

Introduction to Controllers and Compensators

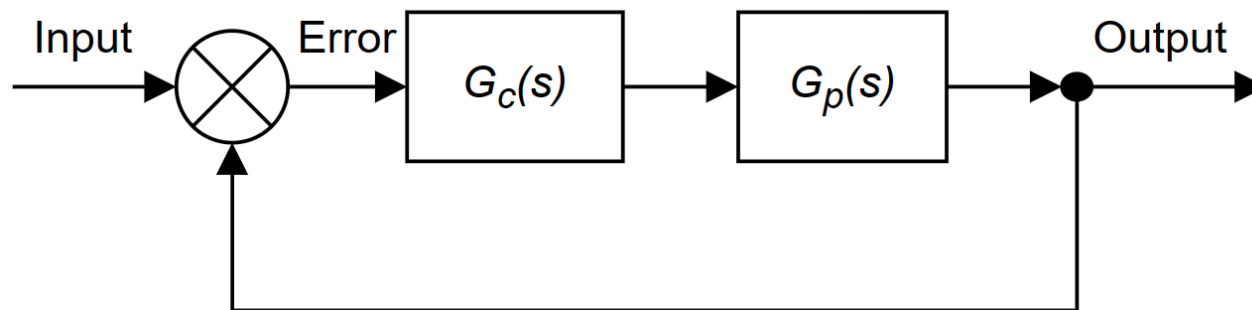
XMUT315 Control Systems Engineering

Topics

- Introduction to Controller and Compensator in the Feedback Control Systems.
- Controllers (e.g. Proportional Controller, Integral Controller, Derivative Controller, Proportional-Integral Controller, Proportional-Derivative Controller, and Proportional-Integral-Derivative Controller).
- Compensators (e.g. Lead Compensator, Lag Compensator, and Lead-Lag Compensator).
- Introduction to Controller/Compensator Design.

Controller and Compensator

- Controller or compensator changes the behaviour of the control system.
 - Controller is an element whose role is to maintain a physical quantity in a desired level.
 - Compensator is an element for modification of system dynamics e.g. to improve characteristics of the open-loop plant so that it can safely be used with feedback control.



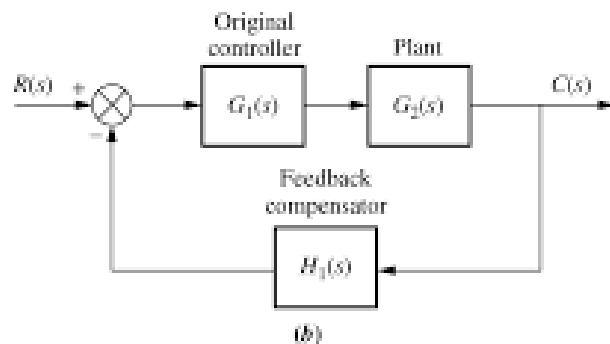
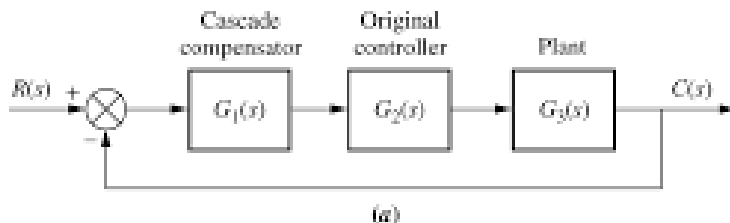
Note: G_c = controller or compensator and G_p = plant.

Controller and Compensator

- Three main types of controller:
 - P (gain or proportional) controller.
 - D (derivative) controller.
 - I (integral) controller.
 - PD (proportional derivative) controller.
 - PI (proportional integral) controller.
 - PID (proportional, integral, and derivative) controller.
- Three main types of compensator:
 - Lag compensator.
 - Lead compensator.
 - Lead-lag compensator.

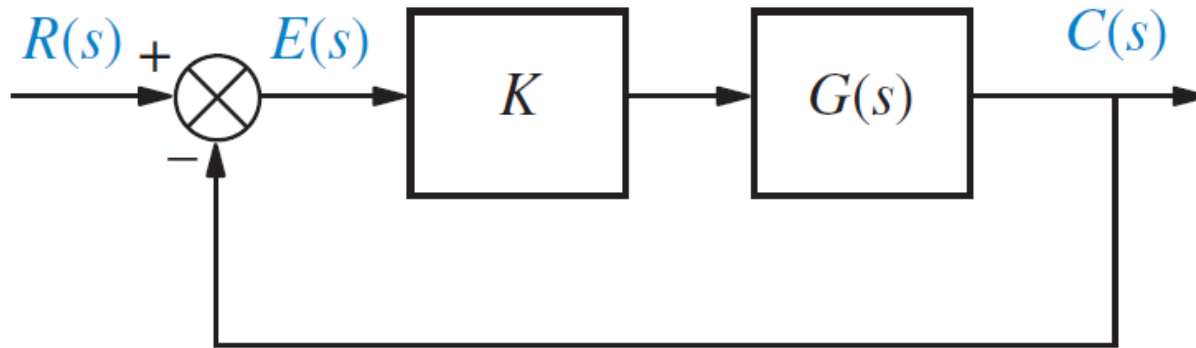
Controller and Compensator

- Controllers or compensators:
 - They change the natural response of the system.
 - They adjust the poles of the system.
 - They help achieve the desired output from a given input.



P Controller

- For a unity feedback system as shown below, a proportional controller ($G_c(s)$) connected in series with the plant ($G(s)$).



- The transfer function of the proportional controller is:

$$G_c(s) = K$$

- Thus

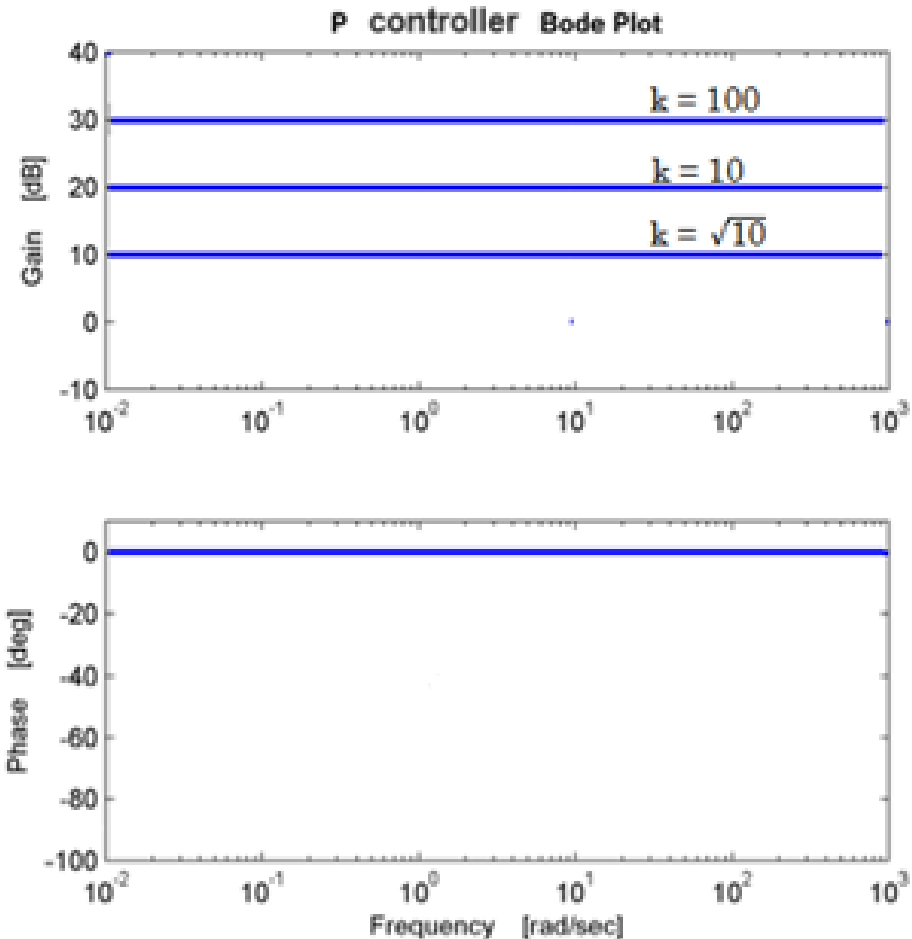
$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{KG(s)}{1 + KG(s)}$$

Characteristics of P Controllers

- The frequency response of a P controller is as shown below.
- Magnitude plot:
 - All frequency:
 $20 \log K$
- Phase-shift plot:
 - All frequency: 0°

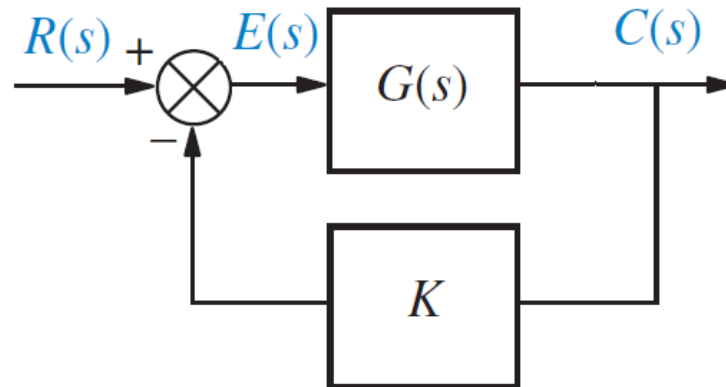
Improve transient response (up to a point):

- Increases the gain of the system, often result in a non-zero steady-state error, and relatively easy to implement.



P Controller

- For a proportional controller ($G_c(s)$) attached in the feedback path, consider a plant in the forward path ($G(s)$).



- The transfer function of the closed-loop feedback system is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 - (-G_c(s))G(s)} = \frac{G(s)}{1 + KG(s)}$$

P Controller

- Unlike previous set up, notice that the controller is placed in the feedback loop of the given control system.
- Notice the negative sign in the equation for the transfer function equation of the feedback system.
- If the size of the loop gain is large, that is if:

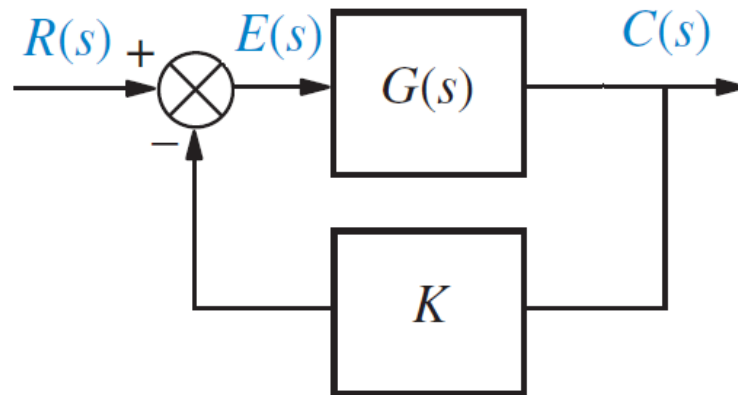
$$|KG(s)| \gg 1$$

- Then, the gain of the closed-loop system approximately is the gain of the controller:

$$T(s) \approx \frac{G(s)}{KG(s)} = \frac{1}{K}$$

P Controller in First-Order System

- For a proportional controller ($G_c(s)$) is attached to the feedback path and the plant in the forward path is a first-order system $G(s)$.



- The transfer function equations of both the controller and the plant are:

$$G_c(s) = K \quad \text{and} \quad G(s) = \left(\frac{A}{1 + sT} \right)$$

P Controller in First-Order System

- In term of $T(s) = C(s)/R(s)$, closed-loop transfer function equation of the system is:

$$T(s) = \frac{G(s)}{1 - (-G_c(s))G(s)} = \frac{\left(\frac{A}{1 + sT}\right)}{1 - (-K)\left(\frac{A}{1 + sT}\right)} = \frac{A}{1 + sT + AK}$$

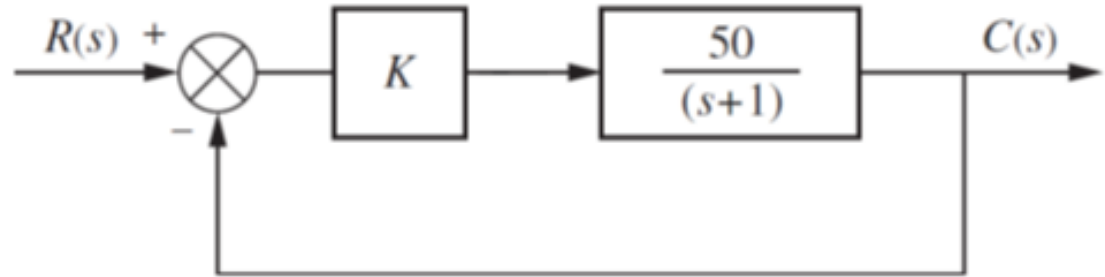
- Thus, we can see that the time constant of the closed-loop first-order system depends on both gains of the plant A and controller K .
- The gain of the closed-loop system depends on the gain of the plant A .

Example of P Controller in First-Order System

The open-loop transfer function equation of a first-order system is given below.

$$G(s) = \frac{50}{s + 1}$$

$$G_c(s) = K$$



- Determine the time constant of the open-loop system.
[2 marks]
- If a proportional controller connected in series with the system as shown below, determine the gain of proportional controller (K) that will change the time constant (τ) of the closed-loop system to become 0.1 second.
[6 marks]

Example of P Controller in First-Order System

- a. The time constant of the open-loop first-order system is:

$$G(s) = \frac{50K}{s + a}$$

Thus

$$\tau = \frac{1}{a} = \frac{1}{1} = 1 \text{ s}$$

- b. The gain of the proportional controller (K) that will change the time constant of the closed-loop first order system to become 0.1 second is determined as follows.

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{K \left(\frac{50}{s + 1} \right)}{1 + K \left(\frac{50}{s + 1} \right)} = \frac{50K}{s + 1 + 50K}$$

Example of P Controller in First-Order System

Thus, equate the time constant with the part of the transfer function equation.

$$\tau = \frac{1}{a} = \frac{1}{1 + 50K}$$

For the time constant of 0.1 second, the gain of the proportional controller is calculated from:

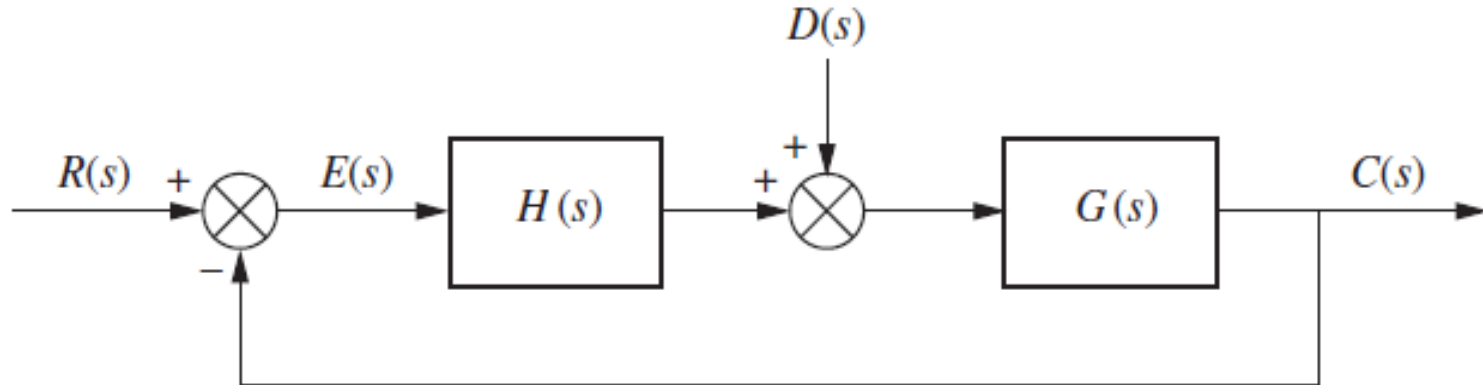
$$0.1 = \frac{1}{1 + 50K}$$

Rearrange the equation above, the value of K is:

$$K = \frac{10 - 1}{50} = 0.18$$

P Controller in Second-Order System

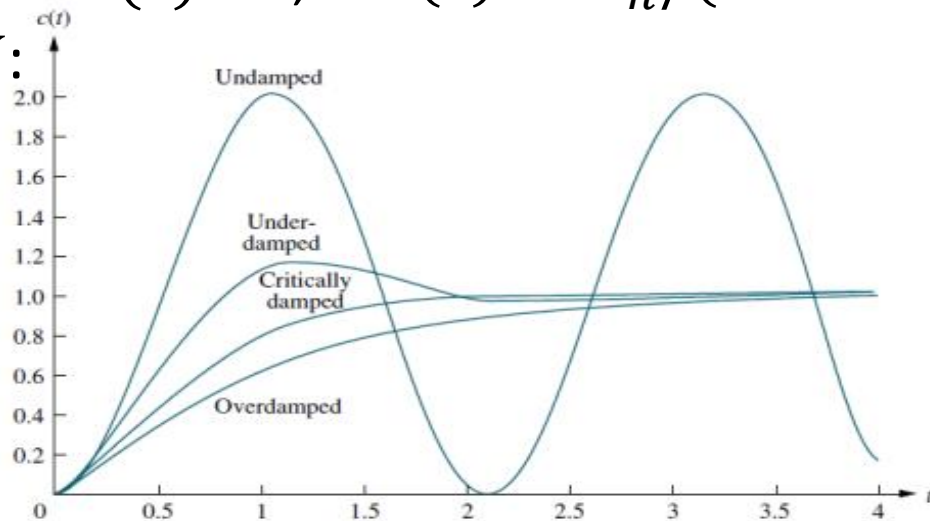
- For a unity feedback second-order system ($G(s)$) with a proportional controller ($H(s)$) added as shown below.



- Unless told otherwise, assume $D(s) = 0$, if $G(s) = \omega_n^2 / (s^2 + 2\omega_n\zeta s + \omega_n^2)$ and $H(s) = K$:

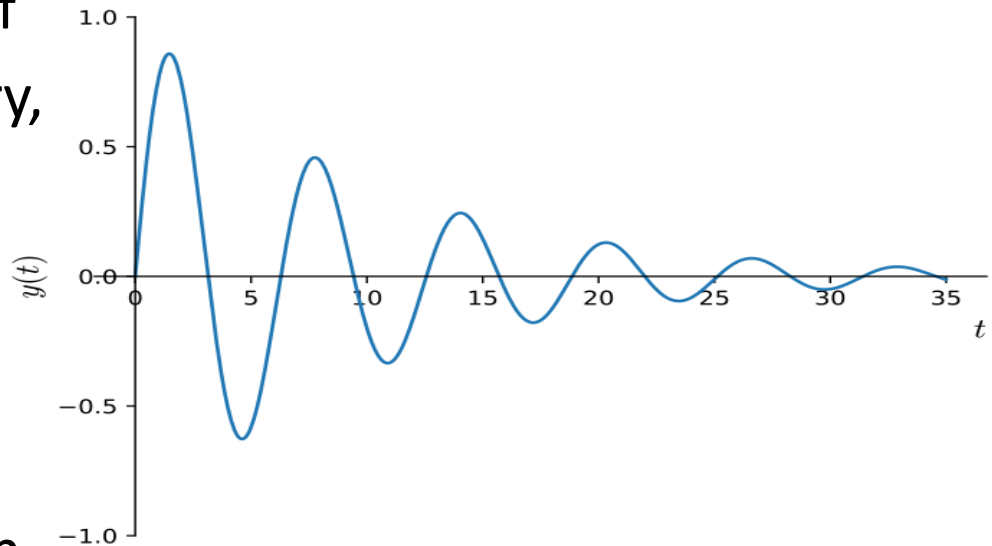
$$T(s) = \frac{H(s)G(s)}{1 + H(s)G(s)}$$

$$= \frac{\omega_n^2 K}{s^2 + 2\omega_n\zeta s + (1 + K)\omega_n^2}$$



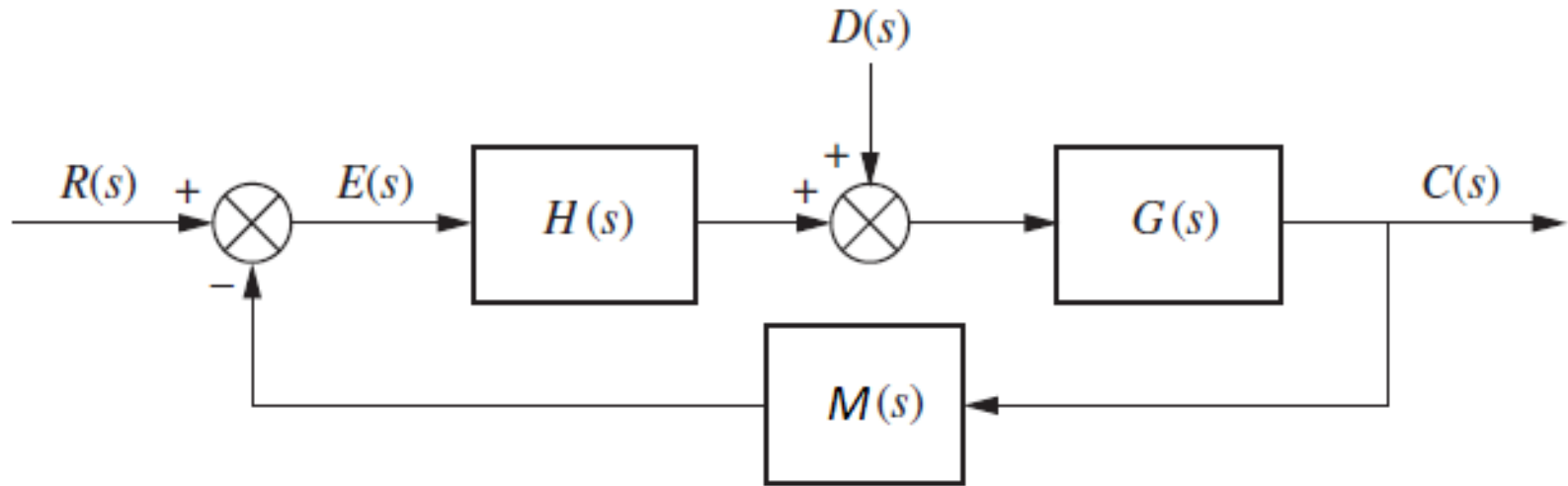
P Controller in Second-Order System

- We can determine values for $1 + K$ to make the system: undamped, underdamped, critically damped, and overdamped.
- As proven for first order system before, higher gain is typically yielding a faster response, but it is at the expense of a more oscillatory response.
- If the transient response of the system is too oscillatory, it will take time before the system settles to its final value.
- We cannot, therefore, just increase the controller gain.



P Controller in Second-Order System

- For a second-order system with a proportional controller ($M(s)$) and non-unity feedback as shown below.



- If $D(s) = 0$, the open-loop gain of the system is:

$$\frac{C(s)}{R(s)} = H(s)G(s)$$

P Controller in Second-Order System

- The transfer function equation of the closed-loop system is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{H(s)G(s)}{1 + M(s)H(s)G(s)}$$

- Focusing on the characteristic equation of the transfer function, it is:

$$1 + M(s)H(s)G(s)$$

- Notice that $M(s)$ influences the characteristic equation.
- As a result, the variable $M(s)$ affects transient response of the closed-loop system as specified above.

P Controller in Second-Order System

- Applying final value theorem, the steady-state equation of the system for a step input is:

$$\begin{aligned} C(\infty) &= \lim_{s \rightarrow 0} s (R(s)) \frac{H(s)G(s)}{1 + M(s)H(s)G(s)} \\ &= \lim_{s \rightarrow 0} s \left(\frac{1}{s} \right) \left(\frac{H(s)G(s)}{1 + M(s)H(s)G(s)} \right) \\ &\cong \frac{1}{M(s)} \end{aligned}$$

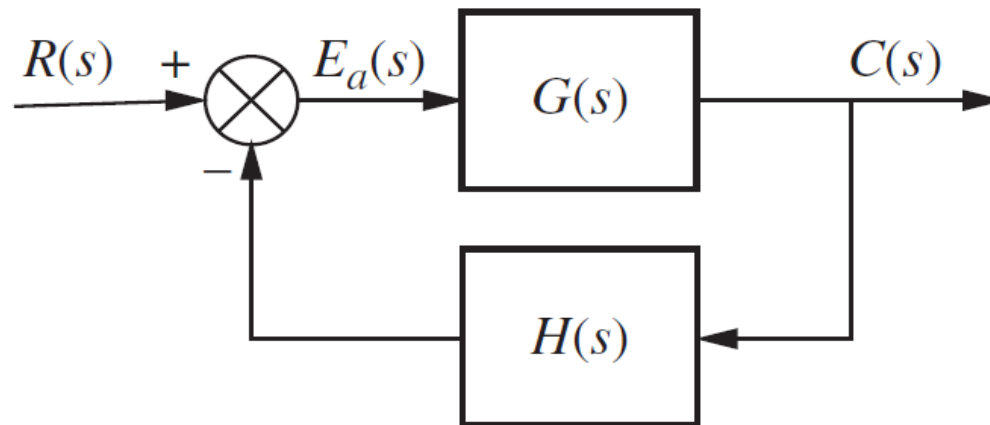
- Thus, the variable $M(s)$ influences also the steady-state response of the closed loop system for a given step input.

Example of P Controller in Second-Order System

For an open-loop control system described as the transfer function equation given below, attempt the following tasks.

$$G(s) = \frac{5}{s^2 + 10s + 5}$$

- a. Derive the transfer function equation of the closed-loop system with a proportional controller $H(s) = M$ added in the feedback loop as shown below. [4 marks]



Example of P Controller in Second-Order System

- b. If M is 9, determine the transient response of the closed-loop system. [6 marks]

- c. As part of design specification for the system, for a step input response, determine the feedback gain (M) if we wish the steady-state error condition of the closed-loop system to be 0.6. [8 marks]

Example of P Controller in Second-Order System

- a. For the given second-order system with non-unity feedback, the transfer function of the closed-loop system is:

$$\begin{aligned} T(s) &= \frac{G(s)}{1 + G(s)H(s)} \\ &= \frac{\left(\frac{5}{s^2 + 10s + 5} \right)}{1 + \left(\frac{5}{s^2 + 10s + 5} \right) M} \end{aligned}$$

Rearrange the equation above, it becomes:

$$T(s) = \frac{5}{s^2 + 10s + 5(1 + M)}$$

Example of P Controller in Second-Order System

- b. The transient response of the closed-loop system when M is 9 is:

$$T(s) = \frac{5}{s^2 + 10s + 5(1 + 9)} = \frac{5}{s^2 + 10s + 50}$$

Evaluating the characteristics equation of the closed loop system, its roots are:

$$\begin{aligned}\text{root}_{1,2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{(10)^2 - 4(1)(50)}}{2(1)} = -5 \pm j5\end{aligned}$$

The roots are complex pair, so the response of the closed-loop system is underdamped.

Example of P Controller in Second-Order System

- c. For a step input, the steady-state error of the closed-loop system is:

$$\begin{aligned} e_{step}(\infty) &= \lim_{s \rightarrow 0} \frac{s(1/s)}{1 + T(s)} \\ &= \lim_{s \rightarrow 0} \frac{1}{1 + \frac{5}{s^2 + 10s + 5(1 + M)}} \\ &= \lim_{s \rightarrow 0} \frac{s^2 + 10s + 5(1 + M)}{s^2 + 10s + 5(2 + M)} \\ &= \frac{1 + M}{2 + M} \end{aligned}$$

Example of P Controller in Second-Order System

To achieve a steady-state error of 0.6 for the step response of the system, the gain of proportional controller M is calculated from:

$$e_{step}(\infty) = \frac{1 + M}{2 + M} = 0.6$$

Thus, the gain of the proportional controller that meets the design specification is:

$$M = \frac{1.2 - 1}{1 - 0.6} = 0.5$$

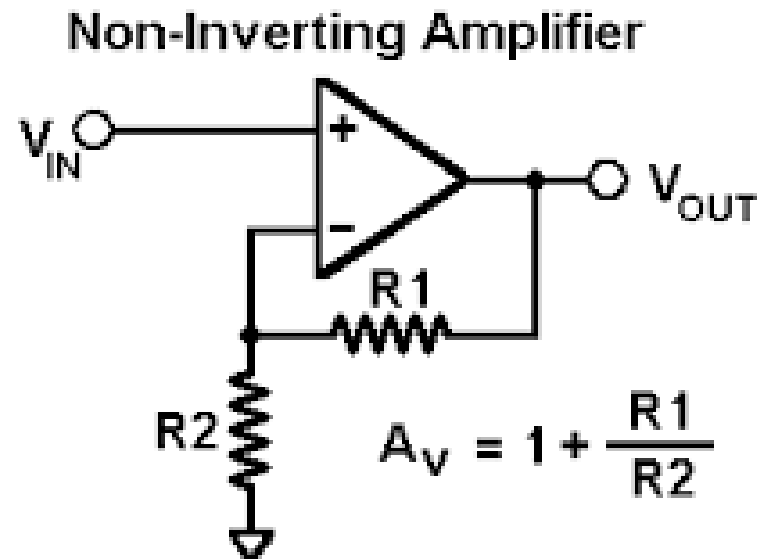
P Controller in Practice

- In practice, the P controller is realised as a non-inverting amplifier with R_1 and R_2 forming the voltage divider part of the circuit.
- Voltage at the non-inverting input is:

$$V_p = V_{IN}$$

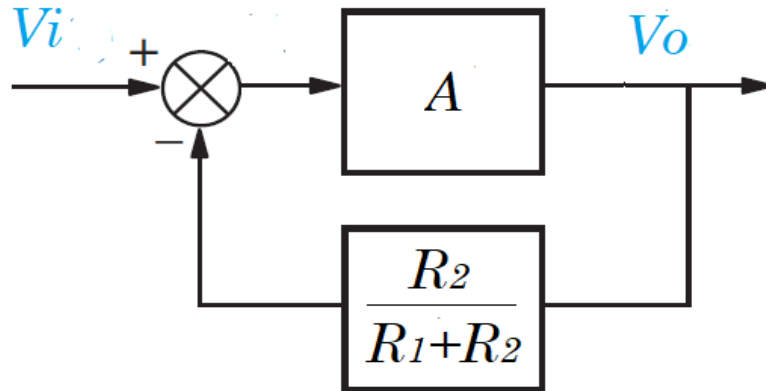
- Due to potential divider arrangement in the circuit, the voltage at the inverting pin of the op amp is:

$$V_n = V_{OUT} \left(\frac{R_2}{R_1 + R_2} \right)$$



P Controller in Practice

- As an example, given the non-inverting amplifier with open-loop gain of A :



- The transfer function equation of the amplifier is:

$$\frac{V_o}{V_i} = \frac{A}{1 - A \left(-\frac{R_2}{R_1 + R_2} \right)} = \frac{A(R_1 + R_2)}{R_1 + R_2 + AR_2}$$

- If the loop gain $AR_2/(R_1 + R_2)$ is large, then:

$$AR_2 \gg R_1 + R_2$$

P Controller in Practice

- As a result, the transfer function of the non-inverting amplifier is:

$$\frac{V_o}{V_i} = \frac{A(R_1 + R_2)}{AR_2} = \frac{R_1 + R_2}{R_2}$$

- In the above equation, the feedback loop path is:

$$\beta = \frac{R_2}{R_1 + R_2}$$

- So, if the loop gain is large:

$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_2} = \frac{1}{\beta}$$

Example of P Controller in Practice

For example, given specification of a non-inverting operational amplifier circuit as shown below, perform the following tasks:

- Gain $A = 10^5$.
 - Feedback resistors: $R_1 = 6 \text{ k}\Omega$, $R_2 = 4 \text{ k}\Omega$.
- a. Derive the transfer function equation of the amplifier. [6 marks]
 - b. Determine whether the forward-loop gain of the amplifier is larger than the feedback loop gain. [4 marks]
 - c. Calculate the gain of the amplifier. [4 marks]

Example of P Controller in Practice

a. The transfer function of the op amp circuit is derived from:

$$V_p = V_i \quad (Eq. 1)$$

And

$$V_n = V_o \left(\frac{R_2}{R_1 + R_2} \right) = V_o \left(\frac{4 \text{ k}\Omega}{6 \text{ k}\Omega + 4 \text{ k}\Omega} \right) = 0.4V_o \quad (Eq. 2)$$

For the non-inverting amplifier with an open-loop gain of A , the output voltage is:

$$V_o = A(V_p - V_n) = A(V_i - 0.4V_o) \quad (Eq. 3)$$

Substituting equations (1) and (2) into equation (3), the transfer function equation of the op amp circuit is:

$$\frac{V_o}{V_i} = \frac{A}{(1 + 0.4A)}$$

Example of P Controller in Practice

- b. Consider whether the feedback loop is large, the feedback loop is:

$$AR_2 = (10^5)(4 \times 10^3) = 4 \times 10^8$$

And

$$R_1 + R_2 = (6 \times 10^3) + (4 \times 10^3) = 10^4$$

As calculated above, the feedback loop is large (e.g. $4 \times 10^8 \gg 10^4$).

- c. As a result, the overall gain of the amplifier is:

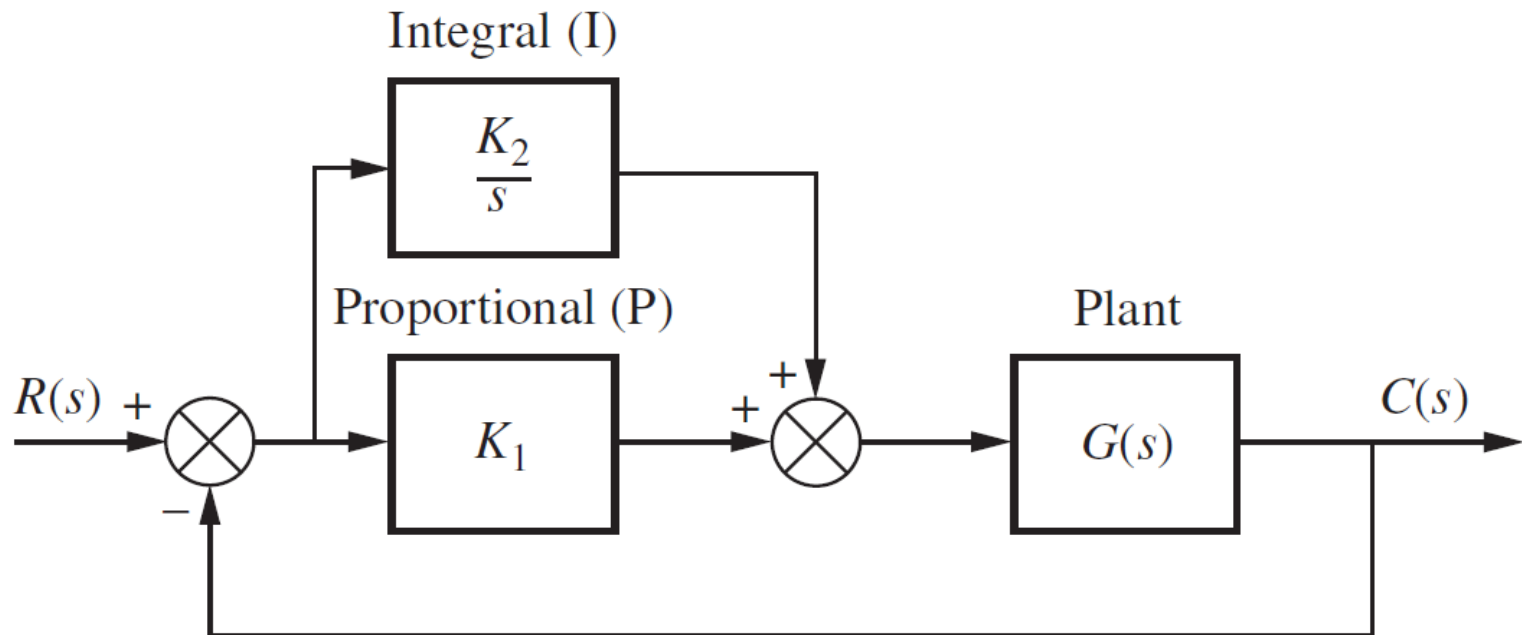
$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_2} = \frac{6 \text{ k}\Omega + 4 \text{ k}\Omega}{4 \text{ k}\Omega} = 2.5$$

PI Controllers

- For a PI Controller, its transfer function can be written as:

$$G_c(s) = P(s) + I(s) = K_1 + \frac{K_2}{s} = \frac{K_1(s + K_2/K_1)}{s}$$

Where: $P(s) = K_1$ and $I(s) = K_2/s$.



Characteristics of PI Controllers

- The frequency response of a PI controller is as shown below.

- Magnitude plot:

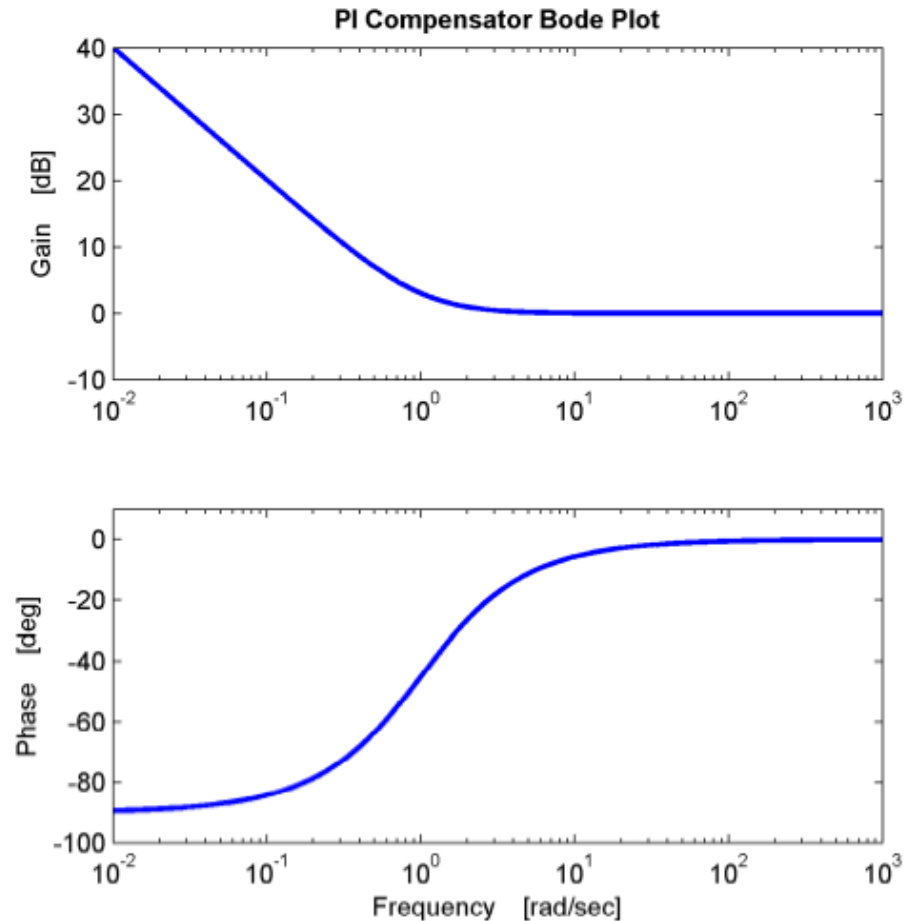
- Low: -slope gain.
- Cut-off: half gain.
- High: zero gain.

- Phase-shift plot:

- Low: -90° .
- Cut-off: -45° .
- High: 0° .

Improve steady-state error:

- Increases system type, error becomes zero, zero at z_c is small and negative, and active circuits are required to implement.



Applications of PI Controllers

- The functions $P(s)$ and $I(s)$ can be chosen so the $(s + K_2/K_1)$ term (e.g. controller zero) cancels plant pole.
- Suppose the plant of a second-order system is:

$$G(s) = \frac{1}{(s + T_1)(s + T_2)}$$

- If we apply PI to this plant, and make $K_2/K_1 = T_2$, then

$$\frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{1}{s(s + T_1)}$$

- So, the closed-loop transfer function equation is:

$$T(s) = \frac{O(s)}{I(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{1}{s^2 + sT_1 + 1}$$

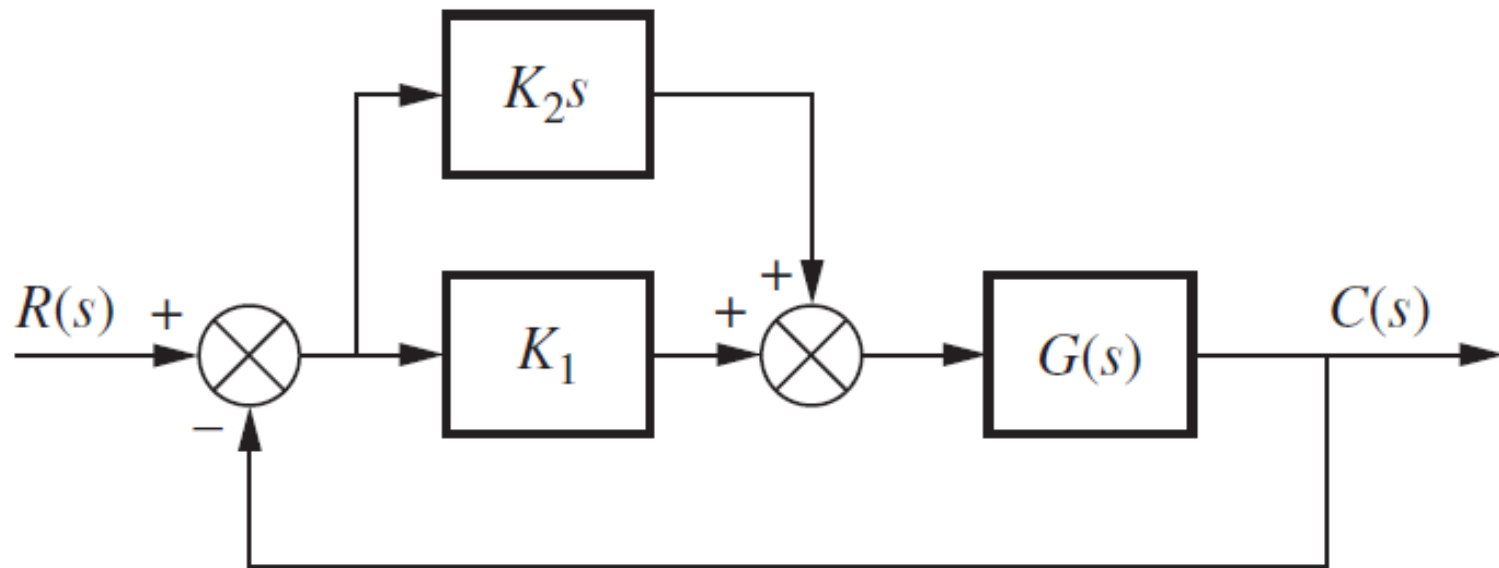
Note, the $I(s)$ term means that the steady-state value is 1.

PD Controllers

- For a PD Controller, its transfer function equation:

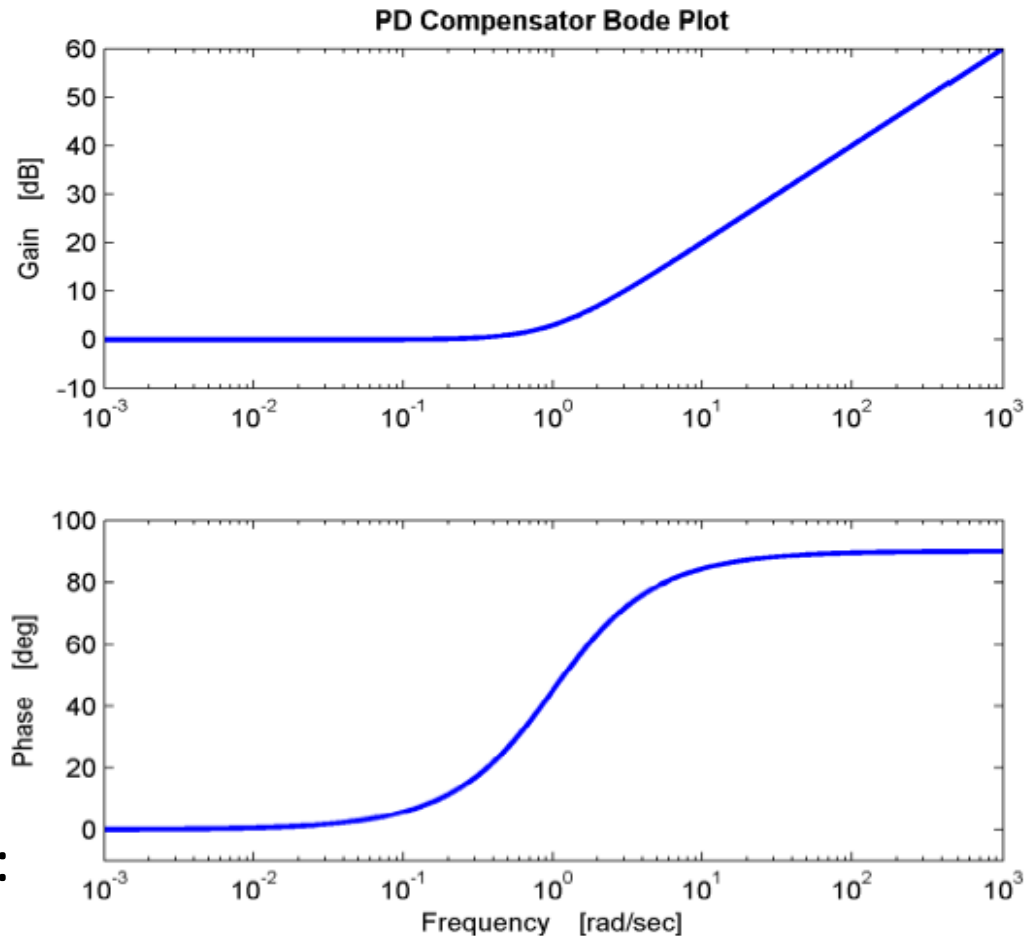
$$G_1(s) = P(s) + D(s) = K_1 + K_2s = K_2(s + K_1/K_2)$$

Where: $P(s) = K_1$ and $D(s) = K_2s$.



Characteristics of PD Controllers

- The frequency response of a PD controller is as shown below.
- Magnitude plot:
 - Low: zero gain.
 - Cut-off: half gain.
 - High: +slope gain.
- Phase-shift plot:
 - Low: 0° .
 - Cut-off: $+45^\circ$.
 - High: $+90^\circ$.



Improve transient response:

Zero at $-z_c$ is selected to indicate the design point, active circuits are required to implement, and it can cause noise and saturation; implement with rate feedback or with a pole (lead).

Applications of PD Controllers

- We could make $s + K_1/K_2$ term to cancel plant pole.
- If transfer function of the plant of a second order system is:

$$G(s) = \frac{1}{s(s + T)}$$

- And PD is applied and assign $K_1/K_2 = T$ then:

$$\frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{1}{s}$$

- So, the closed-loop transfer function equation is:

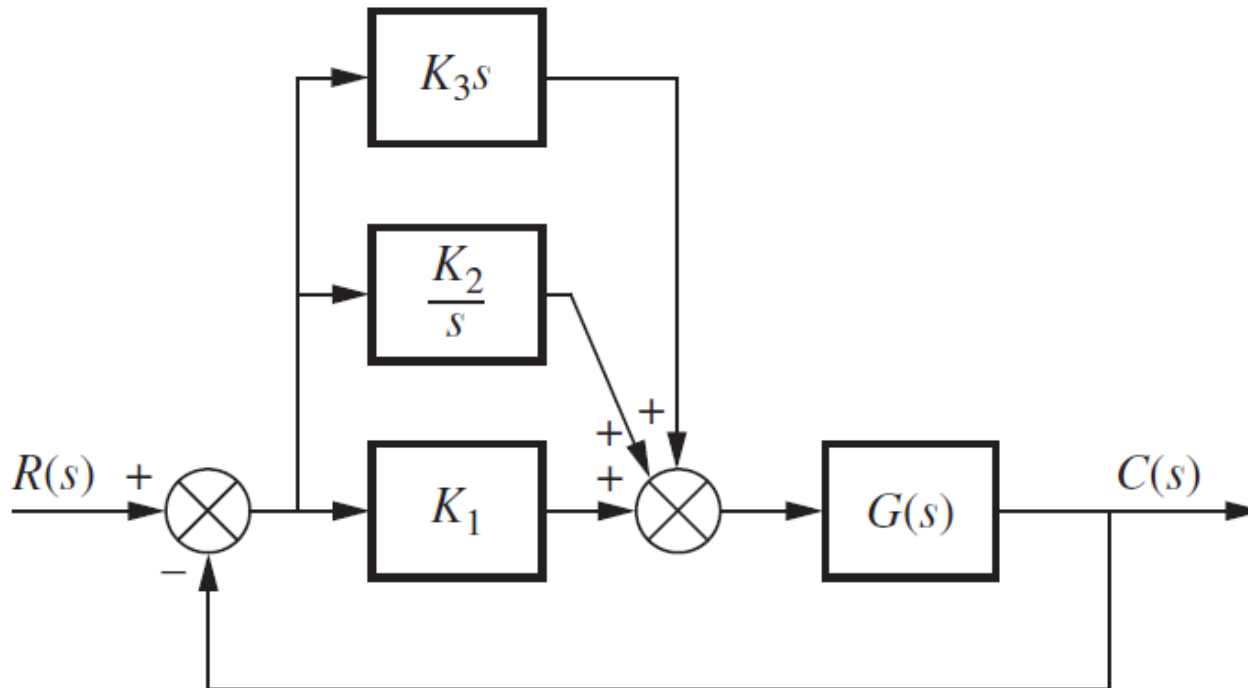
$$T(s) = \frac{O(s)}{I(s)} = \frac{1}{s + 1}$$

PID Controllers

- For a PID Controller, its transfer function equation is:

$$G_c(s) = P(s) + I(s) + D(s) = K_1 + \frac{K_2}{s} + K_3s$$

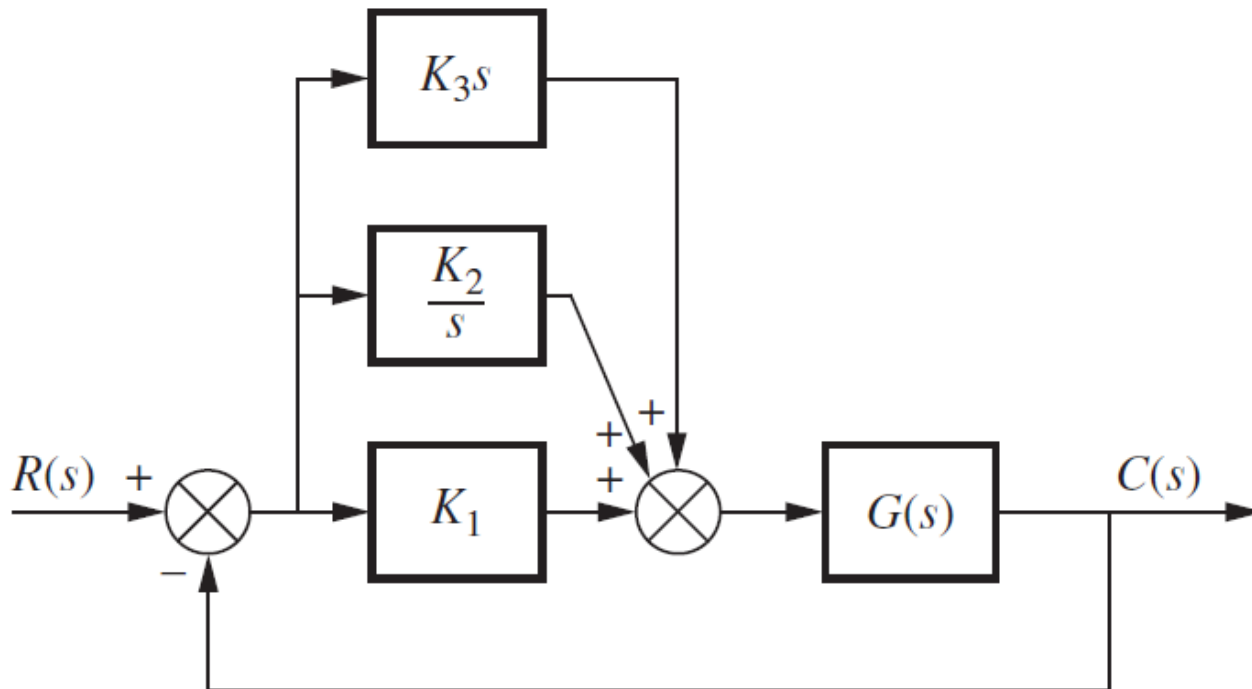
Where: $P(s) = K_1$, $I(s) = K_2/s$, and $D(s) = K_3s$



PID Controllers

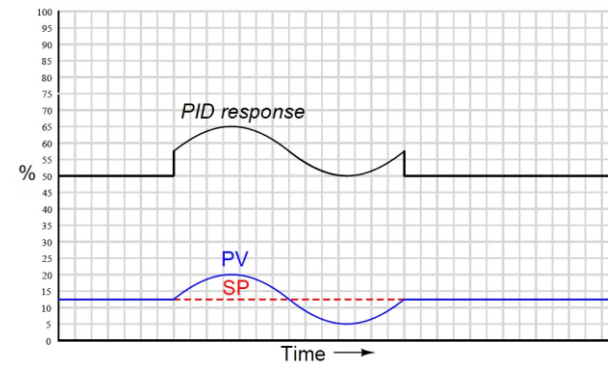
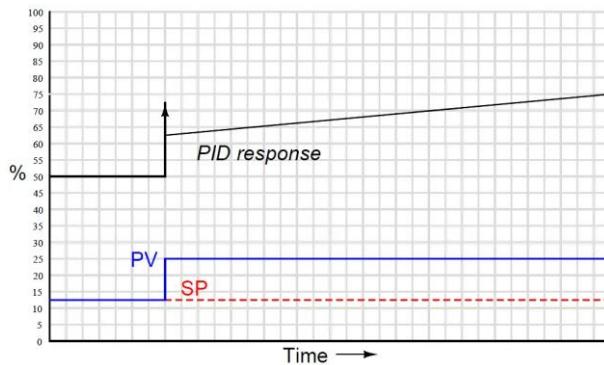
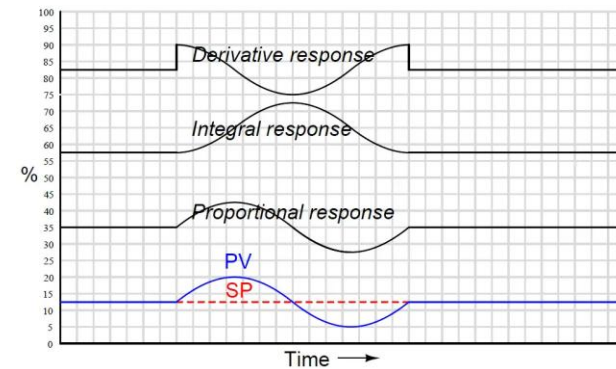
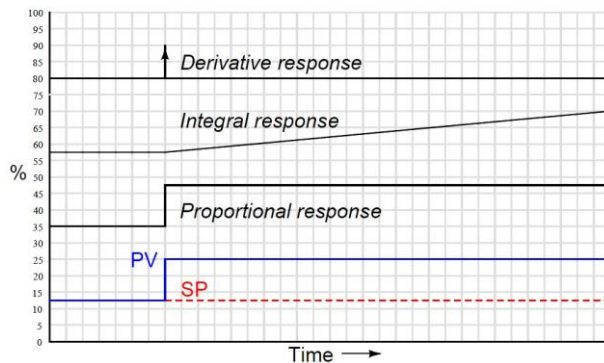
- Alternatively, this gives the transfer function equation of PID controller:

$$G_c(s) = \frac{K_3 s^2 + K_1 s + K_2}{s}$$



Characteristics of PID Controllers

- The response of a PID controller is as shown below over the step and sinusoidal inputs.



Characteristics of PID Controllers

Improve steady-state error and transient response:

- Lag zero at $-z_{lag}$ and pole at the origin improve steady-state error; lead zero at $-z_{lead}$ improves transient response.
- Lag zero at $-z_{lag}$ is close to, and to the left of, the origin.
- Lead zero at $-z_{lead}$ is selected to indicate the design point.
- Active circuits are required to implement.
- It can cause noise and saturation; implement with rate feedback or with an additional pole.

Applications of PID Controllers

- We could apply a PID controller to a plant of second order system:

$$G(s) = \frac{1}{1 + bs + as^2}$$

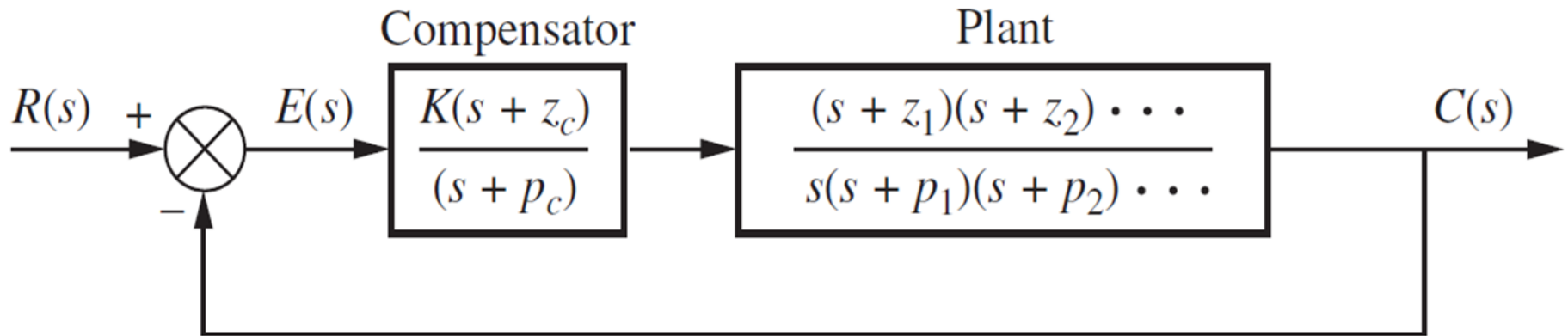
- Thus, set $K_3 = a$, $K_1 = b$, and $K_2 = 1$, the equation is now:

$$T(s) = \frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{1}{s}$$

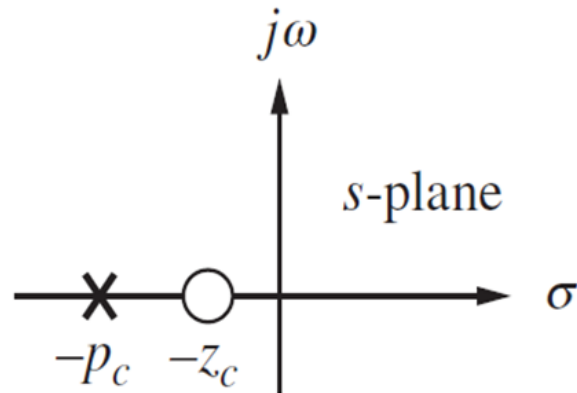
- In all these examples, by careful arrangement, system is either first or second order.
- Cancellation may not give best response, but analysis of systems is easier!

Lead Compensators

- With $P_c > Z_c$, block diagram of lead compensator.



$$G_C(s) = \frac{(s + z_c)}{(s + p_c)}$$



Lead Compensators

- The transfer function of a lead compensator is:

$$G_c(s) = \frac{1}{\beta} \left(\frac{s + \frac{1}{T}}{s + \frac{1}{\beta T}} \right) \quad (\beta < 1)$$

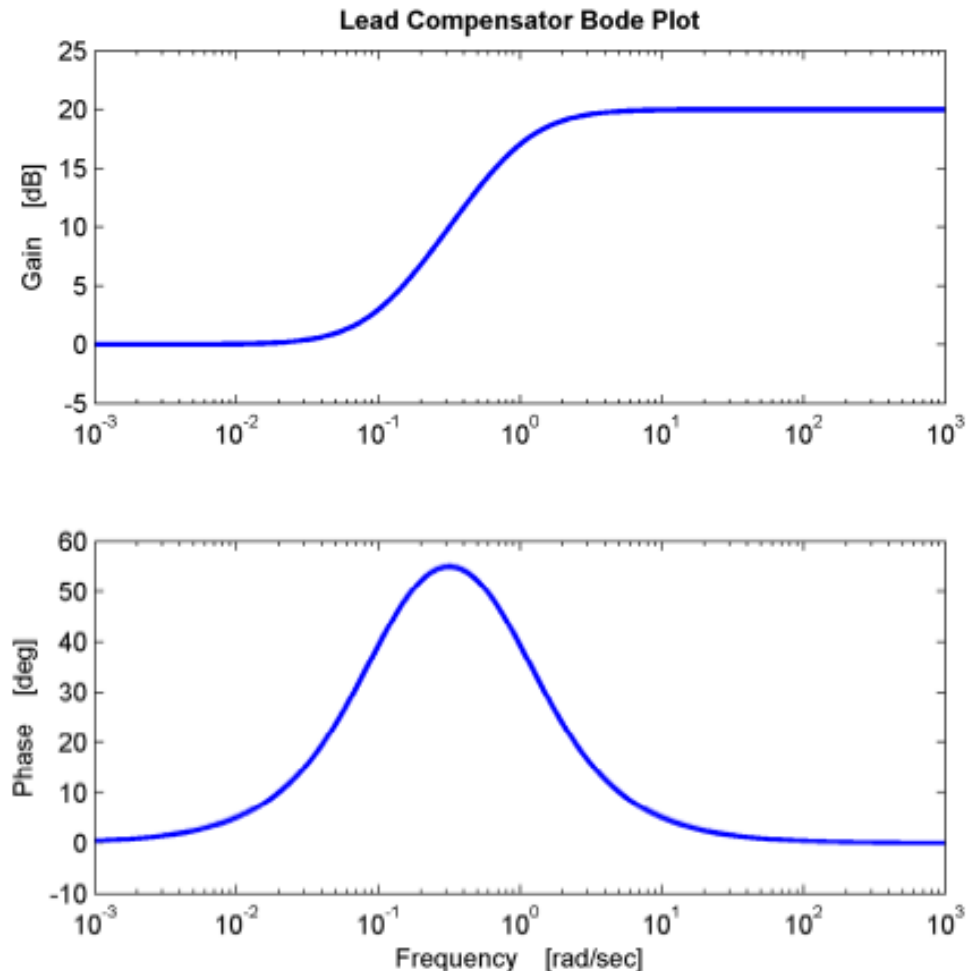
- Or

$$G_{lead}(s) = \frac{s + z_c}{s + p_c} \quad \text{with} \quad |p_c| > |z_c|$$

- This controller consists of 1 pole and 1 zero with $|pole| > |zero|$.

Characteristics of Lead Compensators

- Frequency response plot of lead compensator.
- Magnitude plot:
 - Low: zero gain.
 - Cut-off: half gain.
 - High: +finite gain.
- Phase-shift plot:
 - Low: 0° .
 - Cut-off: $+45^\circ$.
 - High: 0° .

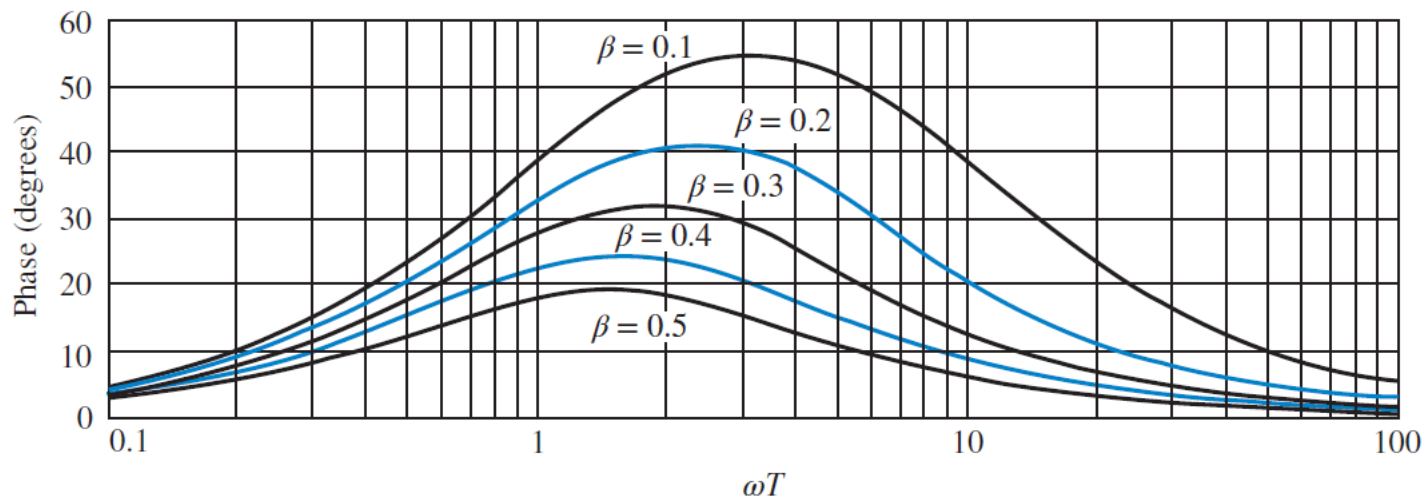
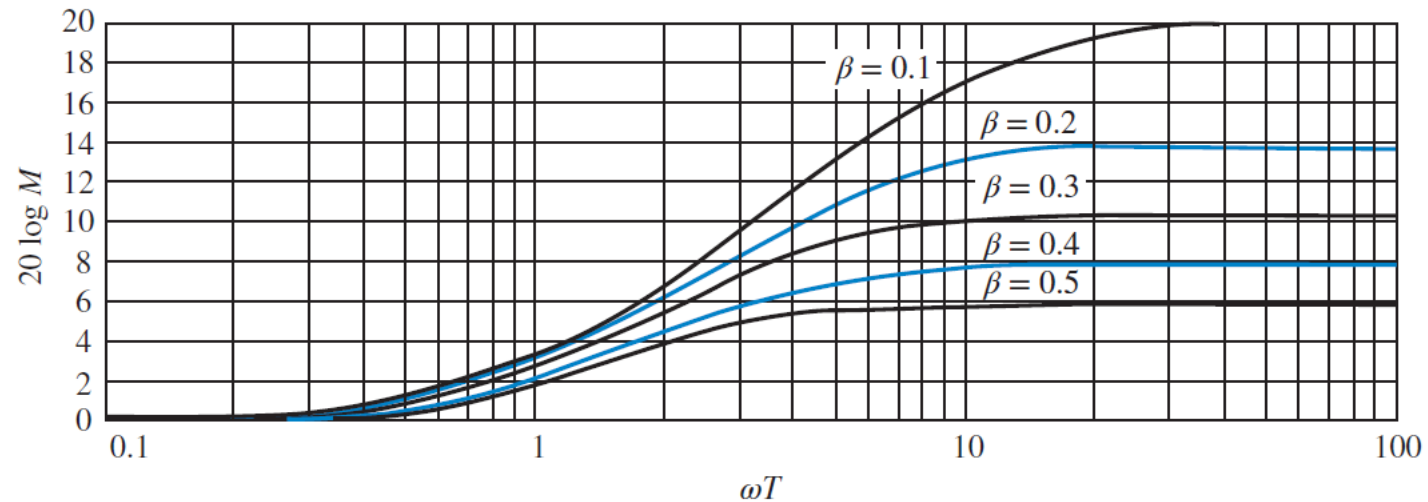


Improve transient response:

Zero at $-z_c$ and pole at $-p_c$ are selected to indicate design point and pole at $-p_c$ is more negative than zero at $-z_c$. Active circuits are not required.

Characteristics of Lead Compensators

- Frequency response plot of lead compensator with varied β .

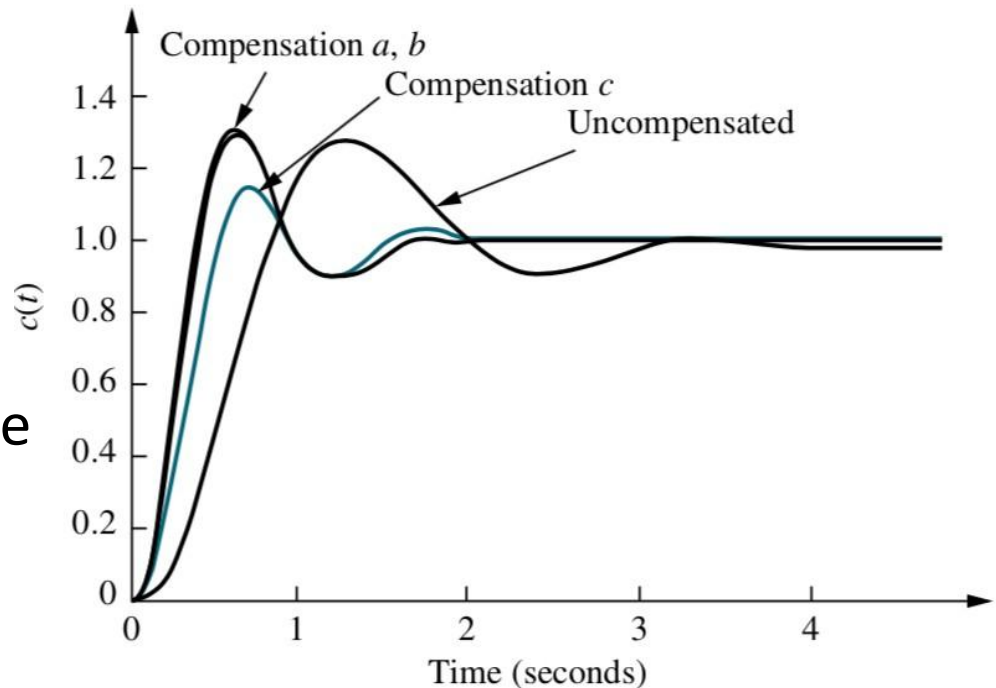


Applications of Lead Compensators

- In lead compensator, the zero is closer to the origin than the pole, that is:

$$z_c < p_c$$

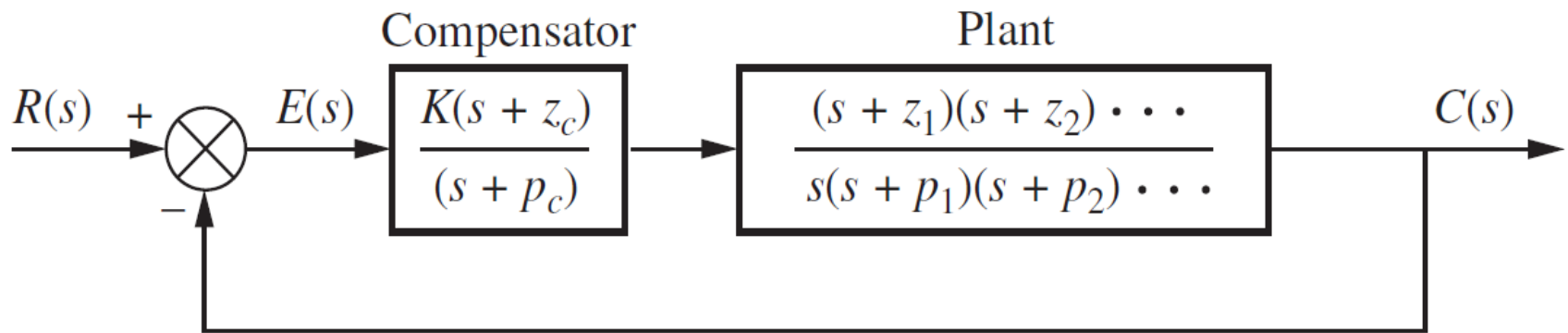
- The lead compensator influences transient response (e.g. percentage overshoot and settling times).



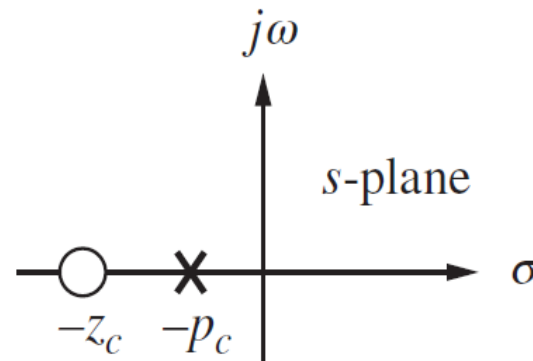
- The uncompensated system is slower compared with the compensated systems (a, b , and c = increasing distance of the poles from origin).

Lag Compensators

- With $Z_c > P_c$, block diagram of lag compensator.



$$G_c(s) = \frac{(s + z_c)}{(s + p_c)}$$



Lag Compensators

- The transfer function of the lag compensator is:

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \quad (\alpha > 1)$$

- Or

$$G_{lag}(s) = \frac{s + z_c}{s + p_c} \quad \text{with} \quad |p_c| < |z_c|$$

- This controller consists of 1 pole and 1 zero with $|pole| < |zero|$.

Characteristics of Lag Compensators

- Frequency response plot of lag compensator.
- Magnitude plot:

- Low: +finite gain.
- Cut-off: half gain.
- High: zero gain.

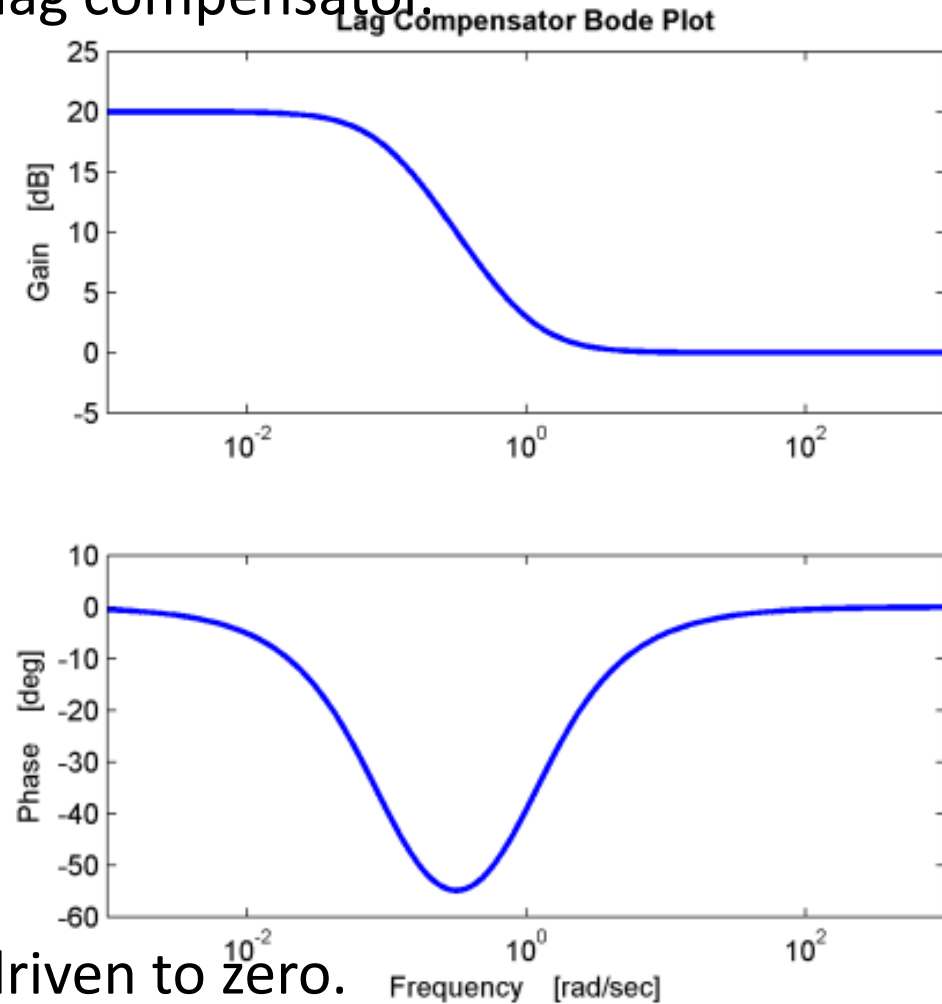
- Phase-shift plot:

- Low: 0° .
- Cut-off: -45° .
- High: 0° .

Improve steady-state error

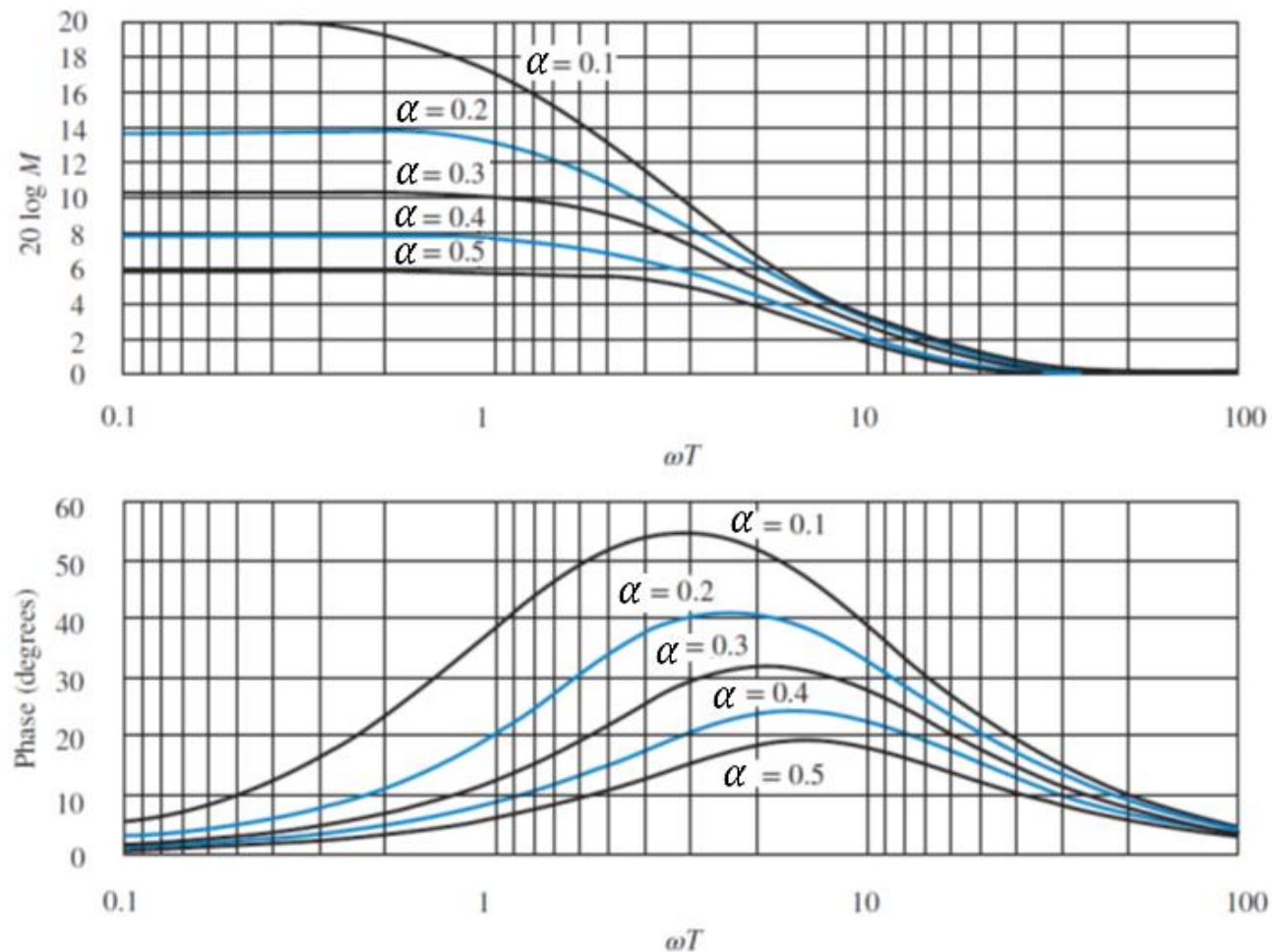
- Error is improved, but not driven to zero.

Pole at $-p_c$ is small and negative; Zero at $-z_c$ is close to, and to the left of the pole at $-p_c$; Active circuits are not required.



Characteristics of Lag Compensators

- Frequency response plot of lag compensator with varied α .

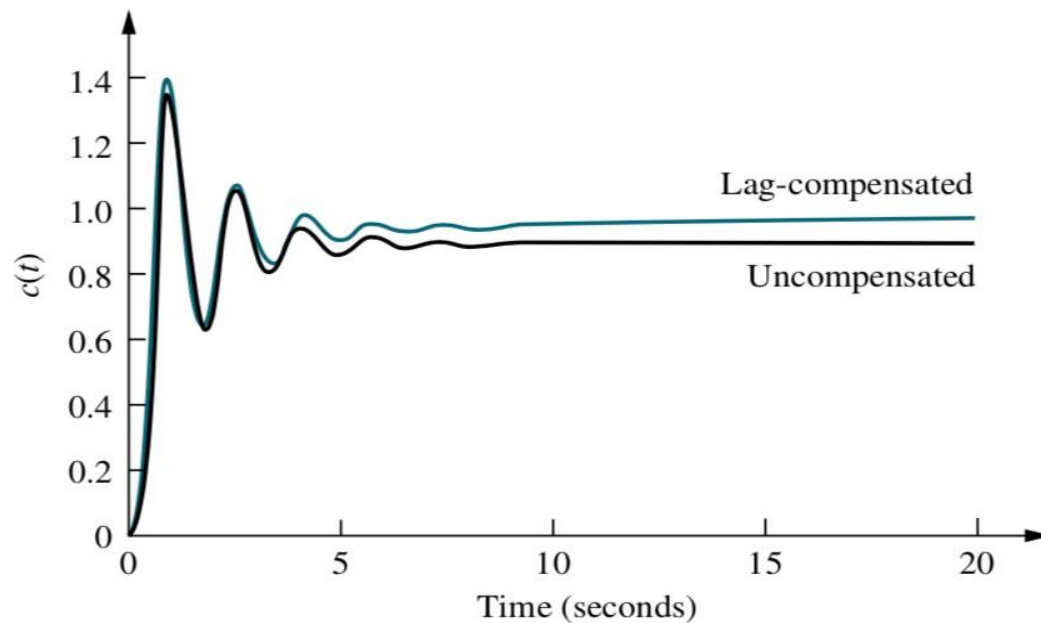


Applications of Lag Compensators

- In lag compensator, the pole is closer to the origin than the zero, that is:

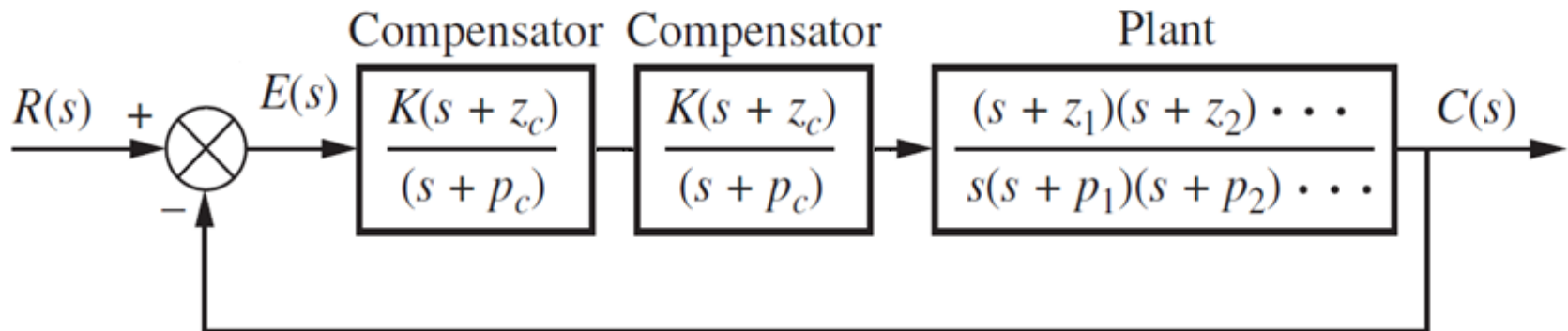
$$z_c > p_c$$

- The lag compensator reduces steady- state error.

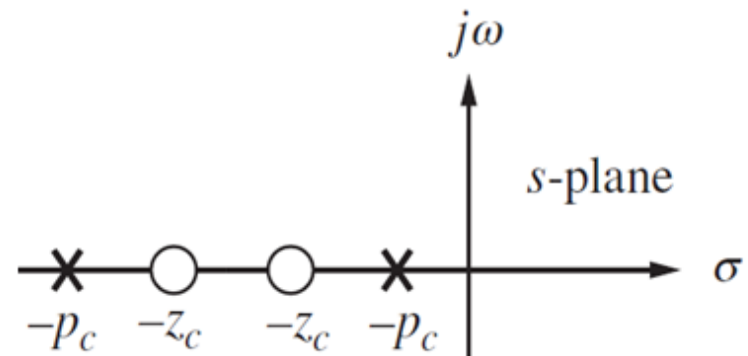


Lead-Lag Compensators

- Block diagram of lead-lag compensator.



$$G_C(s) = \frac{(s + z_c)}{(s + p_c)} \frac{(s + z_c)}{(s + p_c)}$$



Lead-Lag Compensators

- Transfer function of the lead-lag compensator:

$$G_c(s) = \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\gamma T_2}} \right) \quad (\gamma > 1)$$

- Or

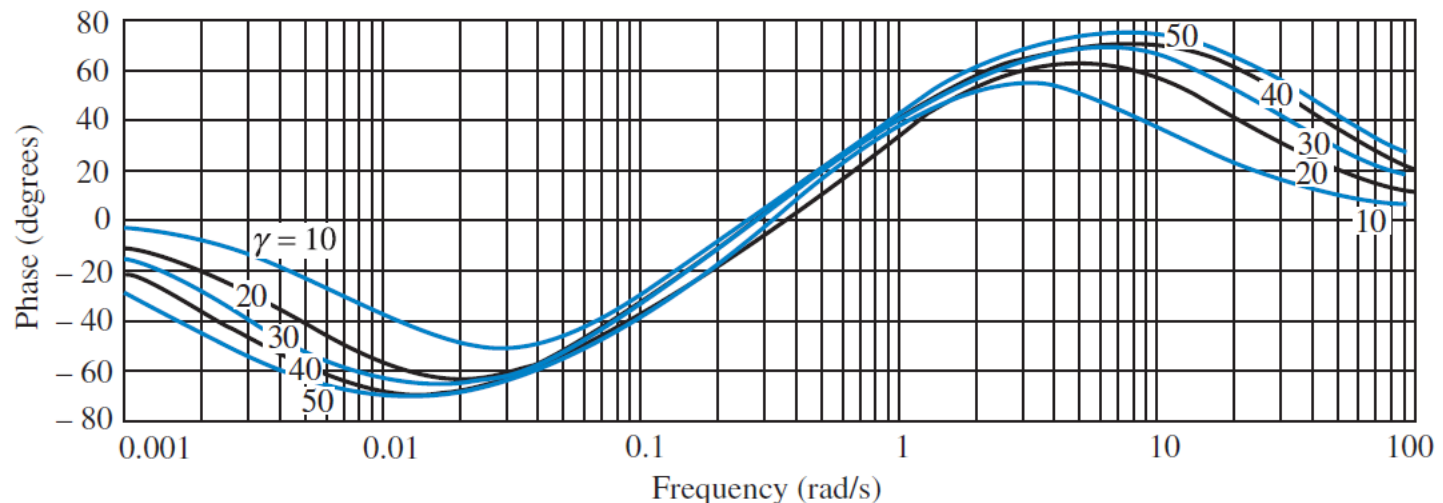
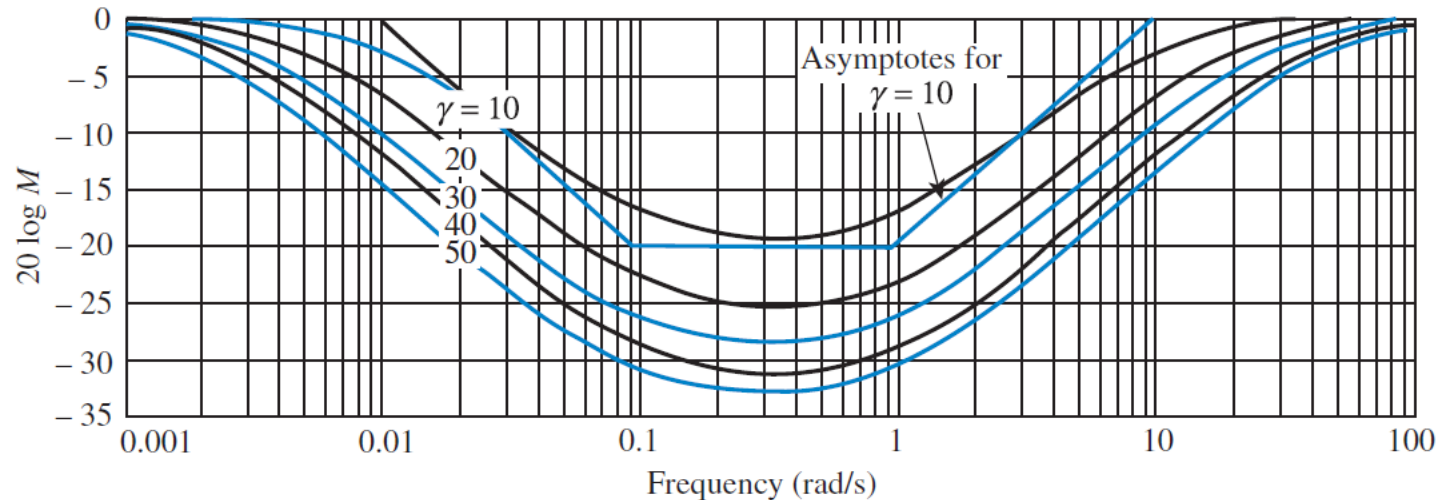
$$\begin{aligned} G_{lead-lag}(s) &= G_{lead}(s)G_{lag}(s) \\ &= \left(\frac{s + z_{c(lag)}}{s + p_{c(lag)}} \right) \left(\frac{s + z_{c(lead)}}{s + p_{c(lead)}} \right) \end{aligned}$$

- The pole and zero of the lag and lead parts of the lead-lag controller are:

$$|p_{c(lag)}| < |z_{c(lag)}| \quad \text{and} \quad |z_{c(lead)}| < |p_{c(lead)}|$$

Characteristics of Lead-Lag Compensators

- The frequency response of lead-lag compensator with varied γ :



Characteristics of Lead-Lag Compensators

Lead-lag compensator improves steady-state error and transient response:

- Lag pole at $-p_{lag}$ and lag zero at $-z_{lag}$ are used to improve steady-state error.
- Lead pole at $-p_{lead}$ and lead zero at $-z_{lead}$ are used to improve transient response.
- Lag pole at $-p_{lag}$ is small and negative.
- Lag zero at $-z_{lag}$ is close to, and to the left of, lag pole at $-p_{lag}$.
- Lead zero at $-z_{lead}$ and the lead pole at $-p_{lead}$ are selected to indicate the design point.
- Lead pole at $-p_{lead}$ is more negative than lead zero at $-z_{lead}$.
- Active circuits are not required to implement.

Applications of Lead-Lag Compensators

- Considering the transfer function equation of the second-order plant is:

$$G(s) = \frac{K}{(s + T_1)(s + T_2)}$$

- For improving transient response, we can make z_c in the $G_c(s)$ to be equal to largest of T_1 and T_2 , say T_2 , to speed up system.
- Then, the open-loop transfer function of the system is:

$$\begin{aligned} \frac{O(s)}{E(s)} &= G_c(s)G(s) \\ &= \left(\frac{s + z_{c(lag)}}{s + p_{c(lag)}} \right) \left(\frac{s + z_{c(lead)}}{s + p_{c(lead)}} \right) \frac{K}{(s + T_1)(s + T_2)} \end{aligned}$$

Applications of Lead-Lag Compensators

- Thus, the transfer function of closed-loop system is:

$$T(s) = \frac{O(s)}{I(s)} = \frac{\text{Forward}}{1 - \text{Loop}}$$

- This gives the closed-loop transfer function equation:

$$\begin{aligned} T(s) = \frac{O(s)}{I(s)} &= \frac{\left[\frac{K}{(s + p_{c(lead)})(s + T_1)} \right]}{1 - (-1) \left(\frac{K}{(s + p_{c(lead)})(s + T_1)} \right)} \\ &= \frac{K}{(s + p_{c(lead)})(s + T_1) + K} \end{aligned}$$

Applications of Lead-Lag Compensators

$$T(s) = \frac{K}{s^2 + s(p_{c(lead)} + T_1) + (p_{c(lead)}T_1) + K}$$

- This case is a further example of pole-zero cancellation for system improvement.
- Note: a pole is like $1 + sT$ term on denominator and a zero is such a term on numerator.
- For improving the steady-state condition of the system, we can make z_c of the lead part in the $G_c(s)$ to be equal to the smaller of T_1 and T_2 , say T_1 , to remove more dominant pole in the system.

Lead-Lag Compensator Characteristics

- Then, the pole in the lead part is used to cancel the zero of the lag part of the compensator.
- This leaves the pole of the lag part to be varied and assign to be very close to the origin e.g. $p_{c(lag)} \cong 0$ (to simulate an integral function like to the system).

$$T(s) = \frac{K}{(s + p_{c(lag)})(s + T_2) + K} = \frac{K}{s(s + T_2) + K}$$

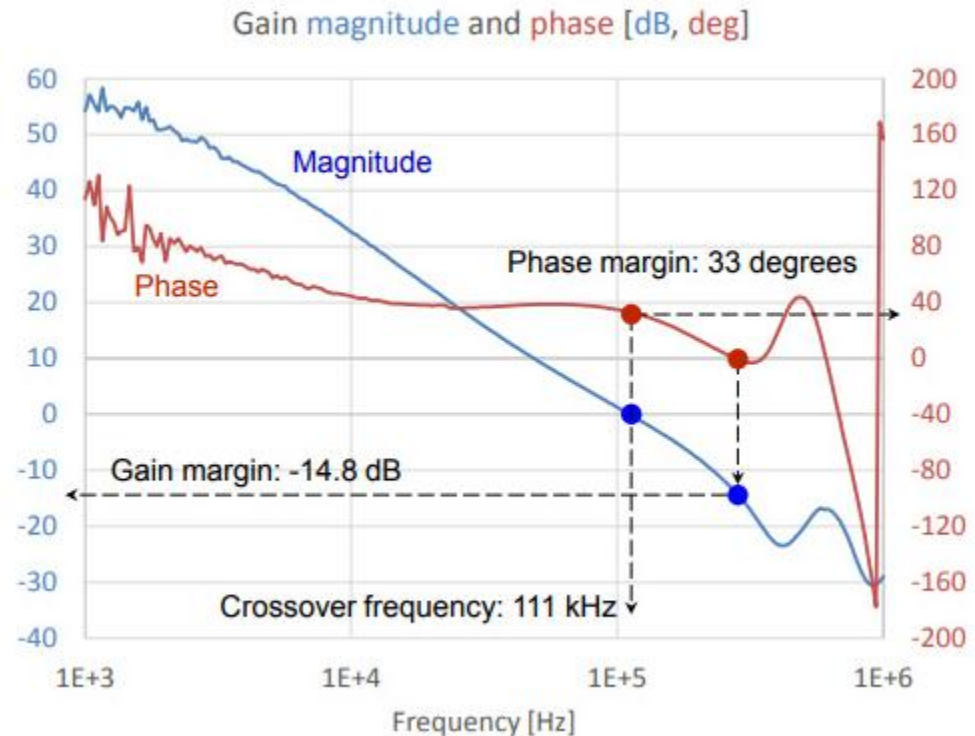
- Thus, this would improve the steady-state condition of the system by reducing or removing the steady-state error in the system.

Intro to Controller/Compensator Design

- We will focus on modifying system characteristics by applying feedback.
- Furthermore, we will be able to tailor the closed-loop transfer function with the addition of a compensator.
- Compensator design is a compromise between two competing goals.
 - Performance: Keeping the open loop gain high reduces system errors and the effects of disturbances.
 - Stability: The closed loop system must be kept stable by carefully managing the gain where the phase approaches -180° .

Intro to Controller/Compensator Design

- Compensator design can often be philosophically reduced to two (inter-related) problems:
 - one operating at low frequencies to achieve the required performance,
 - the other at high frequency to ensure stability.



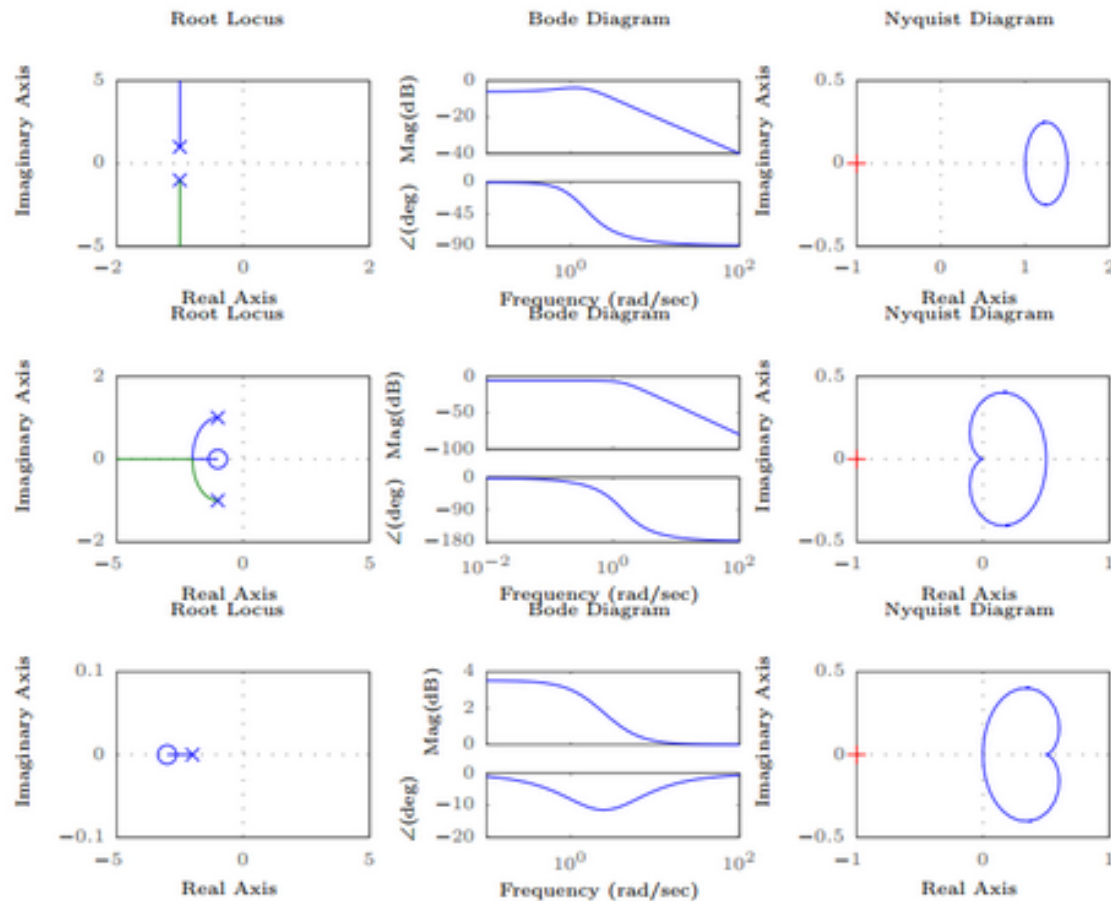
Methods for Controller/Compensator Design

There are a variety of approaches to designing a controller of compensator:

1. Choose a compensator structure and then tune manually.
2. Choose a compensator model and tune using a “recipe” (e.g. Ziegler-Nichols).
3. Use a model and solve for desired pole locations.
4. Measure the system performance and use a graphical technique.
5. Use a mathematical model with a graphical technique.
6. Use mathematical tools to achieve optimal performance (State-Space Analysis).

Controller/Compensator Design in the Course

- In the remaining lectures, we will focus on the graphical methods which form the classical control .
- These are mainly about items 3 and 5 in the list.



Convention for System Topology and Notation

- We will generally design out controller or compensators assuming unity gain feedback with the compensator $C(s)$ placed in the forward path.
- Remember that this is equivalent to a system with the controller or compensator in the feedback path.

