XMUT315 Control Systems Engineering

Demo 7: Nyquist Diagram Analysis

A. Nyquist Diagram Analysis

In this section, we will look into several exercises performing analysis of control systems using Nyquist diagram.

Exercise 1 (Real axis crossing in Nyquist Plot)

Write a program in MATLAB for a unity-feedback system with:

$$G(s) = \frac{K(s+7)}{(s^2+3s+52)(s^2+2s+35)}$$

a. Plot the Nyquist diagram.

b. Display the real-axis crossing value and frequency.

Solution

The MATLAB program for simulating the Nyquist diagram of the given system is:

MATLAB code

```
% demo71.m

% Simulate real axis crossing of the system in Nyquist plot

numg=[1 7]
deng=conv([1 3 52],[1 2 35]);

G=tf(numg,deng)

'G(s)'
```

```
Gap=zpk(G)

w=0:0.1:100;

[re,im]=nyquist(G,w);

nyquist(G,w);

for i=1:1:length(w)

M(i)=abs(re(i)+j*im(i));
A(i)=atan2(im(i),re(i))*(180/pi);

      if 180-abs(A(i))<=1;
            re(i);
            im(i);

            K=1/abs(re(i));

            fprintf('\nw = %g',w(i))
            fprintf(',Re = %g',re (i))
            fprintf(',Im = %g',im (i))
            fprintf(',M = %g',M (i))
            fprintf(',K = %g',K)

            Gm=20*log10(1/M(i));

            fprintf(',Gm=&G',Gm)

            break
      end

end
```

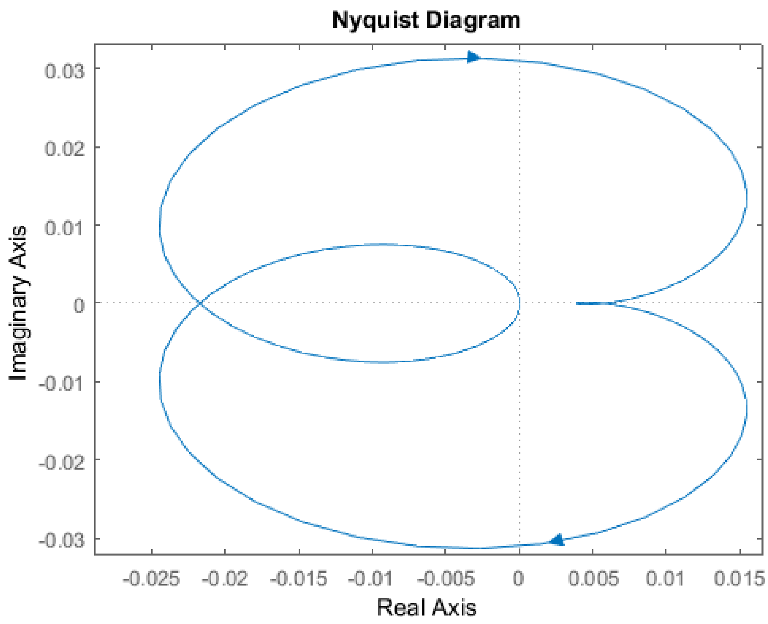
Computer response of the program given above:

```
numg =

     1     7


G =

                s + 7
  -----------------------------------
  s^4 + 5 s^3 + 93 s^2 + 209 s + 1820

Continuous-time transfer function.
```

```
ans =

    'G(s)'


Gap =

                 (s+7)
  -------------------------------
  (s^2 + 2s + 35) (s^2 + 3s + 52)

Continuous-time zero/pole/gain model.
```

w = , Re = , Im = , M =, K =, Gm =

The Nyquist plot is shown in the following figure.



Exercise 2 (Stability in Nyquist Plot)

The open-loop transfer function of a unity-feedback control system is given

$$G(s) = \frac{1}{s^3 + 0.3s^2 + 5s + 1}$$

a. Draw a Nyquist plot of $G(s)$ using Matlab.

b. Determine the stability of the system.

Solution

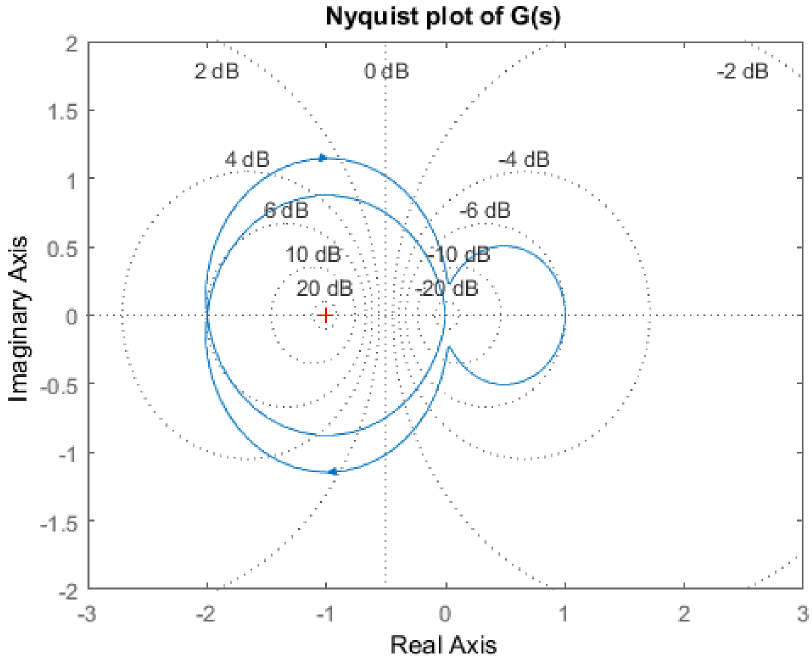
a. The MATLAB program for simulating the Nyquist plot of the system is:

MATLAB code:

```
% demo72.m

% Simulate stability of the system

% Open-loop poles

p=[1 0.3 5 1];

r=roots(p)

% Nyquist plot

num=[0 0 0 1];
den=[1 0.3 5 1];

nyquist(num,den)

v=[-3 3 -2 2]; axis(v); axis('square')

grid

title('Nyquist plot of G(s)')
```

b. The result of the simulation is shown in the figure below. Evaluating the outcome of the simulation, there are two open-loop poles in the right half s plane as there are two encirclements of the critical point, the closed-loop system is unstable.

```
r =

  -0.0496 + 2.2311i
  -0.0496 - 2.2311i
  -0.2008 + 0.0000i
```



Exercise 3 (Nyquist Plots)

The open-loop transfer function of a unity-feedback control system is given by:

$$G(s) = \frac{K(s+3)}{s(s+1)(s+7)}$$

Plot the Nyquist diagram of $G(s)$ for K = 1, 10, and 100 using MATLAB.

Solution

The transfer function of the system becomes as follows:

$$G(s) = \frac{K(s+3)}{s(s+1)(s+7)} = \frac{K}{s^3 + 8s^2 + 7s}$$

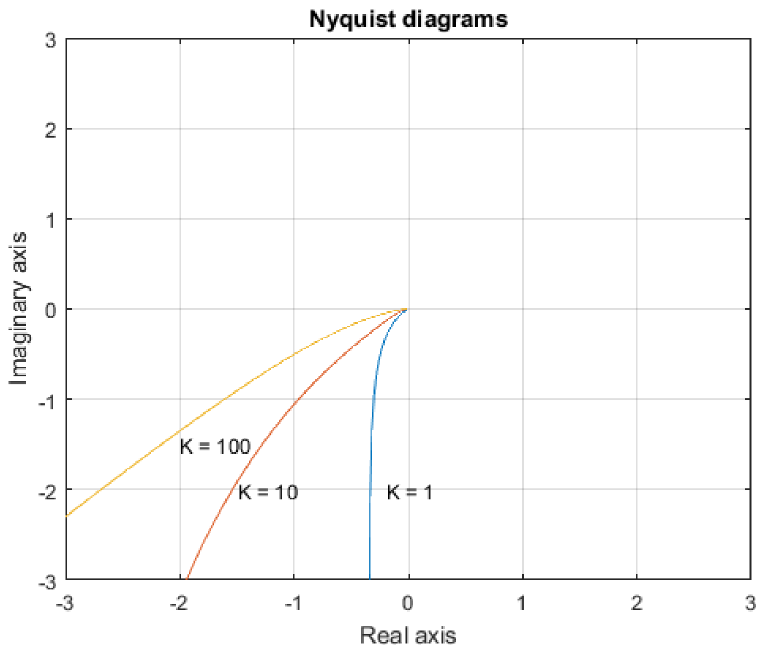
The MATLAB program for simulating the Nyquist diagrams of the given system is shown below:

MATLAB code:

```
% demo73.m

% Simulate Nyquist plots of the systems

num=[1 3];
den=[1 8 7 0];

w=0.1:0.1:100;
```

```
[re1,im1,w]=nyquist(num,den,w);
[re2,im2,w]=nyquist(10*num,den,w);
[re3,im3,w]=nyquist(100*num,den,w);

plot(re1,im1,re2,im2,re3,im3)

v=[-3 3 -3 3]; axis(v)

grid

title('Nyquist diagrams')
xlabel('Real axis')
ylabel('Imaginary axis')
text(-0.2,-2,'K = 1')
text(-1.5,-2.0,'K = 10')
text(-2,-1.5,'K = 100')
```

Referring to the result of the Nyquist plot simulation of the given system for gain, K = 1, 10 and 100, the plot shows that:



Exercise 4 (Nyquist Plot of Positive Feedback System)

The open-loop transfer function of a negative feedback system is given by:

$$G(s) = \frac{5}{s(s+1)(s+3)}$$

Plot the Nyquist diagram for $G(s)$ using MATLAB and the same open-loop transfer function use $G(s)$ of a positive-feedback system using MATLAB.

Solution

The transfer function equation of the system becomes:

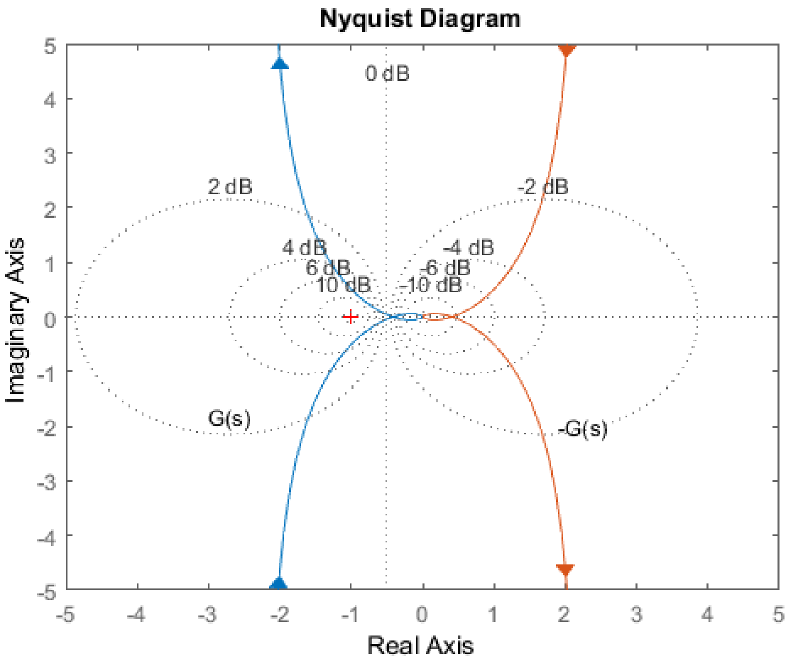
$$G(s) = \frac{5}{s(s+1)(s+3)} = \frac{5}{s^3 + 4s^2 + 3s}$$

The MATLAB program for simulating the Nyquist diagram of the given system is listed below:

MATLAB code:

```
% demo74.m

% Nyquist diagrams of G(s) and - G(s)

num1=[0 0 0 5];
den1=[1 4 3 0];
num2=[0 0 0 -5];
den2=[1 4 3 0];

nyquist(num1,den1)

hold

nyquist(num2,den2)

v=[-5 5 -5 5]; axis(v)

grid

text(-3,-1.8,'G(s)')
text(1.9,-2,'- G(s)')
```

The output of the simulation is shown in the figure below.



B. Nichols Chart Analysis

In this section, we will cover several examples of control system analysis using Nichols chart.

Exercise 5 (Comparing Nyquist Diagram and Nichol Chart)

Write a program in MATLAB to obtain the Nyquist and Nichols plots for the following transfer function for K = 30.

$$G(s) = \frac{K(s+1)(s+3+7i)(s+3-7i)}{(s+1)(s+3)(s+5)(s+3+7i)(s+3-7i)}$$

Determine the stability of the system from the two plots.

Compare at least two of the similarities and two differences between Nyquist diagram and Nichols chart for determining the stability of the system.

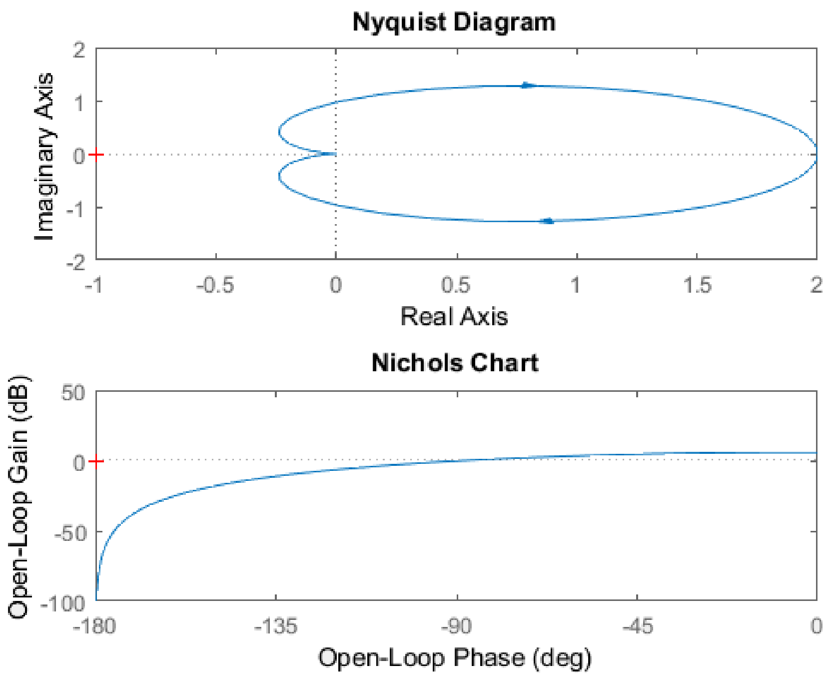
Solution

The MATLAB program for simulating the Nyquist and Nichols diagrams of the given system is shown below:

MATLAB code:

```
% demo75.m

% Simulate Nyquist and Nichols plots

clf

z=[-1 -3+7*i -3-7*i];
p=[-1 -3 -5 -3+7*i -3-7*i];

k=30;

[num,den]=zp2tf(z',p',k);

subplot(211),nyquist(num,den)
subplot(212),nichols(num,den)

% ngrid

% axis([-50 -360 -40 30])
```

Computer response of the Nyquist and Nichols plots are shown in the following figures.



From the results of the simulation, the system is found to be stable e.g. no encirclement at unity gain point in the Nyquist diagram and the gain and phase margins of the system are found to be positive from the Nichols chart.

Comparison of the differences between Nyquist diagram and Nichols chart for determining the stability of the system.

Similarities

- Both graphs are plots that simulate the gain and phase of the systems for given frequency.
- Both are typically used to analyse the stability condition and performance of the system.

Differences

- For determining stability Nyquist diagram is based on encirclement at the unity gain point in the diagram and Nichols chart is from the gain and phase margins of the system at 180° and unity gain point in the chart.
- Simulation with Nyquist diagram of complex system at higher frequency would not be as straight forward as the simulation of the system in the Nichols chart e.g. it can cover to the full extend the characteristics and behaviour of complex system at high frequency.

Exercise 6 (Plotting Nichols Chart)

Plot the Nichols response with and without the grid lines for the following system:

$$H(s) = \frac{-4s^4 + 48s^3 - 18s^2 + 250s + 600}{s^4 + 30s^3 + 282s^2 + 525s + 60}$$

From the plot, describe the characteristics of the system and determine from the simulation the gain and phase of the system at frequency of 10 rad/s.

Simulate the transient step response of the system to prove the result of the stability analysis.

Solution

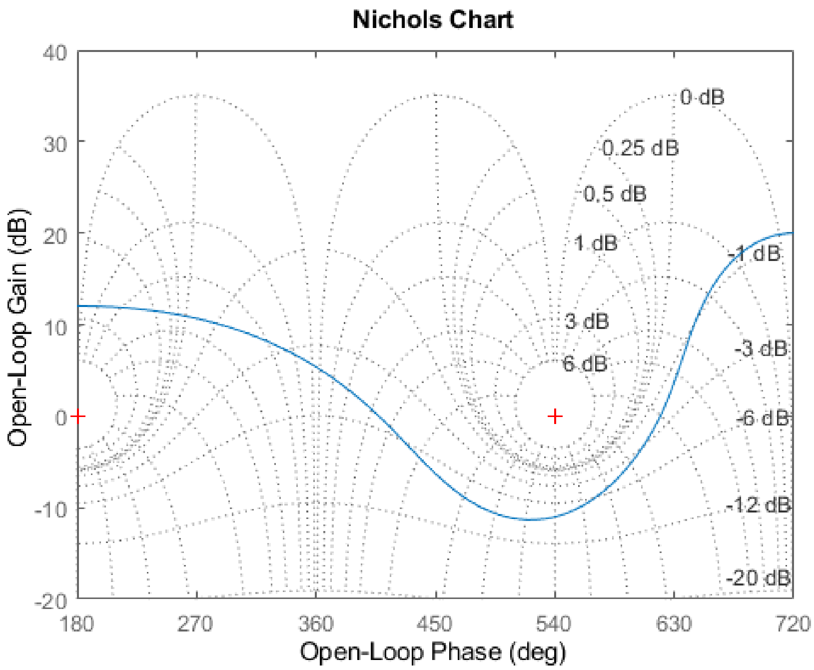
The MATLAB program for simulating the Nichols chart of the given system is shown below:

MATLAB code:

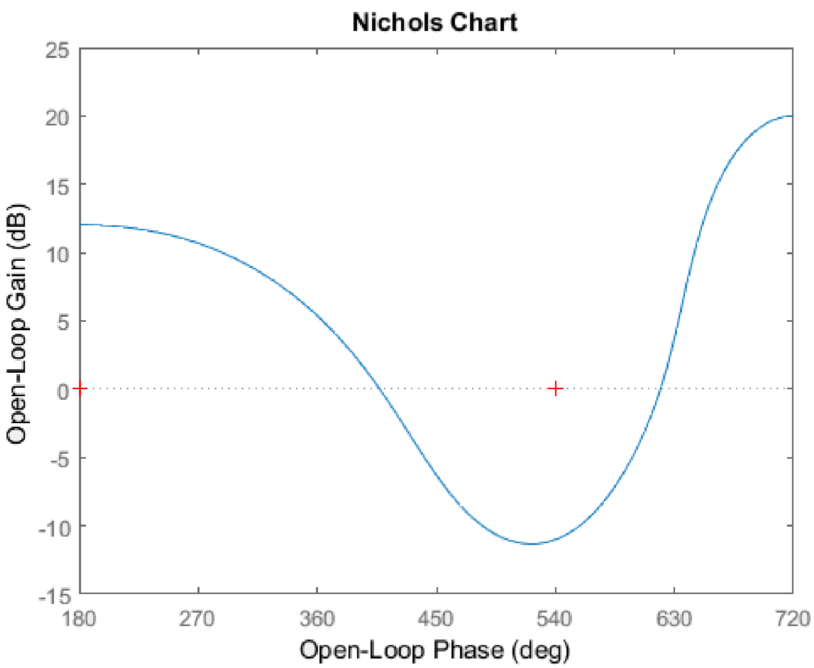
```
% demo76a.m

% Simulate Nichols chart

H = tf([-4 48 -18 250 600],[1 30 282 525 60]);

nichols(H)

ngrid

w = 10;

[mag,phase,w]= nichols(H,w)
```

The result of the simulation is as given in the following figure. The `nichols()` function returns the gain `mag` and phase `phase` (in degrees) of the frequency response at the frequencies `w` (in rad/time unit). The outputs mag and phase are 3-D arrays similar to those produced by bode plot.

The Nichols chart with the ngrid is shown in the figure below.



The Nichols chart without the ngrid is shown in the figure below.



From the simulation, the system is found to be stable as that the gain and phase margins are found to be positive e.g. 12 dB and 220°. Note the plot is inverted due to negative coefficient found in the transfer function of the system.

From the simulation at the frequency of 10 rad/s, the gain and phase (in degrees) of the frequency response of the system are found to be 1.9235 dB at 356.6694°.

The transient response of the system is simulated using the following MATLAB program.

MATLAB code:

```
% demo76b.m

% Simulate step response

H = tf([-4 48 -18 250 600],[1 30 282 525 60]);

step(H);
```

The transient response of the system is given in the plot below. The system is found to be stable and has an exponential step response.