

XMUT315 Control Systems Engineering

Assignment 2: Stability and Time Domain Analysis

Due Date: Friday, 16th May 2025 (online submission to XMUT315 wiki website at VUW)

A. Stability Analysis

- Using the Routh-Hurwitz criterion, analyse the stability of the unity-feedback system given in the figure below.

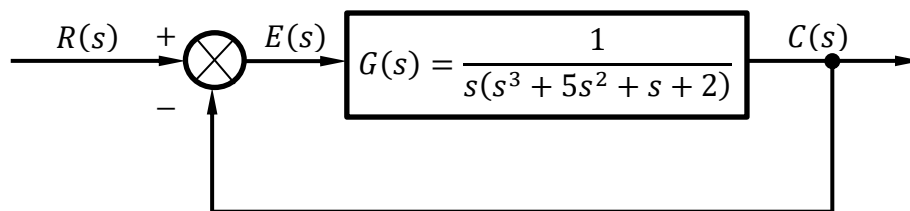


Figure 1: Block diagram of a unity-feedback system

- Determine whether the closed-loop system is stable or not? [10 marks]
- If the system is not stable, describe two reasons that could cause it. [5 marks]
- Suggest at least two appropriate solutions for improving the stability of the system. [5 marks]

B. Transient Response Analysis

- For each of the second-order systems listed below, find its damping ratio, natural frequency (in rads), rise time, time-to-peak, percentage overshoot, and settling time.

- System 1: [10 marks]

$$G_1(s) = \frac{16}{s^2 + 3s + 16}$$

- System 2: [10 marks]

$$G_2(s) = \frac{0.04}{s^2 + 0.02s + 0.04}$$

- c. System 3: [10 marks]

$$G_3(s) = \frac{1.05 \times 10^7}{s^2 + 1.6 \times 10^3 s + 1.05 \times 10^7}$$

3. For each of the transfer functions shown below, perform the following tasks.

i. System 1:

$$G_1(s) = \frac{20}{s^2 + 6s + 144}$$

ii. System 2

$$G_2(s) = \frac{s + 2}{s^2 + 9}$$

iii. System 3

$$G_3(s) = \frac{(s + 5)}{(s + 10)^2}$$

- a. Find the locations of the poles and zeros, plot them on the s-plane, and then write an expression for the general form of the step response without solving for the inverse Laplace transform. State the nature of each response (e.g. overdamped, underdamped, and so on).

[10 marks]

- b. Calculate the exact response of each system using inverse Laplace transform techniques and compare the results to those obtained in that problem.

[10 marks]

- c. Find the damping ratio and natural frequency for each system and show that the value of the damping ratio conforms to the type of response (e.g. underdamped, overdamped, and so on) predicted in that problem.

[10 marks]

C. Steady-State Analysis

4. Given the system in the figure below, find the following:

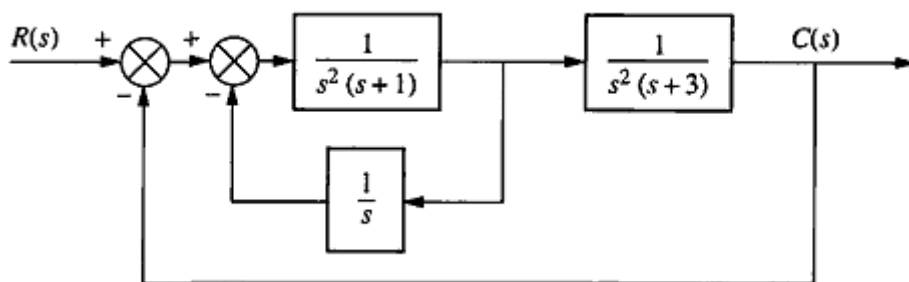


Figure 2: Block diagram of the system

- a. The closed-loop transfer function. [5 marks]
- b. The system type. [2.5 marks]
- c. The steady-state error for an input of $5u(t)$. [5 marks]
- d. The steady-state error for an input of $5tu(t)$. [5 marks]
- e. Discuss the validity of your answers to parts (c) and (d). [2.5 marks]

Marking Schedule

Student ID : _____

Student Name : _____

No	Description	Mark	Your Mark	Remarks
A	Stability Analysis			
1a	Closed-loop system is stable or not?	10		
1b	Two reasons that could cause instability in the system.	5		
1c	Two solutions for improving stability of the system.	5		
B	Transient Response Analysis			
2a	The time-response parameters of system 1.	10		
2b	The time-response parameters of system 2.	10		
2c	The time-response parameters of system 3.	10		
3a	Locations of poles and zeros, s-plane plot, expression for step response and nature of response of system 1.	10		
3b	Locations of poles and zeros, s-plane plot, expression for step response and nature of response of system 2.	10		
3c	Locations of poles and zeros, s-plane plot, expression for step response and nature of response of system 3.	10		
C	Steady-State Analysis			
4a	Closed-loop transfer function.	5		
4b	System type.	2.5		

4c	Steady-state error for input $5u(t)$.	5		
4d	Steady-state error for input $5tu(t)$.	5		
4e	Discussion on the validity of answers in parts (c) and (d).	2.5		
	Total	100		

Comment: