XMUT310 Communication Engineering

Tutorial 1: Signals and Systems

Answers are provided for your convenience.

Most of tutorials can be found in Bilibili https://www.bilibili.com/video/BV1r54y1r7KF/

Q1. Let w(t) be a decaying exponential pulse that is switched on at t = 0. That is,

$$w(t) = \begin{cases} e^{-t}, & t > 0 \\ 0, & t < 0 \end{cases}$$

Establish the Fourier Transform (FT) pair of w(t). Can you also determine the expressions of magnitude and phase of the FT pair?

(Answer: $\begin{Bmatrix} e^{-t}, & t > 0 \\ 0, & t < 0 \end{Bmatrix} \leftrightarrow \dfrac{1}{1 + j2\pi f}$, $|W(f)| = \sqrt{\dfrac{1}{1 + (2\pi f)^2}}$ and $\theta(f) = -\tan^{-1}(2\pi f)$)

Q2. Compute the FT of v(t) = Asin(w_ot) where w_o is 2πf_o.

(Answer: $V(f) = j\dfrac{A}{2}[\delta(f + f_0) - \delta(f - f_0)]$)

Q3. Generalise the result in Q3 for w(t) = Asin(w_ot + $Ø_o$).

(Answer: $W(f) = j\dfrac{A}{2}e^{j\theta_0(f/f_0)}[\delta(f + f_0) - \delta(f - f_0)]$)

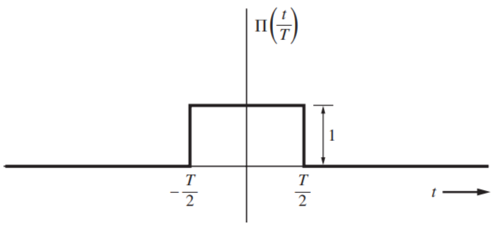
Q4. The spectrum of a waveform can be found by evaluating the FT. Let w(t) be a damped sinusoid as given by

$$w(t) = \begin{cases} e^{-t/T}\sin \omega_0 t, & t > 0,\ T > 0 \\ 0, & t < 0 \end{cases}$$

Obtain the spectrum of the damped sinusoids. Hints: make use of the results of Q1 and Q2 above and some of the FT properties.

(Answer: $\dfrac{T}{2j}\left\{\dfrac{1}{1 + j2\pi T(f - f_0)} - \dfrac{1}{1 + j2\pi T(f + f_0)}\right\}$)

Q5. Evaluate the FT for a rectangular pulse shown here.



(Answer: T sinc(πTf))

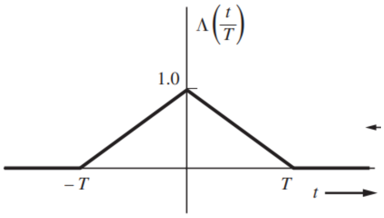
Q6. Compute using analytical method the convolution of the following two signals:

$$x(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{otherwise} \end{cases},$$

$$h(t) = \begin{cases} t, & 0 < t < 2T \\ 0, & \text{otherwise} \end{cases}.$$

(Answer: $$y(t) = \begin{cases} 0, & t < 0 \\ \frac{1}{2}t^2, & 0 < t < T \\ Tt - \frac{1}{2}T^2, & T < t < 2T \\ -\frac{1}{2}t^2 + Tt + \frac{3}{2}T^2, & 2T < t < 3T \\ 0, & 3T < t \end{cases},$$)

Q7. Using the result of Q5 and Q6, explain how the FT of a triangular wave can be computed using rectangular waves and convolution. Sketch the resulting waveform.

(Answer: 

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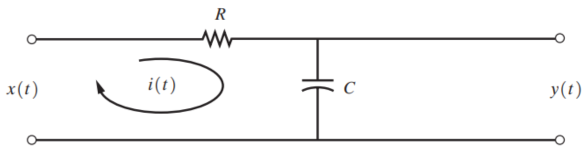
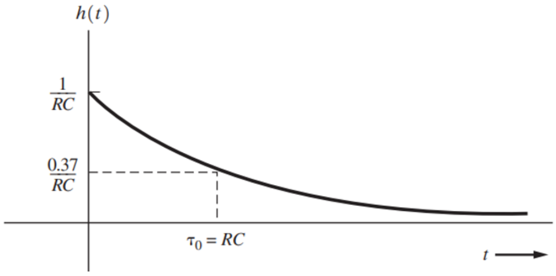
Q8. Show that the set of complex exponential functions $\{e^{jn\omega_0 t}\}$ are orthogonal over the interval a < t < b, where b = a + T_o, $T_o = 1/f_o$, $w_o = 2\pi f_o$ and n is an integer.

Q9. Using FT, find the impulse response of the RC lowpass filter shown here. Sketch the impulse response.



(Answer: $$h(t) = \begin{cases} \frac{1}{\tau_0} e^{-t/\tau_0}, & t \geq 0 \\ 0, & t < 0 \end{cases},$$ 

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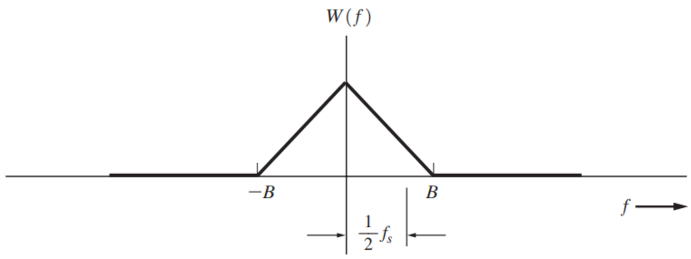
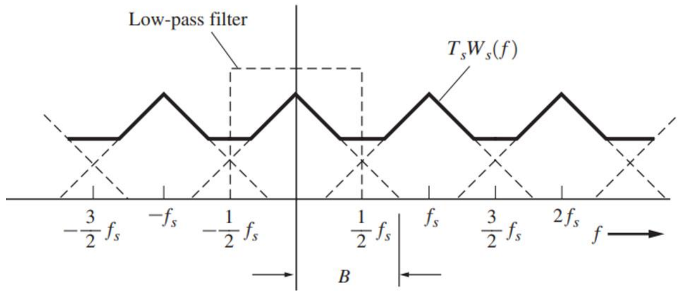
Q10. Given the spectrum of the unsampled waveform shown here, sketch the resulting waveform when sampling is performed with a rate of $f_s < 2B$.





(Answer:)

Q11. Compute the discrete time Fourier Transform for this sequence {1,1,<u>1</u>,1,1} where _represents n = 0. Express your answer as a sum of sines and cosines.

(Answer: $\mathbf{X}(e^{j\Omega}) = 1 + 2\cos(\Omega) + 2\cos(2\Omega)$)