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Demo 1a: Simulations in MATLAB/Simulink

XMUT315 Control Systems Engineering

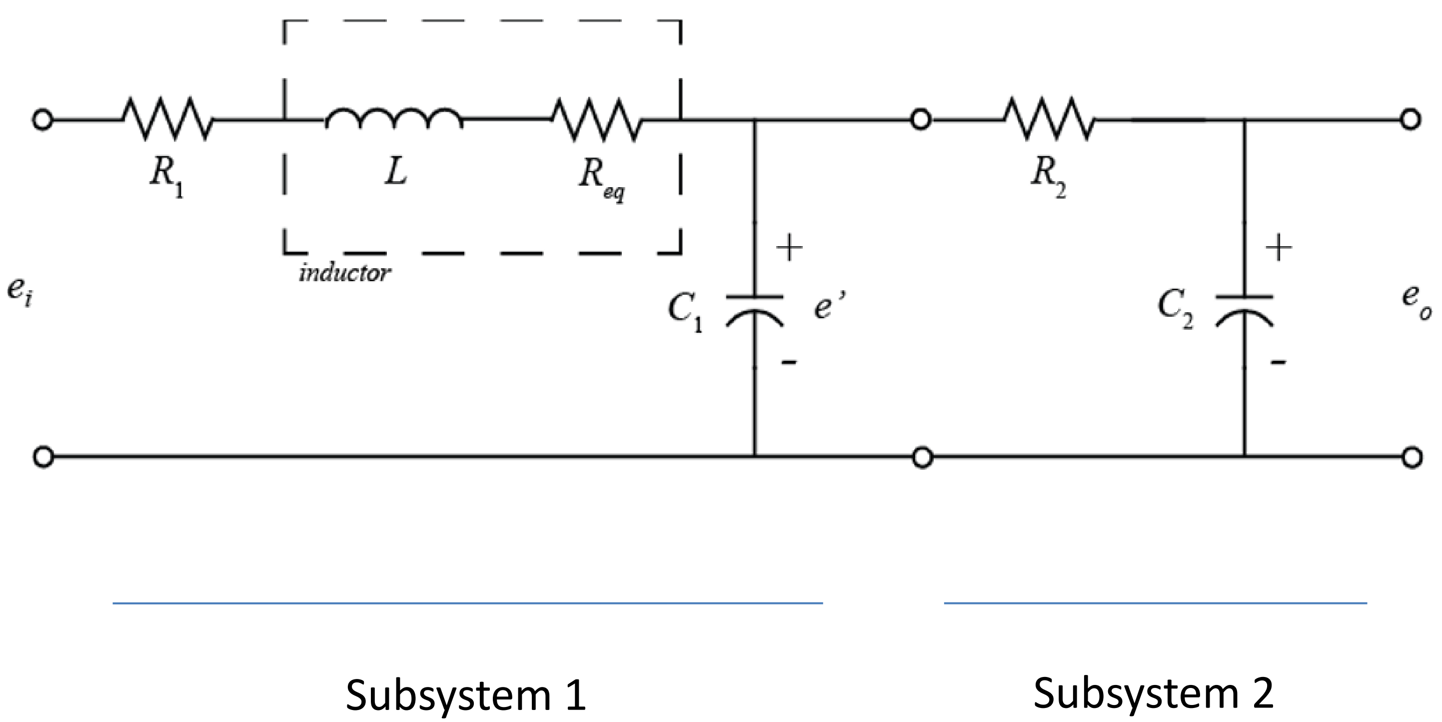
Topics

1. Simulation of electrical systems in MATLAB.
2. Simulation of mechanical systems in MATLAB.
3. Simulation of mechanical systems with differential equation in time domain in Simulink.
4. Simulation of mechanical systems with transfer function in Simulink.
5. Simulation of electromechanical systems in Simulink.

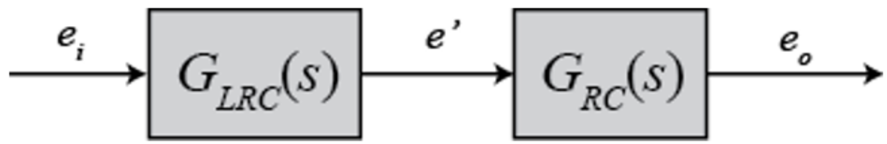
A. Simulation Examples in MATLAB

Experiment 1 (Simulation of Electrical Systems)

You are given the following electronic circuit that consists of series connected LRC circuit and RC circuit.



1. The following transfer function models for the LRC circuit and the RC circuit when the circuit considered separately as subsystems.



Here we introduce an intermediate variable e' representing the voltage drop across the capacitor of the LRC circuit.

$$G_{LRC}(s) = \frac{E'(s)}{E_i(s)} = \frac{1}{LC_1 s^2 + \left(R_1 + R_{eq}\right)C_1 s + 1}$$

$$G_{RC}(s) = \frac{E_o(s)}{E'(s)} = \frac{1}{R_2 C_2 s + 1}$$

2. When modelled as transfer functions, one generally combines systems in series to make up a whole system by multiplying their transfer functions.

3. Throughout the experiment the values of components in the LRC circuit are: $R_1 = 10\ \Omega$, $L = 1$ H, $E_{eq} = 40\ \Omega$, $C_1 = 510$ µF.

For the RC circuit, we have the following component values: R_2 = 10kΩ and $C_2 = 100$ µF.

```
R1 = 10;           % resistance of resistor in LRC circuit
R2 = 10000;      % resistance of resistor in RC circuit
Req = 40;    % inductor equivalent series resistance (ESR)
L = 1;            % inductance of inductor
C1 = 510*10^-6;  % capacitance of capacitor in LRC circuit
C2 = 100*10^-6;  % capacitance of capacitor in RC circuit
ei = 1.53;           % input voltage
tstep = 1.67;        % time step occurred
```

4. We use the MATLAB command step that occurs at time $t = 0$ seconds (also for step appears to occur at time equal to 1.67 seconds). The following graph shows the result of circuit simulation.

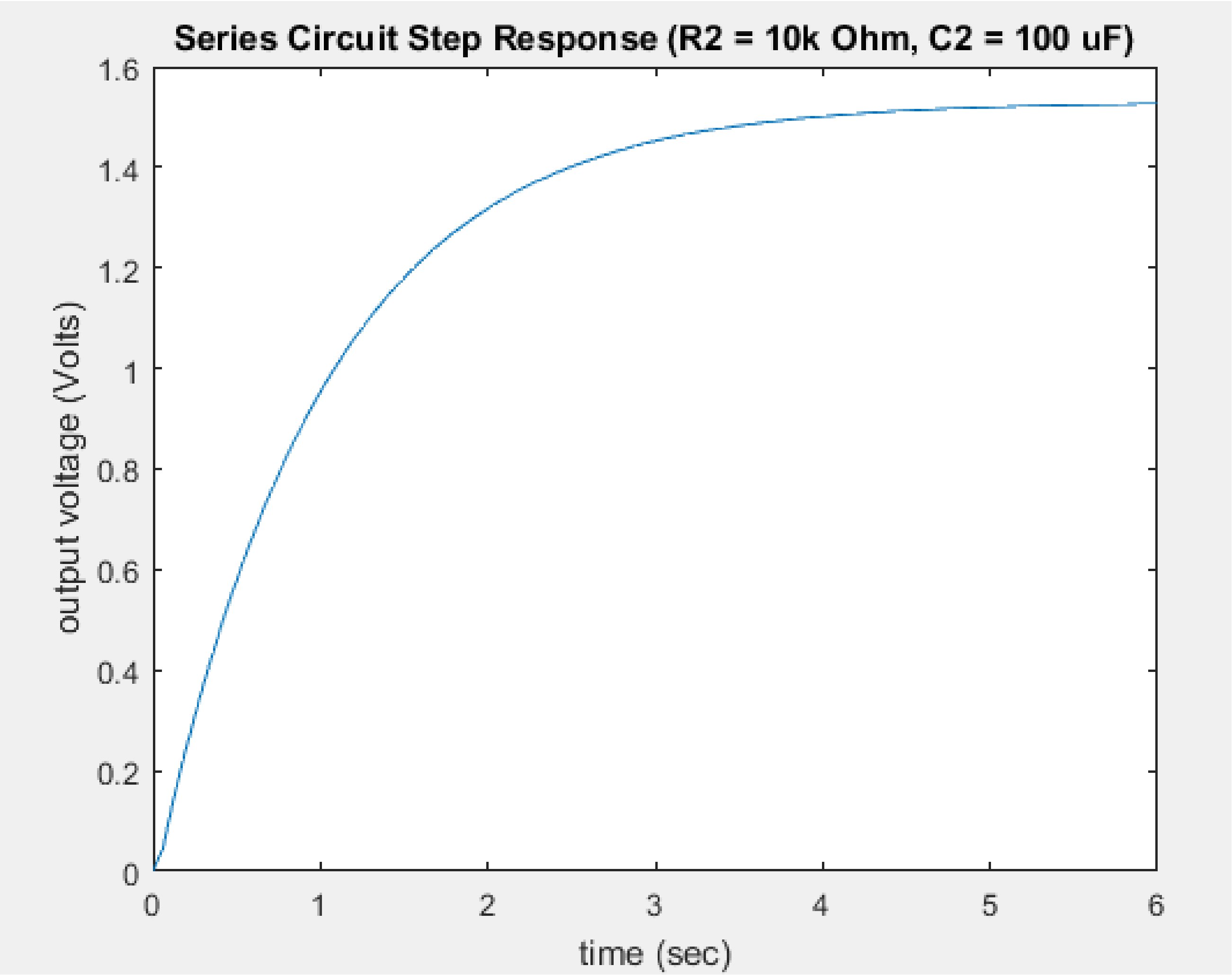
```
G1 = 1/(C1*L*s^2 + C1*(R1+Req)*s + 1);  % LRC Transfer Function
G2 = 1/(C2*R2*s+1);    % RC transfer function
G = G1*G2;     % series transfer function

[y,t] = step(G*ei,6);  % model step response

plot(t,y)
```

MATLAB code for the given electrical system:

```
s = tf('s');

R1 = 10;          % resistance of resistor in LRC circuit
R2 = 10000;      % resistance of resistor in RC circuit
Req = 40;    % inductor equivalent series resistance (ESR)
L = 1;           % inductance of inductor
C1 = 510*10^-6;  % capacitance of capacitor in LRC circuit
C2 = 100*10^-6;  % capacitance of capacitor in RC circuit
ei = 1.53;          % input voltage
tstep = 1.67;       % time step occurred
```

```
G1 = 1/(C1*L*s^2 + C1*(R1+Req)*s + 1);  % LRC TF
G2 = 1/(C2*R2*s+1);    % RC transfer function
G = G1*G2;     % series transfer function

[y,t] = step(G*ei,6);  % model step response

plot(t,y)

xlabel('time (sec)')
ylabel('output voltage (Volts)')
title('Series Circuit Step Response(R2=10kOhm,C2=100uF)')
```

Plot the output of the simulation of the response of the system in the time domain in MATLAB.

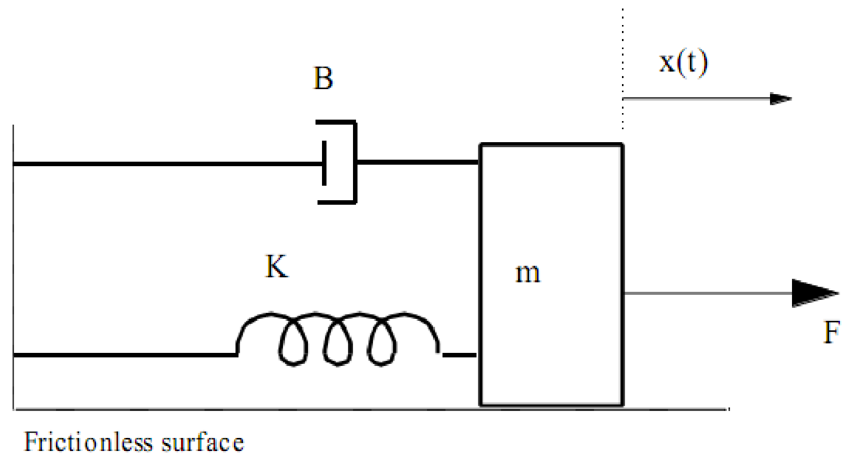


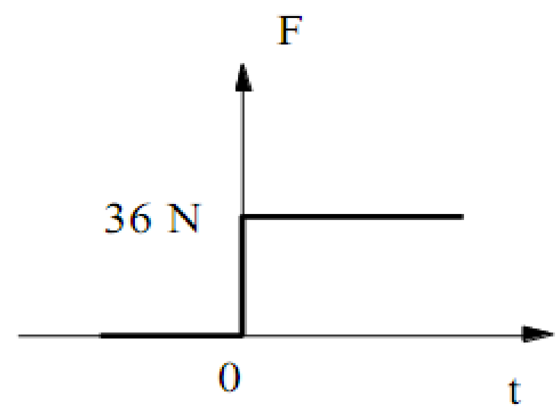
Experiment 2 (Simulation of Mechanical Systems)

For the mass-spring-damper system shown, determine the expression for the motion, $x(t)$ and plot it in MATLAB.

Given that K = 12 N/m, M = 4 kg, F = 36 N and the three examples of damping constant of 24.33 Ns/m, 13.8565 Ns/m, and 8 Ns/m.

Assume zero initial conditions e.g. $x(0) = 0$ and $\dot{x}(0) = 0$





1. Assuming the equilibrium equations and apply Newton law into the mechanical system:

$$F = M\ddot{x} + Kx + B\dot{x}$$

2. Rearranging the equation to become a proper equation (equation (1)):

$$\ddot{x} + \frac{B}{M}\dot{x} + \frac{K}{M}x = F/M$$

3. The homogenous (natural) solution:

$$\ddot{x} + \frac{B}{M}\dot{x} + \frac{K}{M}x = 0$$

4. The characteristic equation of the system by taking the Laplace transform of the above differential equation:

$$s^2X(s) + \frac{M}{M}sX(s) + \frac{K}{M} = 0$$

5. For the above equation, obtain the roots of the equation from (equation (2)):

$$s_{1,2} = \frac{B}{2M} \pm \frac{1}{2}\sqrt{\frac{B^2}{M^2} - \frac{4K}{M}}$$

Case 1: $B = 24.33$ Ns/m (overdamped case)

1. Entering the values $B = 24.33$ Ns/m, K = 12 N/m, M = 4 kg, F = 36 N into the equation (2) above:

$$s_1 = -5.54 \qquad \text{and} \qquad s_2 = -0.54$$

2. Substituting into equation (1) as a result:

$$x(t) = C \qquad \text{and} \qquad C = 3$$

3. Applying inverse Laplace transform

$$x(t) = A_1 e^{-5.54t} + A_2 e^{-0.54t} + 3u(t)$$

4. With $x(0) = 0$ and $\dot{x}(0) = 0$, the two simultaneous equations are

$$A_1 + A_2 = -3$$

$$-5.54A_1 - 0.54A_2 = 0$$

5. Solving for A_1 and A_2 yields

$$A_2 = 0.324 \qquad \text{and} \qquad A_2 = -3.324$$

6. Therefore, this is true for $t \geq 0$:

$$x(t) = 0.324e^{-5.54t} - 3.324e^{-0.54t} + 3$$

Case 2: $B = 13.8565$ Ns/m (critical damped)

1. For value of $B = 13.8565$ Ns/m and other values as per above into the equation (2) as before.

$$s_1 = s_2 = -1.7321$$

2. As the same initial condition as per above still apply then the result of inverse Laplace transform is (for $t \geq 0$):

$$x(t) = -e^{-1.732t}(3 + 5.1963t) + 3$$

Case 3: $B = 8$ Ns/m (underdamped case)

1. For value of $B = 13.8565$ Ns/m and other values as per above into the equation (2) as before.

$$s_{1.2} = -1 \pm j\sqrt{2}$$

2. And the solution is:

$$x(t) = e^{-t}(A_1 \cos(1.414t) + A_2 \sin(1.414t)) + 3u(t)$$

3. This leave the result of the inverse Laplace transform of:

$$x(t) = -3e^{-t}\left(\cos\sqrt{2}t + \sin\sqrt{2}t\right) + 3$$

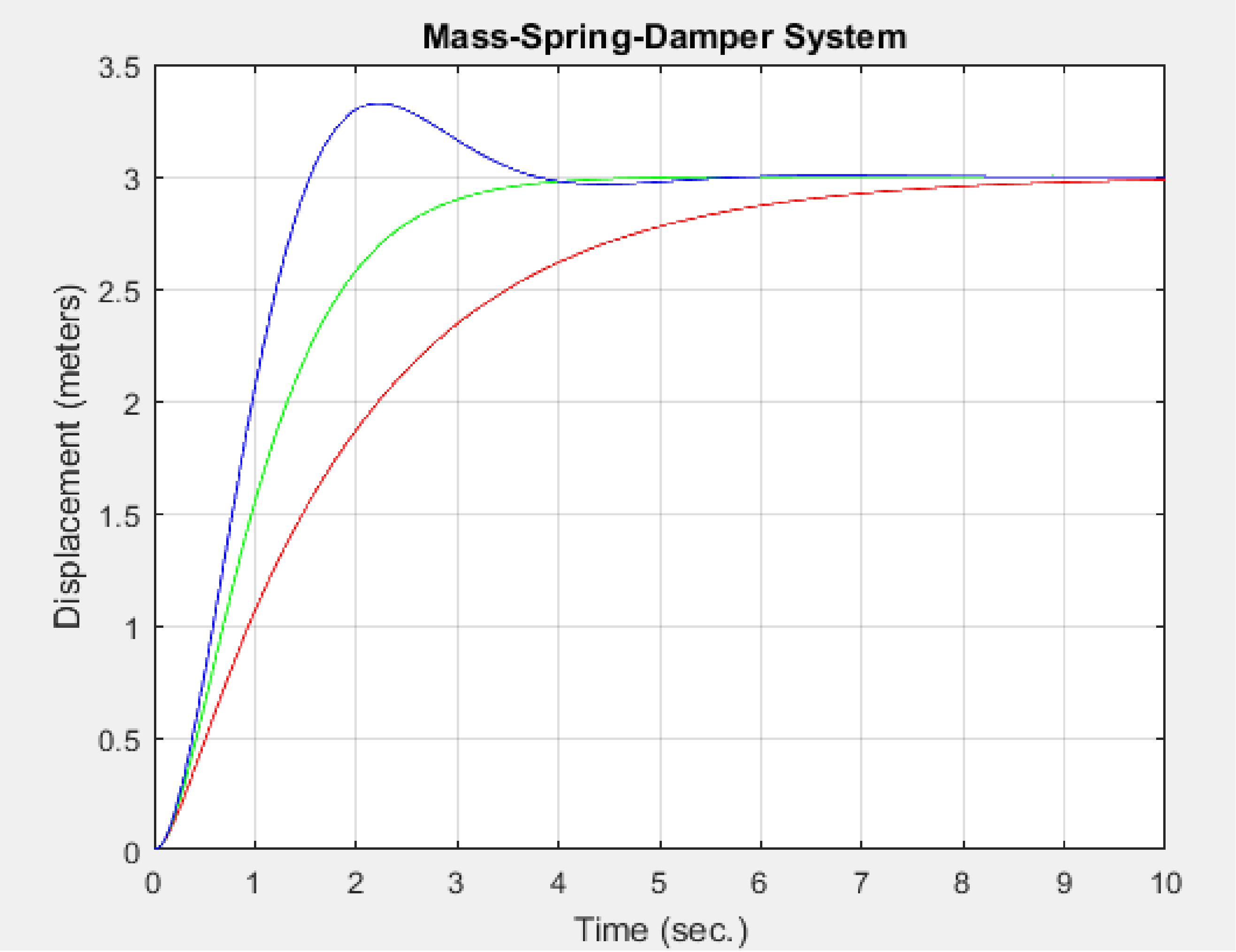
Putting the three responses together is shown below along with the Matlab code.

MATLAB code for the given mechanical system:

```
clear all

F = 36; % Newton (Applied force)
K = 12; % N/m (Spring Constant)
m = 4; % Kg (Mass)

% All cases, B is a vector:

% empty vectors for all cases of x(t)
% X1: case 1, X2: case -2, X3: case -3;
X1=[]; X2=[]; X3=[];
T=[];
```

```
for t = 0:0.01:10;
    x1 = 0.3240*exp(-5.54*t) -3.3240*exp(-0.54*t)+3;
    x2=-(3+5.1963*t)*exp(-1.7321*t)+3;
    x3=-3*exp(-t)*(cos(sqrt(2)*t)+1/sqrt(2)*sin(sqrt(2)*t))+3;
    X1=[X1 x1];
    X2=[X2 x2];
    X3=[X3 x3];
    T=[T t];
end

plot(T,X1,'r',T,X2,'g',T,X3,'b')
xlabel('Time (sec.)')
ylabel('Displacement (meters)')
title('Mass-Spring-Damper System')

grid
```

Plot the output of the simulation of the response of the system in the time domain in MATLAB.



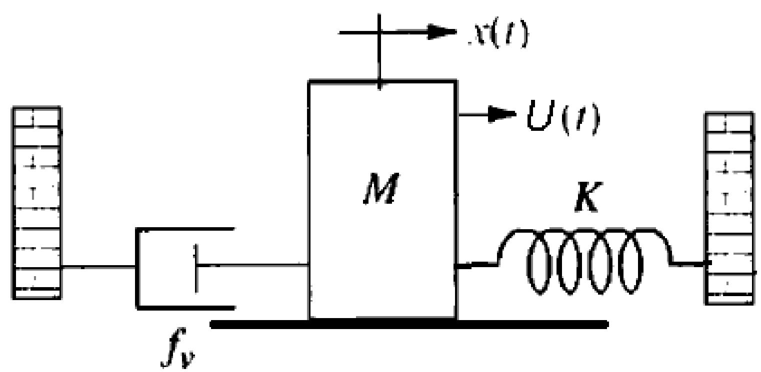
B. Simulation Examples in Simulink

Experiment 1 (Simulation of Mechanical System with Differential Equation in Time Domain)

This experiment is a basic guide to show some of the functions Simulink offers.

You are encouraged to have play around with the model until you feel comfortable with adding blocks, searching the library, creating a subsystem with a mask and running a model to obtain an output.

We will create a model of a damper-spring mechanical system below in the time domain in Simulink.

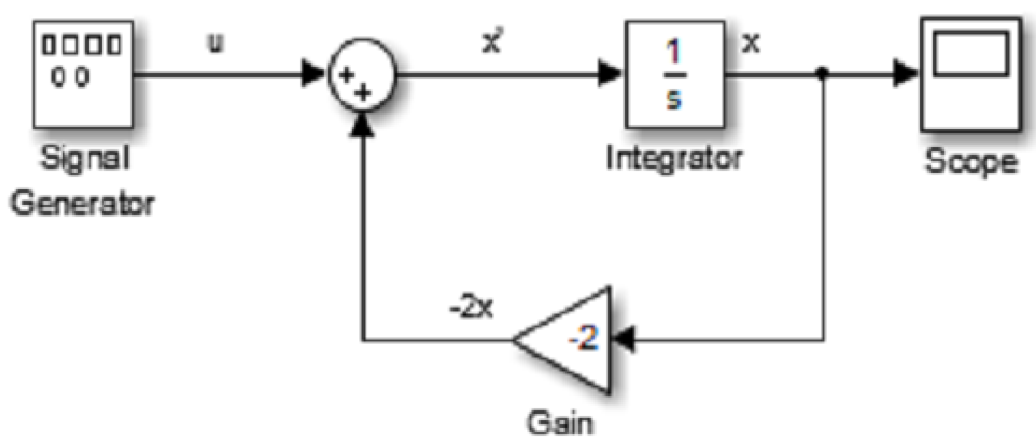


Providing that spring constant K = 2 N/m (force in the spring, $f(t) = Kx(t)$), damper constant f_v = 1 Ns/m (force in the damper, $f(t) = f_v\,\dot{x}(t)$), and assuming frictionless floor, the above system is represented as the following differential equation:

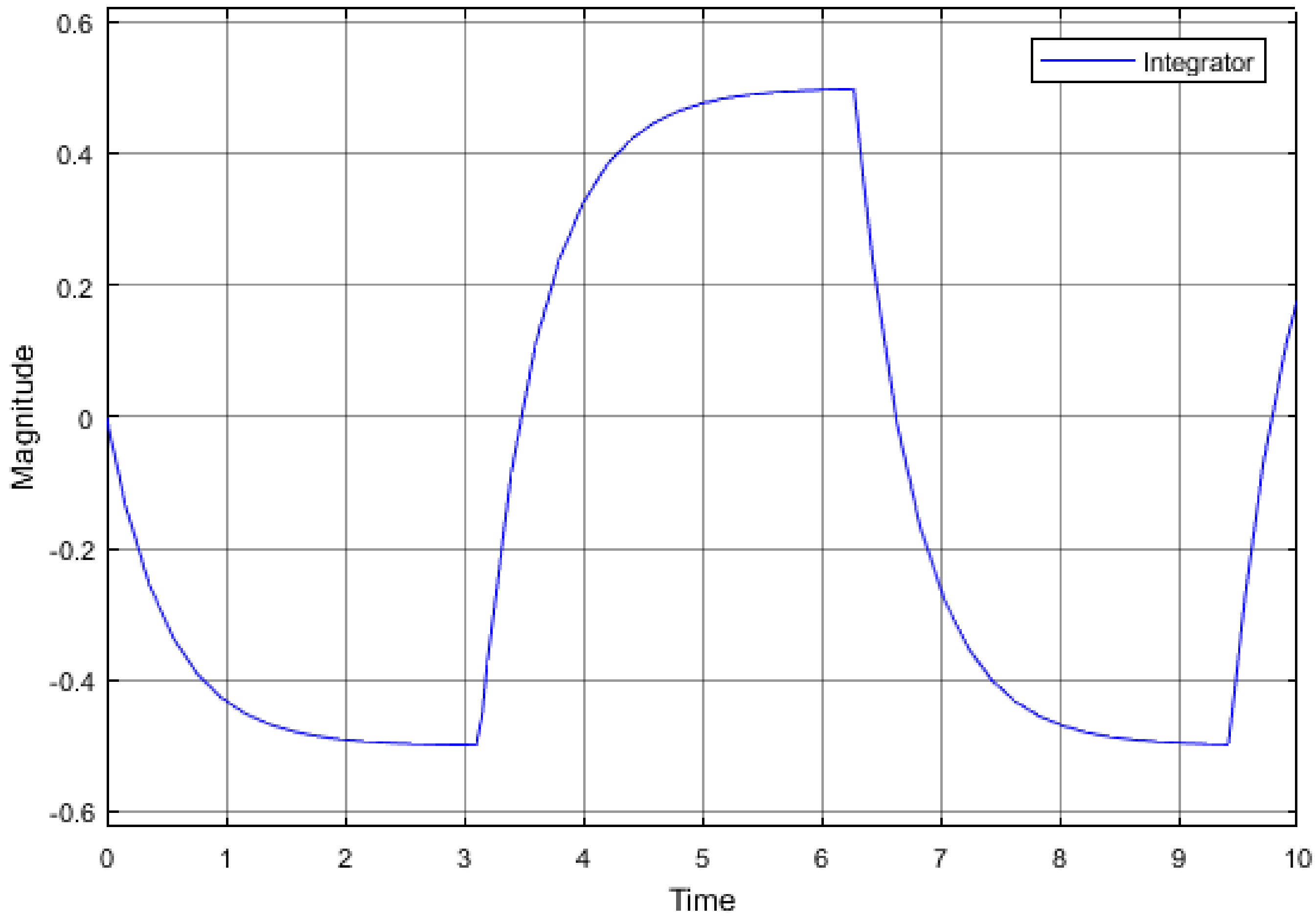
$$\dot{x}(t) = 2x(t) + u(t)$$

Where $u(t)$ is a square wave input with amplitude of 1 N and a frequency of 1 rad/sec

We will create a model of a damper-spring mechanical system in the time domain in Simulink.



Plot the output of the simulation of the mechanical system in the time domain in Simulink.



Experiment 2 (Simulation of Mechanical System with Transfer Function)

This experiment creates a model the differential equation in the frequency domain through use of transfer function.

For the same system as described in the first experiment.

$$\dot{x}(t) = 2x(t) + u(t)$$

The equation you modelled previously in the first experiment can also be expressed as a transfer function.

The model uses the Transfer Fcn block, which accepts u as input and outputs $x(t)$.

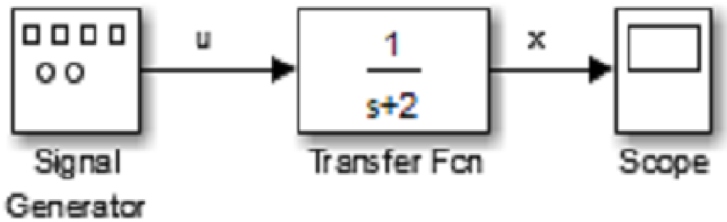
So, the block implements $X(s)/U(s)$. If you substitute $sX(s)$ for $\dot{x}(t)$in the above equation, you get:

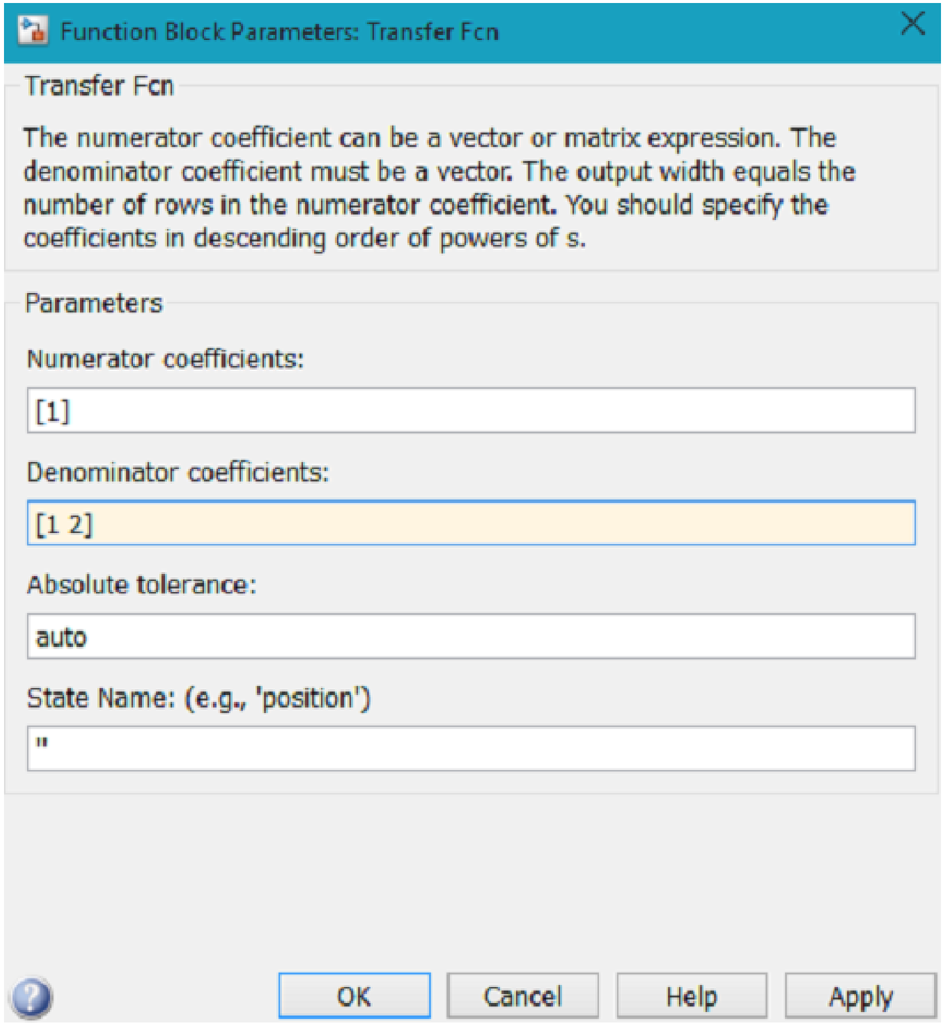
$$sX(s) = -2X(s) + U(s)$$

Solving for $X(s)$ this gives:

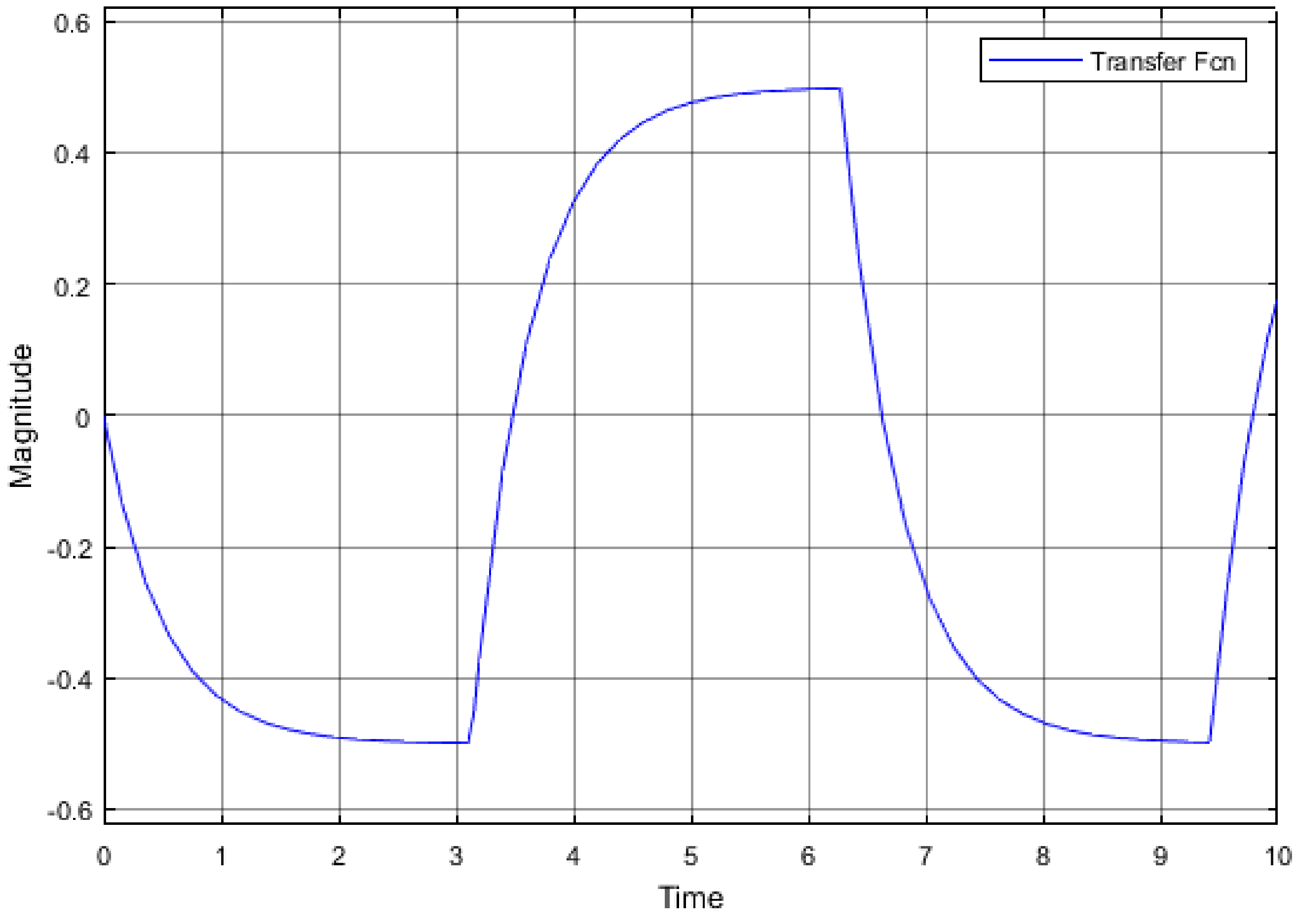
$$X(s) = \frac{U(s)}{s+2} \qquad \text{or} \qquad \frac{X(s)}{U(s)} = \frac{1}{s+2}$$

Create the model the differential equation in the frequency domain in Simulink





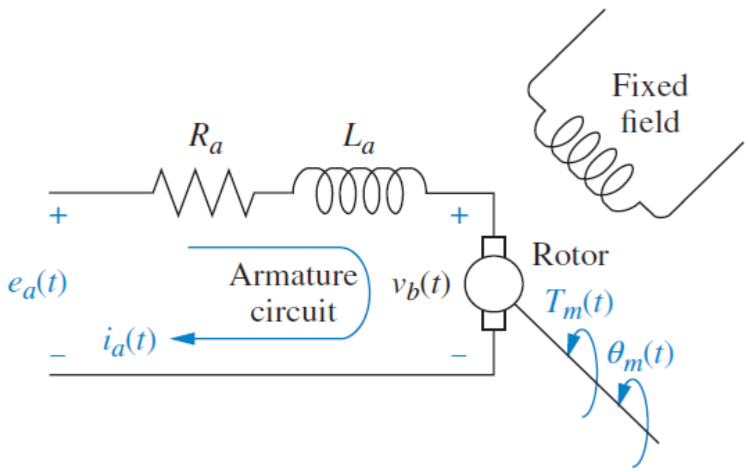
Plot the output of the simulation of the system in the time domain in Simulink (e.g. a graph of displacement vs. time).

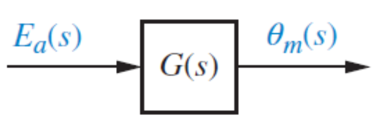


Experiment 3 (Simulation of Electromechanical System)

This experiment will create a model of electromechanical system e.g. a brushless DC motor and its subsystems as given below.

The shown motor drives a mechanical load connected to a damper system.





Note: these are the parameters of the motor: armature inductance (L_a) = 1 H, armature resistance (R_a) = 1 Ω, torque constant (K_t) = 1 N-m/A, motor inertia (J_m) = 1, damping coefficient of motor (D_m) = 1, and back EMF constant (K_b) = 1 V-s/rad.

Electrical subsystem:

$$R_s I_s(t) + L_a I_a(t) + V_b(t) = E_a(t)$$

Mechanical subsystem:

$$T_m(t) = J_m \frac{d^2\theta_m(t)}{dt^2} + D_m \frac{d\theta_m(t)}{dt}$$

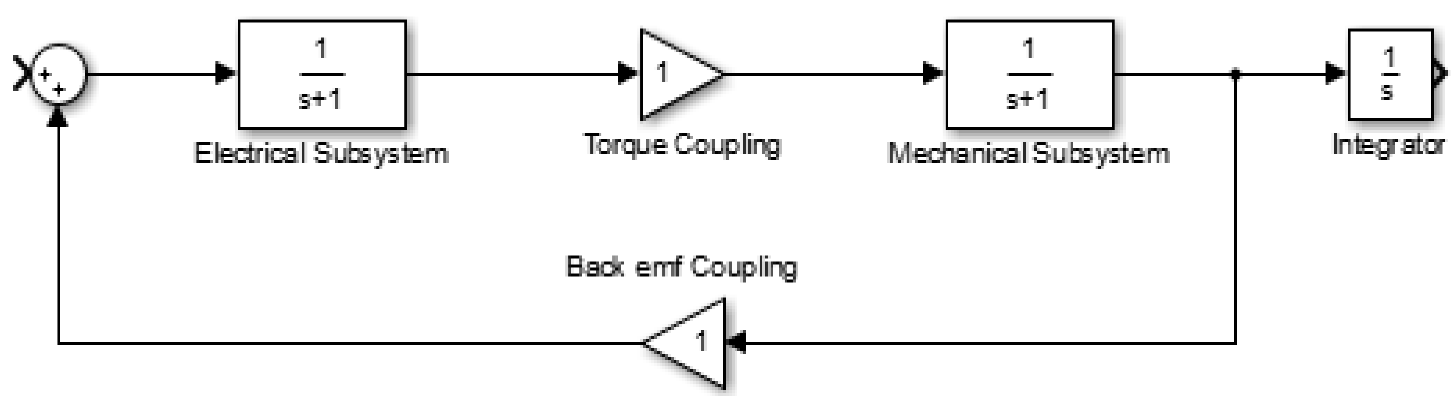
Back EMF coupling:

$$V_b = K_b \frac{d\theta_m(t)}{dt}$$

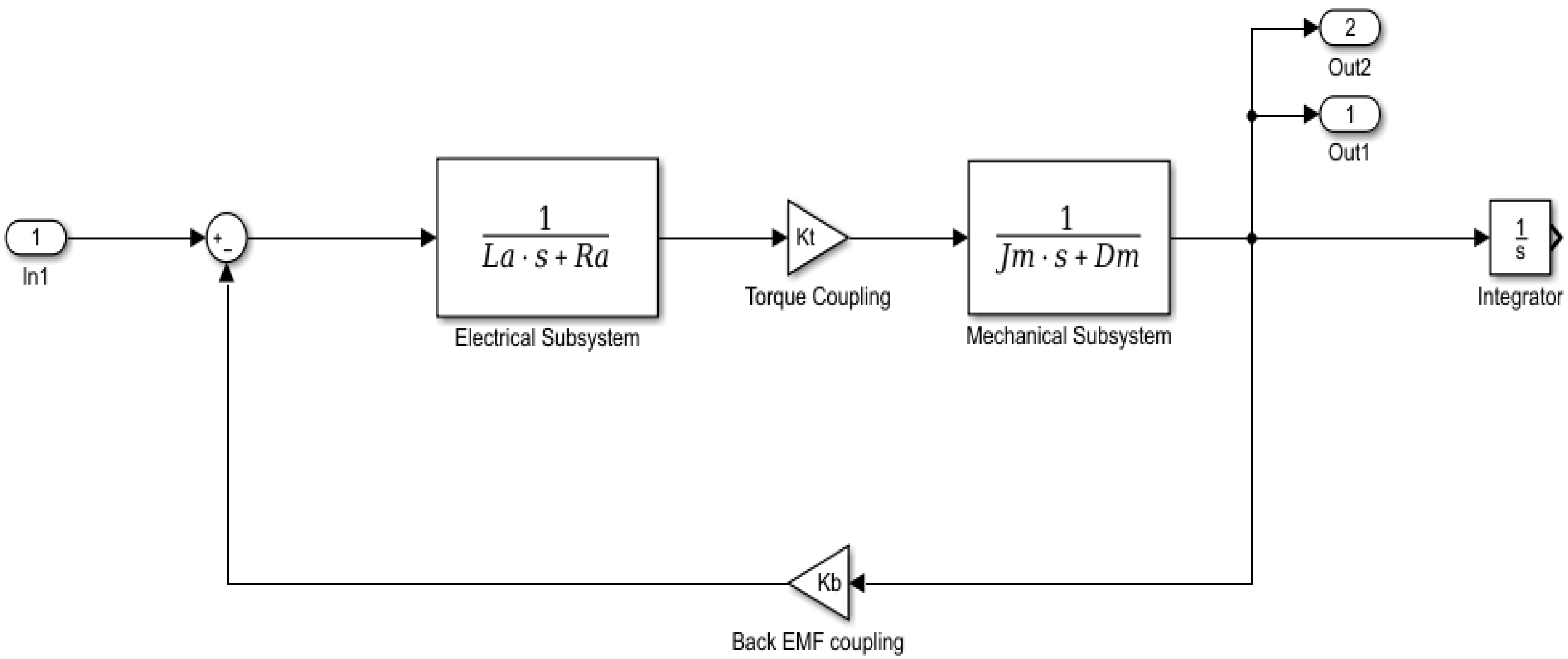
Torque coupling:

$$T_m(t) = K_t I_a(\mathrm{t})$$

Create a model of the electromechanical system in the time domain in Simulink.

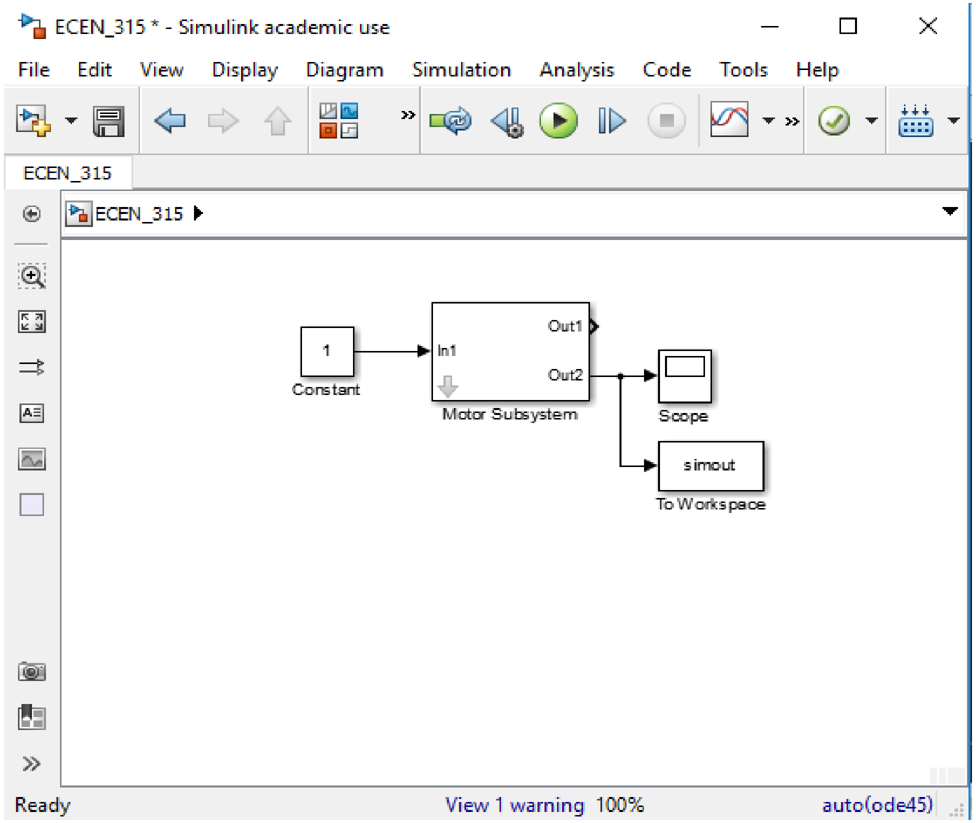


Create a multi tiers model of the given electromechanical system Simulink (e.g. an introduction to modelling of complex systems in Simulink).



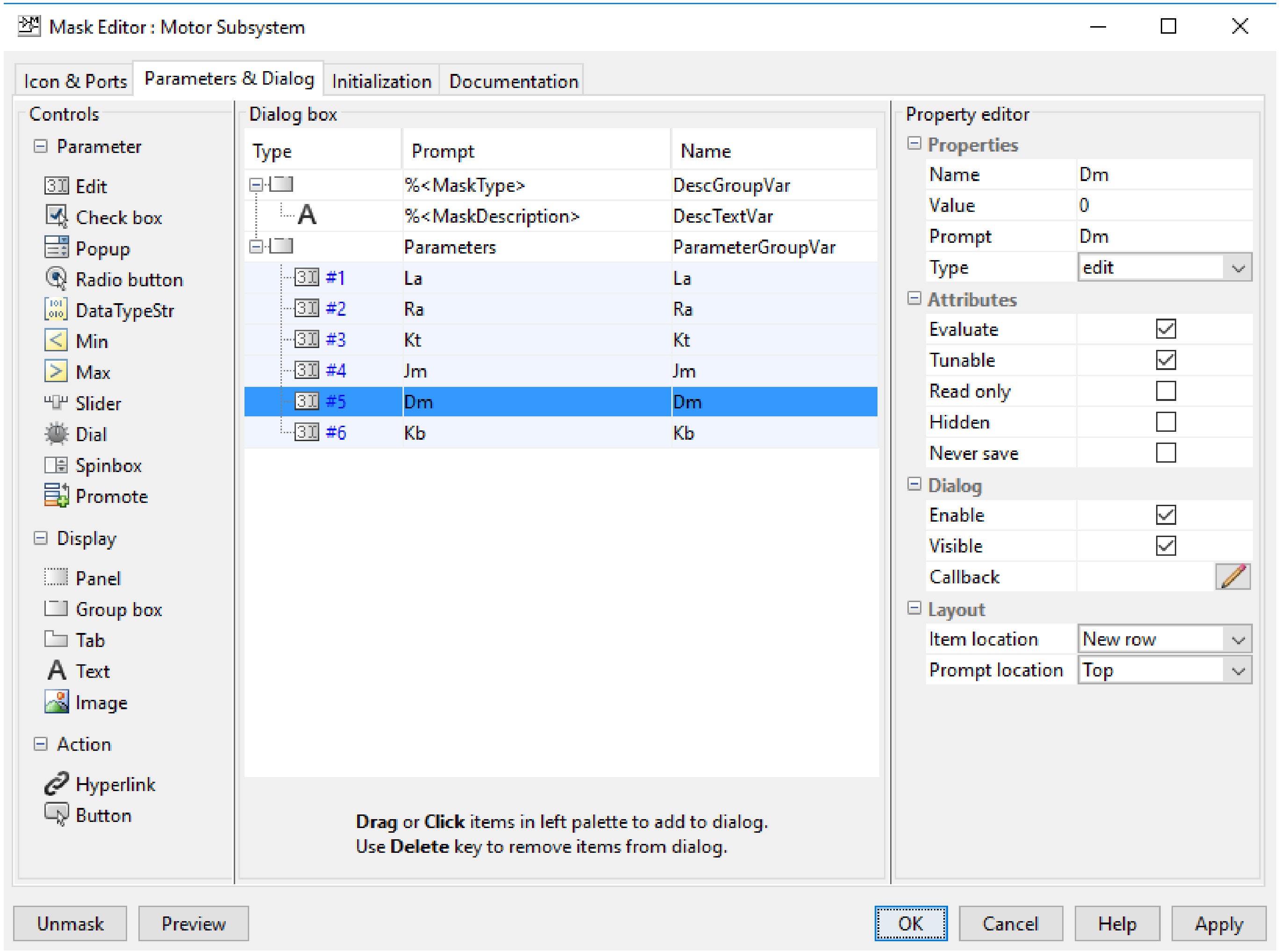
Detail/low level model of the given control system in Simulink

Create a multi tiers model of the given electromechanical system Simulink (e.g. an introduction to modelling of complex systems in Simulink).



Top level model of the given control system in Simulink

Create a mask editor in Simulink for easy user interaction whenever you try to do a number of different simulations for a given control system.



Plot the output of the simulation of the system in the time domain in Simulink (e.g. a graph of angular speed vs. time).