XMUT315 Control Systems Engineering

Table of Laplace Transforms

Note: There are various Laplace transform and its properties. The ones listed here are only those applicable for control systems.

A. Laplace Transform

Time Domain	Laplace Domain
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$

B. Properties of Laplace Transform

Linearity

$$\alpha f_1(t) + \beta f_2(t) \leftrightarrow \alpha F_1(s) + \beta F_2(s)$$

Time Scaling

$$f(at) \leftrightarrow \frac{1}{a} F\left(\frac{s}{a}\right); \qquad a > 0$$

Time Shifting

$$f(t-T)u_s(t-T) \leftrightarrow F(s)e^{-sT}; \quad T \geq 0$$

Frequency Shifting

$$e^{-at} \leftrightarrow F(s+a)$$

Differential Property

$$\frac{df(t)}{dt} \leftrightarrow sF(s) - f(0^-)$$

$$\frac{d^2f(t)}{dt^2} \leftrightarrow s^2F(s) - sf(0^-) - f^{(1)}(0^-)$$

$$\frac{df^n(t)}{dt^n} \leftrightarrow s^nF(s) - s^{n-1}f(0) - s^{n-2}f^{(1)}(0) - \cdots - f^{(n-1)}(0)$$

Final Value Theorem

$$\lim_{t\to\infty} f(t) \leftrightarrow \lim_{s\to 0} sF(s)$$