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Demo 5: Bode Plots Analysis

XMUT315 Control Systems Engineering

Topics

1. Bode plot of the given systems.
2. Gain and phase in Bode plots.
3. Peak gain and bandwidth of Bode plot.
4. Analysis of gain and phase margins in Bode plots.
5. Design of control systems using Bode plots.

Write a program in MATLAB to obtain a Bode plot for the transfer function:

a. The first system:

$$G(s) = \frac{15}{s(s+3)(0.7s+5)}$$

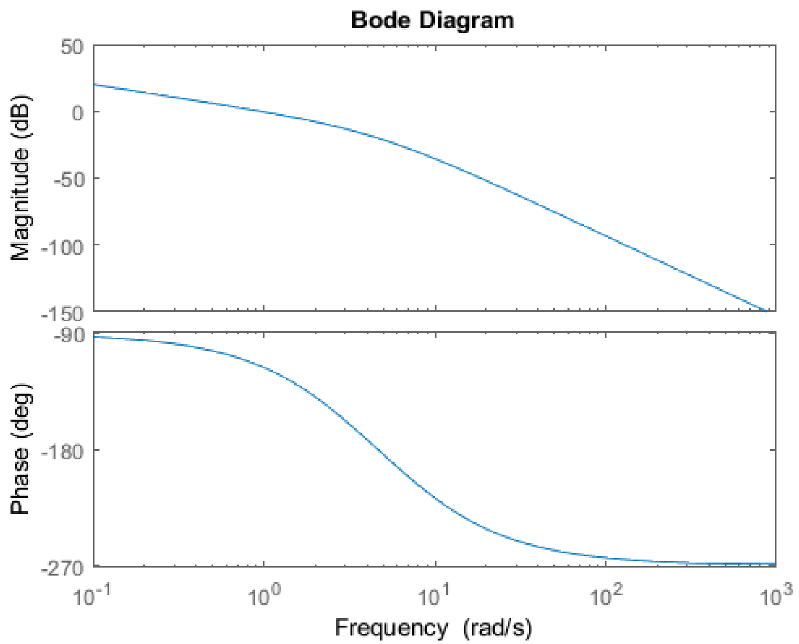
Solution

The MATLAB program for simulating the bode plots of the given system is given below:

MATLAB code:

```
clf

num=15;

den=conv([1 0],conv([1 3],[0.7 5]));

bode(num,den)
```

Computer response of the Bode plot simulation is shown in the figure below.



b. The second system:

$$G(s) = \frac{7s^3 + 15s^2 + 7s + 80}{s^4 + 8s^3 + 12s^2 + 70s + 110}$$

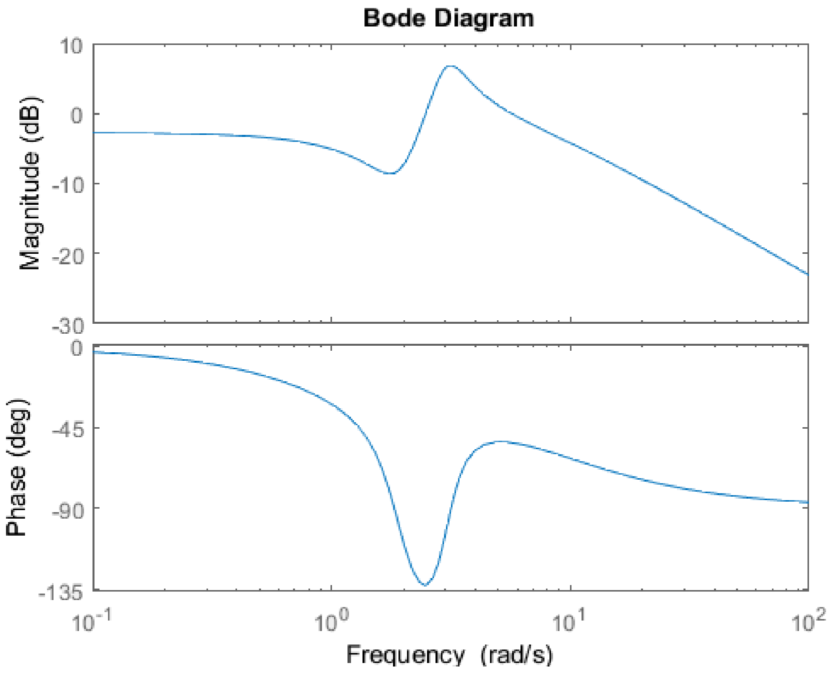
Solution

The MATLAB program for simulating the Bode plots of the given system is given below:

MATLAB code:

```
clf

num=[0 7 15 7 80];

den=[1 8 12 70 110];

bode(num,den)
```

Computer response of the Bode plot is shown in the figure given below.



Exercise 2 (Gain and Phase Margin in Bode Plots)

Write a program in MATLAB for the unity feedback system with the following transfer function:

$$G(s) = \frac{K}{s(s+3)(s+12)}$$

In this system, the value of gain K can be input. Display the Bode plots of the system for an input value of K. Determine and plot the gain and phase margin for an input value of K.

Solution

The MATLAB program for calculating the gain and phase margins of the given system is given below

MATLAB code:

```
% Enter G(s)
numg=1;
deng=poly([0 -3 -12]);

'G(s)'
G=tf(numg,deng)
w=0.01:0.1:100;
```

```
% Enter K
K=input('Type gain, K = ');
bode(K*G,w)

pause


[M,P]=bode(K*G,w);
```

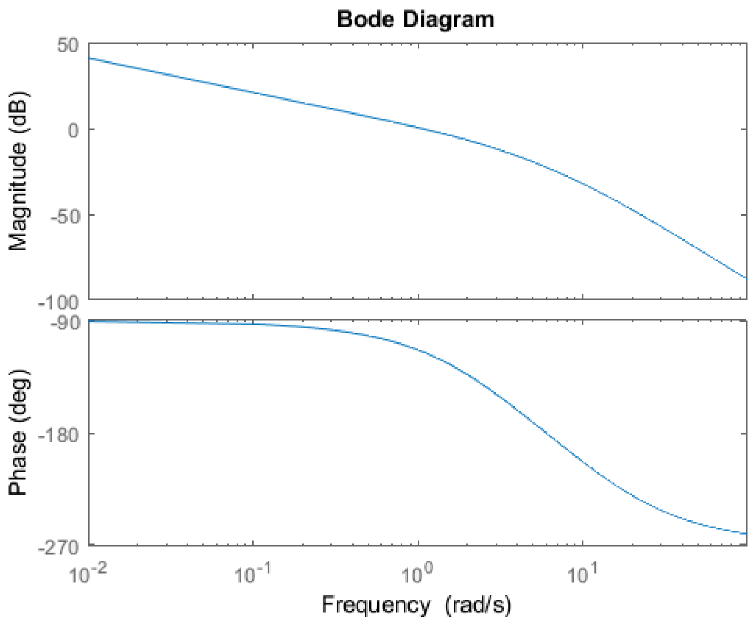
```
% Calculate gain margin

for i=1:1:length(P);
    if P(i)<=-180;
        fprintf('\nGain K = %g',K)
        fprintf(', Frequency (180 deg) = %g',w(i))
        fprintf(', Magnitude = %g',M(i))
        fprintf(', Magnitude(dB) = %g',20*log10(M(i)))
        fprintf(', Phase = %g',P(i))

        Gm=20*log10(1/M(i));

        fprintf(', Gain margin(dB) = %g',Gm)
        break
    end
end
```

```
% Calculate phase margin

for i=1:1:length(M);
    if M(i)<=1;
        fprintf('\nGain K = %g',K)
        fprintf(', Frequency(0 dB) = %g',w(i))
        fprintf(', Magnitude=%g',M(i))
        fprintf(', Magnitude(dB) = %g',20*log10(M(i)))
        fprintf(', Phase = %g',P(i))

        Pm=180+P(i) ;

        fprintf(', Phase margin(dB) = %g',Pm)
        break
    end
end
```

Type gain, K = 40 and the computer response:

```
ans =

    'G(s)'


G =

          1
  -------------------
  s^3 + 15 s^2 + 36 s

Continuous-time transfer function.
```

The Bode plot is shown in the figure given below.




```
Gain K = 40, Frequency (180 deg) = 6.01, Magnitude = 0.0738277, Magnitude(dB) = -22.6356, Phase = -180.076, Gain margin(dB) = 22.6356
Gain K = 40, Frequency(0 dB) = 1.11, Magnitude=0.93481, Magnitude(dB) = -0.585534, Phase = -115.589, Phase margin(dB) = 64.4107
```

The alternative MATLAB program for calculating the gain and phase margins of the given system is shown below:

MATLAB code:

```
% Enter G(s)
numg=1;
deng=poly([0 -3 -12]);

'G(s)'
G=tf(numg,deng)
w=0.01:0.1:100;

% Enter K
K=input('Type gain, K = ');
bode(K*G,w)
[Gm,Pm,Wcp,Wcg]=margin(K*G)

'Gm(dB)'
20*log10(Gm)
```

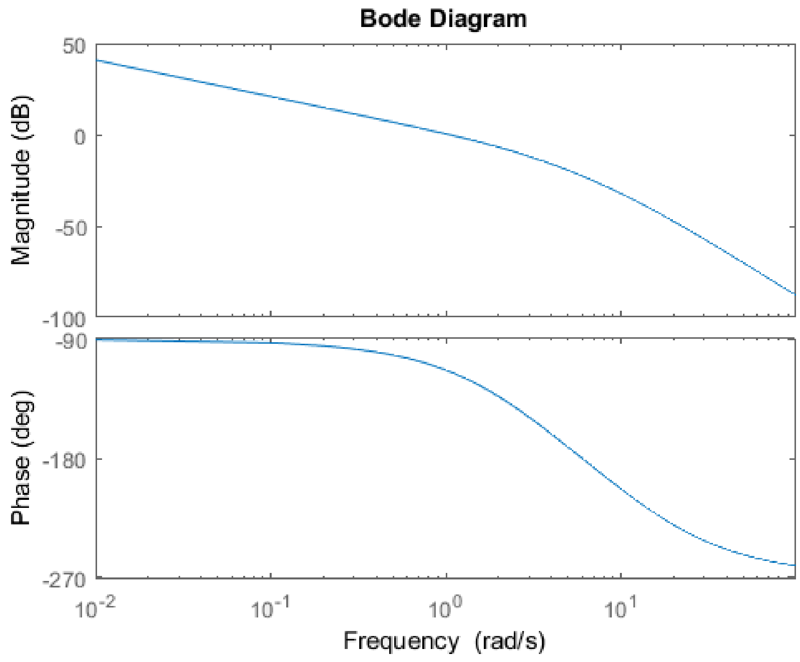
The outcome of the alternative MATLAB program using alternative approach is shown below (Gm = gain margin, Pm = phase margin, Wcp = frequency of phase margin, and Wcg = frequency of gain margin:

```
ans =

    'G(s)'


G =

              1
    -------------------
    s^3 + 15 s^2 + 36 s

Continuous-time transfer function.
```

```
Gm =

   13.5000


Pm =

   65.8119


Wcp =

     6
```

```
Wcg =

    1.0453


ans =

    'Gm(dB)'


ans =

   22.6067
```

The Bode plot of the alternative approach is shown in the figure given below.



Exercise 3 (Peak Gain and Bandwidth of Bode Plot)

Write a program in MATLAB for the system shown below so that the value of K can be input (e.g. entering the value of K = 40 later).

$$\frac{C(s)}{R(s)} = \frac{K(s+5)}{s(s^2+3s+15)}$$

Display the closed-loop gain and phase frequency response for unity feedback system with an open-loop transfer function, $KG(s)$.

Determine and display the peak gain, frequency of the peak gain, and bandwidth for the closed-loop frequency response for the input value of K.

Solution

The MATLAB program for simulating the Bode plot and calculating the frequency response parameters of the system is given below.

MATLAB code:

```
% Enter G(s)

numg=[1 5];
deng=[1 3 15 0];

'G(s)'
G=tf(numg,deng)
% Enter K
K=input('Type gain, K = ');

'T(s)'
T=feedback(K*G,1)

bode(T)
```

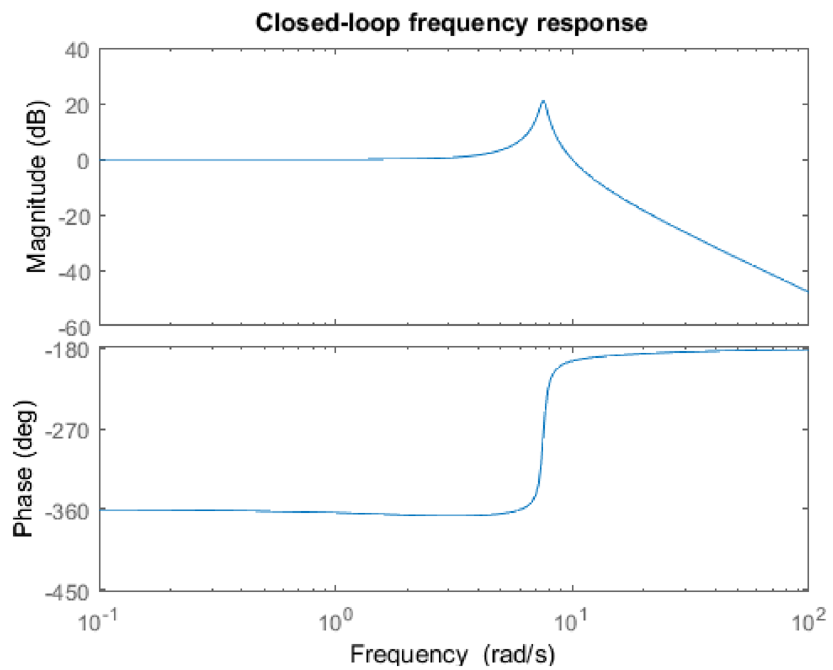
```
title('Closed-loop frequency response')

[M,P,w]=bode(T);

[Mp i]=max(M);

Mp

MpdB=20*log10(Mp)

wp=w(i)

for i=1:1:length(M);
    if M(i)<=0.707;
        fprintf('Bandwidth = %g',w(i))
        break
    end
end
```

Type gain, K = 40 in the command prompt and the computer response is shown below:

```
ans =

    'T(s)'


T =

          40 s + 200
  ------------------------
  s^3 + 3 s^2 + 55 s + 200

Continuous-time transfer function.


Mp =

   11.1162
```

```
MpdB =

   20.9192


wp =

    7.5295

Bandwidth = 11.2549
```

The Bode plot of the system is shown in the figure given below.



Exercise 4 (Analysis of Gain and Phase Margins in Bode plots)

The open-loop transfer function of a unity-feedback control system is given:

$$G(s) = \frac{K}{s(s^2 + 2s + 5)}$$

a. Determine the value of gain K such that the phase margin is 50°.

b. Find the gain margin for the gain K obtained in part (a).

Solution

a. The closed loop transfer function of the system is:

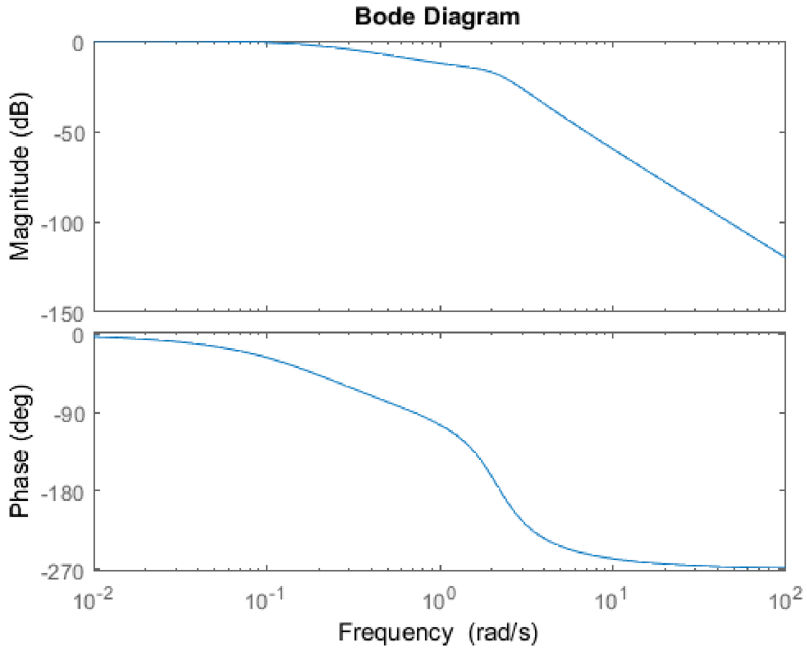
$$\frac{C(s)}{R(s)} = \frac{K}{s^3 + 2s^2 + 5s + K}$$

The MATLAB code for simulating the Bode plot with $K = 1$ is as follows:

MATLAB code:

```
clf

num=[1];

den=[1 2 5 1];

bode(num,den)
```

The Bode diagram is shown in the figure given below from MATLAB program.



b. From the Bode plot on the figure given above, the required phase margin of 50° and occurs at the frequency ω = 1.5 rad/s. The gain of $G(j\omega)$ at this frequency is then –12 dB. The gain K must then satisfy:

$$20 \log K = 12 \; dB$$

As a result, the gain of the system, K=4 for the required phase margin of the system = 50° .

The MATLAB code for simulating the Bode plot with $K = 4$ is as follows:

MATLAB code:

```
clf

num=[4];

den=[1 2 5 4];

bode(num,den)
```

The result of the Bode plot simulation of the system with $K = 4$ given below shows that the phase margin of the system = 50° and the gain margin is given as 0 dB.