XMUT315 Control Systems Engineering

Note 3: Physical Systems Modelling

Topics

- Modelling physical systems.
- Lumped parameters models.
- LTI models.
- Linearization.
- Modelling aspects and process.
- Modelling mechanical systems.
- Modelling electrical systems.
- Modelling electromechanical systems.

1. Modelling Physical Systems

Modelling the physical systems is required in control system engineering to represent the system for its analysis and design.

1.1. How to Model Physical Systems

Often to model a physical system we employ a scaled physical model that is a proportional to the actual model. As illustrated in the figure below, a scaled down version of a car is used to model the car when it undergoes air flow experiment in each car manufacturer's laboratory.

With the miniature model, it is easier to analyse and design the air flow and the model might be able to fit the limited space of the wind tunnel laboratory. But beware that this approach will not be as comprehensive as the experiment with the actual car.

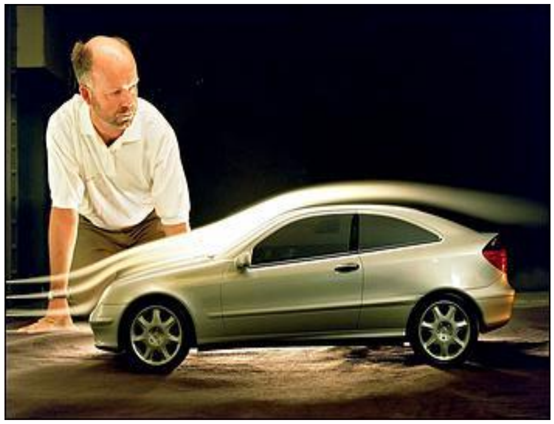


Figure 1: Modelling of a physical system with scaled physical model

On the other hand, to represent the physical system in addition to scaled model, we often use mathematical and numerical models. Mathematical model is described as function and variable in mathematical equation. Whereas numerical model is represented as a set of numbers to describe system characteristic and behaviour.

1.2. Modelling of Physical Systems

We also could develop mathematical models, i.e. ordinary differential equations that describe the relationship between input and output characteristics of a system. These equations can then be used to forecast the behaviour of the system under specific conditions.

All systems can normally be approximated and modelled by one of several models, e.g. mechanical, electrical, thermal, or fluid. We also find that we can translate a system from one model to another to facilitate the modelling.

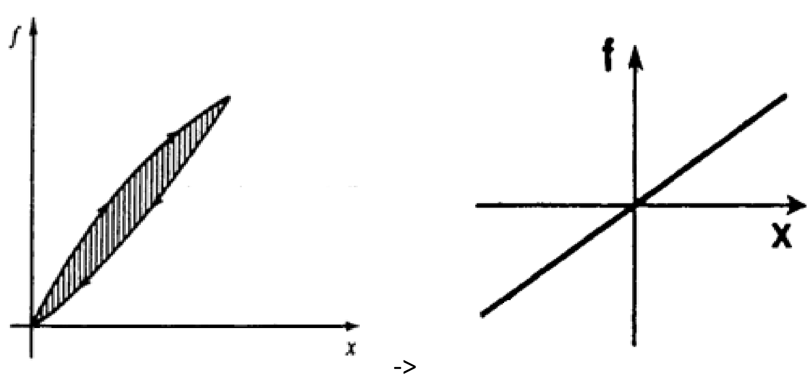


Figure 2: Translation of non-linear model to linear model

1.3. Lumped Parameter Models

Lumped parameter models apply the use of standard laws of physics and break a system down into several building blocks.

Figure 3: Lumped parameter model of a walking robot

Each of the parameters (property or function) is considered independently. Furthermore, the analysis and design will be conducted on these parts. The figure above shows lumped parameter model decomposition of walking robot, whereas the figure given below represents a lumped parameter model for a connection bridge for footpath.

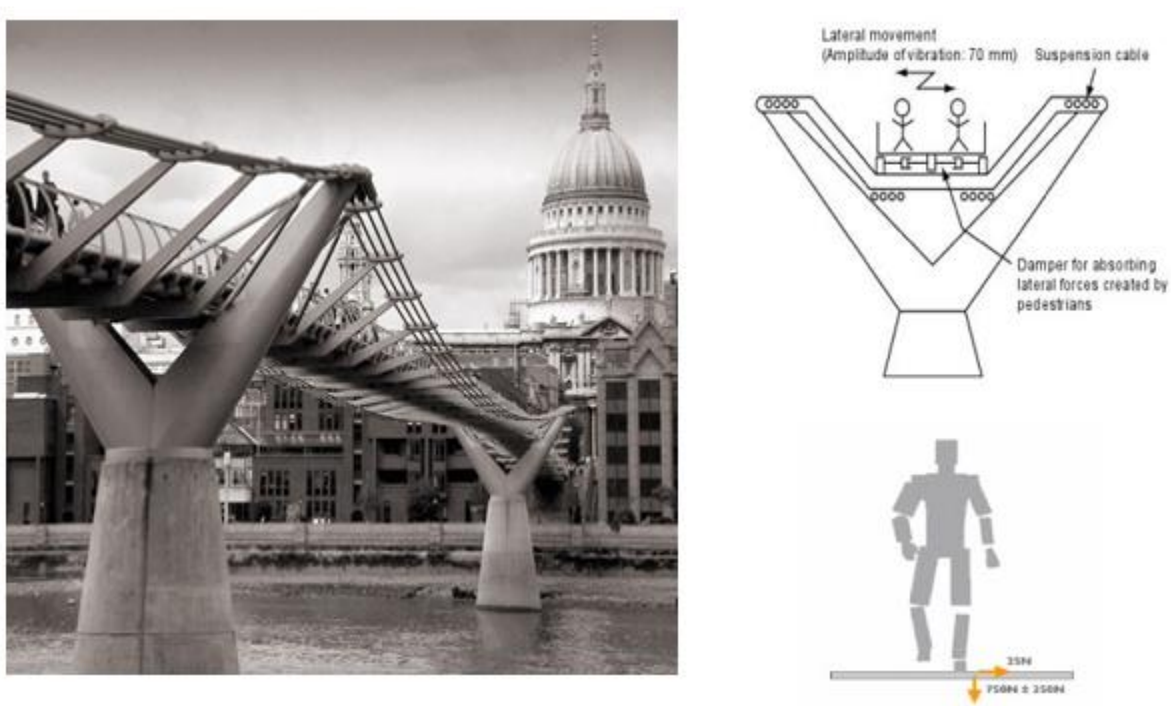


Figure 4: Lumped parameter model of a bridge

The bridge given in the figure above is a millennium bridge that was notoriously wobbly when it was initially constructed, see further for its details: http://www.youtube.com/watch?v=eAXVa__XWZ8

1.4. Linear Time Invariant Models

For modelling of the systems, to simplify the analysis and design, we often assume the property of linearity for these models. A linear system will have two properties:

- Superposition – evaluate parts of the system and conclude the characteristics and behaviour of the overall system as consisted of those of these parts.
- Homogeneity (uniformity of material) – for a given system, there is uniform characteristics and behaviour of the same part of the system.

Then, this allows us to use standard mathematical operations to simplify our models.

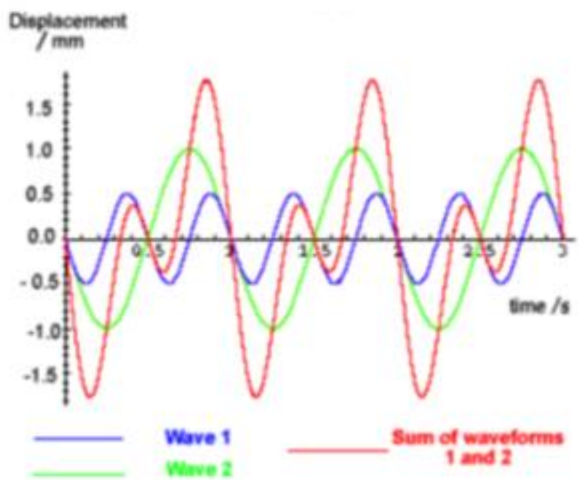


Figure 5: Superposition of several waves

The figure above shows the superposition of several waves to make up a complex signal. The overall characteristics and behaviour of this signal is evaluated based on the characteristics and behaviour of its individual components that constitute itself.

The figure shown below illustrates that the characteristic and behaviour of a given damper is uniform throughout itself. When the damper is stretched, it will stretch uniformly, rather than stretching differently at different parts like illustrated in the figure.

Figure 6: Damper with uniform (left) and non-uniform (right) materials.

Assuming that the system is time-invariant, constants stay constant in the timescales of our model. We acknowledge that proportionality between variables does not change throughout the life span of the system. Note that our shock absorbers do not wear out in our car suspension model as it often happens in practice!

Figure 7: Worn out in the shock absorbers.

1.5. Linearisation

Linearisation is finding the linear approximation to a function at a given point. The linear approximation of a function is the first order Taylor expansion around the point of interest.

In the study of dynamical systems, linearisation is a method of choice for assessing the local stability of an equilibrium point of a system of nonlinear differential equations or discrete dynamical systems.

Intuitively, linearisation is performed with order reduction, tangent, or Taylor series.

Order reduction method: A finite number of terms will give an approximation of the function e.g. the first two terms will give a linear approximation.

$$y = f(x_0) + \left[\frac{dy}{dx}\right]_{x_0}(x - x_0) + \left[\frac{d^2y}{dx^2}\right]_{x_0}\frac{(x - x_0)}{2!} + \cdots$$

Tangent method: Using a linear function evaluated at a given point (i.e. tangent of the curve) instead of higher order function.

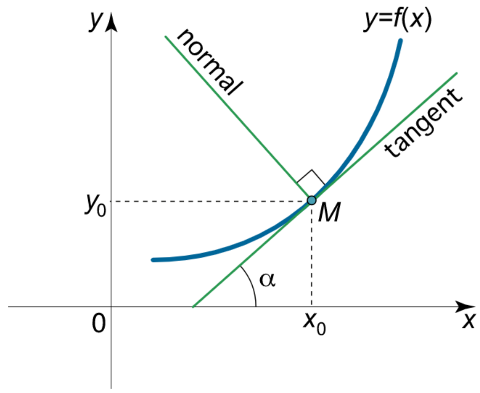


Figure 8: Linearisation using the tangent of a point in the curve method.

Forms of linearisation of a function are typically lines e.g. usually those that can be used for purposes of calculation. Linearisation is an effective method for approximating the output of a function $y = f(x)$ at any $x = a$ based on the value and slope of the function at $x = b$, given that $f(x)$ is differentiable on $[a, b]$ (or) $[b, a]$ and that a is close to b. In short, linearisation approximates the output of a function near $x = a$.

Taylor series method: Suppose we know that y is a function of x and we know the values of y and y' when $x = a$, that is $y(a)$ and $y'(a)$ are known. We can use $y(a)$ and $y'(a)$ to determine a linear polynomial which approximates to $y(x)$. Let this polynomial be:

$$p_1(x) = c_0 + c_1 x$$

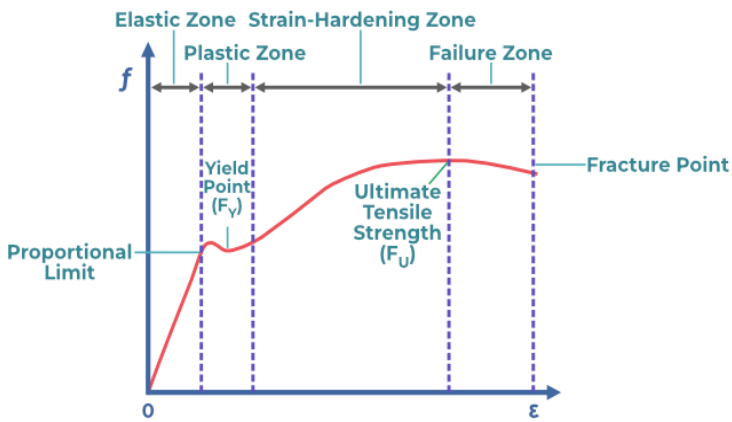
Thus

$$p_1(x) = y(a) + y'(a)(x - a)$$

The $p_1(x)$ is the first-order Taylor polynomial generated by y at $x = a$.

Example for Tutorial 1: Approximation and Linearisation

1. Consider the force acting in a spring during the plastic zone condition that can be described as a third-order function of extension (ε): $f(\varepsilon) = 2\varepsilon + 5\varepsilon^3$.



At its operating point at $\varepsilon = 1$, this function can be approximated as: $f(\varepsilon) \cong -10 + 17\varepsilon$.

Answer

Model the spring force as $f(\varepsilon) = -10 + 17\varepsilon$ around the point $\varepsilon = 1.0$ and the linearised spring forced constant would be given by:

$$\frac{df}{d\varepsilon} = 17(1) = 17\ \mathrm{N/m}$$

2. Find a linear approximation to a function $y(t) = t^2$ near $t = 3$ using Taylor series.

Answer

We require the equation of the tangent to $y = t^2$ at $t = 3$, that is the first-order Taylor polynomial about $t = 3$. Note that $y(3) = 9$ and $y'(3) = 6$.

$$p_1(t) = y(a) + y'(a)(t - a) = y(3) + y'(3)(t - 3)$$
$$= 9 + 6(t - 3) = 6t - 9$$

At $t = 3$, $p_1(t)$ and $y(t)$ have an identical value. Near to $t = 3$, $p_1(t)$ and $y(t)$ have similar values, for example $p_1(2.8) = 7.8$, and on the other hand $y(2.8) = 7.84$.

2. Components of Physical Modelling

Once the physical system or entity is model, we need to represent it for its analysis and design. In control system engineering, the most common approach is to model it in the block diagram modelling.

2.1. Signals

In block diagram modelling, components relate to each other by signals. Signals have many different forms depending on their characteristics and behaviours as shown in the figure below. Signals must also have direction and name when they are modelled in the clock diagram modelling.

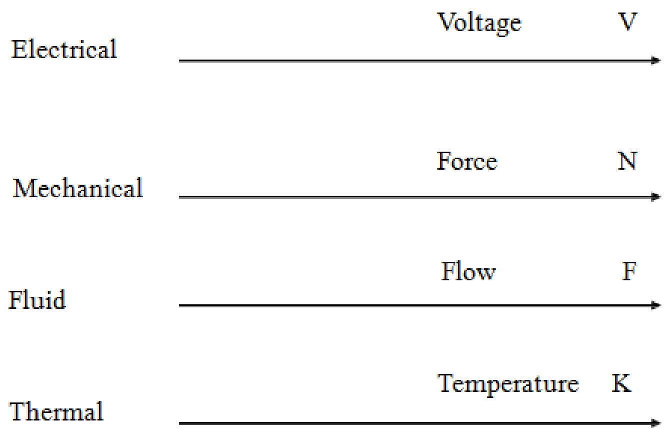


Figure 9: Several examples of common signal in control system

Signals will continue to flow until interrupted. In the modelling, signals and components are considered ideal. We add other signals and components to alter the signals. Often, during modelling, we wish to know how the output signal varies with an input signal for a fixed (invariant) system as illustrated in the figure below. Also, we may plot two signals against each other invariant of time (system relationship).

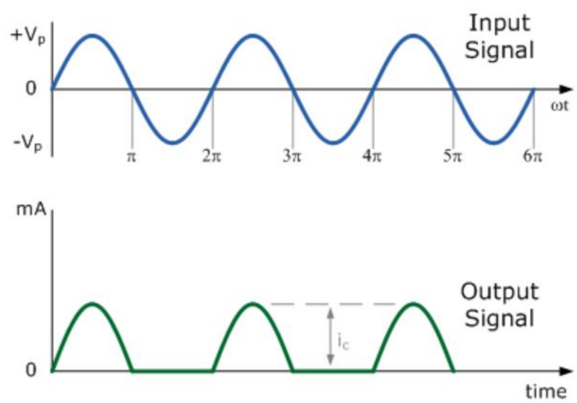


Figure 10: Input and output signals

2.2. Constants

System constants are the time invariant for the given system. For a spring example given in the figure below, the constant of a spring is unique for the given spring. We consider a different system as the spring has been changed. However, the analysis stays the same.

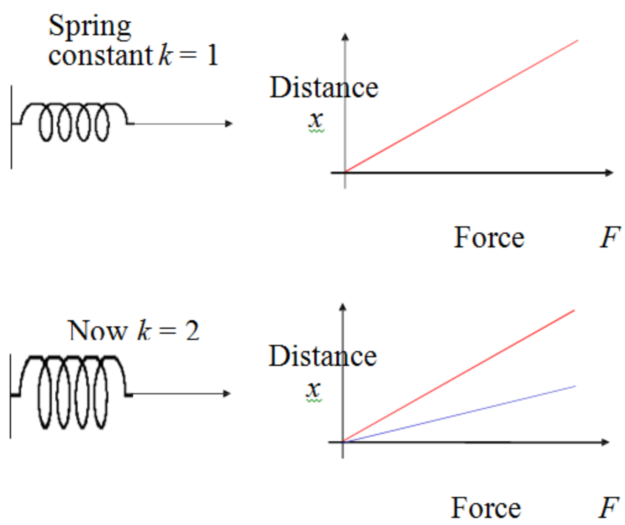


Figure 11: Spring system with different spring constant values

2.3. Differentiation

Consider differentiation as alternative method for modelling. Levels of the water in the smaller tank, L, and water in the larger tank, I, change because of the flow of liquid. Mathematically, change of the level (ΔL) with time (Δt) is calculated as:

$$\frac{\Delta L}{\Delta t} = \frac{dL}{dt}$$

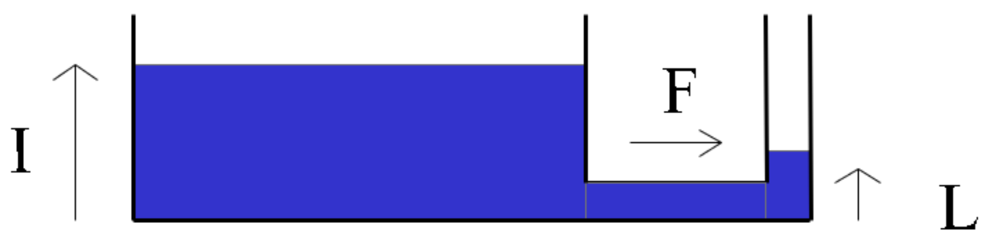


Figure 12: Water tank system

In fact, the change in L is proportional to flow, F and inversely proportional to the cross-sectional area of the connecting conduit, C (e.g.: the pipe):

$$\frac{dL}{dt} = \left(\frac{1}{C}\right)F \qquad \text{and} \qquad F = \frac{I-L}{R}$$

Where: R is the radius of the conduit. Flow is related to difference in levels, combining the two equations given above, thus.

$$\frac{dL}{dt} = \left(\frac{1}{C}\right)\left(\frac{I-L}{R}\right) = \frac{I-L}{CR}$$

Where: CR is the time constant of the system, T.

Note: the above case is a differential equation. It has the differential of L being a function including L.

2.4. Differentiation: Slopes

Consider the graph of level or height of the water (L) against time as shown in the figure given below. At any instant of time, we can see value of L observed in the graph. The change in L is the slope of the graph, which varies with time.

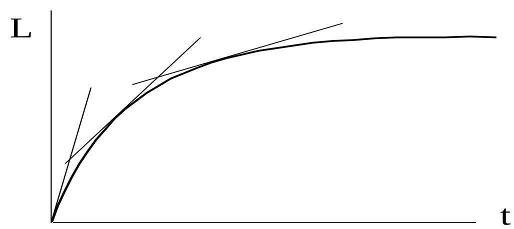


Figure 13: Slopes in graph

But, initially the slope is steep (high value), then becoming less, and becoming finally least. Thus, slope of L is like F, but slope is change of L. In fact, F is proportional to derivate of L with time.

2.5. Integration: Area

The reverse process of differentiation is integration. Its graphical interpretation is the area under a graph. Consider the flow graph of the water level of two tanks: the area at different times is shown as in the figures given below.

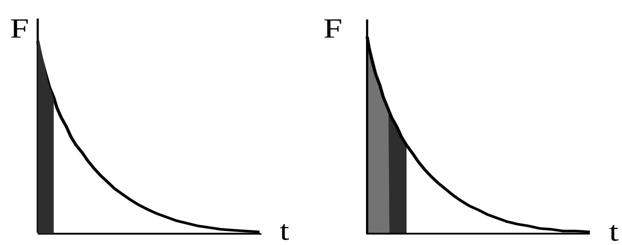


Figure 14: Integration of area under a graph

After a short time, the area is as shown in the figure on the left. Later, area has grown, but by less, etc. Consider the height of water in the tank in the right figure. Thus, L like area under F e.g.:

$$L \propto \int F\,dt$$

The water level, L is proportional to integral of F with time. In fact, for this system, we have:

$$\frac{dL}{dt} = \left(\frac{1}{C}\right) F \qquad \text{and} \qquad L = \frac{1}{C}\int F\,dt$$

The flow, F is differential of the level L and L is integral of F. Differentiation and integration are opposites.

Note: here they are used to model a water system. It can also model electronic circuits, mechanical systems, motors, etc. In fact, the differential equation has the same form, and hence the same exponential response as that for many systems.

There are analogies between water systems and electronics: pipe like a resistor, tank like a capacitor. Also, for thermals, walls have thermal resistance and rooms have thermal capacity.

3. Mechanical System Modelling

Simple mechanical systems can be represented as models from their standard components.

We know that distance $(x(t))$ is related to velocity $(v(t))$ is related to acceleration $(a(t))$ through differentiation.

Displacement:

$$\text{Distance} = x(t)$$

Velocity:

$$v(t) = \frac{dx(t)}{dt}$$

Acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d\left(\frac{dx(t)}{dt}\right)}{dt} = \frac{d^2x(t)}{dt^2}$$

If we derive the model from first principles, it gets messy writing d/dt all the time. Therefore, we use Laplace transform and will write in term of 's' instead.

Displacement:

$$\text{Distance} = X(s)$$

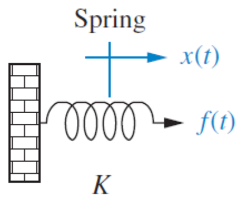
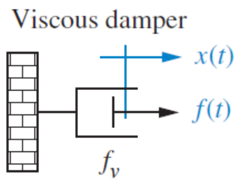
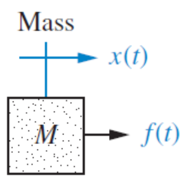
Velocity:

$$V(s) = sX(s)$$

Acceleration:

$$A(s) = sV(s) = s^2X(s)$$

Note: both with respect to the variable.

Component	Force-Velocity	Force Displacement	Impedance
	$f(t) = K\int_0^t v(\tau)\,d\tau$	$f(t) = Kx(t)$	K
	$f(t) = f_v v(t)$	$f(t) = f_v \dfrac{dx(t)}{dt}$	$f_v s$
	$f(t) = M\dfrac{dv(t)}{dt}$	$f(t) = M\dfrac{d^2x(t)}{dt^2}$	Ms^2

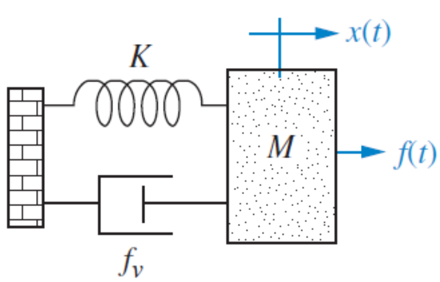
Note: Impedance is $Z_m = F(s)/X(s)$

Table 1: Standard basic mechanical components

Example for Tutorial 2: Modelling of Mechanical System

For the mechanical system given below, it consists of mass, spring, and damper.

- We assume the mass (M) is displaced by $x(t)$ toward the right.
- Note that taking into consideration the zero initial condition, just like the spring (with spring constant, K), the damper (with damper constant, f_v) will also oppose the force $(f(t))$.
- Thus, only the applied force points to the right.
- All other forces impede the motion and act to oppose it e.g. the spring, damper, and the force due to acceleration point to the left.



Determine the transfer function equation of the system in the time domain. [6 marks]

Answer

Write the differential equation of motion using the second Newton's law to sum to zero all the forces that exist in the given mechanical system.

$$M\left(\frac{d^2x(t)}{dt^2}\right) + f_v\left(\frac{dx(t)}{dt}\right) + Kx(t) = f(t)$$

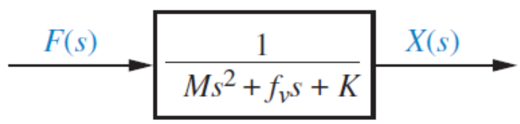
Taking the Laplace transform, assuming zero initial conditions, the equation above becomes:

$$Ms^2X(s) + f_v sX(s) + KX(s) = F(s)$$

As a result, the transfer function equation of the given mechanical system is:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + K}$$

Or, as represented in the block diagram as shown in the figure below.



4. Electrical Systems Modelling

Like mechanical systems, electrical systems can be modelled from their standardised components.

We know that to find the reactance of electrical devices such inductor and capacitor requires integration and differentiation respectively.

- Voltage across resistor:

$$v_R(t) = Ri(t)$$

- Voltage across capacitor:

$$v_C(t) = \frac{1}{C}\int_0^t i(t)$$

- Voltage across inductor:

$$v_L(t) = L\left(\frac{di(t)}{dt}\right)$$

By applying Laplace transform, we have the following:

- Voltage across resistor:

$$V_R(s) = Ri(s)$$

- Voltage across capacitor:

$$V_C(s) = \left(\frac{1}{sC}\right) i(s)$$

- Voltage across inductor:

$$V_L(s) = sLi(s)$$

Note: both components and their Laplace transforms are with respect to the variable.

Component	Voltage-current	Current-voltage	Impedance
Capacitor	$v(t) = \frac{1}{C}\int_0^1 i(\tau)d\tau$	$i(t) = C\frac{dv(t)}{dt}$	$\frac{1}{Cs}$
Resistor	$v(t) = Ri(t)$	$i(t) = \frac{1}{R}v(t)$	R
Inductor	$v(t) = L\frac{di(t)}{dt}$	$i(t) = \frac{1}{L}\int_0^1 v(\tau)\, d\tau$	Ls

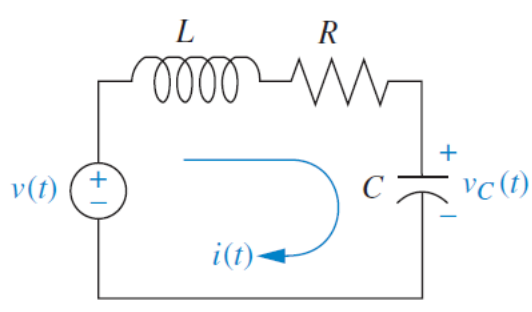
Note: Impedance is $Z(s) = V(s)/I(s)$

Table 2: Standard basic electrical components

Example for Tutorial 3: Modelling of Electrical System

For an electrical system as shown below, it consists of inductor (L), resistor (R) and capacitor (C) that is supplied with a voltage source ($e(t)$). The current that flows in the circuit is $i(t)$.

- It is a series RLC circuit.
- Assume in this case that the capacitor voltage as the output and the applied voltage as the input.
- Assume zero initial conditions (no prior conditions before modelling existed).



Determine the time-domain expression for the output voltage over the input voltage for the given circuit. [12 marks]

Answer

Summing the voltages around the loop, assuming zero initial conditions, yields the integral-differential equation for this network as:

$$L\left[\frac{di(t)}{dt}\right] + Ri(t) + \frac{1}{C}\int_0^t i(\tau)d\tau = v(t)$$

Changing variables from current to charge using $i(t) = dq(t)/dt$ yields:

$$L\left[\frac{d^2q(t)}{dt^2}\right] + R\left[\frac{dq(t)}{dt}\right] + \left(\frac{1}{C}\right)q(t) = v(t)$$

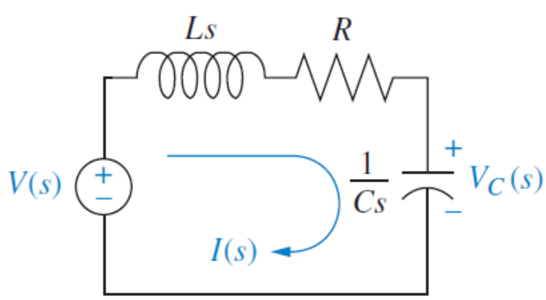
From the voltage-charge relationship for a capacitor, the charge in the capacitor is:

$$q(t) = Cv_C(t)$$

Substituting Eq. (2) into Eq. (1) yields:

$$LC\left[\frac{d^2v_C(t)}{dt^2}\right] + RC\left[\frac{dv_C(t)}{dt}\right] + v_C(t) = v(t)$$

Taking the Laplace transform assuming zero initial conditions, the s-domain equivalent circuit of the RLC circuit is as shown in the figure below.



Rearranging terms and simplifying yields:

$$(LCs^2 + RCs + 1)V_C(s) = V(s)$$

Solving for the output voltage over input voltage transfer function, $V_C(s)/V(s)$, we obtain:

$$\frac{V_C(s)}{V(s)} = \frac{\left(\frac{1}{LC}\right)}{s^2 + \left(\frac{R}{L}\right)s + 1/LC}$$

5. Electromechanical System Modelling

Since it is consisted of mechanical and electrical systems, modelling of electromechanical system could typically be performed using the standardised components of the mechanical and electrical systems. As an example, DC motor is commonly used to illustrate the modelling of the electromechanical systems.

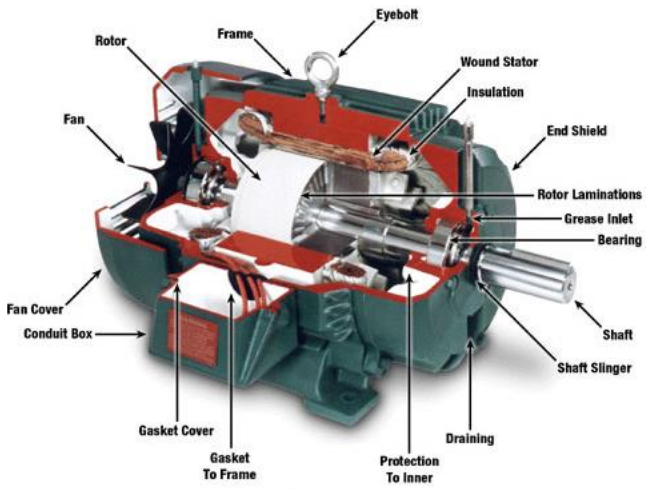


Figure 15: Cross-sectional cut of a typical DC motor

The modelling components of a DC motor are illustrated and derived as outlined in the following sections. It is divided into three parts e.g. electrical system, mechanical system and overall electromechanical system.

5.1. Electrical System of DC Motor

Typically, there are two windings in the DC motor e.g. armature winding and field excitation winding.

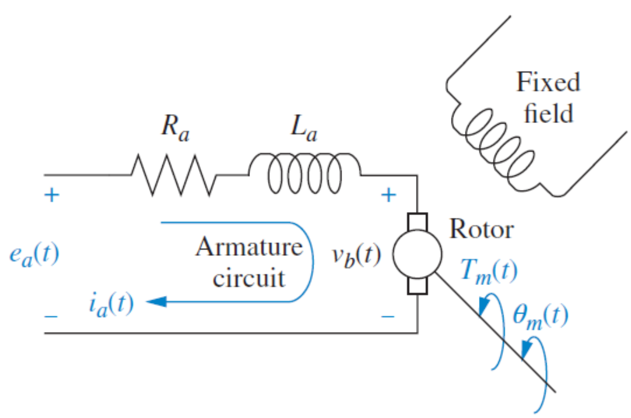


Figure 16: Schematic diagram of a DC motor

Applying the KVL in the armature winding.

$$e_a(t) = R_a i_a(t) + L_a \left[\frac{di_a(t)}{dt}\right] + v_b(t)$$

Using Laplace transform, the equation for the circuit is:

$$E_a(s) = R_a I_a(s) + L_a(s) s I_a(s) + V_b(s) \qquad (Eq. 1)$$

Where K_b is the back EMF constant and $d\theta_m(t)/dt = \omega_m(t)$, for a given motor, the back EMF of the motor is calculated from:

$$v_b(t) = K_b \left[\frac{d\theta_m(t)}{dt}\right]$$

So

$$V_b(s) = K_b s \theta_m(s) \qquad (Eq. 2)$$

The torque developed by motor is proportional to armature current where K_t is a motor torque constant, thus, it is determined from:

$$T_m(t) = K_t i_a(t)$$

So

$$T_m(s) = K_t I_a(s) \qquad (Eq. 3)$$

Thus

$$I_a(s) = \left(\frac{1}{K_t}\right) T_m(s) \qquad (Eq. 4)$$

5.2. Mechanical System of DC Motor

The following figure shows the equivalent mechanical loading that typically connected to a DC motor.

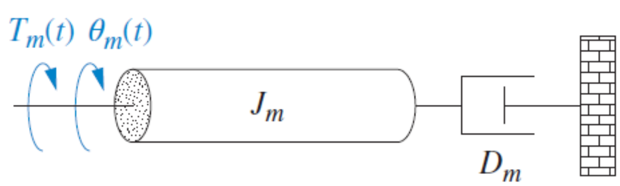


Figure 17: Equivalent mechanical loading on a motor

Where: J_m is the equivalent inertia of the motor (e.g.: both of inertia of the armature and load) and D_m is the vicious damping (e.g.: both of vicious damping of the armature and load).

The torque of the DC motor is calculated from:

$$T_m(t) = J_m \left[\frac{d^2\theta_m(t)}{dt^2}\right] + D_m \left[\frac{d\theta_m(t)}{dt}\right]$$

Thus

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s) \qquad (Eq. 5)$$

For a DC motor connected with a mechanical load as given in the figure below, these are the modelling components of the system:

- Motor is used to drive a mechanical load (J_L) pushing a damper (D_L).
- Motor has inertia (J_a) and damping factors (D_a).
- Gear ratios of the DC motor (N_1) and mechanical load (N_2).

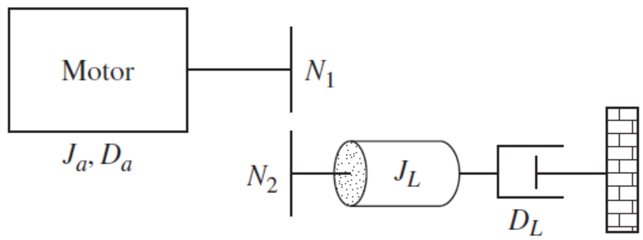


Figure 18: DC motor driving a rotational mechanical load

Knowing inertia (J_m) and damping factor (D_m) of the motor are related through:

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2$$

And

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

Substituting equations (2) and (4) into equation (1), with $L_a = 0$, yields:

$$\left(\frac{R_a}{K_t}\right)T_m(s) + K_b s\theta_m(s) = E_a(s)$$

As $s\theta_m(s) = d\theta_m(t)/d_t = \omega_m(t)$, applying the inverse Laplace transform, we get:

$$\left(\frac{R_a}{K_t}\right)T_m(t) + K_b\omega_m(t) = e_a(t)$$

Rearrange the equation, the equation above becomes:

$$T_m = -\left(\frac{K_b K_t}{R_a}\right)\omega_m + \left(\frac{K_t}{R_a}\right)e_a$$

When the equation above is plotted, it becomes a straight-line graph, T_m vs. ω_m, as shown in the figure below.

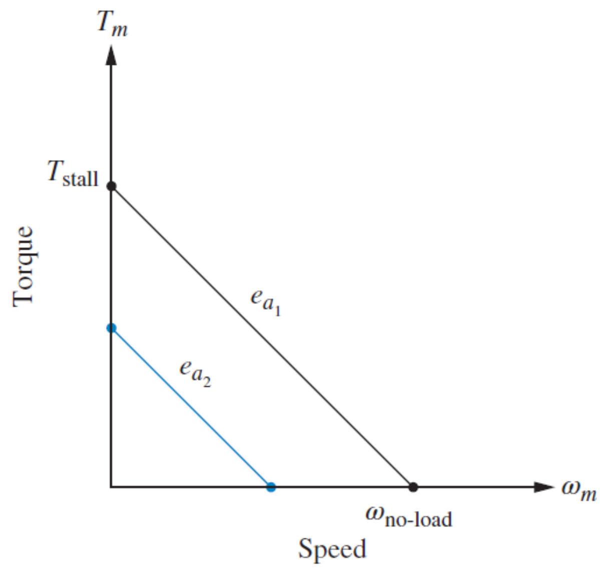


Figure 19: Torque-speed curves with an armature voltage, e_a, as a parameter

From the torques-speed curve diagram, with the armature voltage is set at e_{a_1}, the DC motor is set to operate at the extreme conditions.

In this case, these are the stalling state when $\omega_m = 0$ (e.g.: motor stop rotating and maximum current is drawn) and the no-load state when $T_m = 0$ (e.g.: maximum speed with no load).

The intercepts in the torques-speed curve diagram correspond to these extreme conditions as follows.

Stall torque, T_{stall}:

$$T_{\text{stall}} = \left(\frac{K_t}{R_a}\right) e_a(t)$$

No load speed, $\omega_{\text{no-load}}$:

$$\omega_{\text{no-load}} = \frac{e_a(t)}{K_b}$$

We could obtain the electrical constants, K_t/R_a and K_b from the torques-speed curve diagram given above.

$$\frac{K_t}{R_a} = \frac{T_{\text{stall}}}{e_a(t)}$$

And

$$K_b = \frac{e_a(t)}{\omega_{\text{no-load}}}$$

5.3. Electromechanical System of DC Motor

The transfer function equation of the DC motor, preferably in terms of the angular speed of the given DC motor ($\theta_m(s)$) is determined from the voltage applied across its armature ($E_a(s)$).

Or, in the block diagram shown below, the DC motor is typically illustrated as a block diagram ($G(s)$) with armature voltage, $E_a(s)$ as input and angular speed of the motor, $\theta_m(s)$ as output.

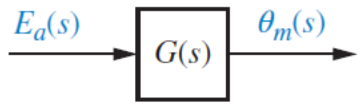


Figure 20: Electromechanical system of a DC motor with load

As a result, substitute equations (4) and (2) into equation (1), the equation becomes:

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \qquad (Eq.6)$$

Then, substitute equation (5) into equation (6), it is:

$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s)$$

Considering that the armature inductance (L_a) is small compared to armature resistance (R_a) which is common for a DC motor, the equation above becomes:

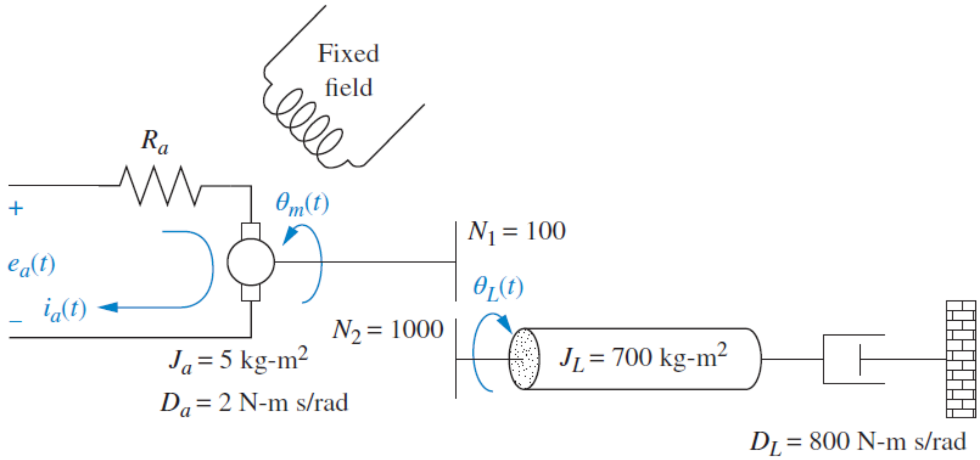
$$\left[\frac{R_a}{L_a}(J_m s + D_m) + K_b\right] s\theta_m(s) = E_a(s)$$

Rearrange the equation above into a ratio of $\theta_m(s)/E_a(s)$, it becomes:

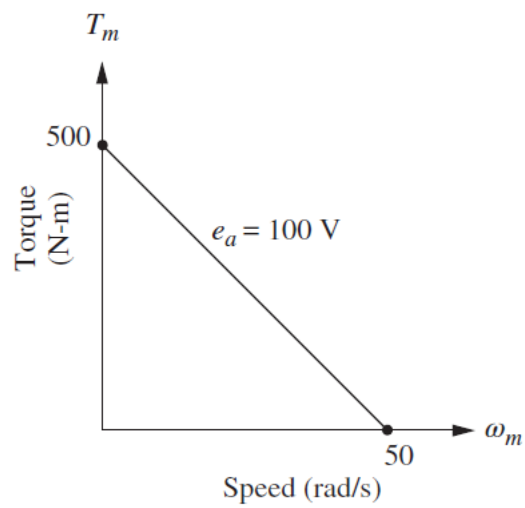
$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

Example for Tutorial 4: Modelling of Electromechanical System

Given the DC motor connected to a mechanical load as shown in part (a) in the figure below used as an example of electromechanical system and torque-speed curve shown in part (b), find the transfer function, $\theta_L(s)/E_a(s)$. [20 marks]



a. DC motor and load



b. The torque-speed

Answer

Begin by finding the mechanical constants, J_m and D_m. From equation given below, the total inertia at the armature of the motor is:

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2 = 5 + 700\left(\frac{1}{10}\right)^2 = 12 \qquad (Eq.1)$$

The total damping at the armature of the motor is:

$$D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 = 2 + 800\left(\frac{1}{10}\right)^2 = 10 \qquad (Eq.2)$$

Now, we will find the electrical constants, $K_t = R_a$ and K_b. From the torque-speed curve of the part (b) in the figure above,

$$T_{stall} = 500 \qquad (Eq.3)$$

$$\omega_{\text{no-load}} = 50 \qquad (Eq.4)$$

$$e_a(t) = 100 \qquad (Eq.5)$$

Hence, the electrical constants are:

$$\frac{K_t}{R_a} = \frac{T_{stall}}{e_a(t)} = \frac{500}{100} = 5 \qquad (Eq.6)$$

and

$$K_b = \frac{e_a(t)}{\omega_{no-load}} = \frac{100}{50} = 2 \qquad (Eq.7)$$

Substitute the values obtained in the equations (1), (2), (6), and (7) into the equation below.

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_t}{(R_a J_m)}}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$= \frac{5/12}{s\left\{s + \frac{1}{12}[10 + (5)(2)]\right\}} = \frac{0.417}{s(s + 1.667)}$$

In order to find $\theta_L(s)/E_a(s)$, we use the gear ratio, $N_1/N_2 = 1/10$, and find:

$$\frac{\theta_L(s)}{E_a(s)} = \frac{0.0417}{s(s + 1.667)}$$

Or, as shown as a block diagram in the figure below.