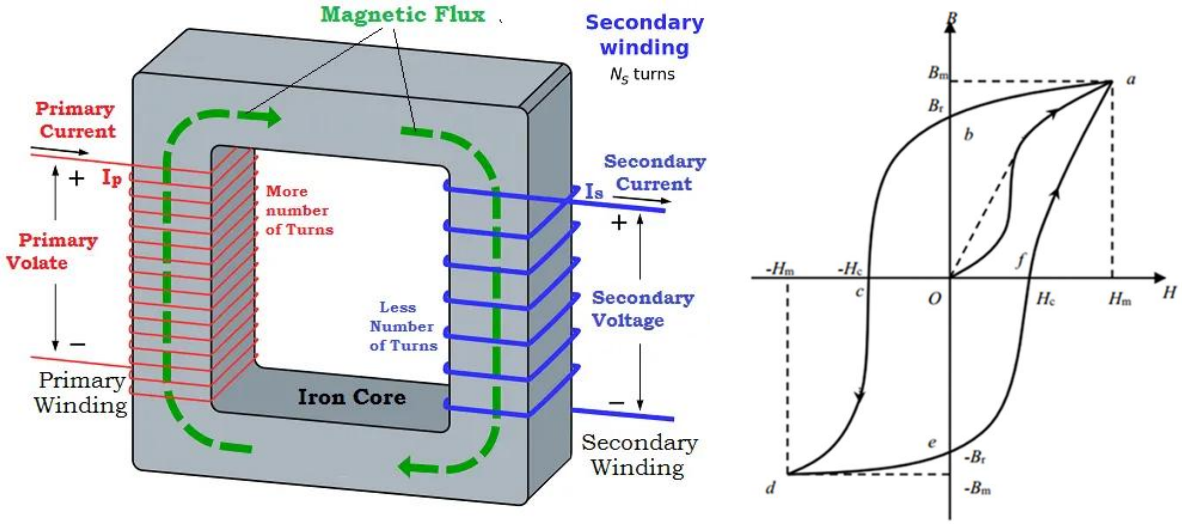


Figure 12.2: Magnetic flux, number of turns, voltages and currents in transformer and flux density (B) vs. magnetic flux (H)

Alternatively, the volts-per-turn is the same in both windings.

$$\frac{e_1}{e_2} = \frac{N_1}{N_2}$$

Under load, current will flow in the secondary, and produce a magnetic flux in opposition to that produced by the primary winding. Equilibrium is reached when:

$$i_1 N_1 = i_2 N_2$$

Or

$$\frac{i_2}{i_1} = \frac{N_1}{N_2}$$

12.1.2. Ideal Transformer in Circuit

Consider the circuit shown in figure below.

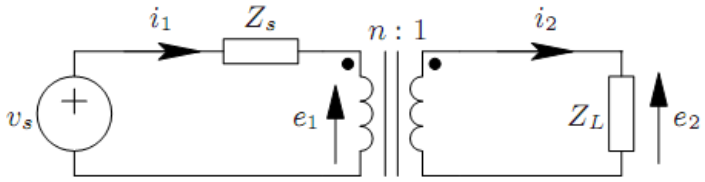


Figure 12.3: Transformer circuit

We know that:

$$e_1 = ne_2 \text{ and } i_2 = ni_1.$$

Also, $e_2 = Z_L i_2$, hence:

$$e_1 = ne_2 = Z_L i_2 = n^2 Z_L i_1$$

For the circuit diagram shown in the figure below, the dots indicate the polarity of the windings.

12.1.3. Equivalent Circuit (Source Side)

For the given circuit, the current that flows in the circuit and voltage of the supply given by:

$$i_1 = \frac{v_s - e_1}{Z_s} = \frac{v_s - n^2 Z_L i_1}{Z_s}$$

And

$$v_s = i_1(Z_s + n^2 Z_L)$$

Hence the load impedance can be considered to have been transformed by n^2, as represented in the figure below.

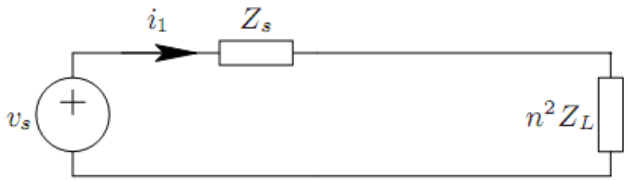


Figure 12.4: Equivalent circuit (source side)

12.1.4. Equivalent Circuit (Load Side)

We could equally consider the circuit from the point of view of the load impedance Z_L to obtain:

$$e_2 = \frac{e_1}{n} = \frac{v_s - Z_s i_1}{n} = \frac{v_s - Z_s\left(\frac{i_2}{n}\right)}{n}$$

$$= \frac{v_s}{n} - \frac{Z_s}{Z_L}\left(\frac{e_2}{n^2}\right) = \frac{\left(\frac{v_s}{n}\right) Z_L}{Z_L + \frac{Z_s}{n^2}}$$

The source can be considered to have been scaled by $1/n$ and the source impedance by $1/n^2$ as represented in figure below.

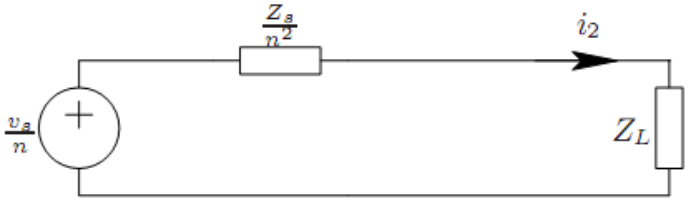


Figure 12.5: Equivalent circuit (load side)

12.1.5. Application of Transformer

There are many applications of transformer. In this section, two examples of applications are discussed e.g. voltage conversions and impedance matching.

12.1.5.1. Voltage Conversions

Voltage conversions: from high to low voltage (step down) or low to high voltage (step up).

$$e_2 = \left(\frac{N_2}{N_1}\right) e_1 \qquad \text{and} \qquad i_2 = \left(\frac{N_1}{N_2}\right) i_1$$

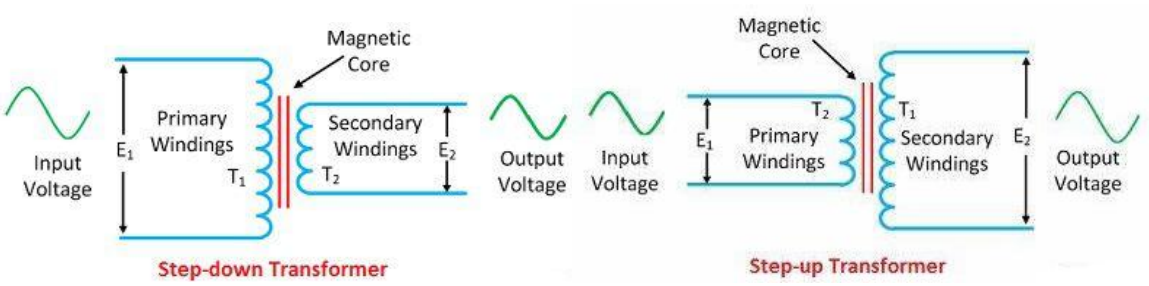


Figure 12.6: Step-down (left) and step up (right) transformers

12.1.5.2. Impedance Matching

Other application of a transformer is for impedance matching.

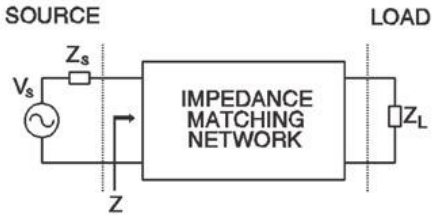


Figure 12.7: Impedance matching network

By carefully choosing and matching the impedance of a load with its source, we can ensure maximum power transfer from the source to the load.

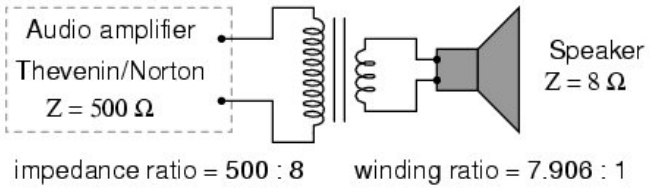


Figure 12.8: Impedance "matching" transformer

To achieve maximum power transfer, we would like the equivalent load $n^2 R_L$ in figure above to be equal to the source resistance R_s.

$$n^2 R_L = R_s$$

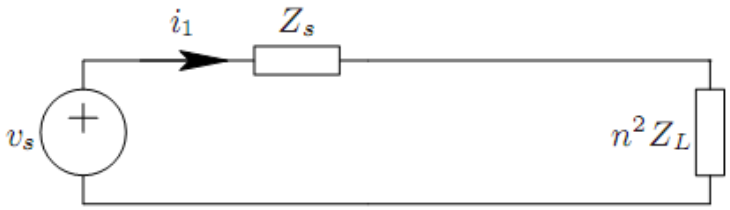


Figure 12.9: Impedance matching circuit using transformer

Hence, we choose the turn ratio, n so that:

$$n = \sqrt{R_s/R_L}$$

The same result is obtained if we consider the equivalent circuit from the load side.

12.2. Transformer Design

The following transformer design example is applied for single-phase transformers. The basic principles, however, are the same for multiphase designs.



Figure 12.10: Single phase transformer (left) and three phase distribution transformer (right)

The first few steps in the design procedure for the transformer are determining and assembling the design inputs. These normally consists of determining:

- Transformer rating: rated primary voltage, current and operating frequency.
- Magnetic design e.g. flux density and inductance of the transformer.
- Transformation ratio e.g. voltage ratio and turns ratio.
- Power determination and thermal constraints e.g. maximum body and ambient temperature.

12.2.1. Transformer Rating

For a given transformer design, the ratings of the transformer are:

- It is for line frequency application (220 V AC, 50 Hz).
- Small power application and losses.
- Use E-shape core laminated magnetic material.
- The windings of primary and secondary coils are with insulated copper wire.

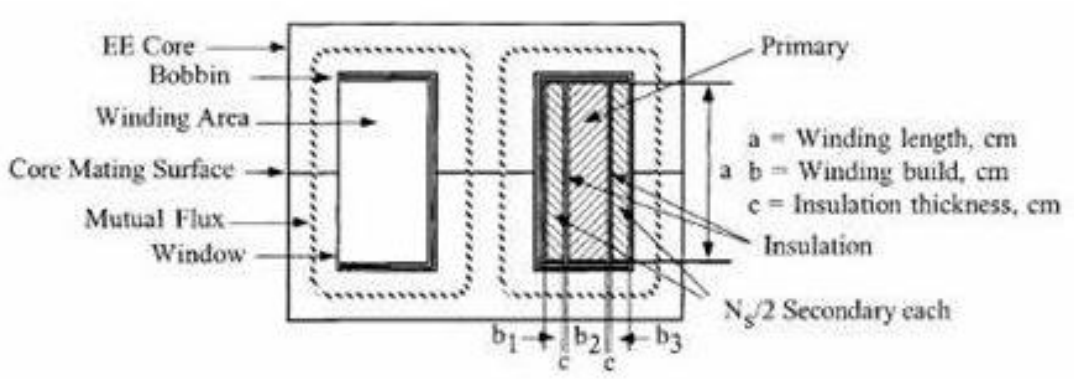
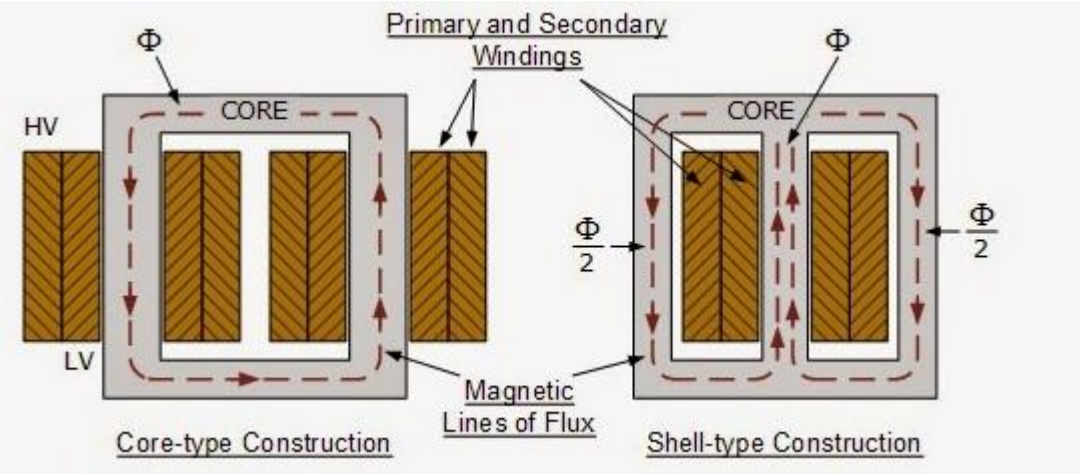


Figure 12.11: Determination of windows of a shell type (E-core) transformer

The following diagram shows other commonly found examples of transformer laminations.



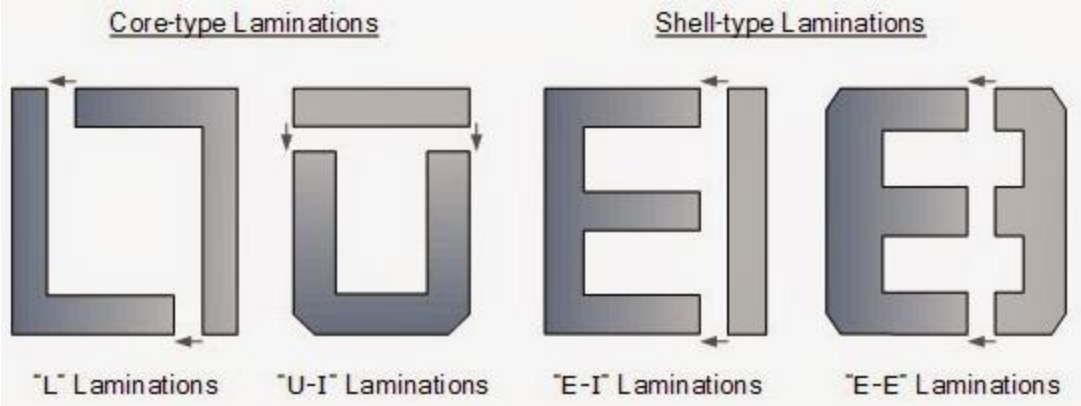


Figure 12.12: Magnetic core, bobbin, and windings of transformer

12.2.2. Magnetic Design

One of the most important parameters imposing limitations on the volts per turn for line frequency application is the maximum flux density in the core of the transformer. For general-purpose applications, typical values of flux density, B are 1 to 1.7 Tesla.

As the frequency of operation, f increases (i.e. above a few hundred Hertz), the core losses will be the limiting factor. The change in flux is the major source for core losses, and thus the flux must be limited in accordance with the manufacturer's specification for the given core and frequency.

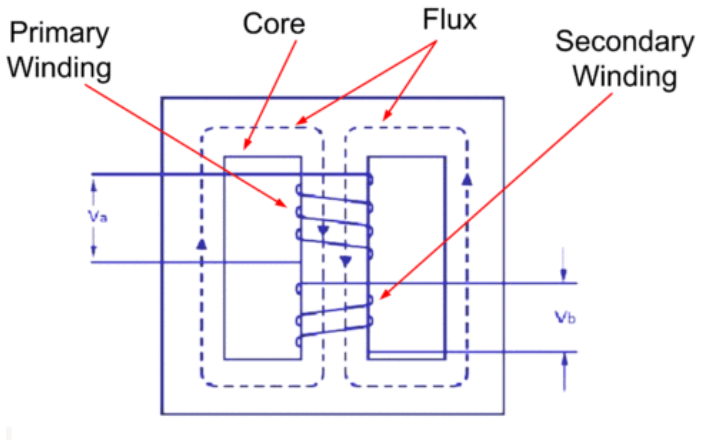


Figure 12.13: Primary and secondary windings in a transformer

For general-purpose applications, typical values of magnetic flux, Φ are 0.05 to 0.2 Weber. The magnetic flux, Φ measured in Weber, in a loop of wire is given by:

$$\Phi = B.A$$

And also

$$\Phi = V.t$$

Where:

- Φ is the magnetic flux.
- B is the magnetic flux density.
- A is the area surrounded by the coil.
- V is the voltage applied to the loop.
- t is the duration (time).

12.2.3. Transformation Ratio

By adding more turns to the coil, the applied voltage (V) will be divided evenly among the turns, hence:

$$\Phi = \frac{V.t}{N}$$

Where:

- V is the voltage applied to the loop.
- t is the duration (time).
- N is the number of turns.

Another way to look at it is that the increased number of turns increases the inductance, hence, the current decreases (and flux depends on current).

$$\Phi = L.I$$

Where:

- L is inductance.
- I is the amount of current that flows in the windings.

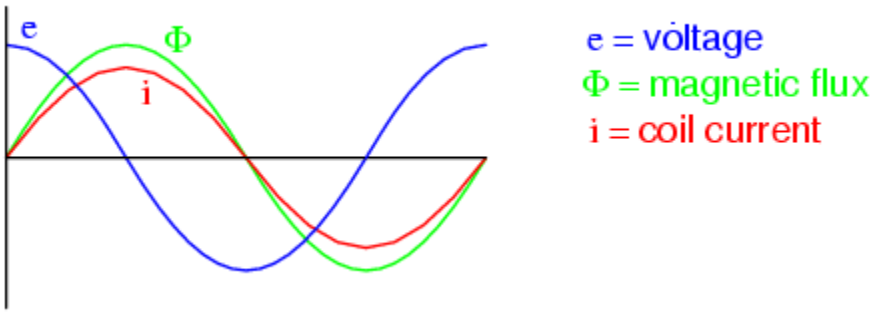


Figure 12.14: Magnetic flux vs. current and voltage of a transformer

The voltage induced in a coil of wire (V) is given by the differential equation:

$$V = -L\left(\frac{di}{dt}\right)$$

Where:

- L is inductance.
- di/dt is rate of change of the current that flows in the windings.

Hence, the inductance of a coil (L) is given by:

$$L = \frac{V.t}{I}$$

Where:

- V is the voltage applied to the loop.
- t is the duration (time).
- I is the current through the wire.

12.2.4. Leakage Inductance

The leakage inductance is caused by the parts of the magnetic flux that does not link the primary and secondary windings. In most transformers, the leakage inductance should be minimized.

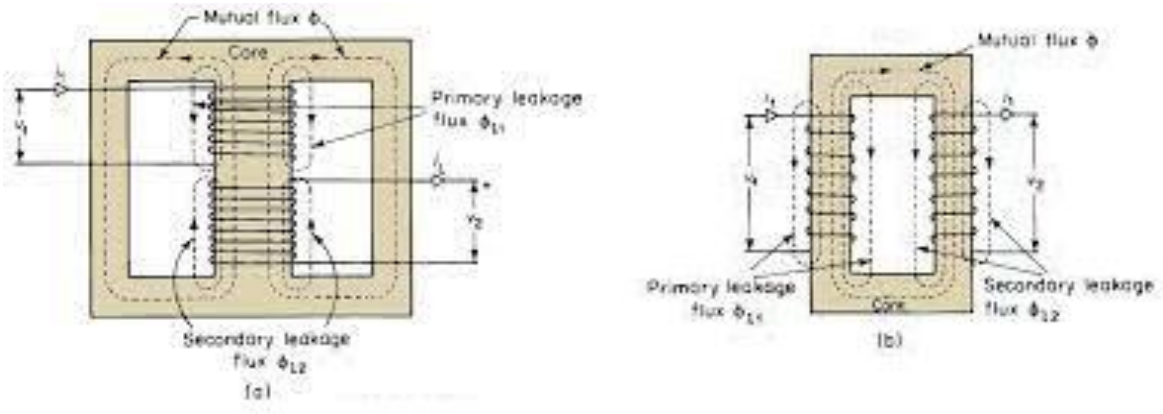


Figure 12.15: Primary and secondary windings in the transformer

An unfavorable leakage inductance may cause “over voltages” in a switching power converter, adding additional requirement to the snubber circuits.

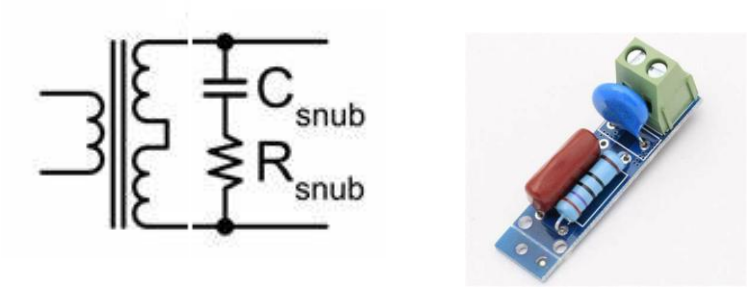


Figure 12.16: RC snubber circuit and components

A notable exception to this rule is microwave oven transformers, where magnetic shunts are added in order to increase the leakage inductance.

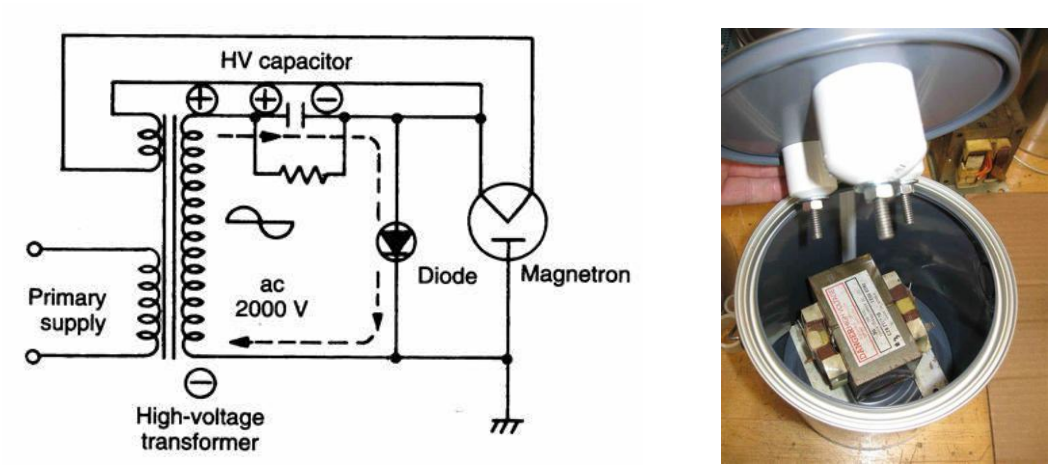


Figure 12.17: Microwave oven transformer

The reason for these shunts is that the transformer is intended to drive a capacitive load. The secondary of a microwave oven transformer is typically connected to a voltage doubler and the capacitive reactance of this doubler should ideally match the inductive reactance of the transformer.

Exact calculation of the leakage inductance is challenging, but approximate analytical formulas are available for various cores and winding configurations. For a shell type transformer that has rectangular shaped (E) core and no interleaving of the windings, the leakage inductance (L_L) may be approximated by:

$$L_L \approx \frac{\mu_0 N_p^2 l_w b_w}{3h_w}$$

Where:

- μ_0 is the permitivity of the magnetic material.
- N_p is the number of primary turns.

- l_w is the mean length of a winding turn.
- b_w and h_w are the width and height of the winding window respectively.

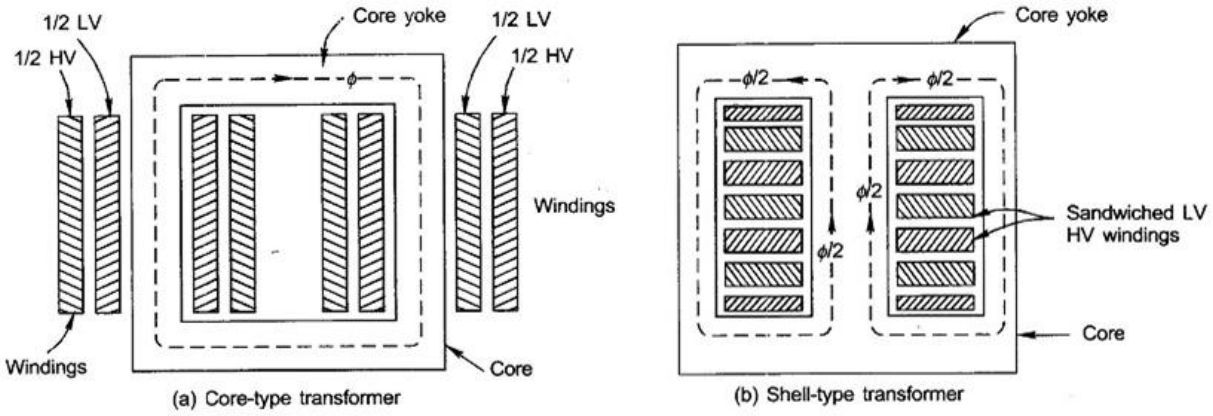


Figure 12.18: Core and shell type for small or low power transformers

For split (interleaved) winding arrangements, the leakage inductance (L_L) may be approximated by:

$$L_L \approx \frac{\mu_0 N_p^2 l_w}{p^2 h_w}\left(\frac{b_{Cu}}{3} + b_i\right)$$

Where:

- μ_0 is the permitivity of the magnetic material.
- N_p is the number of primary turns.
- l_w is the mean length of a winding turn.
- h_w is the height of the winding window respectively.
- p is the number of interlacing sections.
- b_{cu} is the width of the copper in the window of the windings.
- b_i is the inter-winding insulation thickness.

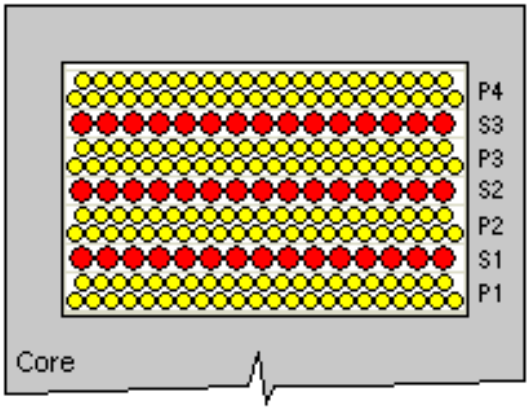


Figure 12.19: Interleaved windings of transformer

12.2.5. Power and Thermal Constraints

For small transformers however, it is usually safe to make some assumptions based on experience.

- An efficiency of 90%, i.e. a loss of 10% is acceptable without the risk of overheating the transformer.
- The primary limiting factor is the heating of the windings that is the I^2R losses.
- Winding areas depend on the type of transformer. Most small transformers are shell type.
- For a given type, e.g. core type or shell type, there will be a given window available for the windings.

The power rating of the transformer is determined by calculating the power losses in the transformer.

Useful copper area and diameter of copper wire are determined from:

$$FF\% = \frac{A_{cu}}{A_{window}} \quad \text{and} \quad A_{wire} = \frac{\left(\frac{A_{cu}}{2}\right)}{N}$$

Power losses in transformer and resistance of winding are:

$$R = \frac{\rho . l}{A_{wire}} \quad \text{and} \quad P_{loss} = \frac{V^2}{R}$$

Finally, the rated power of transformer is calculated as follows:

$$\%\text{loss} = \frac{P_{loss}}{P_{in}} \quad \text{so} \quad P_{in} = \frac{P_{loss}}{\%\text{loss}}$$

12.2.6. A Note about DC Current

DC current in a transformer is normally not a good thing. It will not contribute to the power delivered by the transformer, but it will contribute to the magnetic flux in the core. This means that the core will be

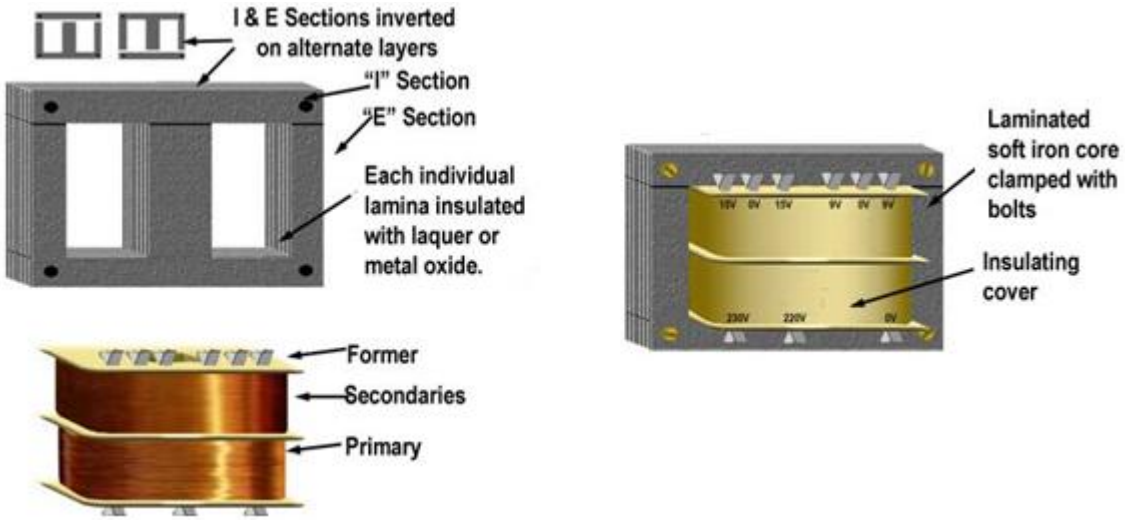
closer to saturation. If the transformer is designed to be at the edge of saturation without DC current, it will saturate.

If you intend to have DC current in the transformer, you should apply the same design constraints as when designing an inductor intended for DC. Typically, this means adding an air gap to the core, in order to reduce the effective permeability.

In the following section, an example of small or low power transformer design process is outlined. High power transformer design might use different order of the design steps although the design concept is the same e.g. high power transformer design might emphasis on the power rating and power dissipation first.

Example for Tutorial 1 - Transformer Design Example

Suppose you want to design a power transformer operating at a primary voltage, V_p of 230 V (RMS), at a frequency of 50 Hz. The secondary voltage, V_s is approximately 18 V. The core for our example has a cross-sectional area, A of 45 mm x 30 mm which is 13.5 cm^2, and a desired maximum flux density, B of 1.3 T. The following figure shows an example of laminated magnetic core and former/bobbin of a transformer for design.



a. Design the magnetic parameters of the transformer. [12 marks]

b. Determine the power parameters of the transformer. [16 marks]

Solution

The steps in the transformer design follow the suggested transformer design process as outlined in the previous sections.

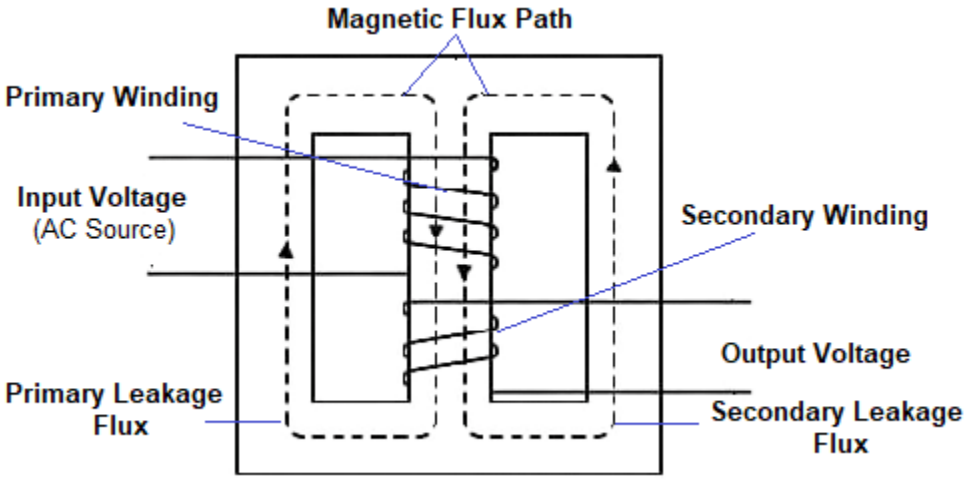
a. Magnetic Design.

The maximum flux in the core ($\Phi_{\max}$) is then calculated as:

$$\Phi_{\max} = B.A = (1.3)(1.35 \times 10^{-3}) = 1.76 \times 10^{-3} \text{ Wb}$$

The maximum flux change in the shell type transformer, however, is double, as the core may be magnetized in both directions, i.e. 3.52 mWb.

The following figure shows the magnetic flux paths in a shell type single-phase transformer.



The number of turns (N) required to obtain this flux is given by:

$$N = \frac{V.t}{\Phi}$$

The time constant (t) is found from:

$$t = \frac{1}{2f} = \frac{1}{2 \times 50 \text{ Hz}} = 0.01 \text{ second}$$

If the input voltage is given by $V(t) = V_p \sin(\omega t)$, then the average input voltage may be derived as follows:

$$V_{ave} = \frac{1}{\pi}\int_0^{\pi} V_p \sin(\omega t)\, dt = \frac{V_p}{\pi}[-\cos(\omega t)]_0^{\pi}$$

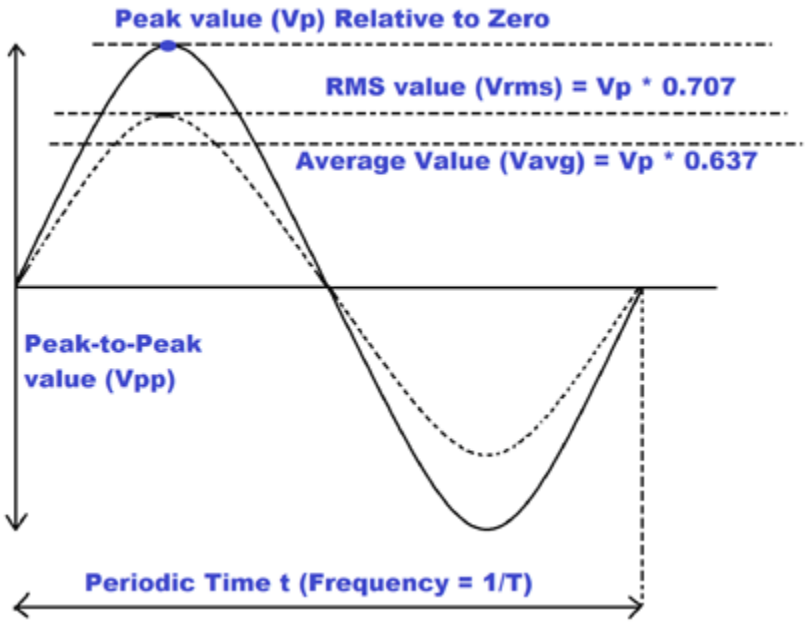
Since $\cos(0) = 1$ and $\cos(\pi) = -1$, hence:

$$V_{ave} = \frac{2V_p}{\pi} = \frac{2\sqrt{2}V_{rms}}{\pi}$$

For our example, the average input voltage (V_{ave}) is computed as:

$$V_{ave} = \frac{(2)(\sqrt{2})(230 \text{ V})}{\pi} = 207 \text{ V}$$

The following figure shows the peak, rms and average values of a sinusoidal signal.



The required number of primary turns, N_p is computed as:

$$N_p = \frac{V_{ave}t}{\Phi} = \frac{(207\text{ V})(0.01)}{3.52\text{ mWb}} = 588\text{ turns}$$

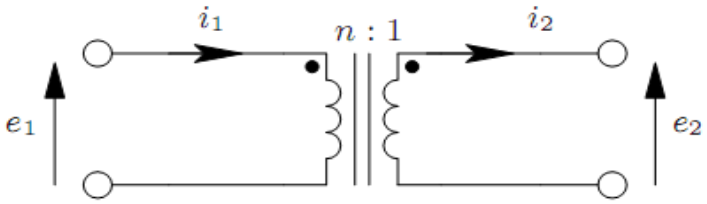
So

$$V/\text{turn} = 230/588 = 0.39 \text{ volts per turn}$$

Or

$$N/\text{volt} = 588/230 = 2.56 \text{ turns per volt}$$

The following diagram shows voltages, currents and number of turns in transformer



Hence, expressed in terms of RMS, we have:

- For the secondary winding, the required number of turns is then simply calculated by multiplying 2.56 by the desired voltage.

- If the required secondary voltage (V_s) is 18 V (which may be suitable for a 12 V linear regulated supply), the required number of secondary turns, N_s will be:

$$N_s = (2.56)(18\ \text{V}) = 46\ \text{turns}$$

b. Power Determination.

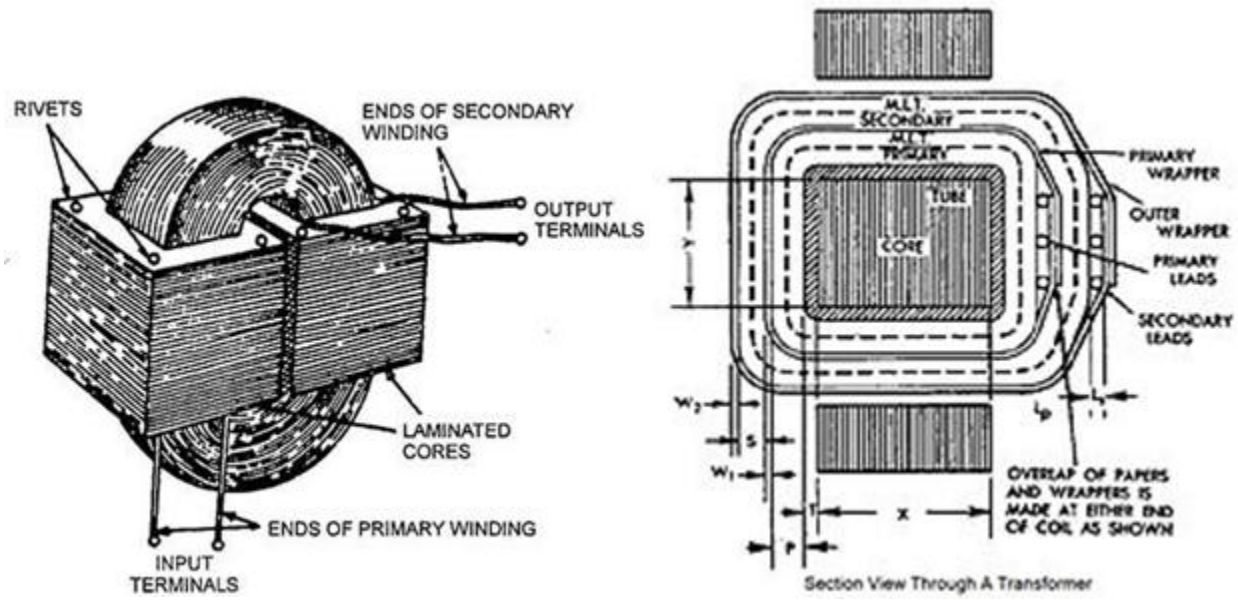
The shell type used in the winding example has a window, A_{window} of:

$$A_{window} = 45\ \text{mm x}\ 15\ \text{mm} = \ 6.75\ \times 10^{-2}\ \text{m}^2$$

Additionally, the distance around the core, L_{core} is:

$$L_{core} = 45 + 45 + 30 + 30 = 150\ \text{mm} = 0.15\ \text{m}$$

The following figure shows the windows and winding of shell type transformer.



It is often assumed a fill factor ($FF\%$) of 40%, that is about 40% of the core window (A_{window}) is used for copper, the rest being used for insulation and air. This gives us a copper area, A_{cu} of:

$$A_{cu} = A_{window} \times FF\% = 6.75 \times 10^{-2} \times 0.4 = 2.7 \times 10^{-4}\ \text{m}^2$$

The primary winding is given half of this area, i.e. the wire cross sectional area, A_{wire} is given as:

$$A_{wire} = \frac{\left(\frac{A_{cu}}{2}\right)}{N} = \frac{1.35 \times 10^{-4}}{588} = 2.3 \times 10^{-7}\ \text{m}^2$$

Copper has a resistivity, $\rho = 1.68 \times 10^{-8}\ \Omega/\text{m}$, and the resistance of the winding (R) is given by:

$$R = \frac{\rho . l}{A_{wire}}$$

If the average turns length ≅ 200 mm = 0.2 m, the total length (l) is about 0.2 x 588 = 117.6 m. The winding resistance (R) is then given as:

$$R = \frac{(1.68 \times 10^{-8})(117.6)}{2.3 \times 10^{-7}} = 8.59\ \Omega$$

As previously stated, we will allow a power loss of 10%, i.e. the primary winding will be allowed a loss of 5%.

$$P_{loss} = \frac{V^2}{R} = \frac{(230\ \text{V})(0.05)^2}{8.59\ \Omega} = 15.4\ \text{W}$$

The rated input power of the transformer, P_{in} is then computed as:

$$P_{in} = \frac{P_{loss}}{\%\ loss} = \frac{15.4\ \text{W}}{0.1} = 154\ \text{W}$$

12.4. Inductor Design

In this section, an example of the inductor design process is outlined. The steps are very similar with the transformer design, but the emphasis is different as inductor has different characteristics compared with transformer e.g. energy storage, quality factor, etc.

12.4.1. Inductor Design Process

Steps in the inductor design process are:

- Calculate the inductance of the inductor.
- Work out the amount of energy storage of the inductor.
- Determine the required quality factor of the inductor.

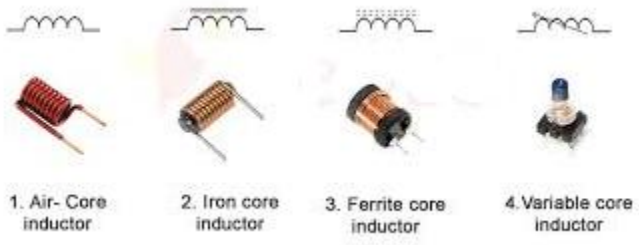


Figure 12.25: Various types of inductors

12.4.2. Inductance of Inductor

Inductance is a parameter of any electric network describing its ability to resist change in current.

$$EMF = -L\left(\frac{di}{dt}\right)$$

Rearranging the equation above

$$L = -EMF\left(\frac{dt}{di}\right)$$



Figure 12.26: Various forms of inductors

12.4.3. Energy Storage

Energy storage is usually not desired in transformers. It is however often the primary purpose of an inductor.

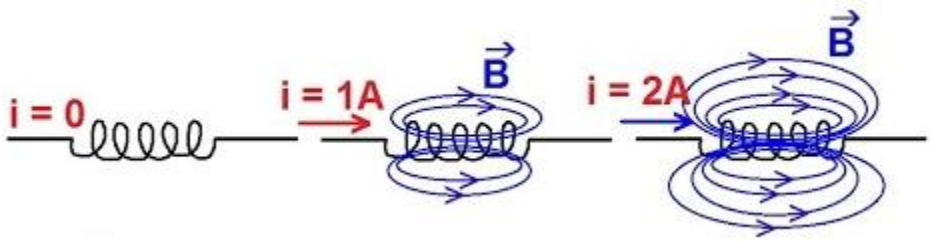


Figure 12.27: Electromagnetic energy stored as fluxes in inductor

It is among other things used in the switching converter circuit. The energy stored in an inductor (E) is given by:

$$E = \frac{1}{2}.L.I^2$$

Where:

- I is the magnetizing current i.e. when computing the energy storage in a transformer. It will be less that the total current.

12.4.4. Quality Factor

The quality factor of an inductor describes the device in relation to an ideal component. It is of particular importance in radio frequency applications. The quality factor of an inductor (Q_L) is given by:

$$Q_L = \frac{X_L}{R_L} = \frac{2\pi f L}{R_L}$$

Hence, the quality factor depends on the frequency (f) under which the inductor is used. It is important to note that due to the skin-effect, the winding resistance R_L is also a function of frequency.

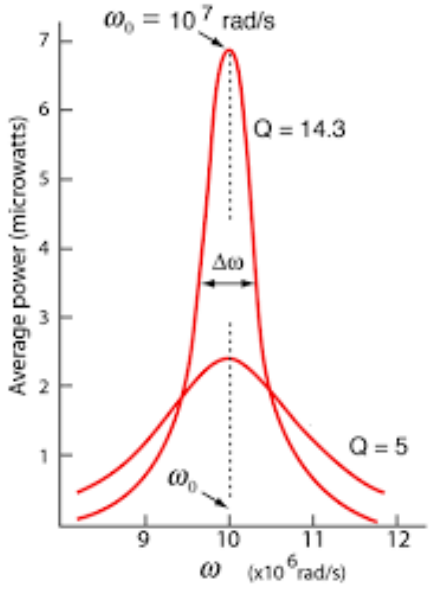


Figure 12.28: Effect of different Q factors in an inductor

12.4.5. Quality Factor Determination

The best way to increase the power capability is to reduce the inductance, and hence increase the current in the core. Remember that the power depends on the current squared e.g. $P = I^2R$. By introducing an air gap in a magnetic core (I_g), the effective permeability (μ) will decrease, and hence the A_L value will also decrease.

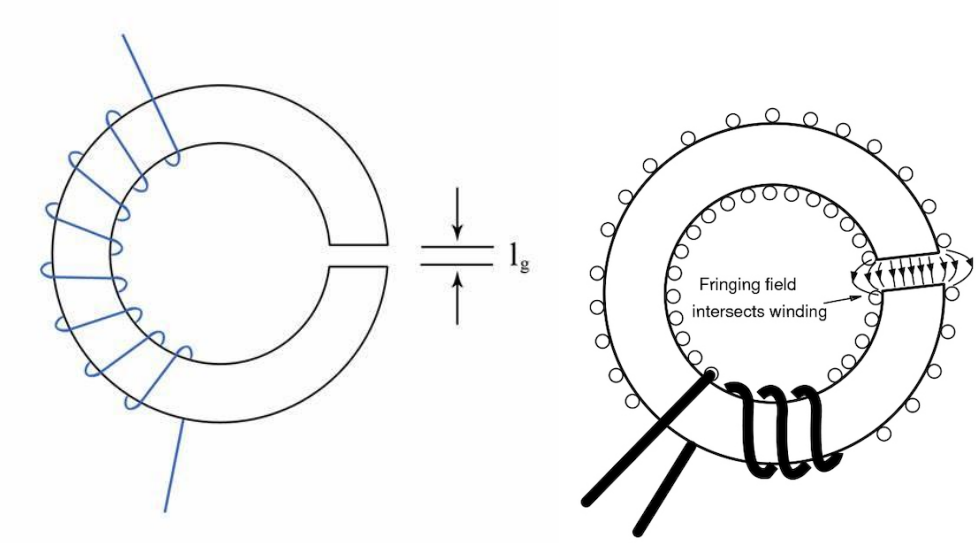


Figure 12.29: Air gap determination in a toroid

12.5. Magnetic Toroid

Toroidal cores can be divided into two main categories, depending on the material from which they are made. There are two toroid types: ferrite and iron powder cores.

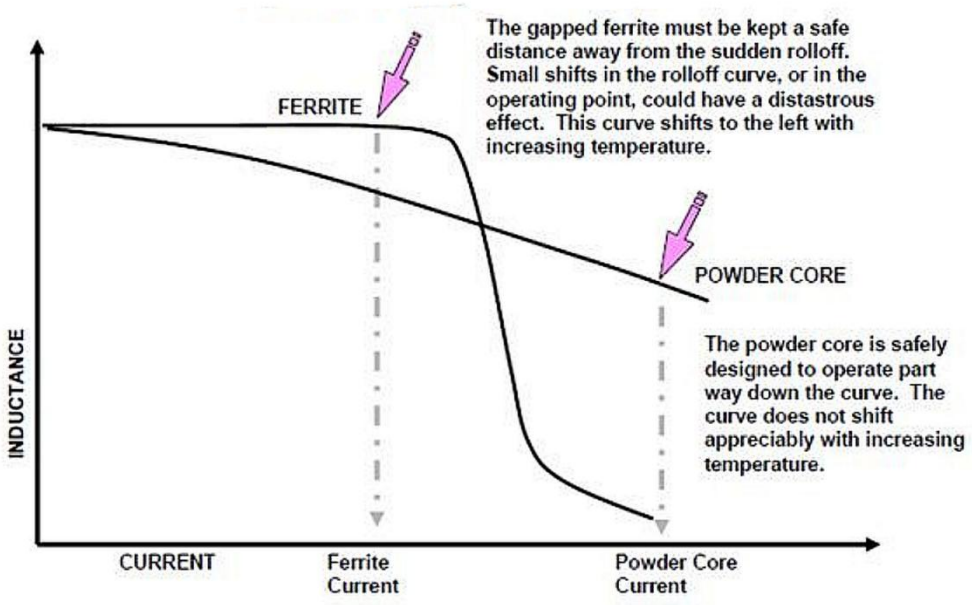


Figure 12.30: Differences between ferrite and powder core magnetic toroid

Ferrite has the highest permeability (μ), but it is not suitable for high frequencies. Additionally, as we have seen, high permeability is not always an advantage.

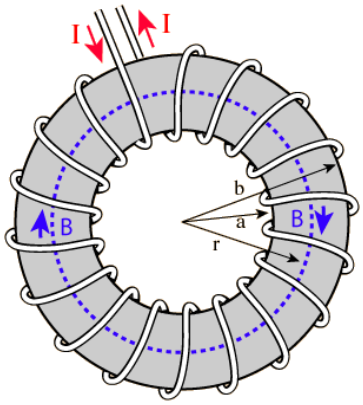


Figure 12.31: Toroid magnetic core

As previously mentioned the flux density must be limited to avoid saturation of the core. For ferrite toroidal cores, this imposes the constraint:

$$B = \frac{V}{2fNA} \leq 0.3$$

Where:

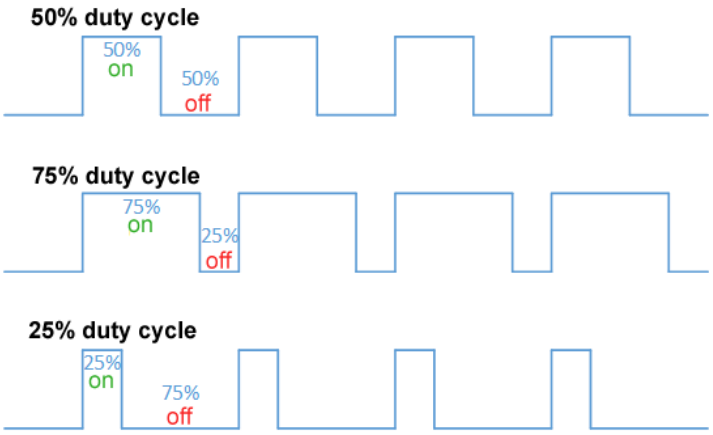
- V is the applied voltage.
- N is the number of windings turns.
- A is the cross-sectional area of the core.

A useful website with specifications for various toroidal cores is located at:

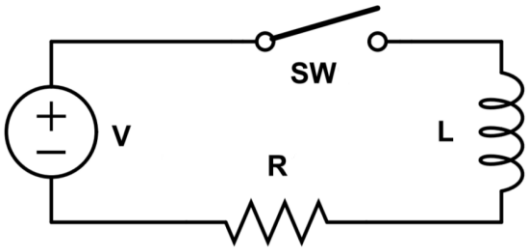
http://toroids.info/

Example for Tutorial 2 - Inductor Design Example

Suppose we want to design an inductor for a switching converter operating at f = 100 kHz, with a 30% duty cycle (d). The following figure shows switching waveforms and duty cycle of the converter.



Our inductor core has a cross-sectional area (A) of 3 cm^2, and a desired maximum flux density (B) of 0.05 Tesla at the frequency of operation. The following diagram shows a switching converter circuit (left) and toroidal magnetic core with its winding (right).





a. Determine the inductance of the inductor. [10 marks]

b. Determine energy storage. [6 marks]

Solution

a. Inductance Determination

The duty cycle of the switching converter (d) is:

$$t = \frac{d}{f} = \frac{0.3}{100 \times 10^3} = 3 \times 10^{-6} \text{ second}$$

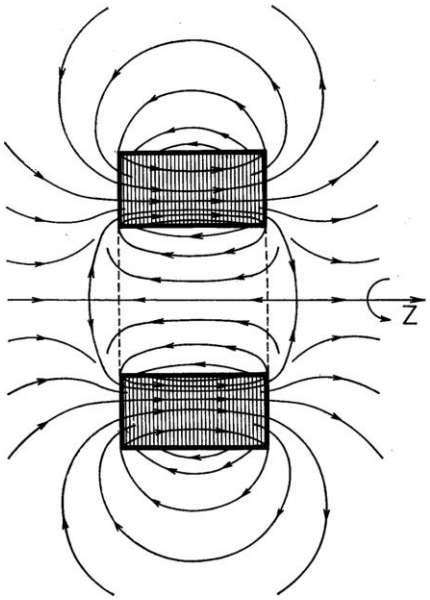
The maximum flux in the core ($\Phi_{\max}$) is given as:

$$\Phi_{\max} = B.A = (0.05)(3 \times 10^{-4}) = 15 \times 10^{-6} \text{ Wb}$$

If the input voltage, V is 230 V (RMS) rectified, i.e. $\sqrt{2} \times 230$ = 325 V (DC), the required number of primary turns (N) is given as:

$$N = \frac{V.t}{\Phi} = \frac{(325)(3 \times 10^{-6})}{(15 \times 10^{-6})} = 65 \text{ turns}$$

Flux generation in the inductor is shown in the following figure.



The energy stored in the core depends on the inductance. Typically, the manufacturer will supply a cross-sectional area of the core, A_c which when multiplied by the turn ratio squared, gives us the inductance. The inductance of a toroid core (L) is approximately given by:

$$L \approx \frac{\mu N^2 A_c}{2\pi r}$$

Where:

- μ is permittivity of free space (8.85×10^{-12} Farad/m).
- N is the number of turns.
- A_c is the cross-sectional area of the core.
- r is the radius to the centerline of the core.

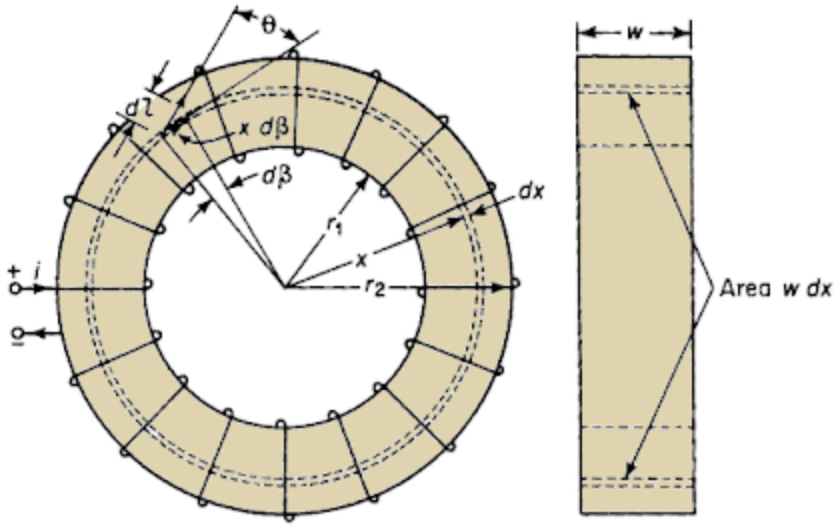
The cross-sectional area of the inductor, A_c is:

$$A_c = (R_2 - R_1)h$$

Where:

- R_2 and R_1 are the outer and inner circumferences of the magnetic core respectively.
- h is the height of the magnetic core.

Physical dimensions of a toroidal magnetic core are shown in the following figure.



As can be seen, all factors, except the number of turns, are constants for the given core. Similar equations may be derived for different core shapes.

Suppose for our example that from datasheet $A_c = 7 \times 10^{-6}$ m² and r = 0.02 m:

$$L \approx \frac{(8.85 \times 10^{-12})(65)^2(7 \times 10^{-6})}{2\pi(0.02\ \text{m})} = 29.6\ \times 10^{-3}\text{H}$$

The inductance of the primary winding (L) will be approximately 29.6 mH.

b. Determine Energy Storage

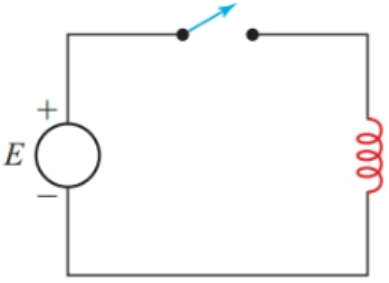
For the given inductor, the current (I) will ramp up linearly, with a peak current at the end of the on-cycle given by:

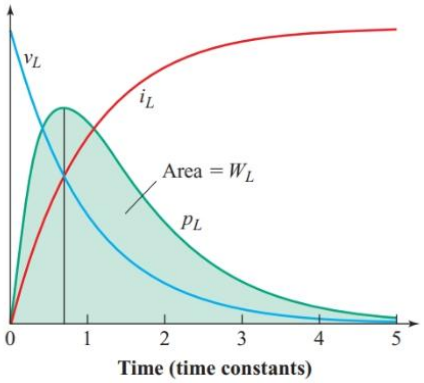
$$I = \frac{V.t}{L} = \frac{(325)(3 \times 10^{-6})}{29.6 \times 10^{-3}} = 0.033\ \text{A}$$

The energy stored in the core (E) at each switching interval is then given as:

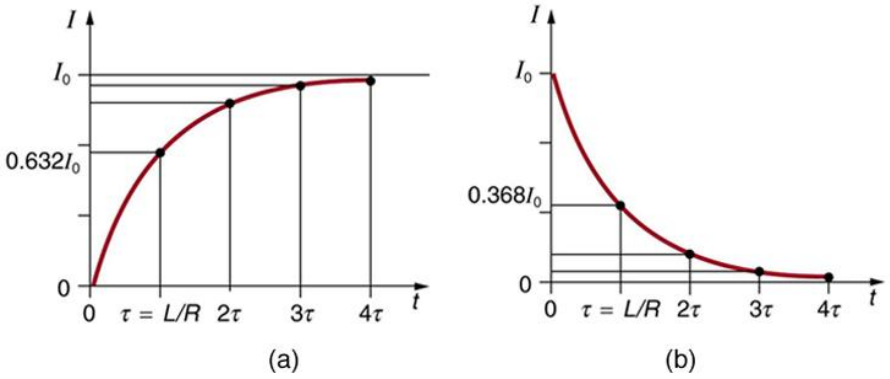
$$E = V.I.t = (325)\left(\frac{0.033}{2}\right)(3 \times 10^{-6}) = 16 \times 10^{-6}\ \text{J}$$

The following figure shows the graph of current, voltage and power in an inductor.





As the frequency, f is 100 kHz, we will have 100,000 = 10^5 such intervals each second. The following figure shows the charging and discharging periods of the inductor circuit.



Hence, the total power transferred (P) each second is given as:

$$P = (16 \times 10^{-6})(10^5) = 1.6 \text{ J/s} = 1.6 \text{ W}$$

This power may seem somewhat low for a core (A) of 3 cm^2 cross-sectional area.