Bode Plots Straight-Line Approximation

(Case Study Revision)

XMUT315 Control System Engineering

Topics

- Pole or zero at origin.
- Simple pole or zero.
- Simple poles or simple zeros.
- Simple double pole(s) and simple zero(s).
- Complex zeros.
- Simple zero and complex poles.

Exercise 1: Zero and poles at origin

- For a system with the following transfer function equation:

$$G_1(s) = s$$

- DC gain of 0 dB at ω = 0 rad/s.

$$|G_1(s)|_{\omega = 0} = 0 = 0 = 0 \text{ dB}$$

- Zero at origin (ω = 0 rad/s):
  - Gain slope of +20 dB/decade.
  - Gain 0 (20 log (1) = 0 dB) at ω = 1 rad/s.
  - Phase shift at +90°.

Exercise 1: Zero and poles at origin

- For a system with the following transfer function equation:

$$G_2(s) = \frac{1}{s}$$

- DC gain at ω = 0 rad/s:

$$|G_2(s)|_{\omega = 0} = \frac{1}{0} = \infty = \infty \text{ dB}$$

- Pole at origin (ω = 0 rad/s):
    - Gain slope of -20 dB/decade.
    - Gain 0 (20 log (1) = 0 dB) at ω = 1 rad/s.
    - Phase shift at -90°.

Exercise 1: Zero and poles at origin

- For a system with the following transfer function equation:

$$G_3(s) = \frac{10}{s}$$

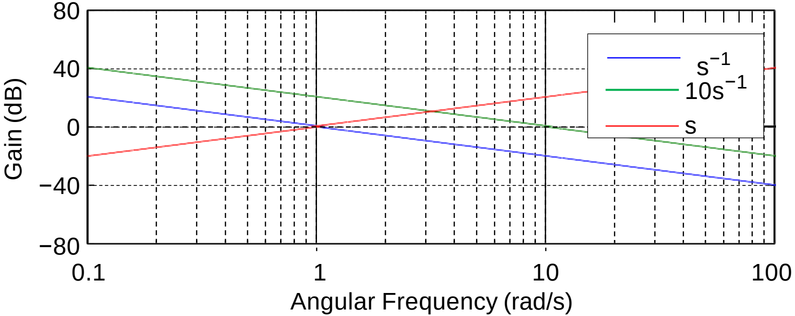
- DC gain at ω = 0 rad/s:

$$|G_3(s)|_{\omega = 0} = \frac{10}{0} = \infty = \infty \text{ dB}$$

- Pole at origin (ω = 0 rad/s):
  - Gain slope of -20 dB/decade.
  - Gain 10 (20 log (10) = 20 dB) at ω = 1 rad/s.
  - Phase shift at -90°.

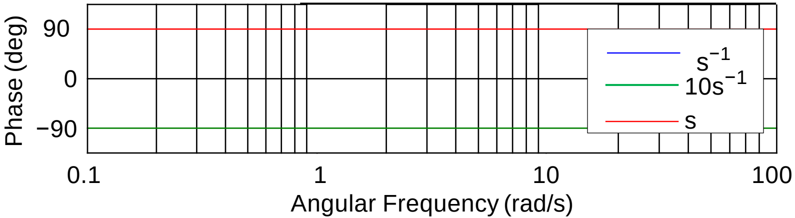
Exercise 1: Zero and poles at origin

- Gain sketches of the systems: $G_1(s) = s$, $G_2(s) = 1/s$, and $G_3(s) = 10/s$.



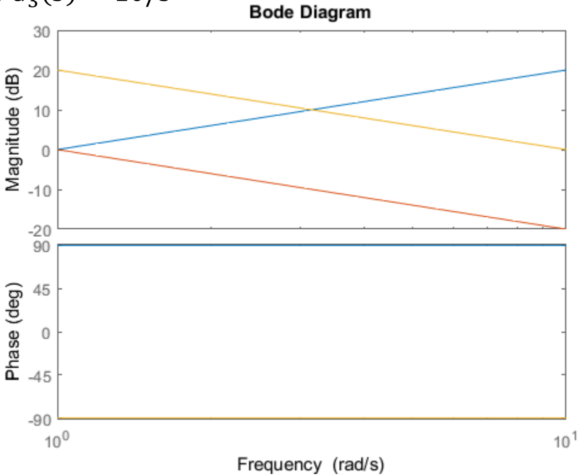
Exercise 1: Zero and poles at origin

- Phase sketches of the systems: $G_1(s) = s$, $G_2(s) = 1/s$, and $G_3(s) = 10/s$.



Exercise 1: Zero and poles at origin

- Matlab simulation of the systems: $G_1(s) = s$, $G_2(s) = 1/s$, and $G_3(s) = 10/s$



Exercise 2: Pole at origin

- For a system with the following transfer function equation:

$$G_4(s) = \frac{100}{s}$$

- DC gain at ω = 0 rad/s:

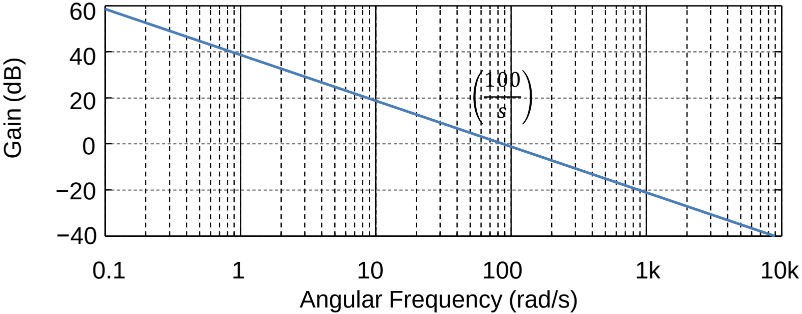
$$|G_4(s)|_{\omega\,=\,0} = \frac{100}{0} = \infty = \infty\ \mathrm{dB}$$

- Pole at origin (ω = 0 rad/s):
    - Gain slope of -20 dB/decade.
    - Gain of 100 (20 log (100) = 40 dB) at ω = 1 rad/s.
    - Phase shift of -90°.

Exercise 2: Pole at origin

- Gain sketch of the system:

$$G_4(s) = \left(\frac{100}{s}\right)$$



Exercise 2: Pole at origin

- Phase sketch of the system:

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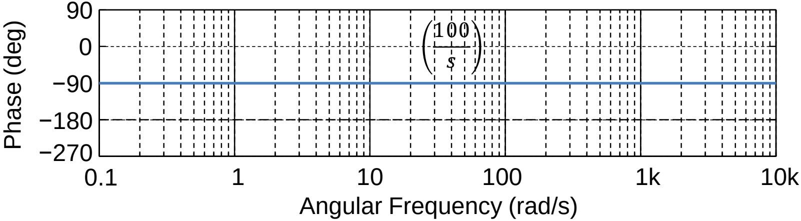
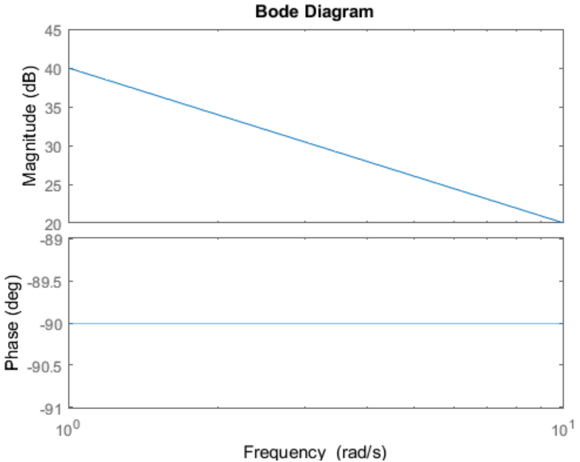


Exercise 2: Pole at origin

- Matlab simulation of the system:

$$G_4(s) = \frac{100}{s}$$



Exercise 3: Pole at origin with negative gain

- For a system with the following transfer function equation:

$$G_5(s) = -\frac{100}{s}$$

- DC gain at ω = 0 rad/s:

$$|G_5(s)|_{\omega = 0} = \frac{100}{0} = \infty = \infty \text{ dB}$$

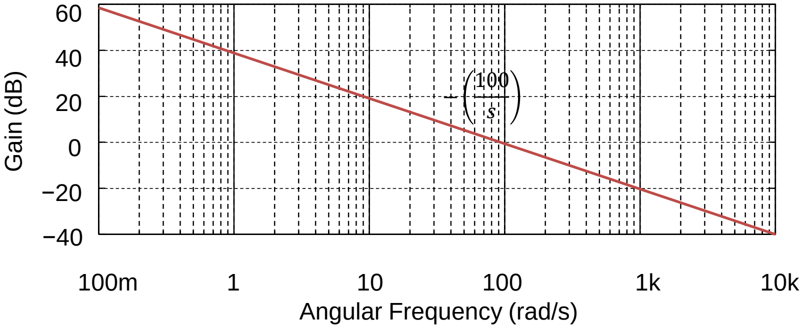
- Pole at origin (ω = 0 rad/s):
  - Gain slope of -20 dB/decade.
  - Gain of 100 (20 log (100) = 40 dB) at ω = 1 rad/s.
  - Phase shift of +90° (θ = 180° - 90°).

Note: to cope with negative gain (<0), you just need to account for the extra 180° phase shift associated with negative gain.

Exercise 3: Real pole at origin with negative gain

- Gain sketch of the system:

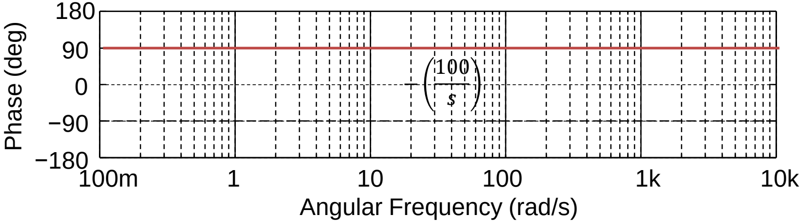
$$G_5(s) = -\frac{100}{s}$$



Exercise 3: Real pole at origin with negative gain

- Phase sketch of the system:

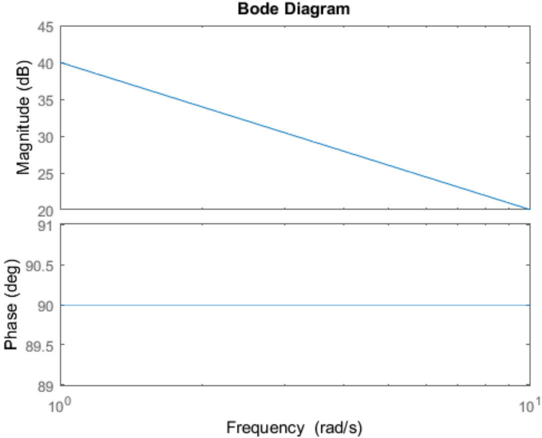
$$G_5(s) = -\left(\frac{100}{s}\right)$$



Exercise 3: Real pole at origin with negative gain

- Matlab simulation of the system:

$$G_5(s) = -\frac{100}{s}$$



Exercise 4: Real pole

- For a system with the following transfer function equation:

$$G_6(s) = \frac{7}{s + 20}$$

- DC gain at ω = 0 rad/s:

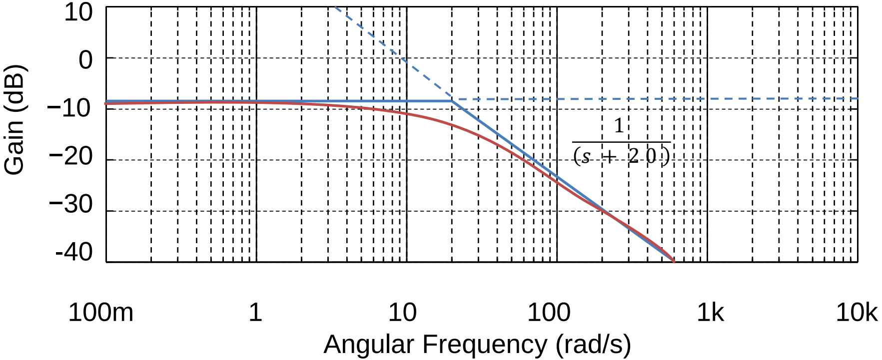
$$|G_6(s)|_{\omega = 0} = \frac{7}{20} = 0.35 = -9 \text{ dB}$$

- Real pole at ω = 20 rad/s:
  - Gain slope of -20 dB/decade after ω = 20 rad/s.
  - Phase shift of 0° at ω = 2 rad/s.
  - Phase shift of 45° at ω = 20 rad/s.
  - Phase shift of -90° at ω = 200 rad/s.

Exercise 4: Real pole

- Gain sketch of the system:

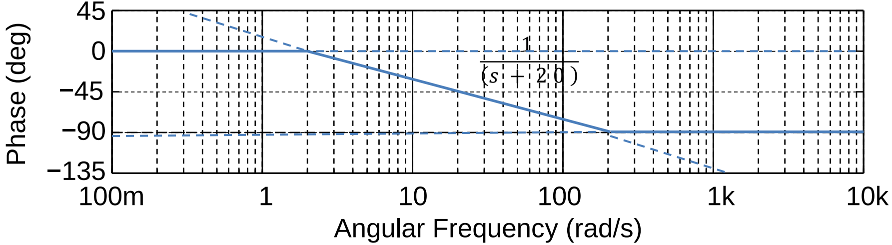
$$G_6(s) = \frac{7}{s + 20}$$



Exercise 4: Real pole

- Phase sketch of the system:

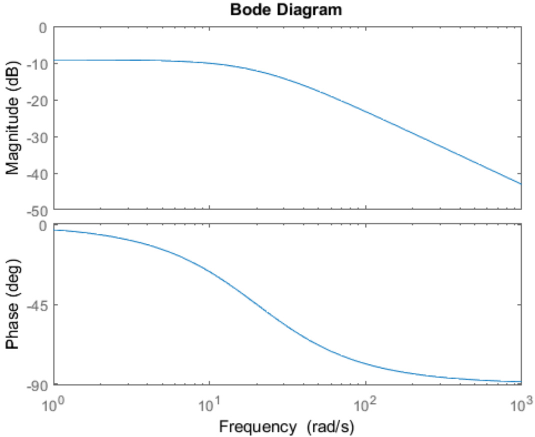
$$G_6(s) = \frac{7}{(s+20)}$$



Exercise 4: Real pole

- Matlab simulation of the system:

$$G_6(s) = \frac{7}{s + 20}$$



Exercise 5: Real zero

- For a system with the following transfer function equation:

$$G_7(s) = (s + 50)$$

- DC gain at ω = 0 rad/s:

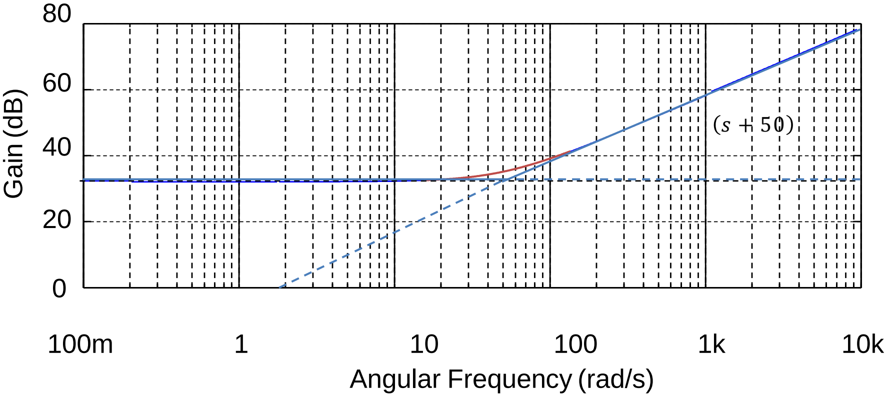
$$|G_7(s)|_{\omega = 0} = 50 = 34 \text{ dB}$$

- Real zero at ω = 50:
  - Gain slope of +20 dB/decade after ω = 50 rad/s.
  - Phase shift of 0° at ω = 5 rad/s.
  - Phase shift of 45° at ω = 50 rad/s.
  - Phase shift of +90° at ω = 500 rad/s.

Exercise 5: Real zero

- Gain sketch of the system:

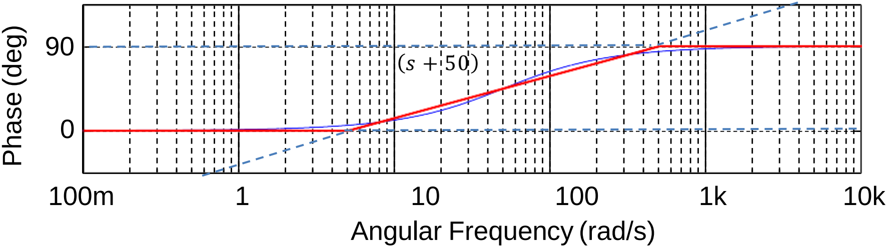
$$G_7(s) = (s + 50)$$



Exercise 5: Real zero

- Phase sketch of the system:

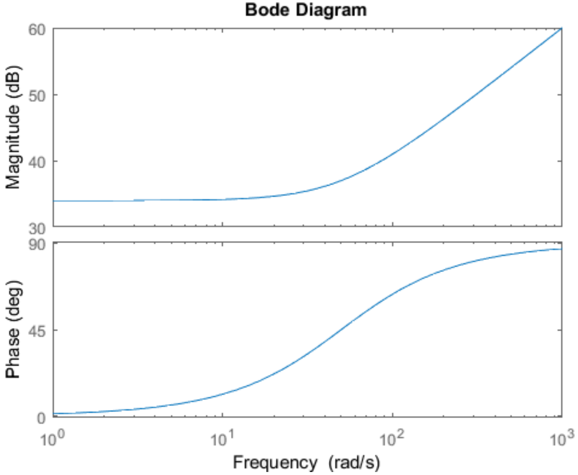
$$G_7(s) = (s + 50)$$



Exercise 5: Real zero

- Matlab simulation of the system:

$$G_7(s) = s + 50$$



Exercise 6: Real poles

- For a system with the following transfer function equation:

$$G_8(s) = \frac{1}{(s+2)(s+20)}$$

- DC gain at ω = 0 rad/s:

$$|G(s)|_{\omega = 0} = \frac{1}{(2)(20)} = 0.025 = -32 \text{ dB}$$

- First real pole at ω = 2 rad/s:
  - Gain slope of 20 dB/decade after ω = 2 rad/s.
  - Phase shift of 0° at ω = 0.2 rad/s.
  - Phase shift of -45° at ω = 2 rad/s.
  - Phase shift of -90° at ω = 20 rad/s.

Exercise 6: Real poles

- Second real pole at ω = 20 rad/s:
  - Gain slope of 20 dB/decade after ω = 20 rad/s.
  - Phase shift of 0° at ω = 2 rad/s
  - Phase shift of -45° at ω = 20 rad/s.
  - Phase shift of -90° at ω = 200 rad/s.

Exercise 6: Real poles

- Overall gain of the system:

$$G_8(s) = \frac{1}{(s+2)(s+20)}$$

  - DC gain of 0.025 (= -32 dB) at ω = 0 rad/s.
  - Gain slope of -20 dB/decade after ω = 2 rad/s.
  - Gain slope of -40 dB/decade after ω = 20 rad/s.

Exercise 6: Real poles

- Overall phase shift of the system:

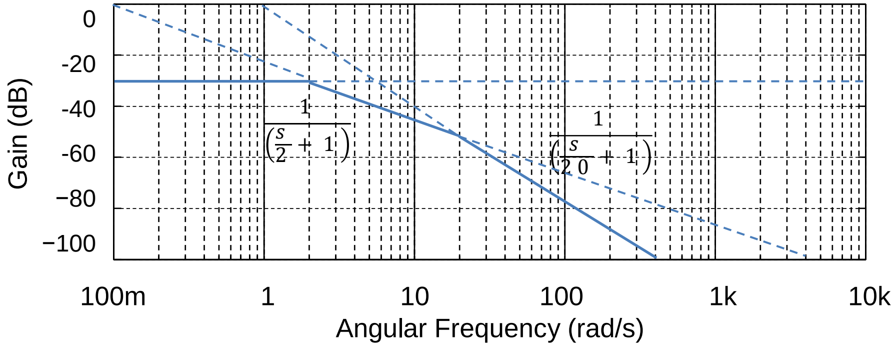
$$G_8(s) = \frac{1}{(s+2)(s+20)}$$

  - Phase shift of 0° at ω = 0 rad/s.
  - Phase shift of -45° at ω = 2 rad/s
  - Phase shift of -90° after ω = 20 rad/s
  - Phase shift of -135° at ω = 20 rad/s.
  - Phase shift of -180° after ω = 200 rad/s.

Exercise 6: Real poles

- Gain sketch of the system:

$$G_8(s) \ = \frac{1}{(s+2)(s+20)}$$



Exercise 6: Real poles

- Phase sketch of the system:

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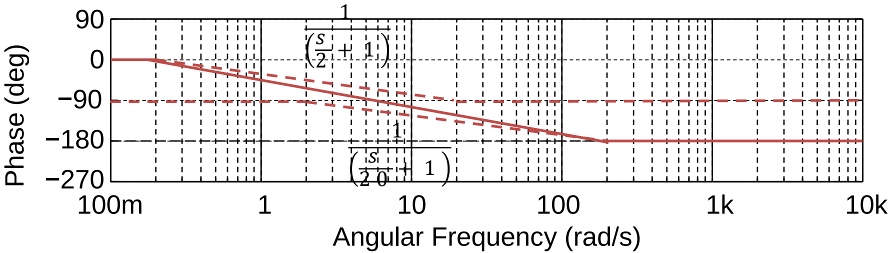
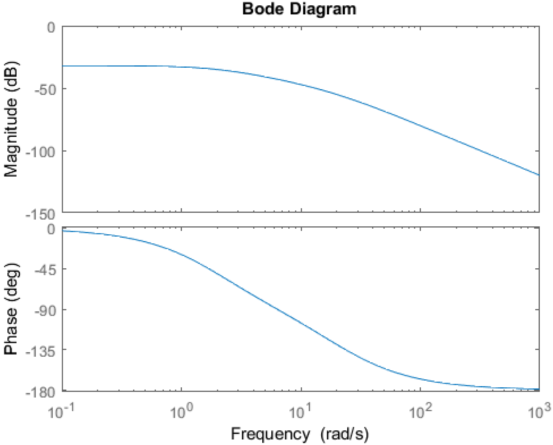


Exercise 6: Real poles

- Matlab simulation of the system:

$$G_8(s) = \frac{1}{(s+2)(s+20)}$$



Exercise 7: Real pole and zero

- For a system with the following transfer function equation:

$$G_9(s) \ = \frac{(s+100)}{(s+5)}$$

- DC gain at ω = 0 rad/s:

$$|G(s)|_{\omega\ =\ 0} = \frac{100}{(5)} = 20 = 26\ \mathrm{dB}$$

- Real pole at ω = 5 rad/s:
  - Gain slope of -20 dB/decade after ω = 5 rad/s.
  - Phase shift of 0° at ω = 0.5 rad/s.
  - Phase shift of -45° at ω = 5 rad/s.
  - Phase shift of -90° at ω = 50 rad/s.

Exercise 7: Real pole and zero

- Real zero at ω = 100 rad/s:
  - Gain slope of +20 dB/decade after ω = 100 rad/s.
  - Phase shift of 0° at ω = 10 rad/s.
  - Phase shift of +45° at ω = 100 rad/s.
  - Phase shift of +90° at ω = 1000 rad/s.

Exercise 7: Real pole and zero

- Overall gain of the system:

$$G_9(s) = \frac{(s+100)}{(s+5)}$$

  - DC gain of 20 (= 26 dB) at ω = 0 rad/s.
  - Gain slope of -20 dB/decade after ω = 5 rad/s.
  - Gain slope of 0 dB/decade after ω = 100 rad/s.

Exercise 7: Real pole and zero

- Overall phase shift of the system:

$$G_9(s) \ = \frac{(s+100)}{(s+5)}$$

  - Phase shift of 0° at ω = 0 rad/s.
  - Phase shift of -45° at ω = 5 rad/s.
  - Phase shift of -90° after ω = 50 rad/s.
  - Phase shift of -45° at ω = 100 rad/s.
  - Phase shift of 0° after ω = 1000 rad/s.

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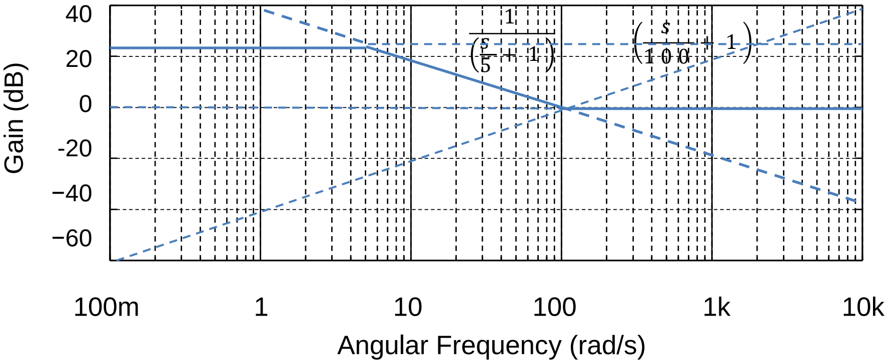

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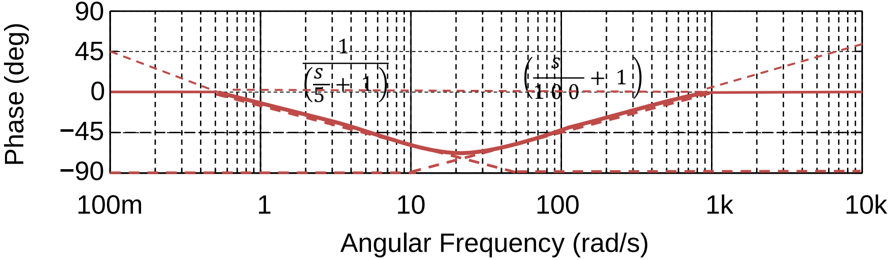
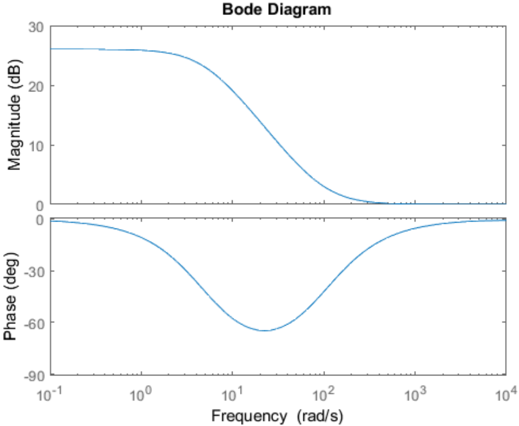


Exercise 7: Real pole and zero

- Matlab simulation of the system:

$$G_9(s) = \frac{(s+100)}{(s+5)}$$



Exercise 8: Real double poles and zero

- For a system with the following transfer function:

$$G_{10}(s) = \frac{(s + 100)}{(s + 5)^2}$$

- DC gain at ω = 0 rad/s:

$$|G(s)|_{\omega = 0} = \frac{100}{(5)^2} = 4 = 12 \text{ dB}$$

- Real double poles at ω = 5 rad/s:
  - Gain slope of -20 dB/decade after ω = 5 rad/s.
  - Phase shift of 0° at ω = 0.5 rad/s.
  - Phase shift of -90° at ω = 5 rad/s.
  - Phase shift of -180° at ω = 50 rad/s.

Exercise 8: Real double poles and zero

- Real zero at ω = 100 rad/s:
  - Gain slope of +20 dB/decade after ω = 100 rad/s.
  - Phase shift of 0° at ω = 10 rad/s.
  - Phase shift of +45° at ω = 100 rad/s.
  - Phase shift of +90° at ω = 1000 rad/s.

Exercise 8: Real double poles and zero

- Overall gain of the system:

$$G_{10}(s) = \frac{(s+100)}{(s+5)^2}$$

  - DC gain of 4 (= 12 dB) at ω = 0 rad/s.
  - Gain slope of -40 dB/decade after ω = 5 rad/s.
  - Gain slope of -20 dB/decade after ω = 100 rad/s.

Exercise 8: Real double poles and zero

- Overall phase shift of the system:

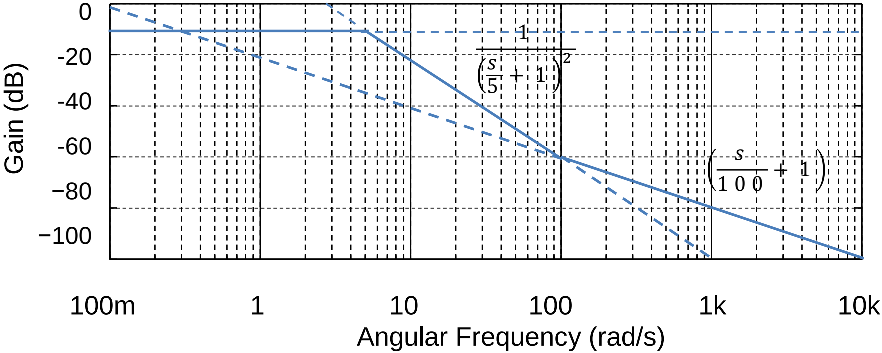
$$G_{10}(s) = \frac{(s+100)}{(s+5)^2}$$

  - Phase shift of 0° at ω = 0 rad/s.
  - Phase shift of -90° at ω = 5 rad/s.
  - Phase shift of about -90° to -180° around ω = 10 rad/s and ω = 50 rad/s.
  - Phase shift of -135° at ω = 100 rad/s.
  - Phase shift of -90° after ω = 1000 rad/s.

Exercise 8: Real double poles and zero

- Gain sketch of the system:

$$G_{10}(s) = \frac{(s+100)}{(s+5)^2}$$



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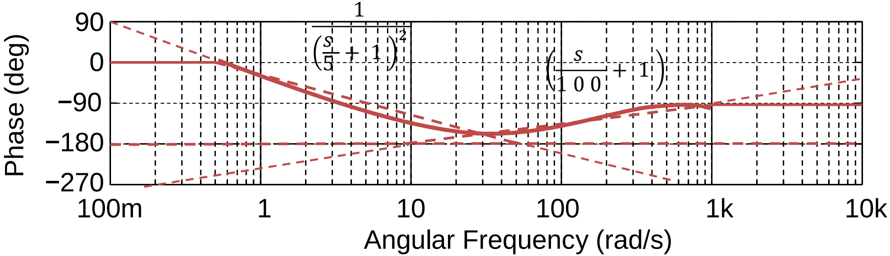
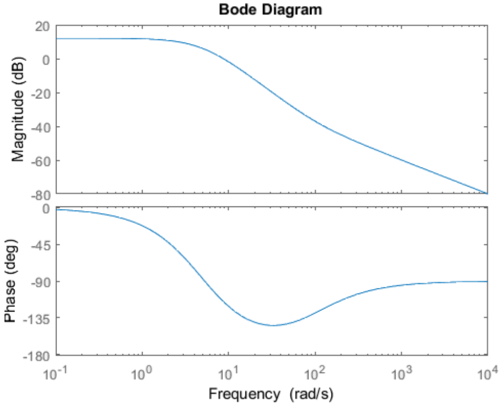


Exercise 8: Real double poles and zero

- Matlab simulation of the system:

$$G_{10}(s) = \frac{(s+100)}{(s+5)^2}$$



Exercise 9: Complex zeros

- For a system with the following transfer function equation:

$$G_{11}(s) = (s + 3 + j4)(s + 3 - j4)$$

- DC gain at $\omega = 0$ rad/s:

$$|G_{11}(s)|_{\omega = 0} = (5)(5) = 25 = 28 \text{ dB}$$

- A pair of complex zeros at $\omega = 3 + j4$ and $3 - j4$ rad/s:
  - Gain slope of +40 dB/decade after $\omega = 5$ rad/s.
  - Phase shift of 0° at $\omega = 0.5$ rad/s.
  - Phase shift of -90° at $\omega = 5$ rad/s.
  - Phase shift of +180° at $\omega = 50$ rad/s.

Exercise 9: Complex zeros

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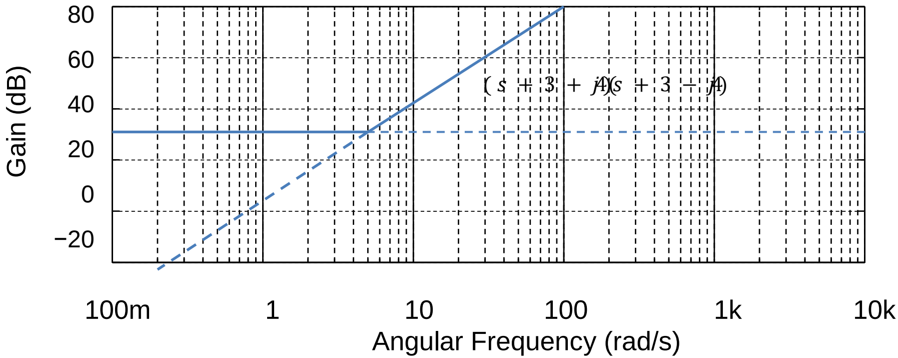
$$G_{11}(s) = (s + 3 + j4)(s + 3 - j4)$$



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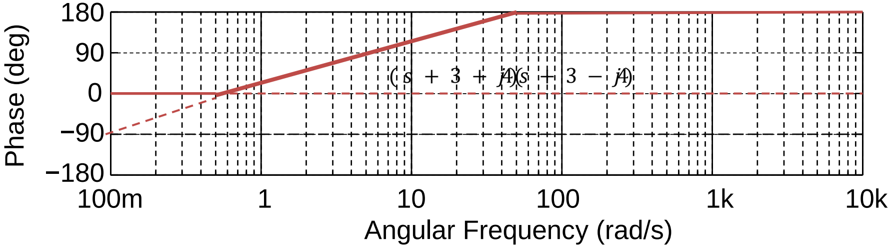
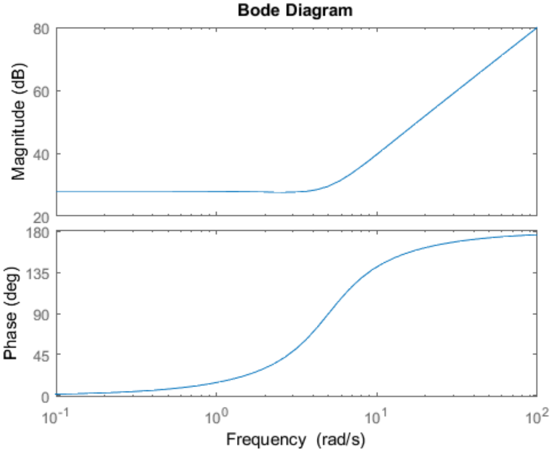


Exercise 9: Complex zeros

- Matlab simulation of the system:

$$G_{11}(s) = (s + 3 + j4)(s + 3 - j4)$$



Exercise 10: Complex poles and real zero

- For a system with the following transfer function equation:

$$G_{12}(s) \; = \frac{(s+100)}{(s+3+j4)(s+3-j4)}$$

- DC gain at ω = 0 rad/s:

$$|G(s)|_{\omega\,=\,0} = \frac{100}{(5)(5)} = 4 = 12\text{ dB}$$

- Complex poles at ω = -3 - j4 and -3 + j4 rad/s:
  - Gain slope of -40 dB/decade after ω = 5 rad/s.
  - Phase shift of 0° at ω = 0.5 rad/s.
  - Phase shift of -90° at ω = 5 rad/s.
  - Phase shift of -180° at ω = 50 rad/s.

Exercise 10: Complex poles and real zero

- Real zero at ω = 100 rad/s:
  - Gain slope of +20 dB/decade after ω =100 rad/s.
  - Phase shift of 0° at ω = 10 rad/s.
  - Phase shift of +45° at ω = 100 rad/s.
  - Phase shift of +90° at ω = 1000 rad/s.

Exercise 10: Complex poles and real zero

- Overall gain of the system:

$$G_{12}(s) \ = \frac{(s+100)}{(s+3+j4)(s+3-j4)}$$

  - DC gain of 4 (= 12 dB) at ω = 0 rad/s.
  - Gain slope of -40 dB/decade after ω = 5 rad/s.
  - Gain slope of -20 dB/decade after ω = 100 rad/s.

Exercise 10: Complex poles and real zero

- Overall phase shift of the system:

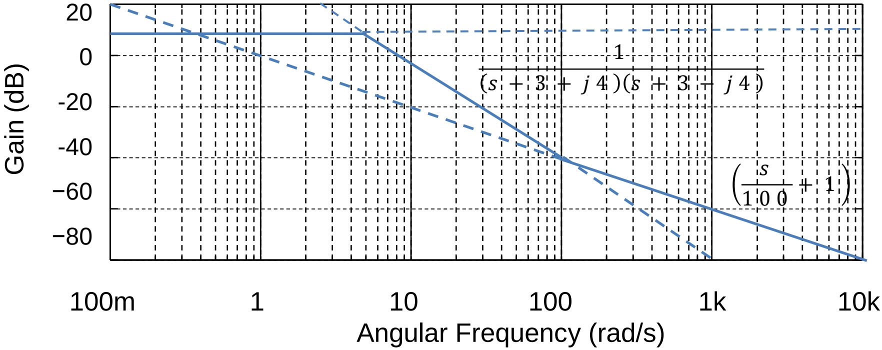
$$G_{12}(s) = \frac{(s+100)}{(s+3+j4)(s+3-j4)}$$

  - Phase shift of 0° at ω = 0 rad/s.
  - Phase shift of -90° at ω = 5 rad/s.
  - Phase shift of about -90° to -180° around ω = 10 rad/s and ω = 50 rad/s.
  - Phase shift of -135° at ω = 100 rad/s.
  - Phase shift of -90° after ω = 1000 rad/s.

Exercise 10: Complex poles and real zero

- Gain sketch of the system:

$$G_{12}(s) = \frac{(s+100)}{(s+3+j4)(s+3-j4)}$$



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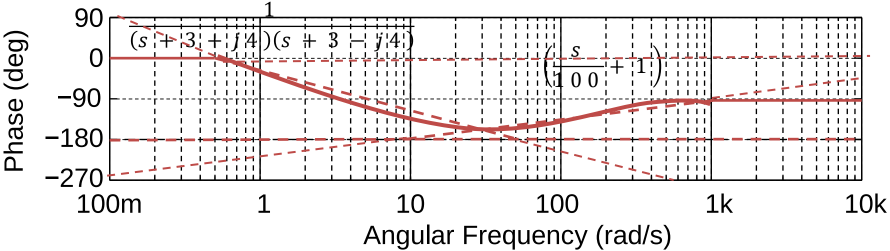

Exercise 10: Complex poles and real zero

- Matlab simulation of the system:

$$G_{12}(s) = \frac{(s+100)}{(s+3+j4)(s+3-j4)}$$