XMUT315 Control Systems Engineering

Demo 1b: Modelling of Physical Systems

A. Modelling of Simple Control Systems

In the following experiment, we are conducting modelling of a mechanical system.

Experiment 1 (Modelling and Simulation of Mechanical System)

For an example of modelling a mechanical system from its physical system, we use a car suspension system in the vehicle system in this exercise. The following diagram illustrates the components and parts that make up a vehicle suspension system.

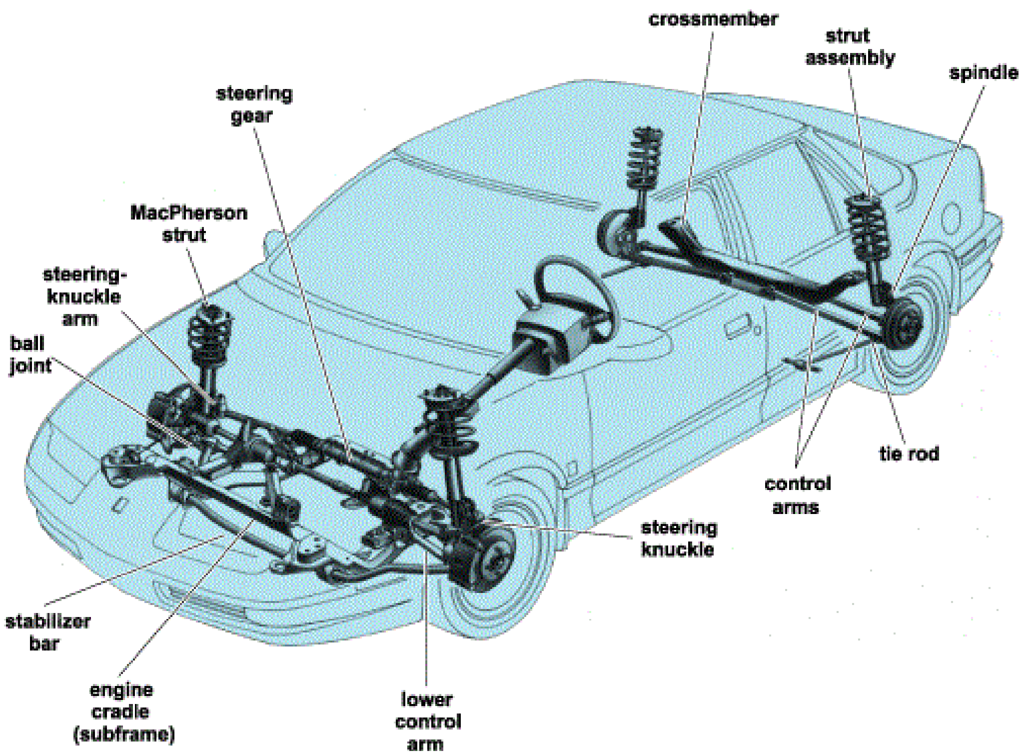


Figure 1: Car suspension system in a typical vehicle system.

As shown in the diagram given below, a typical car suspension system consists of several components e.g. strut rod, transverse rod, coil spring, shock absorber. These components provided the vehicle with the ability for smooth driving on a rough road surface. Notice that the drive axle and drum are parts of the driving system, but in this case, these components become attached to the car suspension system.

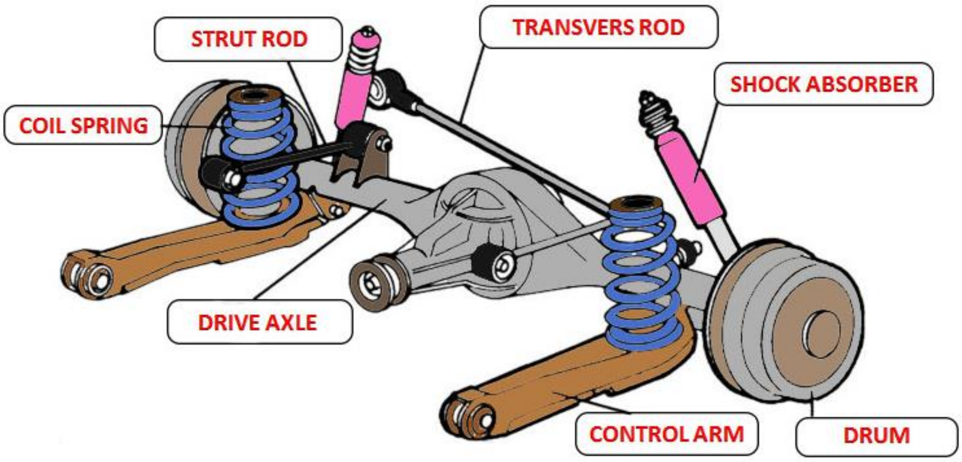


Figure 2: Car suspension system.

Sometimes it is easier, for a complex system such as the example that we have here if we start the modelling by creating a conceptual model of the system rather than creating the model that leads to the derivation of the differential equation for the system from its physical system. The diagram below shows an example of the conceptual model of the given car suspension system.

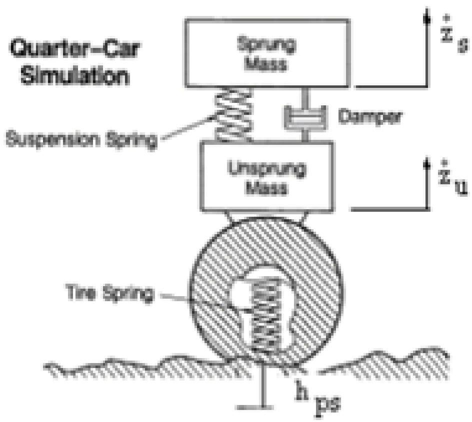


Figure 3: A conceptual model of the vehicle suspension system.

For the given car suspension system, we try to model an event in the actual operation of the system which is associated with this question: *explain what happens in the car suspension system when we have a different shock absorber (damper) installed in the system*?

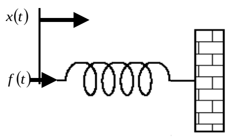
Steps for modelling the mechanical system from its physical system:

- Explain what happens when we have a different damper installed in the system.
- Simplify to the single-input-single-output system.
- Form individual component models.
- Determine their relationships (use physical laws).
- Combine (and simplify if possible).
- This gives us an instantaneous differential equation but want a time-response.

From the conceptual model then you could simplify further the model into a component model. For modelling a simple mechanical system, it is often that we use standardised components to do so.

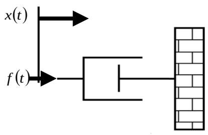
The following shows the elementary standardised components for modelling simple mechanical systems.

Spring



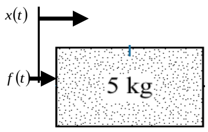
$$f_s(t) = Kx(t)$$

Damper



$$f_c(t) = C\frac{dx(t)}{dt}$$

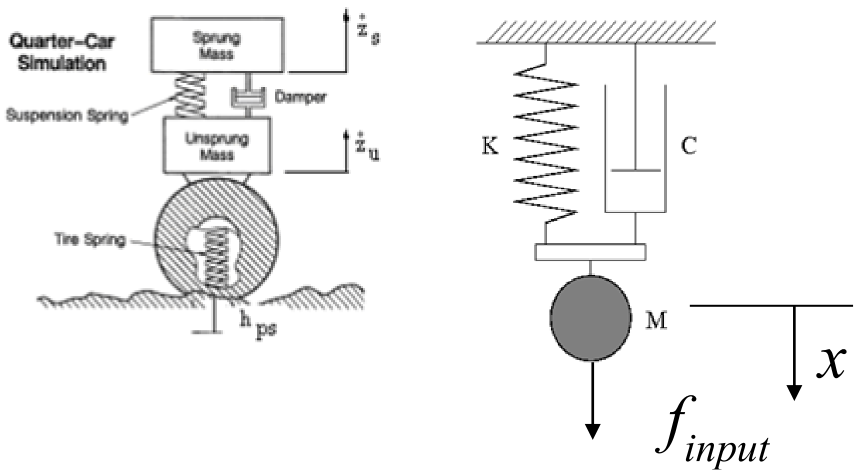
Mass



$$f_m(t) = M\frac{d^2x(t)}{dt^2}$$

Figure 4: Elementary standardised components for modelling simple mechanical systems.

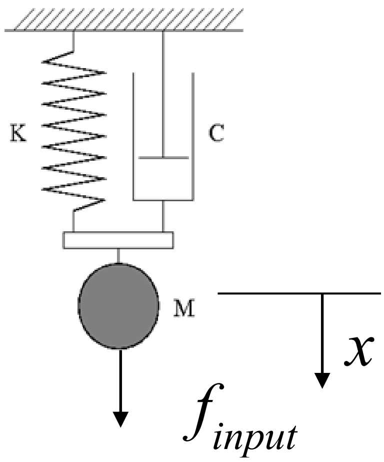
By referring to the conceptual model, we create the component model of the car suspension system using the standardised components as given above.



Conceptual model   Component model

Figure 5: Conversion from the conceptual model to the component model

Notice that now the components and parts that make up our component model are as shown below.



Note:

K = Spring constant

C = Damper coefficient

M = Mass

X = Displacement

F_{input} = Applied force

Figure 6: Parts that constitute a component model of the car suspension system.

As parts of the system that makes up the car suspension system, we have also the following parameters of the system. These parameters are K which is the spring constant, C is the damper coefficient, M is mass, X is defined as the displacement and F_{input} is the applied force.

Applying Newton second law:

$$F(applied) = F(reaction)$$

Hence:

$$F(input) = F(mass) + F(spring) + F(damper)$$

Thus, represent the relationship between the components of the system as a differential equation:

$$f_{input}(t) = f_s(t) + f_d(t) + f_m(t)$$

Forces acting in the spring:

$$f_s(t) = Kx(t)$$

Forces acting in the damper:

$$f_d(t) = C\left(\frac{dx(t)}{dt}\right)$$

Forces acting in the mass:

$$f_m(t) = M\left(\frac{d^2x(t)}{dt^2}\right)$$

Entering each differential equations of the system into the overall system, the differential equation that makes up a car suspension system is:

$$f_{input}(t) = Kx(t) + C\left(\frac{dx(t)}{dt}\right) + M\left(\frac{d^2x(t)}{dt^2}\right)$$

This gives us an instantaneous differential equation. But we want a time-response to evaluate the characteristic and behaviour of the system. So, we end up performing differentiation and integration. These can be done numerically, theoretically, or using tables.

MATLAB Code:

```
% Declaration of parameters and assignment of values into the parameters.

clf; % clear all graphs

K = 10 % Spring constant

C = 3 % Damping constant

m = 1 % Mass (constant)

T = [0: 0.01: 20];% set up the time increments

% Plot and labelling of the graph

xlabel('Time t (s)')

ylabel('Distance x (m)')

hold on % put each graph on top of each other
```

```
% Plot of the graph

for C = 1.0: 1: 10.0

d = tf(9,[m C K])

[y,t]=step(d,T);% step response over 1 second

plot(t,y,'k');

pause(2)

end
```

The transient response of the mechanical system as simulated in MATLAB is shown in the figure below.

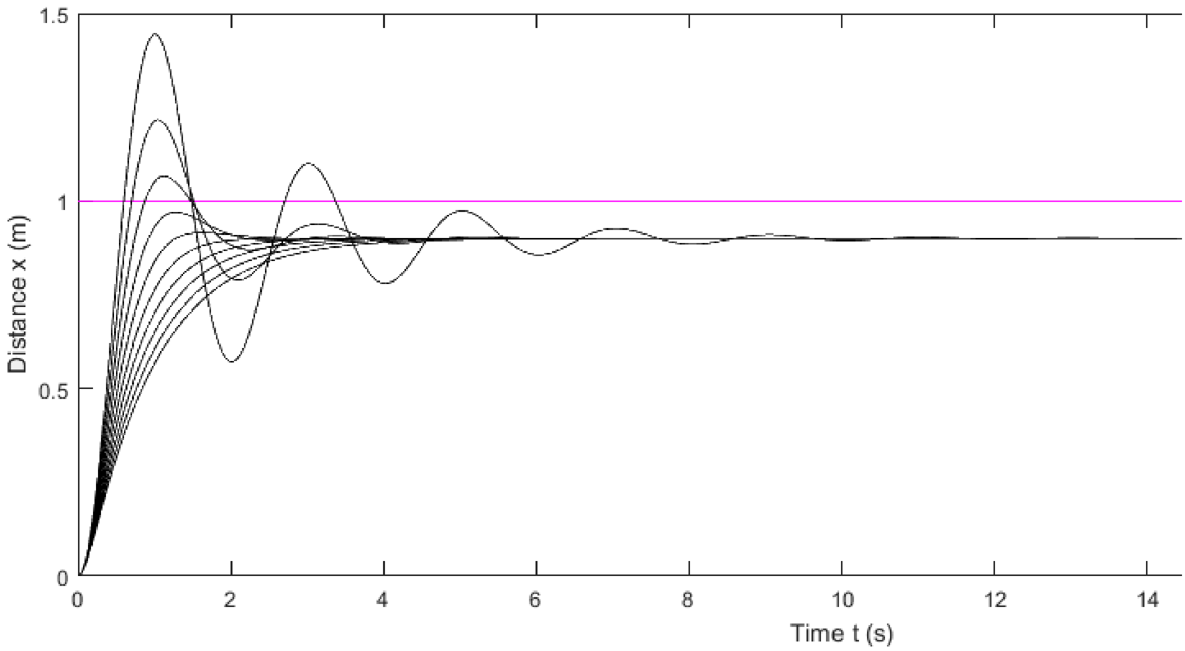


Figure 7: Graph of the result of simulation of the mechanical system

The following second experiment is about modelling of an electrical system from its actual circuit implementation on the breadboard.

Experiment 2 (Modelling of Electrical System)

For the second experiment, consider the following first-order RC circuit realised as capacitor and resistor on the breadboard as shown in the figure below.

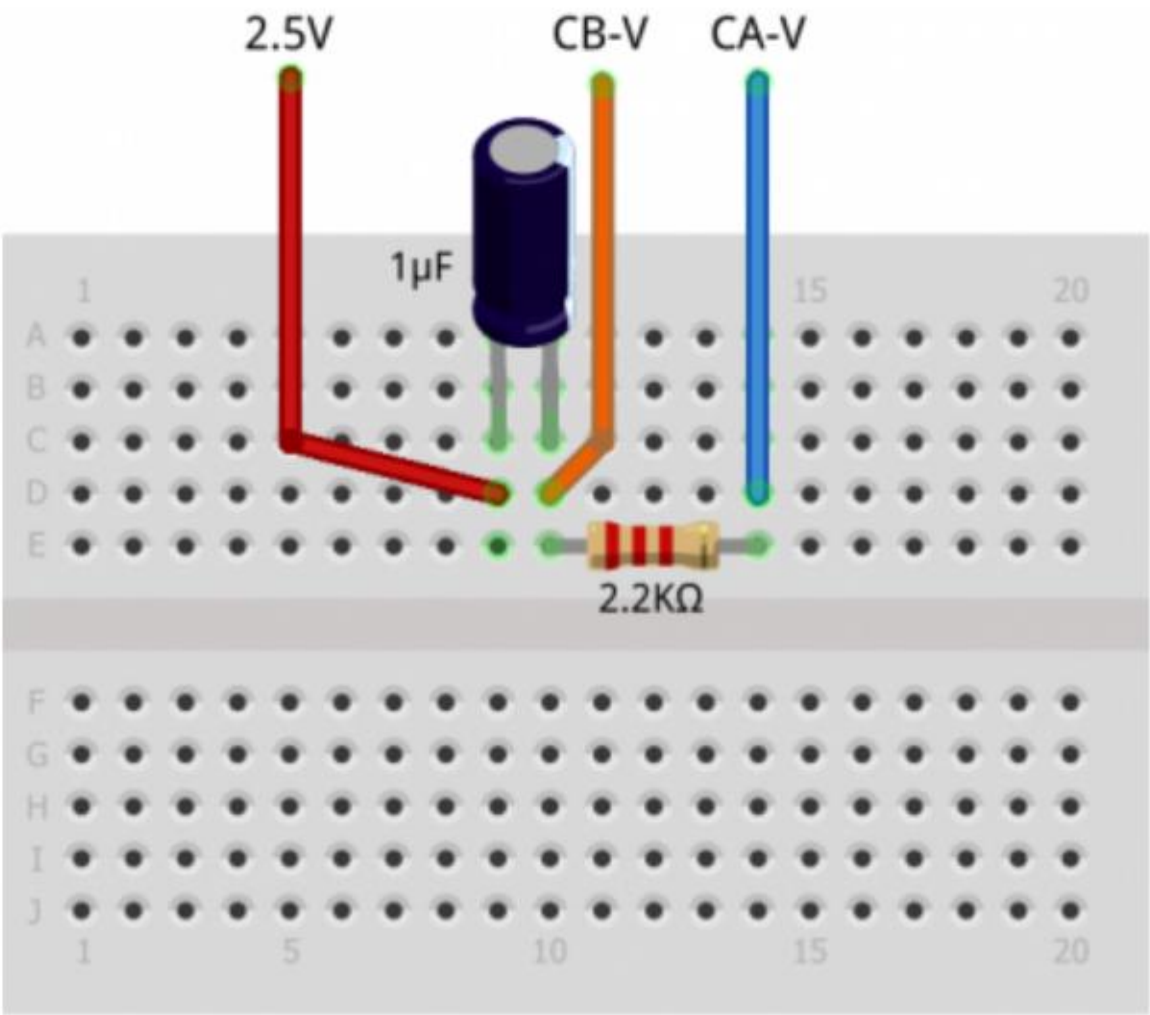


Figure 8: Capacitor and resistor circuit on the breadboard

To simplify the analysis of the given circuit, the connections in the breadboard could be illustrated with a wiring diagram as shown in the figure below.

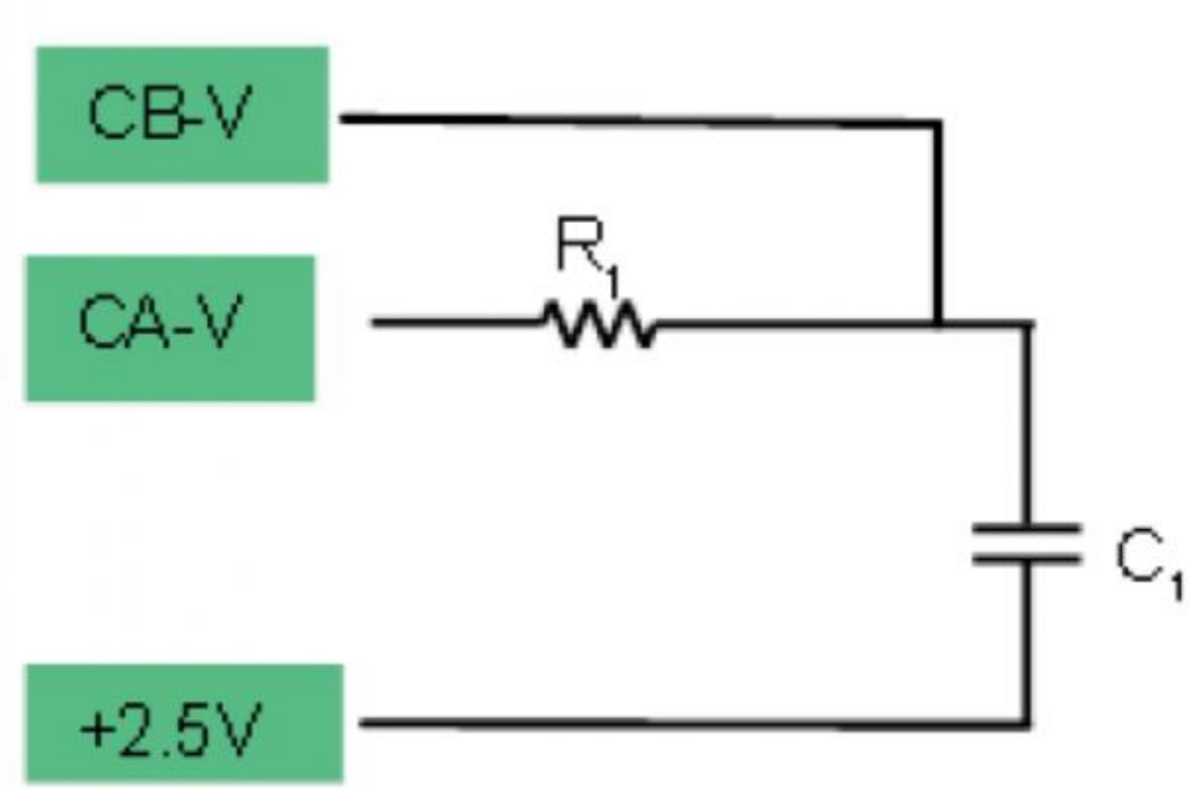


Figure 9: Connection diagram of the capacitor and resistor circuit

Then, the schematic diagram of the given first-order RC circuit is as shown in the figure below.

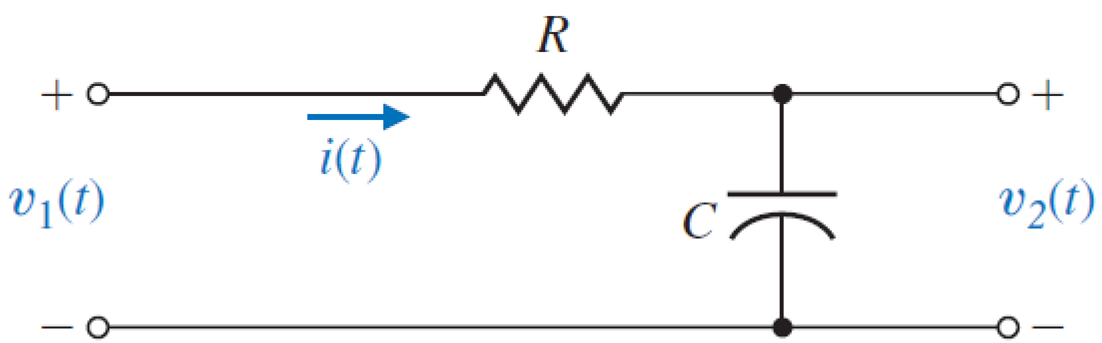


Figure 10: First-order RC circuit

By referring to the schematic diagram of the of the RC network as shown in the figure above, the transfer function of the circuit is obtained by writing the Kirchhoff voltage equation, yielding:

$$V_1(s) = \left(R + \frac{1}{sC}\right) I(s) \qquad (Eq. 1)$$

expressed in terms of transform variables. We shall frequently refer to variables and their transforms interchangeably. The transform variable will be distinguishable by the use of an uppercase letter or the argument (s). The output voltage is:

$$V_2(s) = I(s)\left(\frac{1}{sC}\right) \qquad (Eq.2)$$

Therefore, solving Equation (1) for $I(s)$ and substituting in equation 2, we have:

$$V_2(s) = \frac{\left(\frac{1}{sC}\right)V_1(s)}{R + \frac{1}{sC}}$$

Then, the transfer function is obtained as the ratio $V_2(s)/V_1(s)$,

$$G(s) = \frac{V_2(s)}{V_1(s)} = \frac{1}{RsC + 1} = \frac{1}{\tau s + 1} = \frac{(1/\tau)}{s + (1/\tau)} \qquad (Eq.3)$$

Where: $\tau = RC$, the time constant of the network. The single pole of $G(s)$ is $s = -1/\tau$.

Alternatively, the equation 3 could be immediately obtained if one observes that the circuit is a voltage divider, where:

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

Where: $Z_1(s) = R$, $Z_2 = 1/sC$.

Using MATLAB, we can simulate the transient response of the given first-order RC circuit.

MATLAB Code:

When simulate in MATLAB the circuit shows the following transient response when given a step input.

Figure 11: Transient response of the first-order RC circuit in MATLAB

B. Modelling of Complex Control Systems

In the second experiment, we are performing modelling of electromechanical system.

Experiment 3 (Modelling of Electromechanical System)

For an example of an electromechanical system, we use the electric motor. An electric motor is a good example of systems in control system engineering that has both electrical and mechanical systems.

In this practical exercise, we use, more specific as an example, the electrical motor in an electric vehicle car. The following diagram outlines the parts that make up electrical systems in an electric vehicle. Notice that the electric motor is only one part among other parts in the system e.g. batteries, controller, transmission, etc.

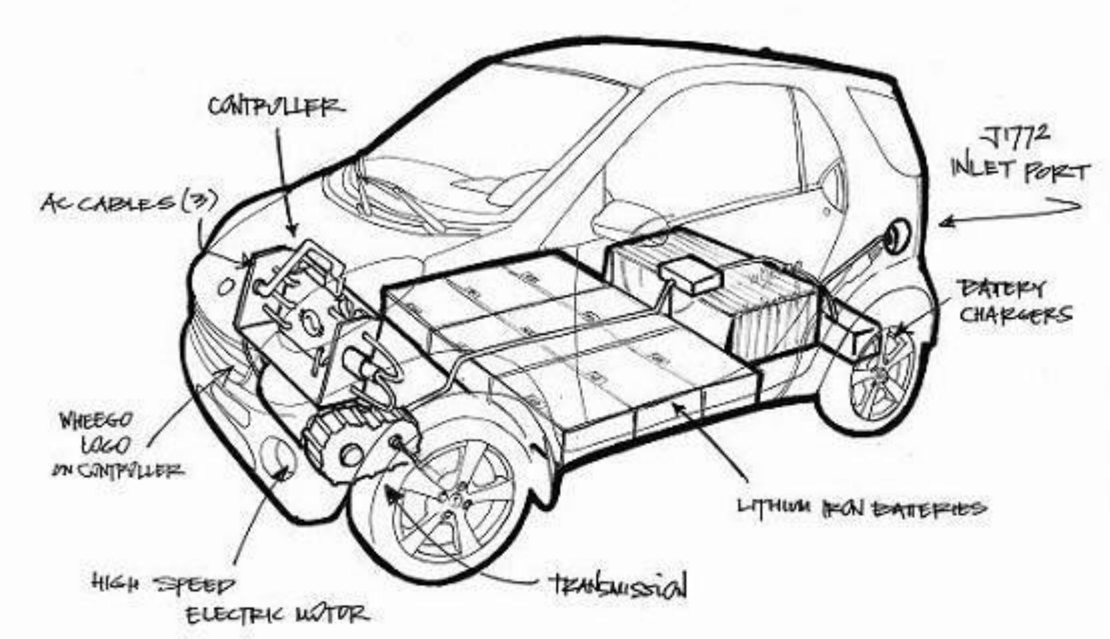


Figure 12: Electrical systems of the given electric vehicle system

The following diagram shows two examples of electric motor e.g. these are DC motor (left) and AC motor (right).

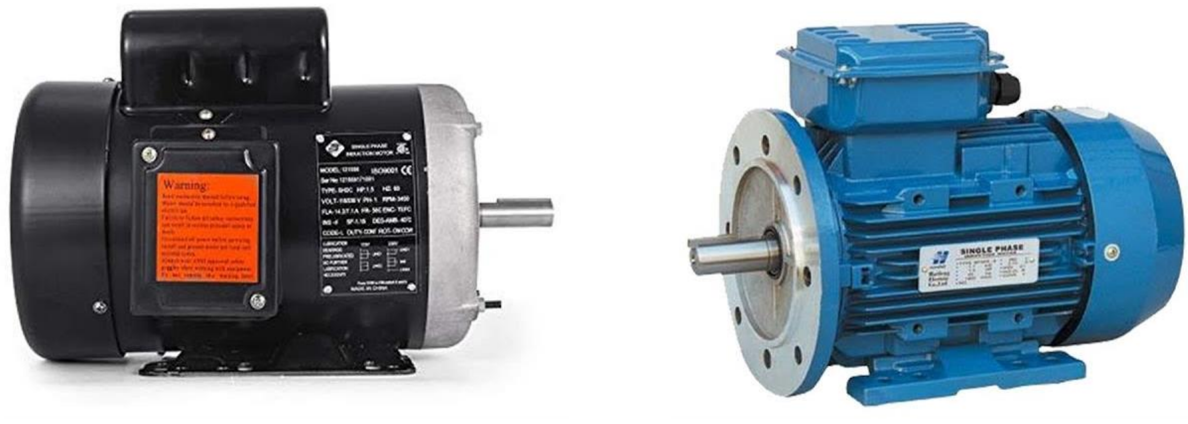


Figure 13: Examples of DC motor (left) and AC motor (right).

In terms of their operations, the AC motor requires an AC power supply whereas the DC motor requires a DC power supply. There are many differences between these motors. By referring to the components inside the motor, you might see also differences between these two motors further in terms of their components. As shown in the diagram below, these are the parts of the DC motor (left) and AC motor (right).

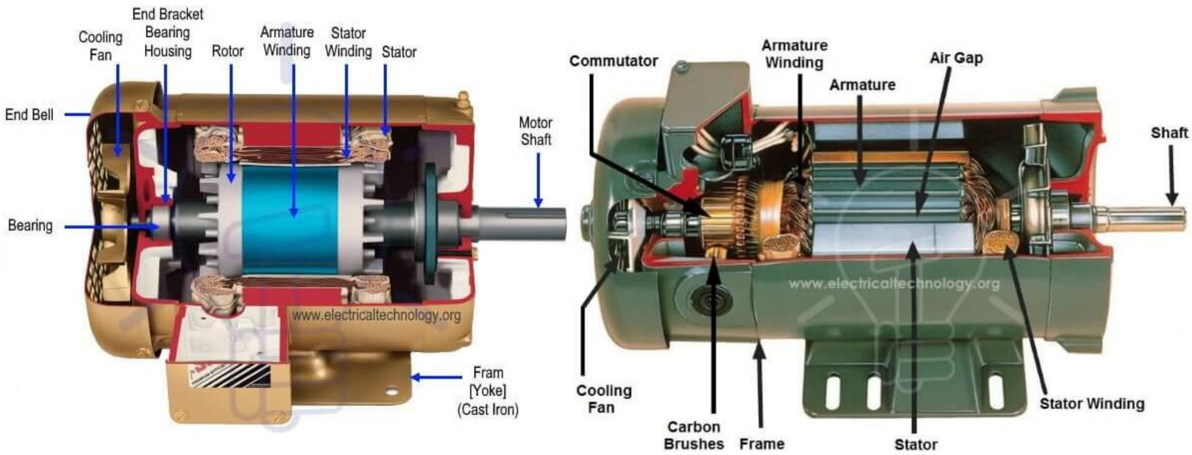


Figure 14: Parts of DC motor (left) and AC motor (right):

If we look closely into the components of the DC motor, you might expect a few commonly found parts such as brush, commutator, rotor, and stator. The following diagram shows (loose) parts of the DC motor. We consider these components later for deriving the differential equation that represents the characteristics and behaviours of a DC motor.

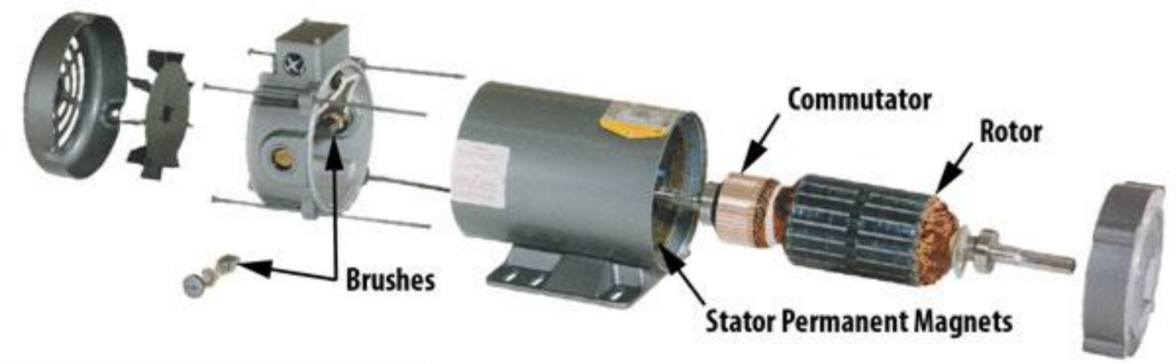


Figure 15: (Loose) parts of a typical DC motor

For this practical exercise, we wish to know *how the DC motor reacts whenever that it is applied a step input e.g. a sudden change in the operation of the motor*.

Steps for modelling the electromechanical system from its physical system:

- Start working with the electrical or mechanical system first.
- Explain what happens when a step input is applied to the system.
- Simplify to the single-input-single-output system.
- Form individual component models.
- Determine their relationships (use electrical circuit laws).
- Combine (and simplify if possible).
- This gives us an instantaneous differential equation but want a time-response.
- Repeat the above steps for the mechanical or electrical system.

For a given example of an electromechanical system i.e. a DC motor system. Its conceptual model is given in the figure below.

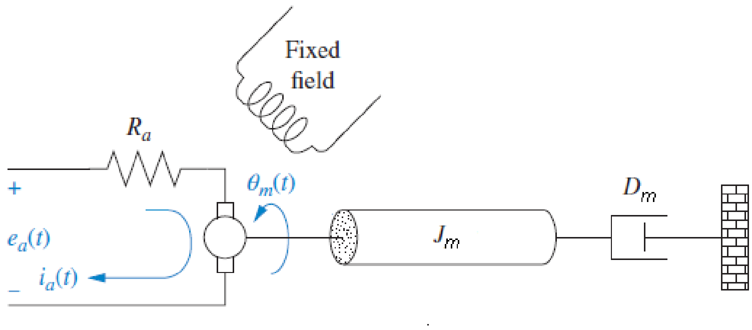
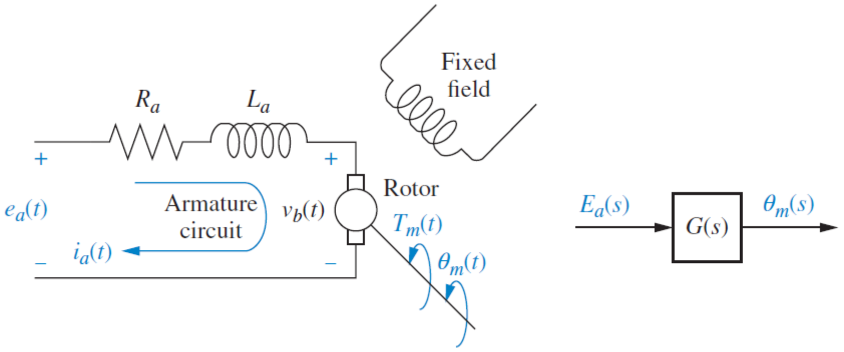


Figure 16: Electromechanical system (i.e. DC motor system)

We will then evaluate the system in terms of its electrical and mechanical subsystems.

Electrical Subsystem:

From the physical system, we might be able to obtain a conceptual model of the DC motor. The following diagram illustrates the electrical components/part of the motor and their interconnections.



Note: R_a = Armature resistance, L_a = Armature inductance, $V_b(t)$ = Back EMF, and $E_a(t)$ = Voltage supply

Figure 17: Electrical system of the DC motor

In a given DC motor, you might find R_a which is the armature resistance of the motor, L_a is the armature inductance, $V_b(t)$ is the back EMF and $E_a(t)$ which is the voltage supply.

Then we can work out the equations that represent the characteristics and behaviour of the DC motor. Start with the modelling of the electrical system of the motor, then we tackle the mechanical system of the motor.

Electrical subsystem:

$$R_a I_a(t) + L_a \frac{dI_a(t)}{dt} + V_b(t) = E_a(t) \qquad (1)$$

Back EMF coupling:

$$V_b = K_b \frac{d\theta_m(t)}{dt} \qquad (2)$$

Substituting $V_b(t)$ in equation (1) with equation (2) and applying Laplace transform, this gives the differential equation of the electrical system of the DC motor:

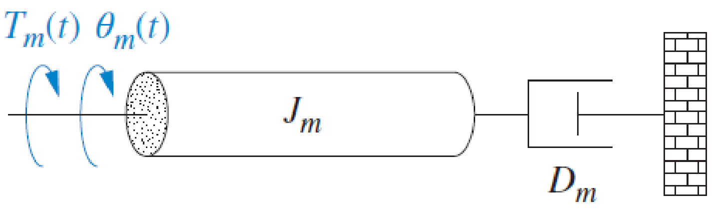
$$(R_a + L_a s) I_a(s) = E_a(s) - K_b s \theta_m(s)$$

Or

$$I_a(s) = \frac{E_a(s) - K_b s \theta_m(s)}{(R_a + L_a s)} \qquad (3)$$

Mechanical Subsystem:

For the given mechanical system, the rotor of the motor usually has some form of inertia as it can be classified as a stiff body due to its weight and size. Furthermore, when the rotor is turning, you might expect some sort of viscous friction or damping existed in the system due to the movement of the rotor.



Note: J_m = Inertia of motor shaft, D_m = Damping of motor shaft, $T_m(t)$ = Applied torque, $\theta_m(t)$ = Angular displacement, and $\omega_m(t)$ = $d\theta_m(t)/dt$ = Angular speed

Figure 18: Mechanical system of the DC motor

As illustrated in the above diagram, for the parts that make up the mechanical system of the given DC motor, we have the following parameters of the system.

These are J_m which is the inertia of the motor shaft, D_m which is the damping of the motor shaft, $T_m(t)$ which is the applied torque, and $\theta_m(t)$ = Angular displacement. Notice also that we can derive also $\omega_m(t) = d\theta_m(t)/dt$ which is the angular speed of the rotor. Then we start working on the mechanical system of the DC motor.

Mechanical subsystem:

$$T_m(t) = J_m \frac{d^2\theta_m(t)}{dt^2} + D_m \frac{d\theta_m(t)}{dt} \qquad (4)$$

Torque coupling:

$$T_m(t) = K_t I_a(t) \qquad (5)$$

Substituting T_m in equation (4) with equation (5) and apply Laplace transform, this gives:

$$s(J_m s + D_m)\theta_m(s) = K_t I_a(s) \qquad (6)$$

Overall Electromechanical System:

By considering both equations (3) and (6), we can eliminate $I_a(s)$.

$$s(J_m s + D_m)\theta_m(s) = K_t \left[\frac{E_a(s) - K_b s\theta_m(s)}{(R_a + L_a s)}\right]$$

Rearrange the equation above:

$$(J_m s + D_m)(R_a + L_a s)s\theta_m(s) + K_t K_b s\theta_m(s) = K_t E_a(s)$$

As a result, we can obtain the following open-loop transfer function where the rotational speed is the output and the voltage at the armature is the input:

$$\frac{\omega_m(s)}{E_a(s)} = \frac{K_t}{\left(J_m s + D_m(s)\right)(R_a + L_a s) + K_b K_t}$$

Note: $\omega_m(s) = s\theta_m(s)$

For a DC motor circuits with the following specification: R_a = 2 Ω, L_a = 0.5 H, J_m= 0.02 kg-m^2, D_m = 0.2 N-m s/rad, K_t = 0.1 N-m-A, and K_b = 0.1 V-s/rad.

Find the relationship $\omega_m(t)/E_a(t)$ by simulating the DC motor system in Matlab. Figure out what happens to the system when it is injected with a step input.

Entering the values as given above into the differential equation of the DC motor

$$\frac{\omega_m(s)}{E_a(s)} = \frac{0.1}{(0.02s + 0.2)(0.5s + 2) + (0.1)^2}$$

$$= \frac{0.1}{0.01s^2 + 0.14s + 0.41}$$

MATLAB code:

```
s=tf('s');

% defining of parameters of simulation

Ra=2;

La=0.5;

Jm=0.02;

Dm=0.2;

Kt=0.1;

Kb=0.1;

% transfer function of the system

 G=Kt/[(Jm*s+Dm)*(Ra+La*s)+Kt*Kb];

% apply step function to the system

step(G);

title('Step Response for the Open Loop System');
```

Graph of simulation results:

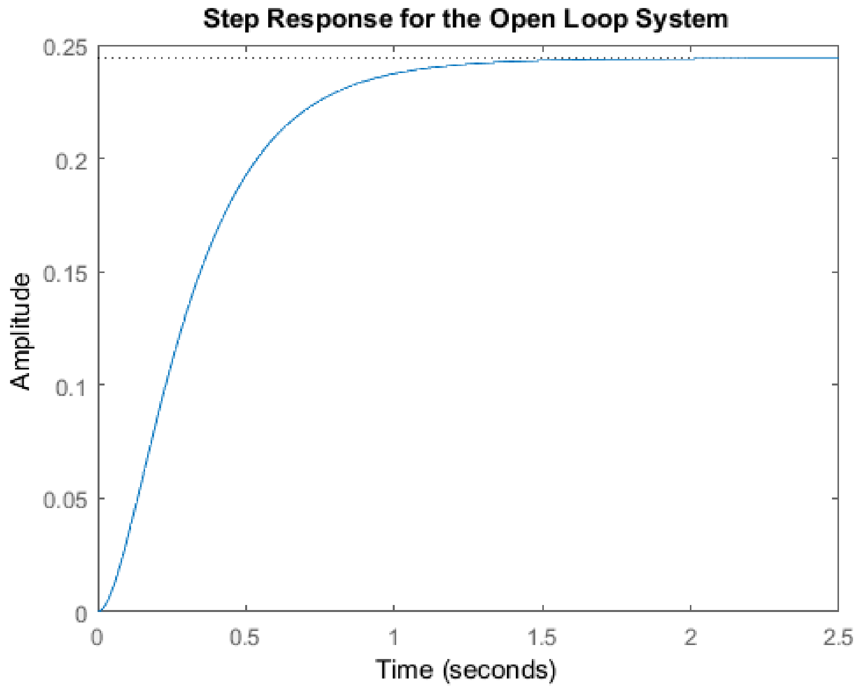


Figure 19: Graph of simulation results of the example electromechanical system