

XMUT310 Communication Engineering

Tutorial 4: Analog to Digital Solutions

Answers are provided for your convenience.

Q1.

Soln. A continuous time signal is sampled at Nyquist rate

$$\text{i.e. } f_s = 2f_m \quad \text{Samples/sec}$$

It can be fully recovered.

The spectrum $F(\omega)$ repeats periodically without overlap. The signal can be recovered by passing sampled signal through a low pass filter with sharp cut-off at frequency f_m .

Option (d)

Q2.

Soln. The given power spectrum has main lobe and other side lobes. We observe that the maximum frequency of main lobe is 1KHz.

practically about 90% of the total signal lies in the major lobe. So maximum frequency of the signal can be taken as 1 KHz.

Minimum sampling rate (Nyquist rate) = $2f_m$

$$\text{Nyquist rate} = 2 \times 1\text{KHz} = 2\text{KHz}$$

Option (b)

Q3.

Soln. This is the case of band pass sampling

$$f_H = 1800 \text{ Hz}$$

$$f_L = 300 \text{ Hz}$$

$$\text{Bandwidth} = f_H - f_L = 1800 - 300 = 1500 \text{ Hz}$$

$$\text{Now, } m = \frac{f_H}{BW} = \frac{1800}{1500} = 1.2, \text{ Integer} = 1.0$$

$$(f_s)_{\min} = \frac{2f_H}{m} = \frac{2 \times 1800}{1} = 3600 \text{ samples/sec}$$

Ans. 3600 samples/sec

Q4.

Soln. For flat top sampling the spectrum is of sinc function, where the amplitude of high frequency component is reduced. This effect is called aperture effect.

As the sampling aperture increases i.e. increase of pulse width ----- there will be more loss in high frequency components.

Option (a)

Q5.

Soln. Given

$$f_m = 1\text{KHz}$$

$$f_s = 1.8\text{ k samples/sec}$$

The frequency components in the sampled signal are

$$nf_s \pm f_m$$

$$\text{For } n = 0, f_m = 1\text{KHz} = 1000\text{Hz}$$

$$n = 1, \text{ frequencies are } 1.8 \pm 1 \text{ i.e. } 800\text{Hz} \text{ \& } 2800\text{ Hz}$$

$$n = 2, \text{ frequencies are } 3.6 \pm 1 \text{ i.e. } 2600\text{Hz} \text{ \& } 4600\text{ Hz}$$

Cutoff frequency of LPF 1100 Hz

So, 800 Hz & 1000 Hz components will appear at the output

Option (c)

Q6.

Soln. Signal is given as

$$\sin c(700t) + \sin c(500t)$$

$$= \frac{\sin(700\pi t)}{700\pi t} + \frac{\sin(500\pi t)}{500\pi t}$$

The maximum frequency component is

$$2\pi f_m = 700\pi$$

$$\text{Or } f_m = 350\text{Hz}$$

$$\text{The Nyquist rate is } f_s = 2f_m$$

$$= 2 \times 350\text{ Hz}$$

$$= 700\text{ Hz}$$

$$\text{Sampling interval } \left(\frac{1}{f_s}\right) = \frac{1}{700}\text{ sec}$$

Option (c)

Q7. Note: $\text{sinc } x = \sin x / x$ or $\text{sinc } x = \sin \pi(x)/\pi(x)$

$\text{Sinc}(200t)$. Remember that the Fourier transform of a sinc function is a rectangle. In this case, the rectangle is bandlimited to 100 Hz. Therefore, the Nyquist frequency is 200 Hz corresponding to a Nyquist interval of 5 ms. $\text{Sinc}(200t) = \sin(\pi 200t)/(\pi 200t)$. So, $\pi 200t = 2\pi \cdot f \cdot t$, so $f = 100$ Hz.

$g(t) = \text{sinc}^2(200t)$: From Fourier transform properties, we know that the Fourier transform of product of two signals in time domain is the convolution of their Fourier transforms in Frequency domains. Therefore, when two sinc functions are multiplied, two rectangles are convolved in the frequency domain. Then, the result of the convolution will be bandlimited to 200 Hz and therefore, the Nyquist frequency and the corresponding Nyquist interval are 400 Hz and 2.5 ms, respectively.

$g(t) = \text{sinc}(200t) + \text{sinc}^2(200t)$: This is similar to the previous part. Due to the terms $\text{sinc}^2(200t)$, the signal will be bandlimited to 400 Hz.

Soln. The signal given is

$$6 \times 10^4 \sin^2(400t) * 10^6 \sin^3(100t)$$

Let the given function is $f(t)$ and

$$f_1(t) = 6 \times 10^4 \sin^2(400t)$$

$$f_2(t) = 10^6 \sin^3(100t)$$

We know that

$$f_1(t) * f_2(t) \Rightarrow F_1(\omega) \cdot F_2(\omega)$$

Convolution in time domains is product in frequency domain

Bandwidth of

$$F_1(\omega) = 2 \times 400 \text{ rad/sec} = 800 \text{ rad/sec or } 400 \text{ Hz}$$

Due to $\sin^2$ term

Bandwidth of

$$F_2(\omega) = 3 \times 100 = 300 \text{ rad/sec or } 150 \text{ Hz}$$

Due to $\sin^3$ term

Then $F_1(\omega) \cdot F_2(\omega)$ will have bandwidth of 150 Hz only (since product of two spectrums)

$$\text{Sampling frequency is } 2 \times 150 \text{ Hz} = 300 \text{ Hz}$$

Option (b)

Q8.

Soln. Four messages are sampled at the Nyquist rate

$$\text{I - message } f_{s1} = 200 \text{ Hz}$$

II – message $f_{s2} = 200Hz$

III – message $f_{s3} = 400Hz$

IV – message $f_{s4} = 800Hz$

So there are 1600 samples in 1 sec

So, the speed of commutator is 1600 samples per sec

Option (a)



Q9.

Soln. Signal frequency $\frac{24\pi \times 10^3}{2\pi} = 12 \text{ KHz}$

Sampling period $T_s = 50 \mu\text{sec}$

So $f_s = \frac{1}{T_s} = \frac{1}{50 \times 10^{-6}} = 20 \text{ KHz}$

After sampling, signal will have frequency f_m and $f_s \pm f_m$

i.e. 12 KHz and $20 \text{ KHz} - 12 \text{ KHz} = 8 \text{ KHz}$

so, after filter the frequency present will be 12 KHz and 8 KHz

Option (d)

Note that sampling frequency (20 KHz) is less than $2 f_m$ (24 KHz) Nyquist rate, so there will be aliasing error.

Q10.

Soln. Given $T = 10^{-3}$ so frequency $= \frac{1}{10^{-3}} = 1 \text{ KHz}$ of pulse train.

To determine frequency components in pulse train we find Fourier series Coefficient C_n

$$C_n = \frac{1}{T_0} \int_{-T_0/6}^{T_0/6} A e^{-jn\omega_0 t} dt = \frac{1}{T_0} \cdot \frac{A}{(-jn\omega_0)} [e^{-jn\omega_0 t}]_{-T_0/6}^{T_0/6}$$
$$= \frac{A}{T_0(-jn\omega_0)} [e^{-jn\omega_0 T_0/6} - e^{jn\omega_0 T_0/6}]$$

$$C_n = \frac{A \cdot 2 \cdot j}{T_0(jn\omega_0)} \sin\left(\frac{n\pi}{3}\right) = \frac{A}{n\pi} \sin\left(\frac{n\pi}{3}\right)$$

From C_n we see that harmonics present are 1,2,3,4,5,7-----

So $p(t)$ has 1 KHz, 2 KHz, 4 KHz-----

The signal has frequency components 0.4 KHz and 0.7 KHz

The sampled signal will be $x(t) \times p(t)$ will have

$$1 \pm 0.4 \quad \text{and} \quad 1 \pm 0.7 \text{ KHz}$$

$$2 \pm 0.4 \quad \text{and} \quad 2 \pm 0.7 \text{ KHz}$$

$$3 \pm 0.4 \quad \text{and} \quad 3 \pm 0.7 \text{ KHz etc}$$

In the frequency range of 2.5 KHz to 3.5 KHz

The frequency present are

$$2 + 0.7 = 2.7 \text{ KHz}$$

$$4 - 0.7 = 3.3 \text{ KHz}$$

Option (d)

Q11.

Soln. Given

$$\begin{aligned} s(t) &= \frac{\sin(500\pi t)}{\pi t} \times \frac{\sin(700\pi t)}{\pi t} \\ &= \frac{1}{2} \left[\frac{2 \sin(500\pi t) \cdot \sin(700\pi t)}{\pi^2 t^2} \right] \\ &= \frac{1}{2} [\cos(700\pi t - 500\pi t) - \cos(700\pi t + 500\pi t)] \\ &= \frac{1}{2\pi^2 t^2} [\cos(200\pi t) - \cos(1200\pi t)] \end{aligned}$$

Maximum frequency component

$$f_m = \frac{1200\pi}{2\pi} = 600 \text{ Hz}$$

Nyquist sampling rate

$$f_{s_{min}} = 2f_m = 1200 \text{ Hz}$$

Option (c)

Q12.

Soln. Given,

Dynamic range or voltage range = 0 to 8 V

Number of levels = 16

Maximum quantization error

$$Q_e = \frac{\text{step size } (\Delta)}{2}$$

Where

$$\Delta = \frac{\text{Dynamic range}}{L}$$

$$= \frac{8}{2^4} = \frac{8}{16} = 0.5$$

$$Q_e = \frac{0.5}{2} = 0.25 \text{ V}$$

Quantization error is 0.25 V

Q13.

Soln. In PCM, Companding results in making SNR uniform irrespective of signal amplitude level.

In formed from two words ‘compressing’ and ‘expanding’

In PCM, analog signal values are rounded on a non-linear scale. The data is compressed before it is sent and then expanded at the receiving end using same non-linear scale.

So right option is compression of higher signal amplitudes

Option (a)

Q14.

Soln. Signal to quantization noise in an n bit PCM is

$$\left(\frac{S}{N_q}\right) = (SQNR) = \frac{3}{2} \cdot 2^{2n} \text{ --- (1A)}$$

Where n is number of bits in the word of binary PCM.

It can be written in dBs

$$(SQNR)_{dB} = \left(\frac{S}{N_q}\right)_{dB} = 1.76 + 6n \text{ --- (1B)}$$

From above equations (S/N) increases with n

Option (c)

Q15.

Soln. Note $\left(\frac{S}{N_q}\right)_{dB} = (1.76 + 6n)_{dB}$

$$(SQNR)_1 = 1.76 + 6n$$

$$(SQNR)_2 = 1.76 + 6n(n+1) = 1.76 + 6n + 6$$

$$(SQNR)_2 - (SQNR)_1 = 1.76 + 6n + 6 - 1.76 - 6n = 6dB$$

So for every one bit increase in bits per sample will result is 6 dB improvement in signal to quantization ratio

Option (b)

Q16.

Soln. $(Bandwidth)_{PCM} = nf_m$

Where n – number of bits in PCM code

f_m – signal bandwidth

$$n = \log_2 L$$

$$n_1 = \log_2 4 = 2$$

$$n_2 = \log_2 64 = 6$$

$$(BW)_1 = n_1 f_m = 2f_m$$

$$(BW)_2 = n_2 f_m = 6f_m$$

$$\frac{(BW)_2}{(BW)_1} = \frac{6f_m}{2f_m} = 3 \text{ times}$$

$$(BW)_2 = 3(BW)_1$$

So, increase is 3 times

Q17.

Soln. $SQNR = \frac{3}{2} 2^{2n}$

For $n_1 = n$

$$n_2 = n + 1$$

$$(SQNR)_1 = \frac{3}{2} 2^{2n}$$

$$(SQNR)_2 = \frac{3}{2} 2^{2(n+1)} = \frac{3}{2} 2^{2n+2} = \frac{3}{2} [2^{2n} \cdot 2^2]$$

$$\frac{(SQNR)_2}{(SQNR)_1} = \frac{2^2}{1} = 4$$

So, $(SQNR)_2 = 4(SQNR)_1$

Note that signal to quantization noise increases by factor of 4. So this is improvement in SQNR is independent of n

Option (d)

Q18.

Soln. Quantization noise in PCM is given by

$$N_q = \frac{\Delta^2}{12}$$

$$\text{stepsize } (\Delta) = \frac{\text{voltage range}}{2^n} = \frac{V_{p-p}}{2^n}$$

$$N_q = \frac{V_{p-p}^2}{12 \times 2^{2n}} \quad \text{so, } N_q \propto \frac{1}{2^{2n}}$$

$$\frac{(N_q)_2}{(N_q)_1} = \frac{2^{2n_1}}{2^{2n_2}} = \frac{2^{2 \times 8}}{2^{2 \times 9}} = \frac{2^{16}}{2^{18}} = \frac{1}{2^2} = \frac{1}{4}$$

$$\text{or } (N_q)_2 = \frac{(N_q)_1}{4}$$

Quantization noise reduces by a factor of 4

Option (c)

Q19.

Soln. Given

$$V_{p-p} = 2 \text{ volts}$$

No. of bits = 8

$$n = \log_2 L$$

$$\text{Or } 8 = \log_2 L \text{ or } L = 2^7 = 128 \text{ levels}$$

$$\left(\frac{S}{N_q} \right)_{dB} = 1.76 + 6n$$

$$= 1.76 + 6 \times 7 = 43.8 \text{ dB}$$

Option (b)

Q20.

Soln.

Given sampling rate = 8 KHz

$$\text{Then bit rate} = nf_s = 8 \times 8\text{kHz} = 64 \text{ kbps}$$

$$SNR_q = (1.76 + 6.02 \cdot n) \text{ dB}$$

$$= 1.76 + 6.02 \times 8$$

$$= 49.8 \text{ dB}$$

Option (b)

Q21.

Soln. Given

$$\text{Signaling frequency (f)} = 3500 \text{ Hz}$$

$$\text{Excess bandwidth used is } B = 0.75 \times 3500 = 2625 \text{ Hz}$$

We know that

$$\text{Bandwidth} \geq \frac{R_b}{2}$$

Where R_b is the data rate

$$\text{Minimum bandwidth required (B)} = \frac{R_b}{2}$$

$$\text{or, } B = \frac{R_b}{2}$$

$$\text{or, } R_b = 2B = 2 \times 2625 = 5250 \text{ Hz}$$

Option (d)

Q22.

Soln. Signal is band limited to 4 KHz

$$\text{Nyquist rate} = 2 \times f_m = 8\text{KHz}$$

$$\text{Levels} = 4 \text{ i.e. } 2^n = L \text{ or } n = 2$$

The number of bits = 2

Each sample requires 2 bits.

Two samples per second are transmitted so, the number of bits per second

$$2 \times 2 = 4 \text{ bits/sec}$$

Option (d)