XMUT303 Analogue Electronics

Note 5: Dynamic Limitations

Topics

- Dynamic limitations and imperfections.
- Gain bandwidth of op amp.
- Transient performances of op amp (e.g. finite gain, rise time, slew rate and bandwidth of op amp).

5.1. Dynamic Imperfections

Dynamic limitations of the op amp circuit are imperfections arise from op amp device and circuit arrangements. Just like the static limitation, it is about the real vs. ideal performance.

Since it is specific to the kind of use of these op amp circuits, their details are complex and application specific. As a result, we need to use generic overview of basic building blocks for op-amp based circuit applications e.g. amplifier, filter, comparator, oscillator, etc.

Furthermore, we will look more specifically the following issues in op amp circuit: gain-bandwidth product (GBP), finite gain, rise time, slew rate, and bandwidth.

As these limitations and imperfections are circuit and application specific, there are many other dynamic limitations that are not covered in this chapter.

5.2. Gain Bandwidth Product

The forward gain, A_{oc} is defined as the gain of the op-amp when a signal is fed differentially into the amplifier with no negative feedback applied.

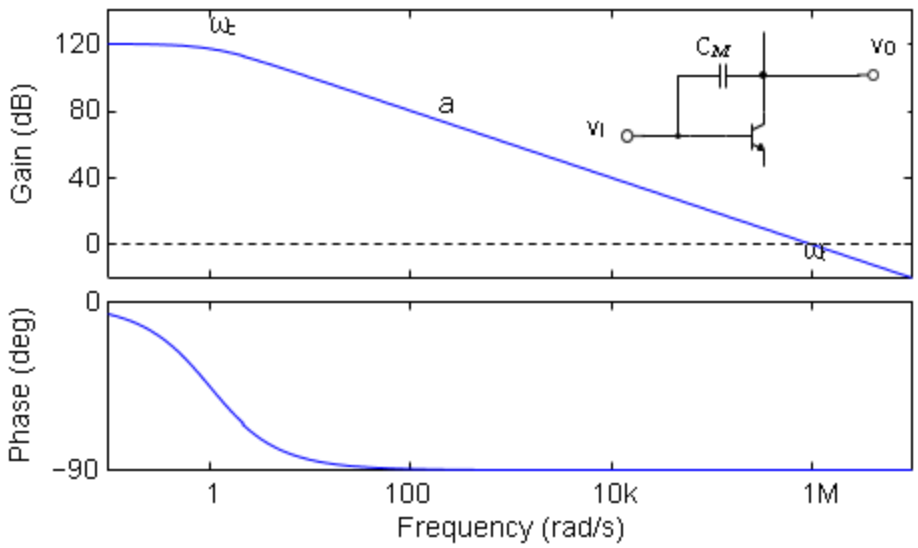


Figure 5.1: Bandwidth profile of an op-amp amplifier

As illustrated in the figure given above, this gain is ideally infinite at all frequencies, but in a real op-amp is finite and depends on the frequency.

At DC (f = 0 Hz) and low frequency, the gain is maximum and decreases linearly with increasing frequency. It has a value of one at the frequency commonly referred to as the unity gain or cut-off frequency f_0 (in equation form, the gain at this frequency, A_{cl} = 1).

The phase shift of the op-amp based amplifier circuit shows case the phase shift of a first order RC circuit, e.g. it is formed by the Miller capacitance and parasitic impedance at the input of the op amp. It is 0° at low frequency and settling down to -90° at higher frequency.

Gain bandwidth product (GBP) is the (constant) ratio of the gain and the bandwidth of an op-amp based amplifier. The GBP is defined as the gain-bandwidth product and for all input frequencies, this product is constant and equal to the closed loop gain, A_{cl} and the cut-off frequency of the op amp, f_0.

$$\text{Bandwidth}, \omega_c = \frac{GBP}{A_{cl}} \qquad (Eq.1)$$

With bandwidth is proportional to cut-off frequency,

$$GBP \propto f_0 \qquad (Eq.2)$$

As above, f_0 is the unity gain cut-off frequency. The gain can be specified as a simple number (magnitude) or in decibel (dB).

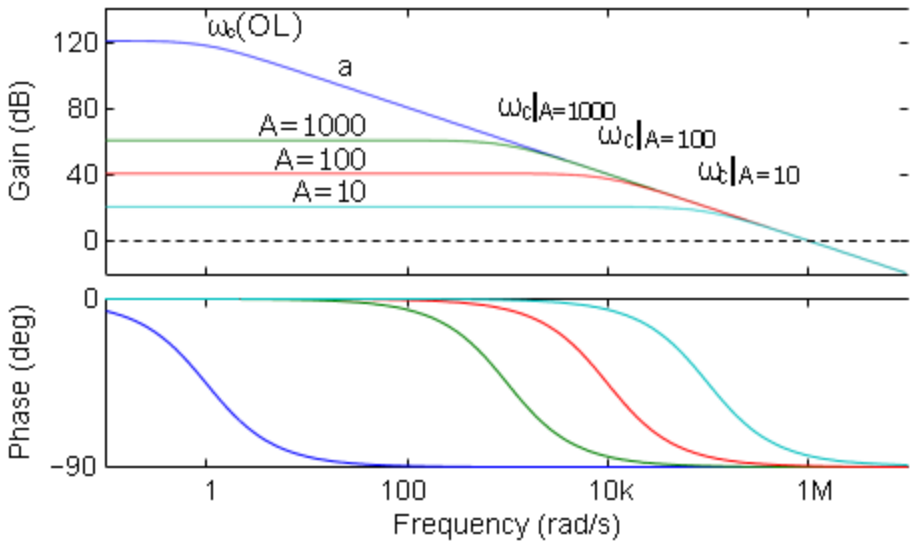


Figure 5.2: Bandwidth of an op amp amplifier with varied open-loop gain

The graph above shows the bandwidth of an op-amp based amplifier as its open-loop gain is varied. Notice that the product of frequency and bandwidth is the gain-bandwidth product, and it is found to be constant.

At gain, $A_{2\text{ kHz}}$ = 1000, the cut-off frequency is around 2 kHz. At $A_{20\text{kHz}}$= 100, the cut-off frequency is a little higher at 20 kHz and at $A_{200\text{ kHz}}$ = 10, the gain is around 200 kHz.

$$GBP = A_{2\text{ kHz}}\omega_{2\text{ kHz}} = (1000)(2\times 10^3) = 2\times 10^6\text{ Hz}$$

$$GBP = A_{20\text{ kHz}}\omega_{20\text{ kHz}} = (100)(20\times 10^3) = 2\times 10^6\text{ Hz}$$

$$GBP = A_{200\text{ kHz}}\omega_{200\text{ kHz}} = (10)(200\times 10^3) = 2\times 10^6\text{ Hz}$$

The GBP for each arrangement is found to be constant around 2 MHz. In fact, the GBP for this op amp is 2 MHz from its manufacturer datasheet.

Every open-loop op amp has limited bandwidth, the gain of the op amp at a given frequency is determined from the following equation:

$$A_{cl}(s) = \frac{A_{oc}}{1+\frac{s}{\omega_c}} \qquad (Eq.3)$$

As we decrease closed-loop gain, we obtain larger ω_c. So, the gain is found as:

$$A_{cl} \propto \frac{1}{\omega_c} \qquad (Eq.4)$$

In this example, GBP, gain and operating frequency of the op amp are related through:

$$A_{cl}\times\omega_c = \text{constant} \qquad (Eq.5)$$

Example for Tutorial 1 – Bandwidth, Gain, and Frequency of Operation

Referring to the datasheet for the OP97 op-amp, answer the following questions:

a. Determine the open loop gain and bandwidth at unity gain frequency and bandwidth of the closed-loop gain. [2 marks]

b. Determine the gain, bandwidth and cut off frequency of the OP97 op amp at various operating frequencies. [6 marks]

c. Use the graph in the datasheet of the op amp to illustrate the GBP relationship. How does this limit gain of the feedback? [6 marks]

Solution

a. For the OP97 op-amp, the unity gain frequency is 900 kHz, the open-loop gain at this frequency is simply one.

$$GBP = A_{cl} \times \omega_c$$

Thus, bandwidth of the op amp (ω_c) when it is operated at 900 kHz is approximately 900 kHz. This is also the closed-loop bandwidth or the maximum frequency when the feedback is configured with a closed loop gain of 1.

b. The gain, bandwidth and cut-off frequency of the OP97 op-amp based amplifiers are measured at 1 Hz, 1 kHz, and 900 kHz:

- $GBP_{1\text{ Hz}}$: A_{cl} = 900,000, ω_c = 1 Hz, and f_c = 1 Hz

$$GBP_{1\text{ Hz}} = A \times \omega_c = (900{,}000)(1)$$

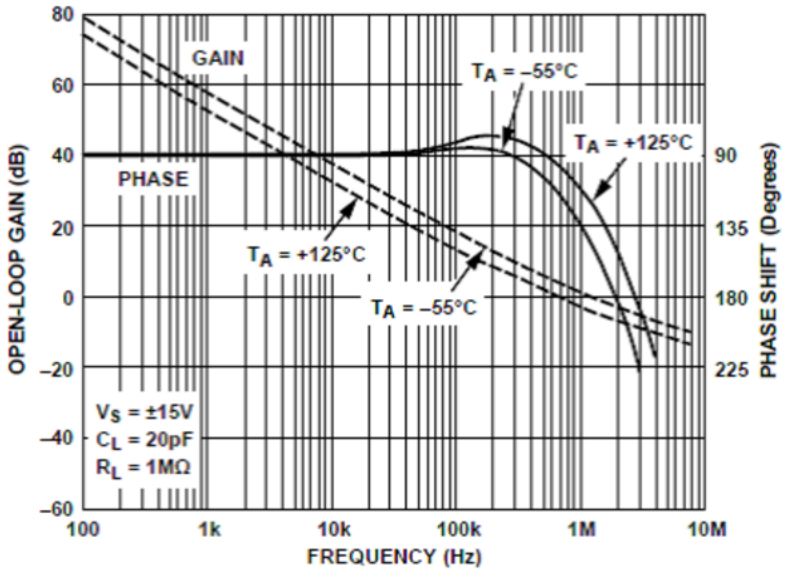
- $GBP_{1\text{ kHz}}$: A_{cl} = 900, ω_c = 1 kHz, and f_c = 1 kHz

$$GBP_{1\text{ kHz}} = A \times \omega_c = (900)(1000)$$

- $GBP_{900\text{ kHz}}$: A_{cl} = 1, ω_c = 900 kHz, and f_c = 900 kHz

$$GBP_{900\text{ kHz}} = A \times \omega_c = (1)(900{,}000)$$

c. Shown in the figure below is the frequency response (gain and phase plots) of OP97 op-amp, from its datasheet, it illustrates graphically this relationship.



When feedback is provided, as in an inverting amplifier, the gain is given by:

$$A = -\left(\frac{R_2}{R_1}\right)$$

However, it must be recognised that the magnitude of this gain can never exceed the gain as given by the gain-bandwidth product.

Note:

The OP97 amplifier has a pin (e.g. pin 5) which allows the user to connect an external frequency compensation capacitor to effectively lower the gain-bandwidth product.

As shown in the figure below, it shows the change in bandwidth for different external capacitors.

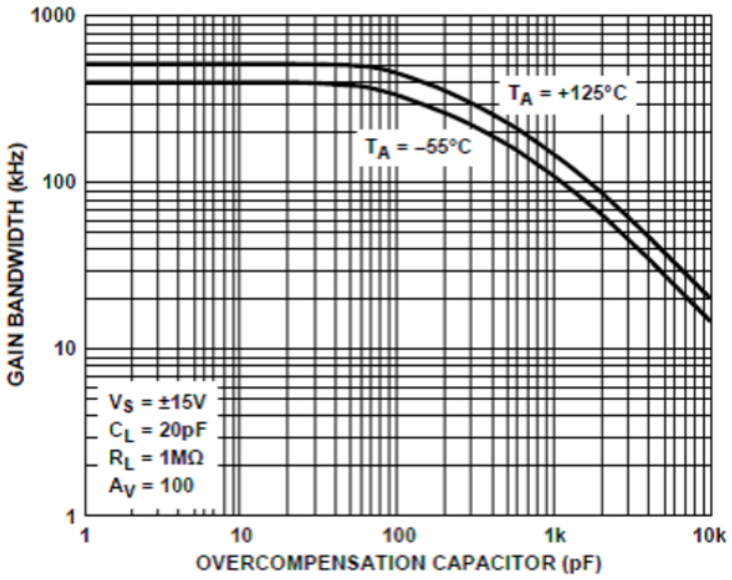


Figure 5.3: OP97 op-amp amplifier with external compensation capacitor

As shown in the figure above, the decision depends on bandwidth and gain requirements.

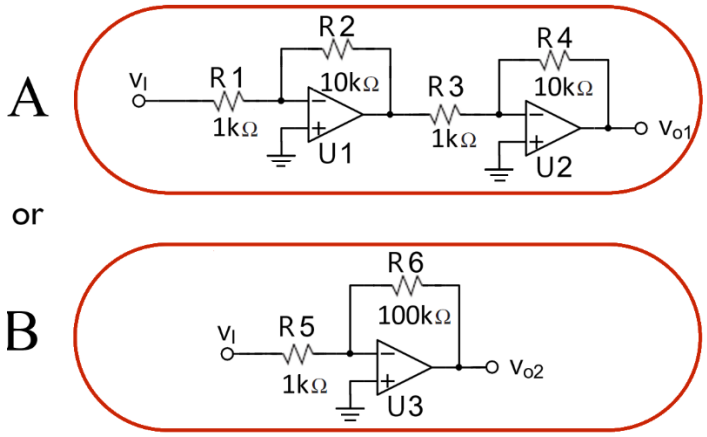


Figure 5.4: Series or parallel combination of op-amp based amplifier

If you could not meet both the gain and bandwidth requirement of the design from a single amplifier circuit, then spread the circuit into a multistage amplifier circuit e.g. a number amplifier stages with lower gains and the resulting wider bandwidths.

- For an amplifier circuit that has large bandwidth requirements, choose option B.
    - Single stage amplifier has wider bandwidth compared with multistage or cascaded amplifier. But, with less overall gain of the amplifier circuit.
- For an amplifier circuit that has large gain requirements, then choose option A.

- Multistage or cascaded amplifier can afford overall higher gain than single stage amplifier. But this is at the expense of less wide bandwidth available for its operating frequency.

Example for Tutorial 2 – Gain Bandwidth Product

An op-amp amplifier circuit with the following specifications: f_{signal} = 100 kHz and desired gain, A_{cl} = 20.

a. Find required minimum GBP. [2 marks]

b. Compared with the GBP of ADA4004 op amp. [2 marks]

Solution

a. Simple gain equations only apply at 'low' frequencies. The bandwidth is calculated from:

$$\omega_c = \frac{GBP}{A_{cl}}$$

Different op amps have different GBP. Then the required (minimum) GBP is:

$$GBP = \omega_c A_{cl} = (100\ \text{kHz})(20) = 2\ \text{MHz}$$

Later, we consulted the datasheets of potential op amp as we need an op amp that has the GBP as specified above. Any op amp that has GBP to be 2 MHz or above would be able to realise the desired amplifier circuit.

b. In case of ADA4004 op amp with a GBP of 12 MHz, at a gain of 1, the bandwidth is 12 MHz. With open-loop gain (maximum) of 500,000, its open-loop bandwidth is:

$$\omega_c = \frac{GBP}{A_{ol}} = \frac{12\ \text{MHz}}{500{,}000} = 24\ \text{Hz}$$

In this example, the cut-off frequency of open loop op amp has a very low bandwidth to be used for any circuit applications. So, electronic engineers tend to design it at lower gain to obtain wider bandwidth.

5.3. Finite Gain

In our analysis of ideal op-amp circuits this far, we have considered them to have an infinite gain and an infinite bandwidth. This is not true for physical or practical op-amps as these op amps have typically finite gain as illustrated in the figure below.

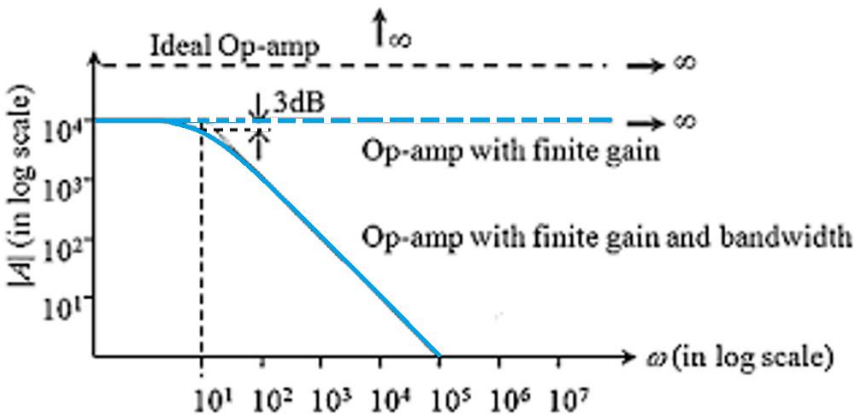


Figure 5.5: Bandwidth profile of a typical op amp amplifier with finite gain

The following figure shows the gain vs. frequency graph of a typical 741 op amp from the manufacturer datasheet.

At DC, the gain of op amp is around 130 dB and a bandwidth of 0.1 Hz. But, at gain of 80 dB, its bandwidth is 100 Hz, and at gain of 20 dB its bandwidth increases to around 100 kHz.

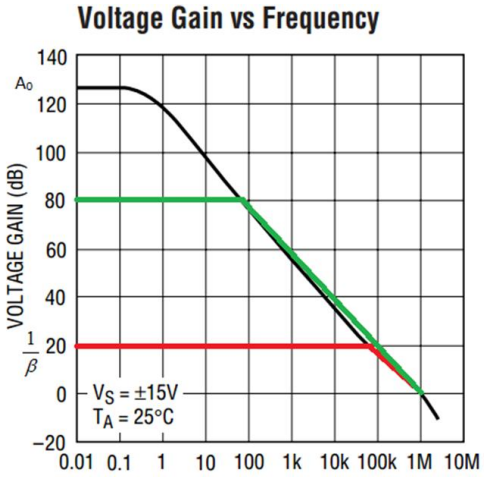


Figure 5.6: Graph of voltage gain vs. frequency of a typical op amp amplifier

5.3.1. Non-Inverting Amplifier

For a given non-inverting amplifier circuit as shown in the figure below, knowing a = open loop gain, the voltage ratio of output voltage and input voltage is given as:

$$V_N = \left(\frac{R_1}{R_1 + R_2}\right) V_O \qquad (Eq. 6)$$

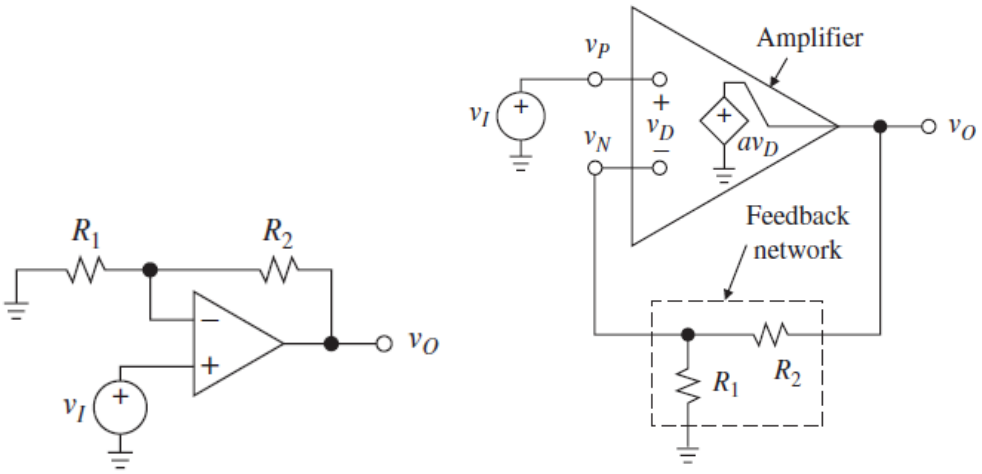


Figure 5.7: Op amp based non-inverting amplifier and its circuit model for analysis

From the circuit given above, the output voltage of the op amp is:

$$V_O = a(V_I - V_N) \qquad (Eq.7)$$

Combining equation 1 into equation 2 gives:

$$V_O = a\left[V_I - \left(\frac{R_1}{R_1 + R_2}\right)V_O\right]$$

Rearranging the equation above, it becomes:

$$aV_I = \left[1 + \left(\frac{aR_1}{R_1 + R_2}\right)\right]V_O$$

Thus, the ratio of output voltage and input voltage is:

$$\frac{V_O}{V_I} = \frac{a}{\left[1 + \left(\frac{aR_1}{R_1 + R_2}\right)\right]} \qquad (Eq.8)$$

Simplifying the equation further, notice that the transfer function of the practical non-inverting amplifier with a finite open-loop gain (a) is reduced slightly from the ideal op amp case.

$$\frac{V_O}{V_I} = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{1}{1 + \left(\frac{1}{a}\right)\frac{R_1 + R_2}{R_1}}\right) \qquad (Eq.9)$$

Notice that finite gain of the op amp reduces the transfer function of the amplifier. The bigger the gain of the op amp, the smaller the difference from the gain of ideal op amp.

5.3.2. Inverting Amplifier

For an inverting amplifier circuit shown in the figure below, the transfer function (ratio of output voltage and input voltage) of the amplifier circuit is given as:

$$\frac{V_O}{V_I} = -\left(\frac{R_2}{R_1}\right)\left(\frac{1}{1+\left(\frac{1}{a}\right)\frac{R_1+R_2}{R_1}}\right) \qquad (Eq.10)$$

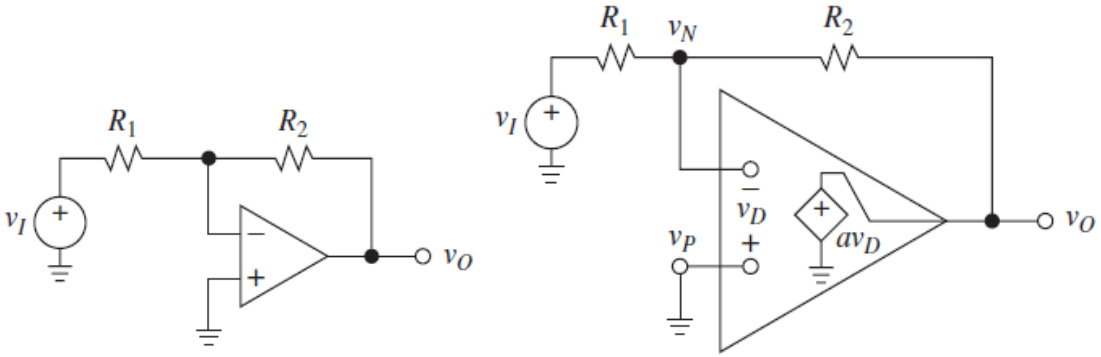
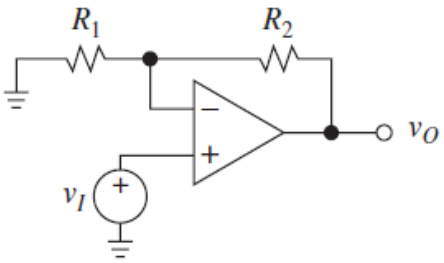


Figure 5.8: Op amp based inverting amplifier and its circuit model for analysis

Notice that the transfer function of the practical inverting amplifier with a finite open-loop gain is also slightly less than the ideal inverting amplifier.

Example for Tutorial 3 – Finite Gain

For a non-inverting op-amp based amplifier, determine the voltage gain of the amplifier when R_1 = 1 kΩ and R_2 = 10 kΩ and the op amp is ADA4004 with an open-loop gain of 500,000. [2 marks]



Solution

The voltage gain of the non-inverting based amplifier with a finite gain is calculated form:

$$\frac{V_O}{V_I} = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{1}{1+\left(\frac{1}{a}\right)\frac{R_1+R_2}{R_1}}\right)$$

$$= \left(\frac{1\ \text{k}\Omega + 10\ \text{k}\Omega}{1\ \text{k}\Omega}\right)\left(\frac{1}{1 + \left(\frac{1}{500{,}000}\right)\frac{1\ \text{k}\Omega + 10\ \text{k}\Omega}{1\ \text{k}\Omega}}\right) = 10.99$$

Thus, the voltage gain of the amplifier is 10.99.

5.4. Gain Non-Linearity

Changes in the output voltage level and output loading are the most common causes of changes in the open-loop gain of op amps.

Notice some non-linearity on the curves in the graph given below for a OP177 op amp.

As shown in the graph, the output voltage is not entirely linear for a given range of operating conditions of this op-amp based amplifier circuit.

Notice also as the values of the load is changes, the non-linearity is also slightly changed.

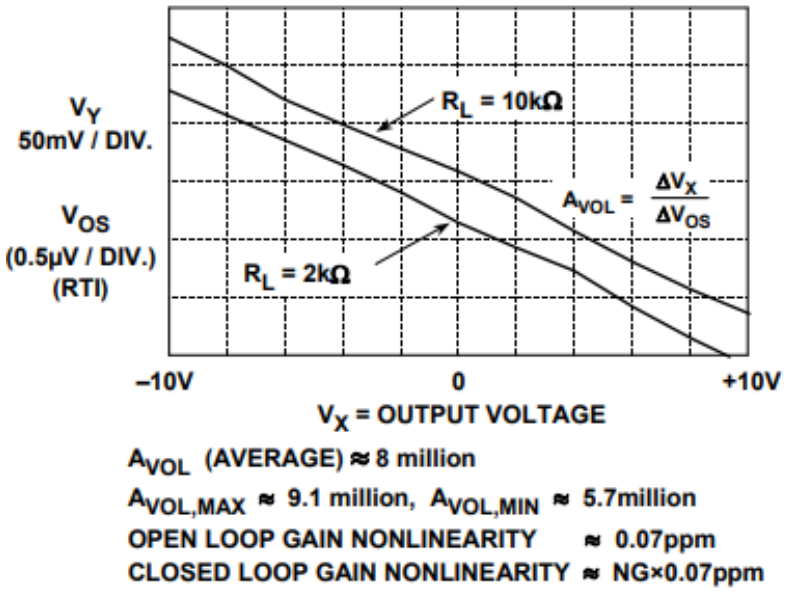


Figure 5.9: Gain non-linearity of OP177 op-amp amplifier

A change in open-loop gain with signal level produces a nonlinearity in the closed-loop gain transfer function, which also cannot be removed during system calibration.

The severity of this non-linearity varies widely from one device type to another and generally is not specified on the data sheet.

5.5. Rise Time

We know for a fact that op amps have finite bandwidth. As a result, output cannot change arbitrarily quickly. As shown in the figure below, the rise time of a step response is defined as the time it takes for the waveform to transition from 10% to 90% of the final value. Sometimes rise time is defined using 20/80%.

Note that rise time is defined as a percent of the waveform size and is independent of the voltages involved. The rise time is derived from the standard first-order circuit exponential equation.

$$V_o = A_0\left(1 - e^{\frac{t}{\tau}}\right) \qquad (Eq.11)$$

Referring to the figure below, it shows the transient response of an op amp-based amplifier circuit. The waveform has approximately a rise time of about 2 time constant.

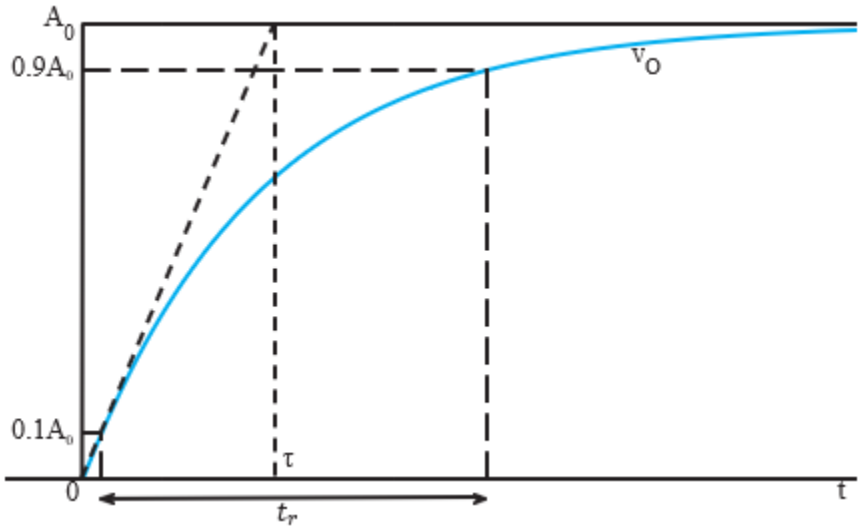


Figure 5.10: Rise time of op-amp amplifier

5.6. Slew Rate

Slew rate is not the same as the rise time. For large input, we cannot ever achieve rise time-limited output. This is due to the Miller capacitance requires current to charge.

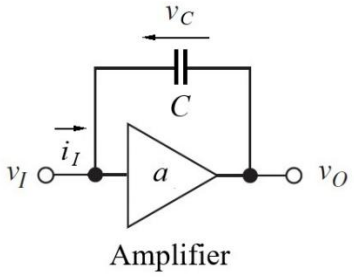


Figure 5.11: (Parallel) parasitic Miller capacitance in an op amp amplifier

If current is limited, output goes linear. As shown below, the current at the input depends on the rate of change of the voltage across the parasitic capacitance.

$$i_I = C\left(\frac{dV_c}{dt}\right) \qquad (Eq.12)$$

Graph below shows the slew rates of an op-amp based amplifier. Notice the delay due to times taken for changing the levels (up and down level). Most of the time these delays are not the same for up and down level.

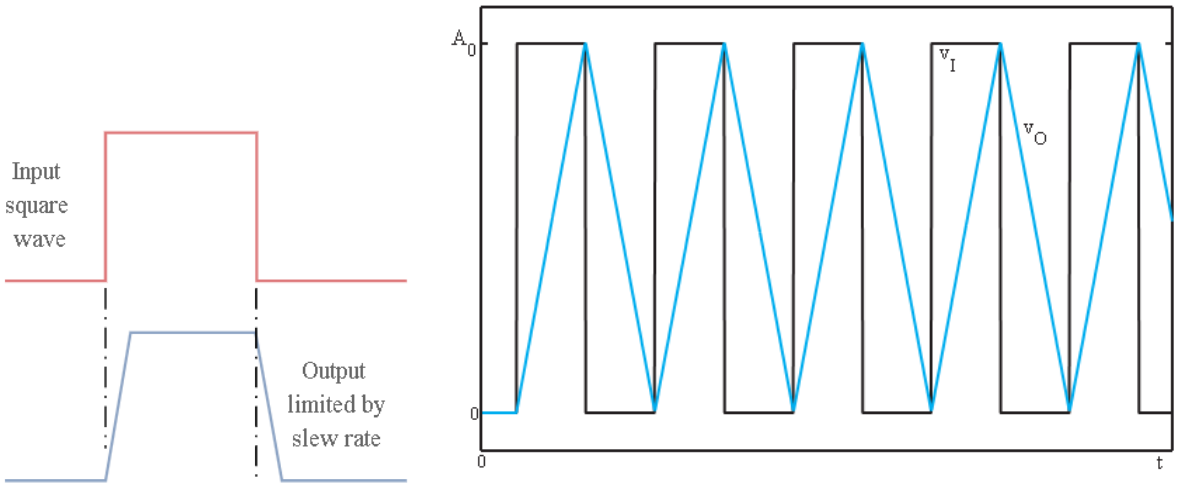


Figure 5.12: Slew rates of op-amp based amplifier

Slew rate is defined as the rate of change of a waveform, which is the same as the slope. It can be calculated using $\Delta V/\Delta t$ as shown in the figure below. Figure below shows the $\Delta V/\Delta t$ calculation at the 10% and 90% points, but the calculation can be performed at any location on the waveform.

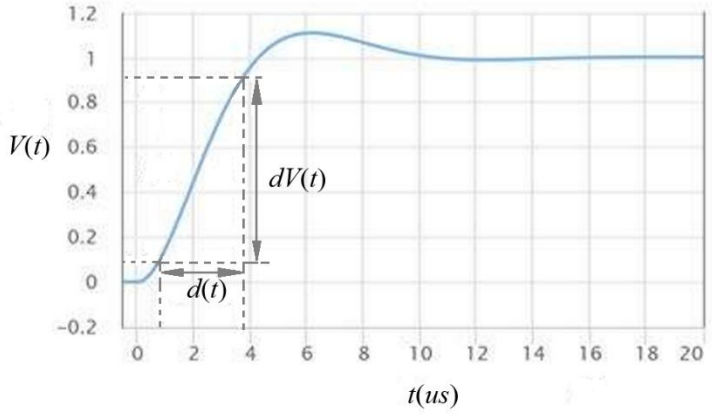


Figure 5.13: Calculation of slew rate of op-amp based amplifier

Really, we are just looking for the slope. Note that slew rate is in units of volts/second (or more commonly volts/µs). It is a measure of how much the voltage changes per unit time. From given figure, we can calculate the slew rate:

$$\text{Slew Rate (SR)} = \frac{\text{Change of voltage}}{\text{Change of time period}} = \frac{\Delta V}{\Delta t} \qquad (Eq.13)$$

This highlights the key difference between rise time and slew rate:

- The rise time ignores the absolute voltage level and just represents the time it takes for the waveform to transition.
- The slew rate describes the rate of voltage change.

For the sine wave case (as shown below), we can apply some basic calculus to find the slew rate.

$$v(t) = V_p \times \sin(2\pi f t) \qquad (Eq.14)$$

The sine wave has various slew rates at various node locations in the waveform, e.g. a maximum slew rate at the zero crossings.

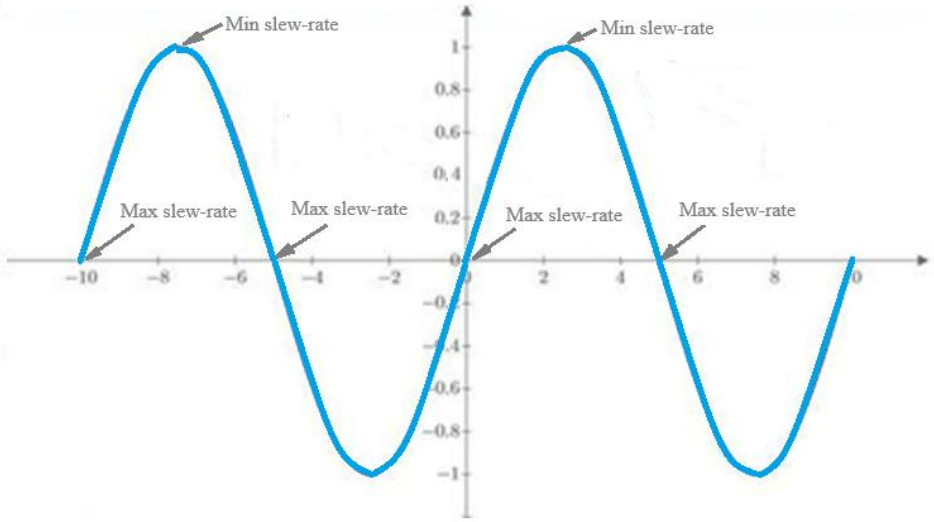


Figure 5.14: Slew rates at various node locations in the sinusoidal waveform

The slew rate (SR) is the slope or derivative of the waveform.

$$SR = \frac{dV}{dt} = V_p \times 2\pi f \times \cos(2\pi f t) \qquad (Eq.15)$$

The slew rate will be maximum when the cosine function reaches its maximum of 1. The same thing occurs but with opposite sign when the cosine function hits -1. Figure below also shows that these maximum slew rate points occur at the zero crossings of the sine wave.

$$SR(\max) = 2\pi f V_p \qquad (Eq.16)$$

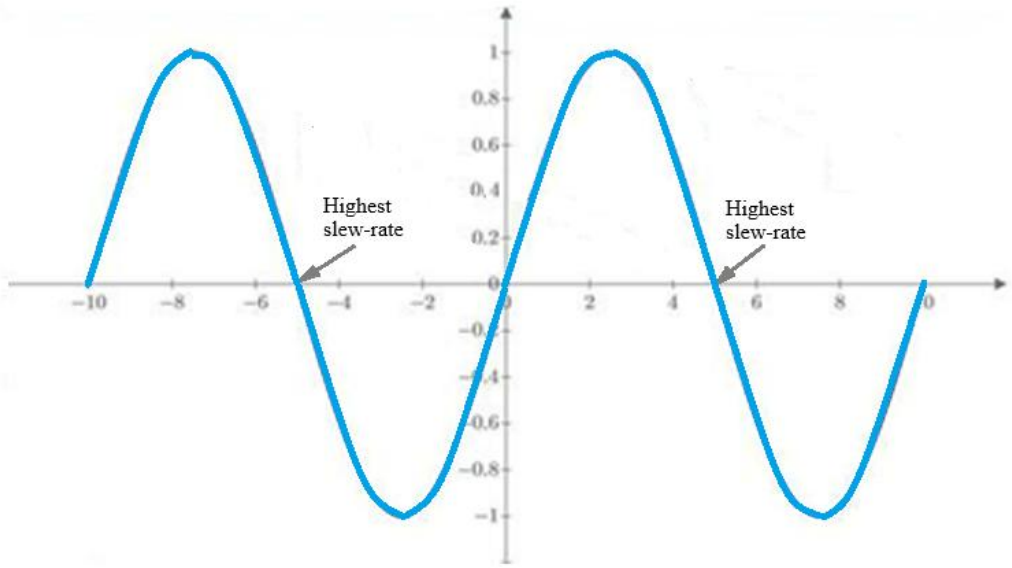


Figure 5.15: Highest slew rates the crossover nodes in sinusoidal waveform

No surprise, the max slew rate is proportional to the frequency of the waveform and its amplitude.

- Bigger waveforms slew faster.
- Higher frequency waveforms also slew faster.

The slew rate issues arise from the internal circuitry within the op amp. There are various reasons for the limitations of most op amps:

Frequency compensation: The capacitors used within the chip to reduce the high frequency response have a marked effect on the slew rate.

Output driver limitations: Within the chip, and particularly within the output driver, the low current levels limit the rate at which change can occur. This limits the slew rate of the op amp

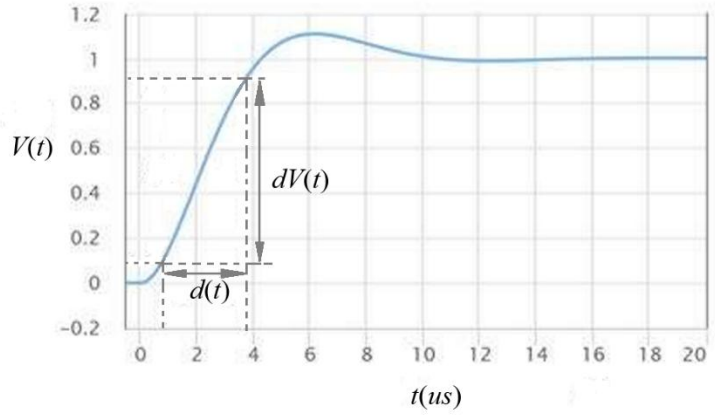
High gain input stages: Operational amplifiers use high gain differential input stages. The high gain, and the fact that they are transconductance amplifiers where a voltage input produces a current output means that there is a possibility that the signals can saturate causing the amplifier to act as a constant current source. When this happens the output change rate of the amplifier is severely limited.

Example for Tutorial 4 – Slew-Rate

For a practical op amp with a noticeable slew rate as shown in the figure below, perform the following tasks.

a. Determine the slew rate of the op amp if for a step input pulse of 0.8 V, it takes 3 μs for the waveform to change state. [2 marks]

b. If the step response of the circuit is ten times the one in part (a), determine the slew rate of the op amp. Does it change from before? [4 marks]



Solution

a. The slew rate calculation for the given transient response of the op-amp based amplifier in the figure above is:

$$\text{SR} = \frac{\Delta V}{\Delta t} = \frac{0.8\ \text{V}}{3\ \mu\text{s}} = 0.267\ \text{V}/\mu\text{s}$$

b. For the case where the step response in given figure is 10 times larger, going from zero volts to 10 volts with the same 3 μs rise time. Sure enough, even though the rise time is the same, the slew rate is 10 times larger because the voltage is changing at a much higher rate.

5.7. Full-Power Bandwidth

Full power bandwidth (FPB) is the frequency where op amp can swing through its maximum output range without suffering slew rate limitations.

$$\omega_{FPB} = \frac{SR}{A_{max}} \qquad (Eq.17)$$

The full-power bandwidth is the range of frequencies where the op amp has the most gain. It should not be confused with "half-power" bandwidth, only used in conjunction with filter frequency response curves, where it refers to -3 dB points in the frequency response of a typical filter.

Data sheets for operational amplifiers often use the term (full-) power bandwidth to indicate the highest frequency at which the achievable peak-to-peak output voltage swing is still equal to the DC output voltage range. This is also sometimes described as the slew-rate-limited bandwidth.

As a result, the full-power bandwidth (ω_{FPB}) is then related to the slew rate (SR) in Volts per micro second and the voltage swing (V_{pp}). Since the bandwidth is approximately also the cut-off frequency or the operating frequency of the op amp, it is expressed as in the equation shown below:

$$\omega_{FPB} \cong f_c = \left(\frac{SR}{2}\right) V_{pp} \qquad (Eq. 18)$$

Where:

- ω_{FPB} is full-power bandwidth expressed in radian/second.
- f_c is the cut off or operating frequency.
- V_{pp} is in Volts.

In the manufacturer data sheets for commonly available operational amplifiers, slew rate is usually given in Volts per microsecond.

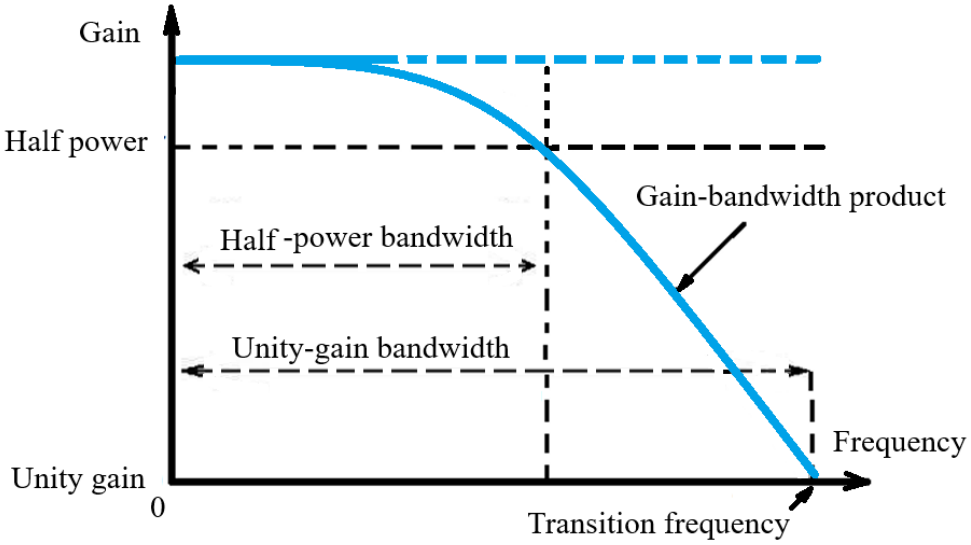


Figure 5.16: Graph of gain vs. frequency of a typical op-amp based amplifier

As illustrated in the section given above, the full-power bandwidth is different from the bandwidth of the op amp determined from the cut-off point (e.g. when it drops 3 dB from its maximum gain). This is then considered as the half-power point. After this, the gain of the op amp falls at a steady, constant rate called the gain-bandwidth product, until it reaches 0.

For the bandwidth of an op-amp based amplifier, as illustrated in the figure given below, for the bandwidth of the op amp, the cut-off frequency is where half-power closed loop gain is determined (after you work out the closed-loop gain of the op amp circuit). With this cut-off frequency of the op amp circuit, we can specify the frequency range of the op amp circuit that is available for application use, hence its difference with the full-power bandwidth of the op amp.

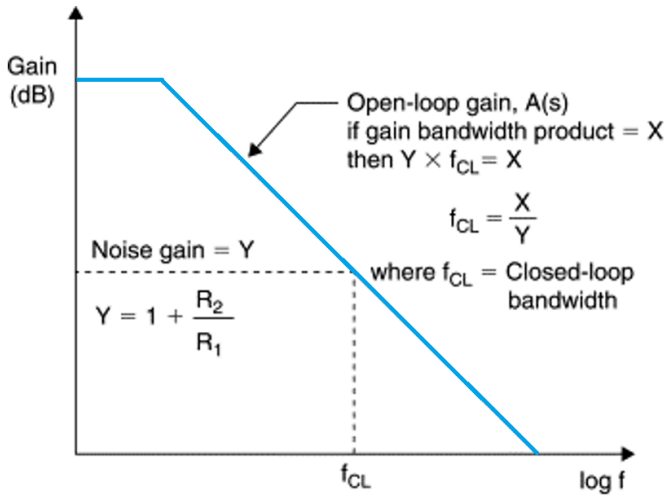
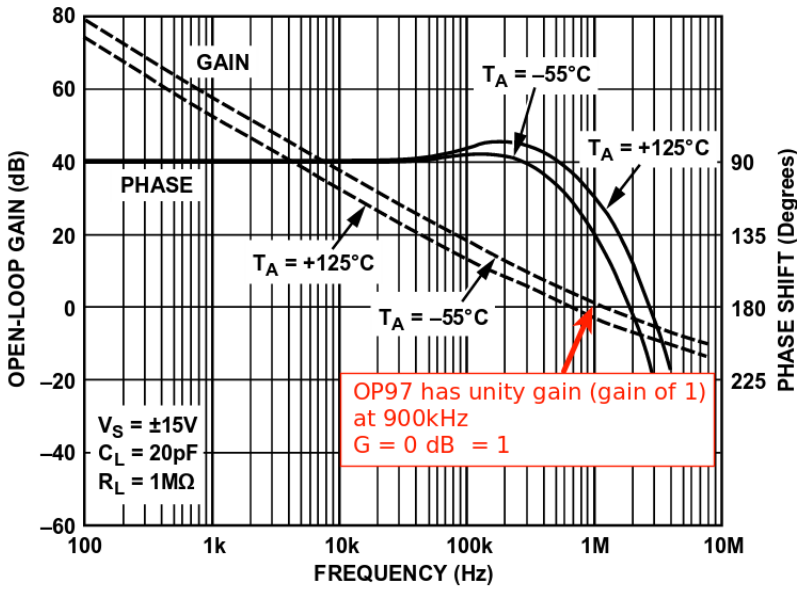


Figure 5.17: Bandwidth of an op-amp based amplifier

Example for Tutorial 5 – Full-Power Bandwidth

The following figure from the datasheet shows the frequency response of OP97 op-amp based amplifier.



a. Determine the full-power bandwidth of the OP97 op amp at maximum gain of 100. [4 marks]

b. Calculate its maximum operating frequency with slew-rate considered, if the amplifier circuit is supplied with ±12 V power supply. Note: find the slew-rate figure of OP97 op amp from its datasheet. [2 marks]

Solution

a. This OP97 op amp has a unity gain at 900 kHz, but its half-power closed-loop gain of the op amp could be calculated after we determine the closed-loop gain of the op amp.

Furthermore, the maximum operating frequency of the amplifier based on this op amp is calculated from:

$$f_c = \left(\frac{\text{SR}}{2}\right) V_{pp}$$

For an OP97 op-amp based amplifier, refer to the datasheet of this op amp for its specifications.

From its datasheet, OP97 has slew rate of 0.2 V/μs. the full-power bandwidth of the op amp at maximum gain of 100 is calculated from:

$$\omega_{FPB} = \frac{\text{SR}}{A_{max}} = \frac{0.2\ \text{V/μs}}{100} = 2{,}000\ \text{Hz}$$

b. Assuming that maximum swing of the output voltage of the amplifier circuit is limited by the supply voltage, the maximum operating frequency is:

$$f_c = \left(\frac{\text{SR}}{2}\right) V_{pp} = \left(\frac{0.2\ \text{V/μs}}{2}\right)[12 - (-12)] = 2.4 \times 10^6\ \text{Hz}$$