

XMUT310 Mock Exam 2025 T2

Solutions

Question 1: Digital Baseband and BER

a) Determining the New BER

Step 1: Identify the correct formula.

For a binary Polar NRZ scheme, the Bit Error Rate (BER) is given by:

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

where E_b is the energy per bit and $N_0/2$ is the two-sided noise power spectral density.

Step 2: Find the original argument of the Q-function.

We are given that the original $P_e = 10^{-6}$. From the table, we see that $Q(4.76) \approx 10^{-6}$. Therefore:

$$\sqrt{\frac{2E_b}{N_0}} = 4.76$$

Squaring both sides:

$$\frac{2E_b}{N_0} = (4.76)^2 \approx 22.59$$

Step 3: Account for the doubled signal power.

Signal power S is related to E_b by $S = E_b/T$, where T is the bit duration. If the signal power is doubled ($S' = 2S$), then the new energy per bit $E'_b = 2E_b$, assuming T remains constant.

Step 4: Find the new argument and the new BER.

The new argument for the Q-function becomes:

$$\sqrt{\frac{2E'_b}{N_0}} = \sqrt{\frac{2(2E_b)}{N_0}} = \sqrt{2 \cdot \frac{2E_b}{N_0}} = \sqrt{2 \times 22.59} = \sqrt{45.18} \approx 6.722$$

We now find $Q(6.722)$, where the new P_e' will be 0.9×10^{-11} . We can state that the new BER is **significantly better than** 10^{-8} .

b) Conclusion

The calculation shows that increasing the transmitted signal power dramatically improves the Bit Error Rate. This is because the argument of the Q-function, $\sqrt{2E_b/N_0}$, increases with the square root of the power, leading to an exponential decrease in P_e . This highlights a fundamental trade-off between power and performance in digital communication systems.

Question 2: BER Performance of Digital Modulations

a) Explanation of the BER Plot

- i) **Fixed BER of 10^{-5} :** To achieve a BER of 10^{-5} , BPSK requires the least E_b/N_0 , followed closely by QPSK, and then 16-PSK requires the highest E_b/N_0 . This shows that for a given quality of service (BER), BPSK is the most power-efficient.
- ii) **Fixed E_b/N_0 of 10 dB:** At 10 dB, BPSK and QPSK will have the lowest (best) BER, while 16-PSK will have a significantly higher (worse) BER. This shows that for a given power budget, BPSK/QPSK provide better reliability.

b) Curve Identification and Reasoning

1. **Curve with best performance (leftmost):** BPSK and QPSK
2. **Curve overlapping/very close to BPSK:** 8-PSK.
3. **Curve with worst performance (rightmost):** 16-PSK.

Reasoning: For a given BER, BPSK requires the least E_b/N_0 because it has the smallest constellation and greatest distance between constellation points for a given power. QPSK has the same power efficiency as BPSK (the distance between constellation points is identical for the same energy per symbol, and $E_b = E_s/2$). 16-PSK has a more crowded constellation, requiring higher power to maintain the same minimum distance and thus the same BER.

c) Identical Performance of BPSK and QPSK

BPSK and QPSK have the same relationship between bit energy E_b and the distance between constellation points. The BER for both is primarily determined by the probability of mistaking a symbol for its nearest neighbor. For the same E_b/N_0 , this probability is identical for both BPSK and QPSK, leading to nearly identical BER curves. QPSK is essentially two orthogonal BPSK streams.

d) Comparison of 16-QAM and BPSK

- **Bandwidth Efficiency:** 16-PSK is more bandwidth-efficient. It transmits 4 bits per symbol, while BPSK transmits 1 bit per symbol. For the same symbol rate, 16-PSK has four times the bit rate of BPSK.
 - **Error Performance:** BPSK has superior error performance (power efficiency). For example, at a BER of 10^{-4} , let's assume from the plot that BPSK requires ~ 8.5 dB, while 16-PSK requires ~ 12.5 dB. Therefore, 16-PSK requires approximately **4 dB more** E_b/N_0 than BPSK to achieve the same BER. This demonstrates the classic trade-off: higher data rates (16-PSK) come at the cost of requiring more power for the same reliability.
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Question 3: Pulse Code Modulation and Line Coding

a) PCM Levels and SQNR

Step 1: Relate SQNR to the number of bits.

The Signal-to-Quantization-Noise Ratio (SQNR) for a PCM system is approximately:

$$\text{SQNR} \approx (6.02n + 1.76) \text{ dB}$$

where n is the number of bits per sample.

Step 2: Find the minimum number of bits n .

We require $\text{SQNR} \geq 40$ dB.

$$\begin{aligned} 6.02n + 1.76 &\geq 40 \\ 6.02n &\geq 38.24 \\ n &\geq \frac{38.24}{6.02} \approx 6.35 \end{aligned}$$

Since n must be an integer, we choose $n = 7$ bits.

Step 3: Calculate the exact SQNR using $n = 7$.

$$\text{SQNR} \approx 6.02 \times 7 + 1.76 = 42.14 + 1.76 = 43.9 \text{ dB}$$

b) Line Code Waveforms

For the sequence 1 0 1 1 0 0 1 0:

- **Unipolar RZ (50% duty cycle):** '1' is a positive pulse that returns to zero halfway through the bit period. '0' is no pulse (zero voltage).
- **Manchester code:** '1' is a low-to-high transition in the middle of the bit period. '0' is a high-to-low transition in the middle of the bit period.
- **Polar NRZ:** '1' is a positive voltage (+V) for the entire bit period. '0' is a negative voltage (-V) for the entire bit period.

(A hand-drawn waveform diagram would be inserted here, showing the three distinct patterns for the given sequence. SEE lecture ppt.)

Question 4: Maximum Bit Rate Calculation

Step 1: Identify the correct BER formula.

For Unipolar NRZ signaling, the average probability of error is:

$$P_e = Q\left(\sqrt{\frac{E_b}{N_0}}\right)$$

We are given $P_e \leq 10^{-5}$.

Step 2: Find the required E_b/N_0 .

From the Q table, we find that at $P_e \leq 10^{-5}$, corresponds to Q(4.27). Therefore:

$$\sqrt{\frac{E_b}{N_0}} = 4.27$$

$$\frac{E_b}{N_0} = (4.27)^2 \approx 18.19$$

Step 3: Express E_b in terms of bit rate R_b .

Energy per bit $E_b = P_s T = \frac{P_s}{R_b}$, where P_s is the average signal power.

For unipolar NRZ (+2V for '1', 0V for '0'), and equally likely bits, the average power is:

$$P_s = \frac{1}{2}(2^2) + \frac{1}{2}(0^2) = \frac{1}{2} \times 4 = 2 \text{ Watts}$$

We are given the noise PSD: $N_0/2 = 5 \times 10^{-6}$, so $N_0 = 1 \times 10^{-5}$ W/Hz.

Step 4: Solve for the maximum bit rate R_b .

We have:

$$\frac{E_b}{N_0} = \frac{P_s/R_b}{N_0} = 18.19$$

$$\frac{2/R_b}{1 \times 10^{-5}} = 18.19$$

$$\frac{2}{R_b} = 18.19 \times 1 \times 10^{-5} = 1.819 \times 10^{-4}$$

$$R_b = \frac{2}{1.819 \times 10^{-4}} \approx 10,995 \text{ bits per second (bps)}$$

Therefore, the maximum bit rate is approximately 11 kbps.