Demo 4: Time Domain Analysis

XMUT315 Control Systems Engineering

Topics

1. Transient response analysis.
2. Steady-state analysis.

Exercise 1 (Transient Response Analysis - Unit Step Response)

A higher-order system is defined by:

$$\frac{C(s)}{R(s)} = \frac{7s^2 + 16s + 10}{s^4 + 5s^3 + 11s^2 + 16s + 10}$$

a. Plot the unit-step response curve of the system using MATLAB.

b. Obtain the rise time, peak time, maximum overshoot, and settling time using MATLAB.

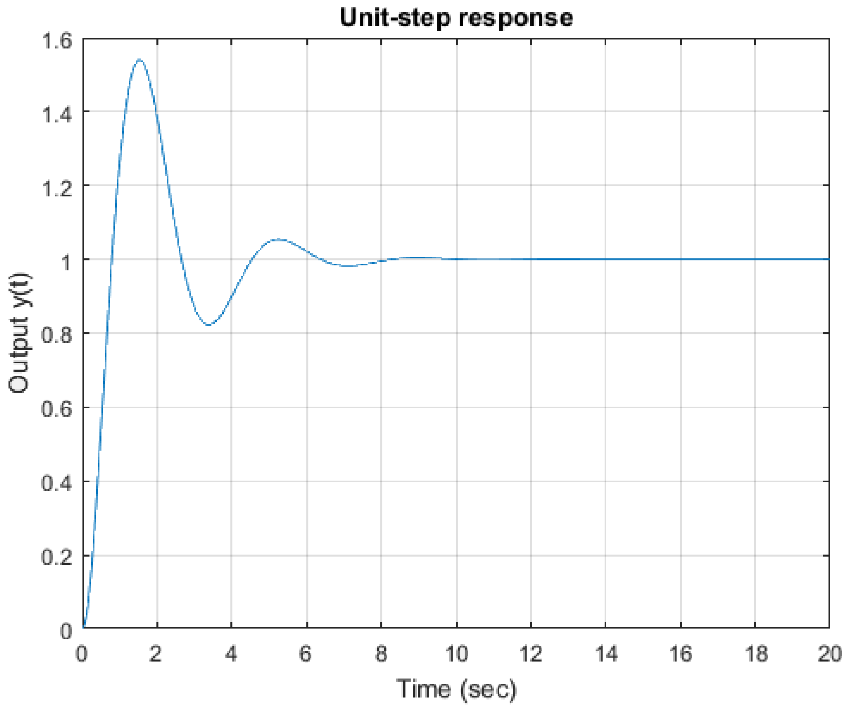
Solution

a. The MATLAB code for simulating the unit step response curve of the system is given below.

MATLAB code:

```
% demo41a.m
% Unit-step response curve

num=[0 0 7 16 10];
den=[1 5 11 16 10];

t=0:0.02:20;

[y,x,t]=step(num,den,t);

plot(t,y)
grid
title('Unit-step response')
xlabel('Time (sec)')
ylabel('Output y(t)')
```

The result of the simulation is given in the diagram below.



b. Obtain the rise time, peak time, maximum overshoot, and settling time using MATLAB.

The rise time, peak time, maximum overshoot, and settling time of the system is simulated using the following MATLAB code.

MATLAB code:

```
% demo41b.m

% Response to rise from 10% to 90% of its final value

r1=1; while y(r1) < 0.1, r1=r1+1; end

r2=1; while y(r2) < 0.9, r2=r2+1; end

rise_time=(r2-r1)*0.02;
[ymax,tp]=max(y);
peak_time=(tp-1)*0.02;
max_overshoot=ymax-1;
s=1001; while y(s)>0.98 & y(s)<1.02; s=s-1; end
settling_time=(s-1)*0.02;
```

Type the following variables in the command prompt: rise_time, peak_time, max_overshoot, and settling_time. The outcomes of the simulation are as shown below.

```
rise_time =

    0.5400


peak_time =

    1.5200


max_overshoot =

    0.5397


settling_time =

    6.0200
```

Exercise 2 (Transient Response Analysis – Second Order Systems)

For each of the second order systems below, find ζ, ω_n, T_s, T_p, T_r, $\%OS$, and plot the step response using MATLAB.

a. System 1:

$$T_1(s) = \frac{130}{s^2 + 15s + 130}$$

b. System 2:

$$T_2(s) = \frac{0.045}{s^2 + 0.025s + 0.045}$$

c. System 3:

$$T_3(s) = \frac{10^8}{s^2 + 1.325 \times 10^3 s + 10^8}$$

Comment on the results of the step response simulation.

Solution

a. The first system's ζ, ω_n, T_s, T_p, T_r, $\%OS$, and plot the step response.

The MATLAB code for simulating the first system's time response parameters and plotting the step response.

MATLAB code:

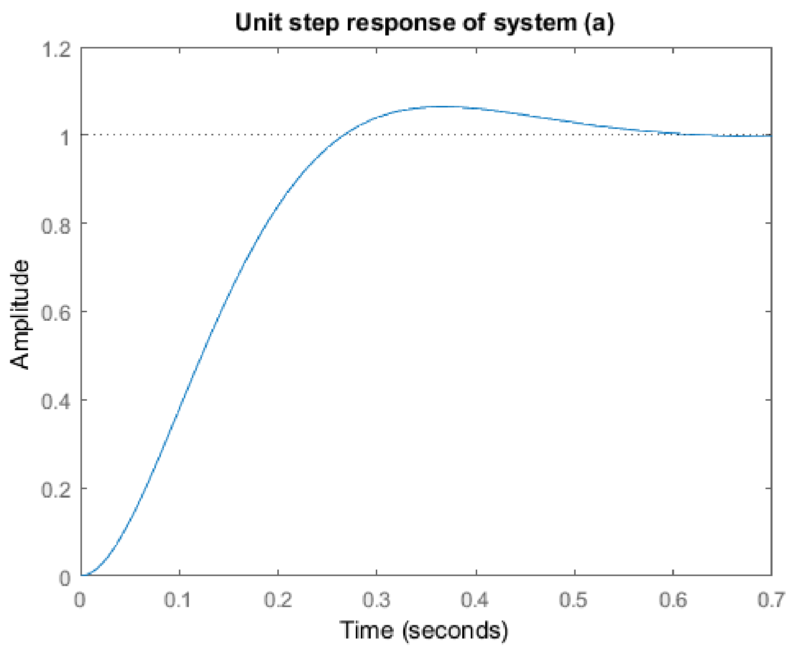
```
% demo42a.m
% Time domain responses of the first system
clf
numa=130;
dena=[1 15 130];
Ta=tf(numa,dena)
omegana=sqrt(dena(3))
zetaa=dena(2)/(2*omegana)
Tsa=4/(zetaa*omegana)
Tpa=pi/(omegana*sqrt(1-zetaa^2))
Tra=(1.76*zetaa^3-0.417*zetaa^2+1.039*zetaa+1)/omegana
percenta=exp(-zetaa*pi/sqrt(1-zetaa^2))*100
step(Ta)
title('Time response of system (a)')
```

The outcomes of the MATLAB simulation and plot are as shown below:

```
Ta =

        130
  ----------------
  s^2 + 15 s + 130

Continuous-time transfer function.


omegana =

   11.4018


zetaaa =

    0.6578
```

```
Tsa =

    0.5333


Tpa =

    0.3658


Tra =

    0.1758


percenta =

    6.4335
```

The time response of the first system is as shown in the diagram given below.



b. The second system's ζ, ω_n, T_s, T_p, T_r, $\%OS$, and plot the step response

The MATLAB code for simulating the second system's time response parameters and plotting the step response.

MATLAB code:

```
% demo42b.m
% Time domain responses of the second system
clf
numb=0.045;
denb=[1 0.025 0.045];
Tb=tf(numb,denb)
omeganb=sqrt(denb(3))
zetab=denb(2)/(2*omeganb)
Tsb=4/(zetab*omeganb)
Tpb=pi/(omeganb*sqrt(1-zetab^2))
```

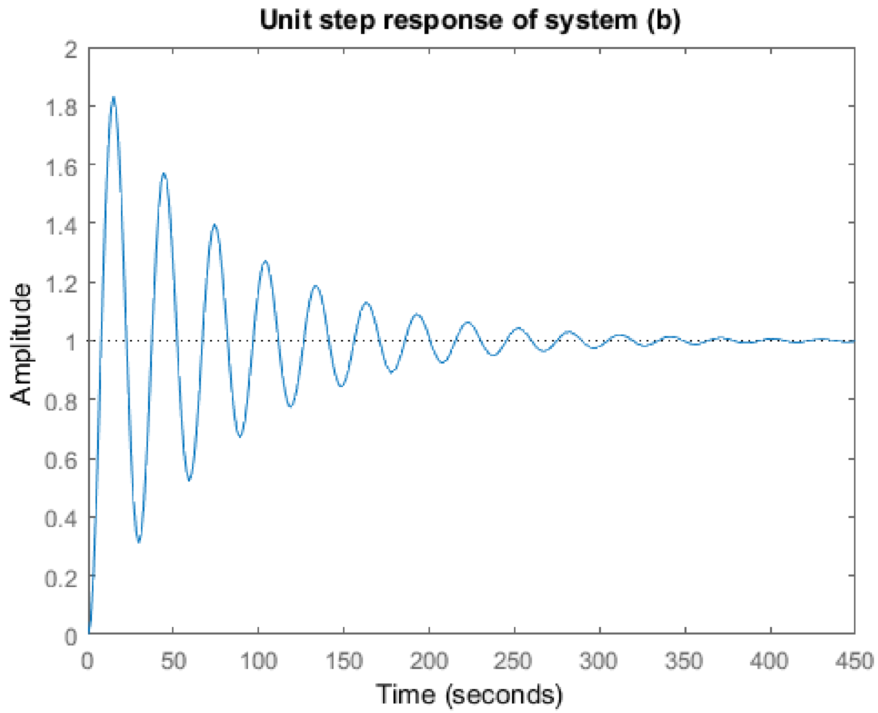
```
Trb=(1.76*zetab^3-0.417*zetab^2+1.039*zetab+1)/omeganb
percentb=exp(-zetab*pi/sqrt(1-zetab^2))*100
step(Tb)
title('Unit step response of system (b)')
```

The outcomes of the MATLAB simulation and plot are as shown below:

```
Tb =

          0.045
  ---------------------
  s^2 + 0.025 s + 0.045

Continuous-time transfer function.


omeganb =

    0.2121


zetab =

    0.0589
```

```
Tsb =

   320


Tpb =

   14.8354


Trb =

    4.9975


percentb =

   83.0737
```

The time response of the second system is as shown in the diagram given below.



c. The third system's ζ, ω_n, T_s, T_p, T_r, $\%OS$, and plot the step response

The MATLAB code for simulating the third system's time response parameters and plotting the step response.

MATLAB code:

```
% demo42c.m
% Time domain responses of the third system
clf
numc=1e8;
denc=[1 1.325e3 1e8];
Tc=tf(numc,denc)
omeganc=sqrt(denc(3))
zetac=denc(2)/(2*omeganc)
Tsc=4/(zetac*omeganc)
Tpc=pi/(omeganc*sqrt(1-zetac^2))
```

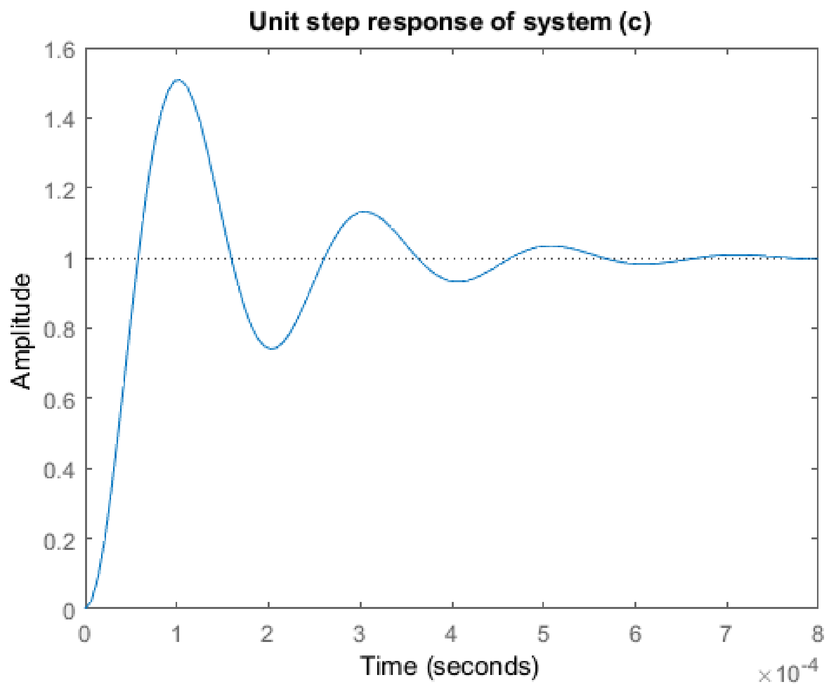
```
Trc=(1.76*zetac^3-0.417*zetac^2+1.039*zetac+1)/omeganc
percentc=exp(-zetac*pi/sqrt(1-zetac^2))*100
step(Tc)
title('Unit step response of system (c)')
```

The outcomes of the MATLAB simulation and plot are as shown below:

```
Tc =

          1e09
  --------------------
  s^2 + 13250 s + 1e09

Continuous-time transfer function.


omeganc =

   3.1623e+04


zetac =

    0.2095
```

```
Tsc =

   6.0377e-04


Tpc =

   1.0160e-04


Trc =

   3.8439e-05


percentc =

   51.0123
```

The time response of the third system is as shown in the diagram given below.



The results of step responses simulations show that the first, second and third systems are underdamped responses. The first system settles down quicker than other systems which is followed by second and third systems.

Exercise 3 (Transient Response - System with Delay)

For a unit feedback system with the forward-path transfer function:

$$G(s) = \frac{K}{s(s+5)(s+12)}$$

This system has a delay of 0.5 second, estimate the percent overshoot for $K = 40$ using a second-order approximation.

Model the delay using MATLAB function `pade(T,n)`.

Determine the unit step response and check the second-order approximation assumption made.

Solution

The MATLAB code that simulates a unit step response of the given feedback system is as follows.

MATLAB code:

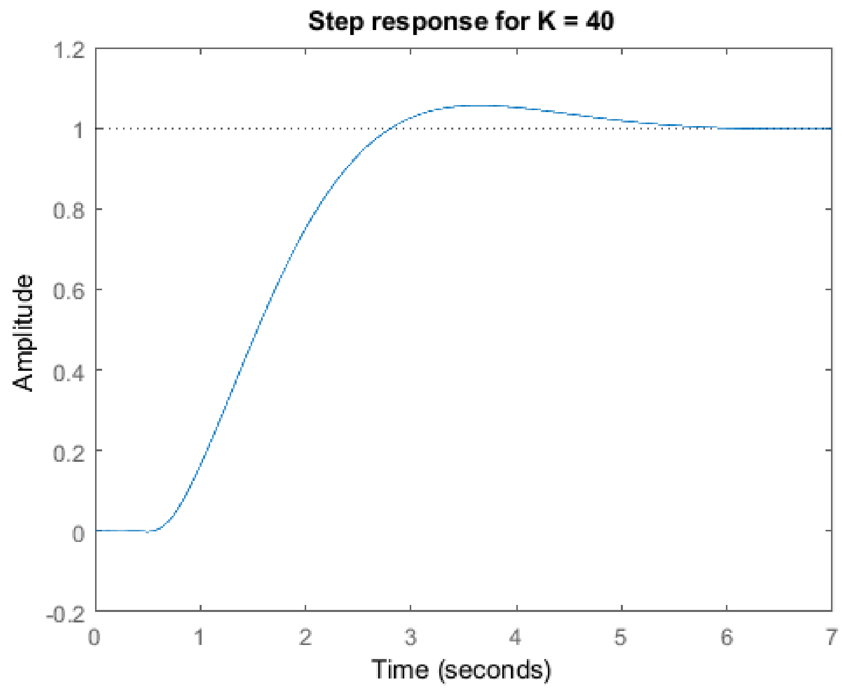
```
% demo43.m

% Unit step response of a system with delay with pade(T,n)

% Enter G(s)

numg1=1;

deng1=poly([0 -5 -12]);

'G1(s)'

G1=tf(numg1,deng1)

[numg2,deng2]=pade(0.5,5);
```

```
'G2(s)'

G2=tf(numg2,deng2)

'G(s)=G1(s)G2(s)'

G=G1*G2

% Enter K

K=input('Type gain, K: ');

T=feedback(K*G,1);

step(T)

title(['Step response for K = ', num2str(K)])
```

Output of this program is as follows:

```
    'G1(s)'


G1 =

            1
  -------------------
  s^3 + 17 s^2 + 60 s

Continuous-time transfer function.


ans =

    'G2(s)'


G2 =

  -s^5 + 60 s^4 - 1680 s^3 + 2.688e04 s^2 - 2.419e05 s + 9.677e05
  ---------------------------------------------------------------
  s^5 + 60 s^4 + 1680 s^3 + 2.688e04 s^2 + 2.419e05 s + 9.677e05

Continuous-time transfer function.


ans =

    'G(s)=G1(s)G2(s)'


G =

               -s^5 + 60 s^4 - 1680 s^3 + 2.688e04 s^2 - 2.419e05 s + 9.677e05
  ------------------------------------------------------------------------------------------
  s^8 + 77 s^7 + 2760 s^6 + 5.904e04 s^5 + 7.997e05 s^4 + 6.693e06 s^3 + 3.097e07 s^2 + 5.806e07 s

Continuous-time transfer function.
```

When the simulation is running, type Gain, $K = 40$, as a result the following figure is obtained.



Exercise 4 (Steady-State Analysis - Ramp Input)

Determine the unit-ramp response of the following system using MATLAB and `lsim()` function.

$$\frac{C(s)}{R(s)} = \frac{1}{3s^2 + 2s + 1}$$

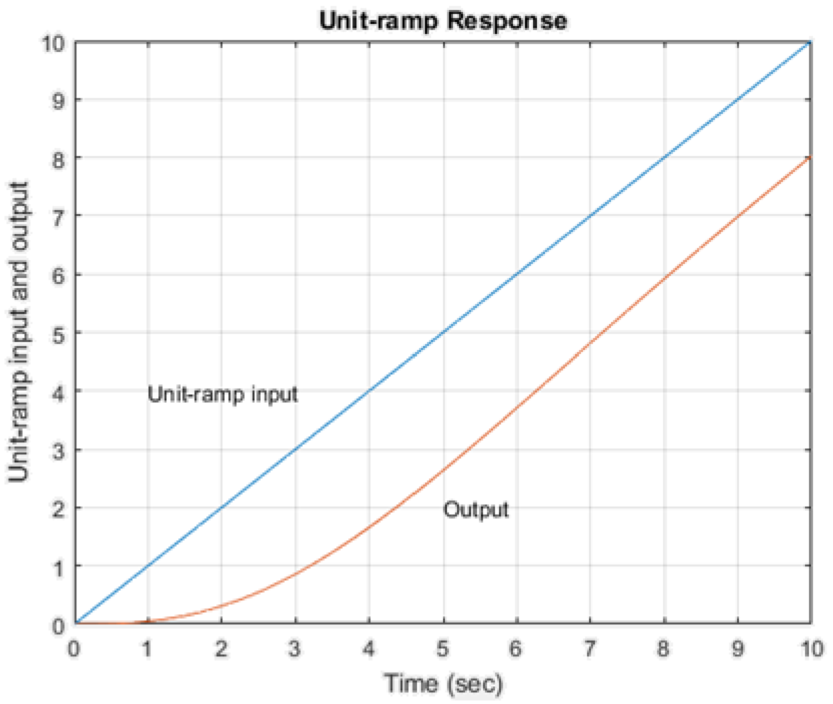
Answer

The MATLAB code for simulating a unit ramp response of the system given system is given as follows.

MATLAB code:

```
% demo34.m
% Unit-ramp response of a system with lsim() function
num=[0 0 1];
den=[3 2 1];
t=0:0.1:10;
r=t;
y=lsim(num,den,r,t);
plot(t,r,t,y)
grid
title('Unit-ramp Response')
xlabel('Time (sec)')
ylabel('Unit-ramp input and output')
text(1.0,4.0,'Unit-ramp input')
text(5.0,2.0,'Output')
```

The result of the MATLAB simulation is given in the diagram below.



Exercise 5 (Steady-State Analysis – Ramp and Exponential Inputs)

For the following closed-loop control system whose closed-loop transfer function is given by:

$$\frac{C(s)}{R(s)} = \frac{s + 12}{s^3 + 5s^2 + 8s + 12}$$

a. Obtain the unit-ramp response of the system using `lsim()` function in MATLAB.

b. Determine also the response of the system when the input is given by:

$$r = e^{-0.7t}$$

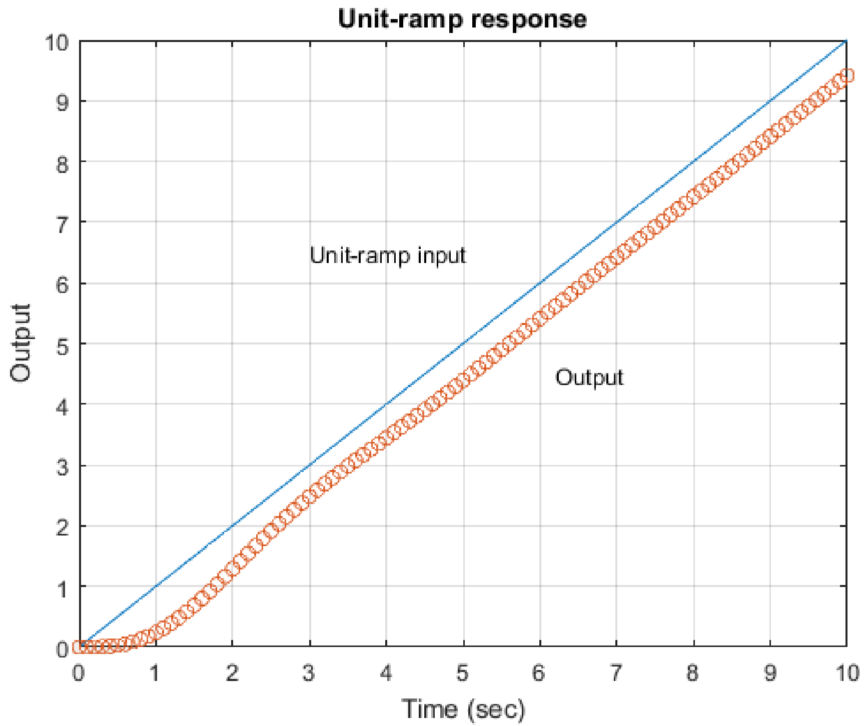
Answer

a. The MATLAB code for simulating a unit-ramp response of the given closed-loop control system is as follows:

MATLAB code:

```
% demo45a.m

% Unit-ramp response of a system with lsim() function

num=[0 0 1 12];
den=[1 5 8 12];

t=0:0.1:10;

r=t;

y=lsim(num,den,r,t);

plot(t,r,'-',t,y,'o')

grid

title('Unit-ramp response')

xlabel('Time (sec)')

ylabel('Output')

text(3.0,6.5,'Unit-ramp input')

text(6.2,4.5,'Output')
```

The unit-ramp response output of the MATLAB simulation is given in the following diagram below.



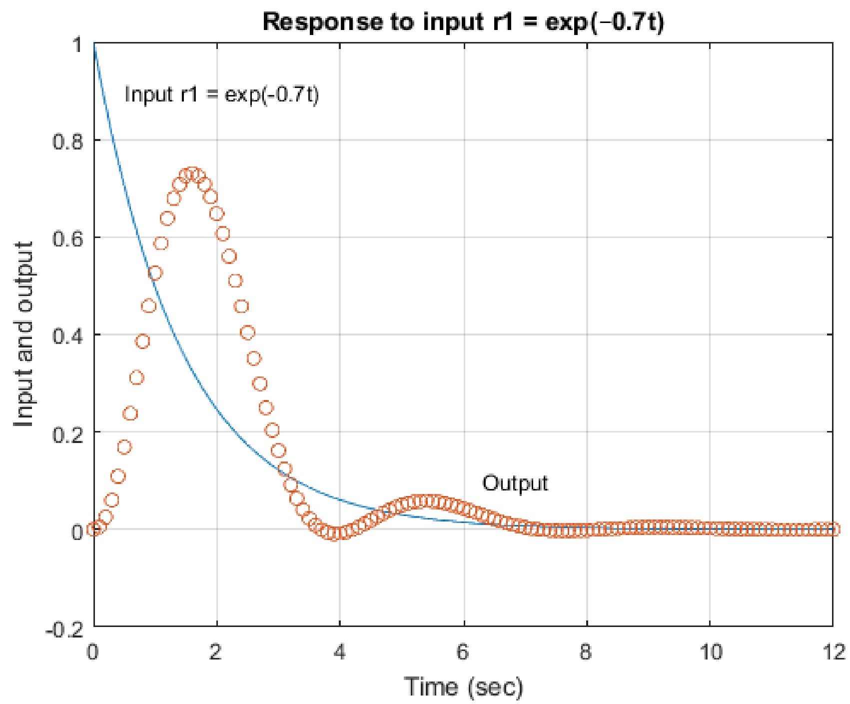
b. The following MATLAB code is used for simulating the response of the system whenever it is subjected to input of `r=exp(-0.7t)`.

MATLAB Code:

```
% demo45b.m

% Response of a system with exponential input

% Input r1=exp(-0.7t)

num=[0 0 1 12];
den=[1 5 8 12];

t=0:0.1:12;

r1=exp(-0.7*t);

y1=lsim(num,den,r1,t);

plot(t,r1,'-',t,y1,'o')

grid
```

```
title('Response to input r1 = exp(-0.7t)')

xlabel('Time (sec)')

ylabel('Input and output')

text(0.5,0.9,'Input r1 = exp(-0.7t)')

text(6.3,0.1,'Output')
```

The time response to the given input outcome of the MATLAB simulation is given in the following diagram below.



Exercise 6 (Steady-State Analysis – Step and Ramp Inputs)

The closed-loop system is defined by:

$$\frac{C(s)}{R(s)} = \frac{7}{s^2 + 2 + 7}$$

Obtain the response of the closed-loop system using MATLAB when it is subjected to input `r(t)` which is a step input of magnitude 3 plus unit-ramp input, `r(t)= 3+t`.

Answer

The following MATLAB code is for obtaining the response of the system whenever it is given a step input of magnitude 3 plus a unit ramp input (e.g. `r(t)=3+t`).

MATLAB code:

```
% demo46.m

% Unit step and ramp response of a system

num=[0 0 7];
den=[1 1 7];

t=0:0.05:10;

r=3+t;

c=lsim(num,den,r,t);

plot(t,r,'-',t,c,'o')

grid

title('Response to input r(t)=3+t')

xlabel('Time (sec)')

ylabel('Output c(t) and input r(t)=3+t')
```

The response to the given input result of the simulation if given in the diagram below.