XMUT315 Control Systems Engineering

Note 4: Block Diagram Modelling

Topic

- Introduction to block diagram modelling.
- Feedback systems in block diagram.
- Block diagram modelling.
- Block diagram manipulation and reduction.
- Block diagram modelling and physical modelling of control systems.

1. Introduction to Block Diagram Modelling

The block diagram is the chosen method used in this course for modelling the control systems.

1.1. System Modelling

To analyse a system, it is important that we could perform the following steps:

- We identify an input signal [a variable].
- Using block diagram components [basic block, summing junction, and take-off point].
- We combine internal signals [modified variables].
- To produce the output signal [another variable].

With all these steps given above carried out, the input-output relationship and hence the transfer function of the system may then be determined.

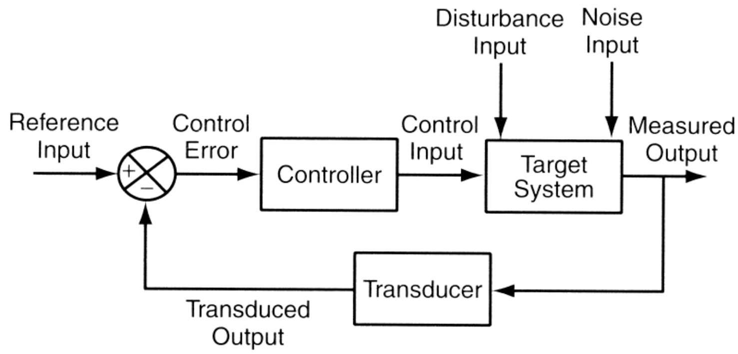


Figure 1: An example of a feedback control system.

1.2. Components

There are several components that will be needed in the block diagram modelling e.g. block, summing junction and take-off point.

1.2.1. Block

Block is a representation or feature of a system or function of the system signal. The following diagram outlines a resistance system in the electrical circuit e.g. voltage (V) is a product of current (I) with resistance (R). Also shown in the figure below is a spring function where the force acting in a spring (F) is equal to distance (x) multiplied by $1/k$, where k is the spring constant.

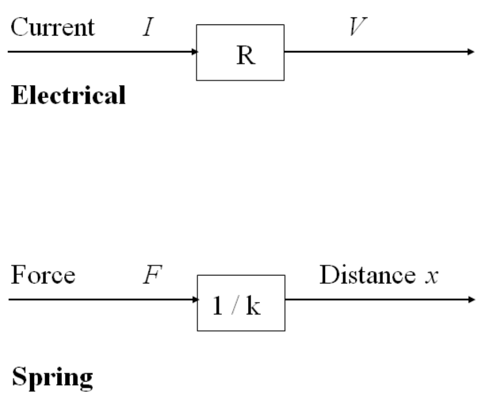


Figure 2: Block in the block diagram modelling.

1.2.2. Summing Junction

Summing junction is used to combine several signals. Depending on its types, it is used to do addition (+) and/or subtraction (–) of the system signals. In the block diagram modelling, we could have up to three inputs and only one output for a given summing junction.

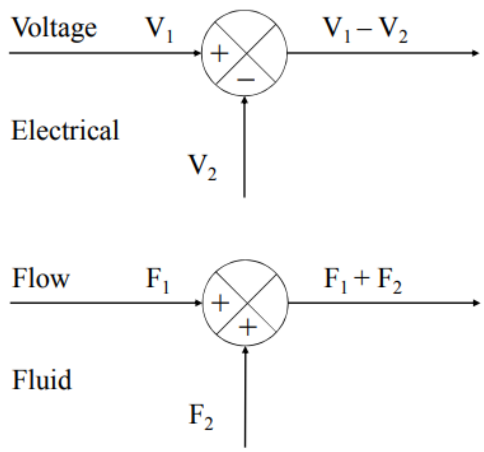


Figure 3: Summing junction in block diagram modelling.

The diagram given above illustrates the use of summing junction to subtract two voltages (V_1 and V_2) in an electrical system and part of the diagram on the bottom shows the addition of two flows (e.g. F_1 and F_2) in a fluid system.

1.2.3. Take-off Point

Take-off point is used to split a signal into several of the same signals. The system signal can be used elsewhere but is not affected by the split. For a given take-off point, we have only one input and many outputs as shown in the figure below. As shown below, we split voltages in an electrical system.

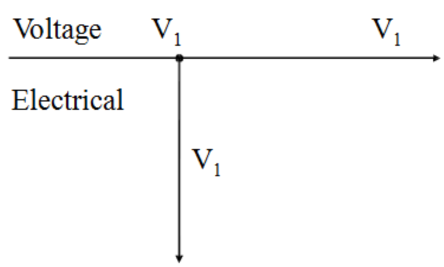


Figure 4: Take-off point in block diagram modelling.

2. Feedback System in Block Diagram

It is important to be able to model feedback system in the block diagram as we often deal with feedback system in analysis and design of control systems.

2.1. Negative in the Feedback Loop

For a given feedback system, useful paths in the feedback mechanism are the forward path and the feedback loop path as shown in the figure below.

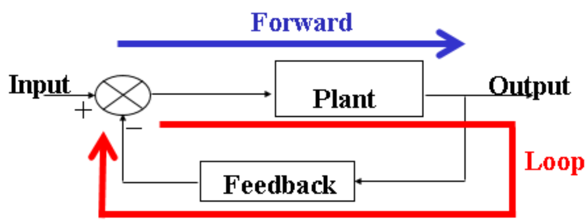


Figure 5: Main paths in the feedback system.

To model feedback system, create transfer function from the variables (input and output) and constants (bits inside the boxes).

$$\frac{\text{Output}}{\text{Input}} = \frac{\text{Forward}}{1 - \text{Loop}}$$

Common useful components in the feedback system are input, output, feedback and error signals.

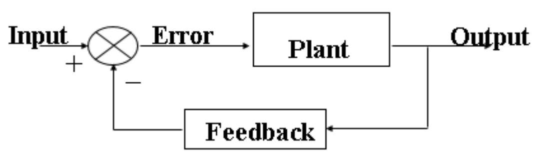


Figure 6: Components of feedback system.

From the feedback system given in the figure above, the error of the feedback system is:

$$\text{Error} = \text{Input} - \text{Feedback}$$

Output of the feedback system is found from the following equation:

$$\text{Output} = \text{Error} \times \text{Plant}$$

2.2. Derivation of Equation for Feedback System

A feedback system can be expressed as functions in respect to the changing variables as illustrated in the figure below:

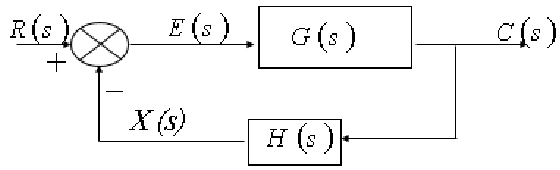


Figure 7: Variables in the feedback system.

Where:

$E(s)$ = Error function, $C(s)$ = Output function, $X(s)$ = Feedback function, $R(s)$ = Input (Reference) function, $G(s)$ = Plant system, and $H(s)$ = Feedback system.

From all of the variables given above, we can form three equations:

$$E(s) = R(s) - X(s) \qquad (1)$$

$$C(s) = G(s)E(s) \qquad (2)$$

$$X(s) = H(s)C(s) \qquad (3)$$

Note: this derivation example is for negative feedback. For the positive feedback, change the sign in front of parameter $X(s)$ in equation (1) above to (+) positive.

We need to form a relationship between input & output by removing the intermediate variables:

Combine (1) + (3):

$$E(s) = R(s) - H(s)C(s) \qquad (4)$$

Combine (2) + (4):

$$C(s) = G(s)[R(s) - H(s)]$$

We need to form a relationship between input and output by removing the intermediate variables.

Multiply out bracket:

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

Collect output terms:

$$C(s) + G(s)H(s)C(s) = G(s)R(s)$$

Rearrange the equation:

$$C(s)[a + G(s)H(s)] = G(s)R(s)$$

As a result, the transfer function of a feedback system is:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\text{Forward}}{1 - \text{Loop}}$$

Notice that for positive feedback loop is (+), and for negative feedback loop is (-).

3. Block Diagram Manipulation

For a complex system, it is important that we could simplify the in-block diagram for analysing and designing the systems. To perform the simplification, we need to do manipulation of the components used in the diagram.

3.1. Combining blocks in series

In the block diagram modelling, blocks in series can be combined to form a bigger block. As shown in the figure below, three blocks (G_1, G_2, and G_3) in series are combined to form a single block.

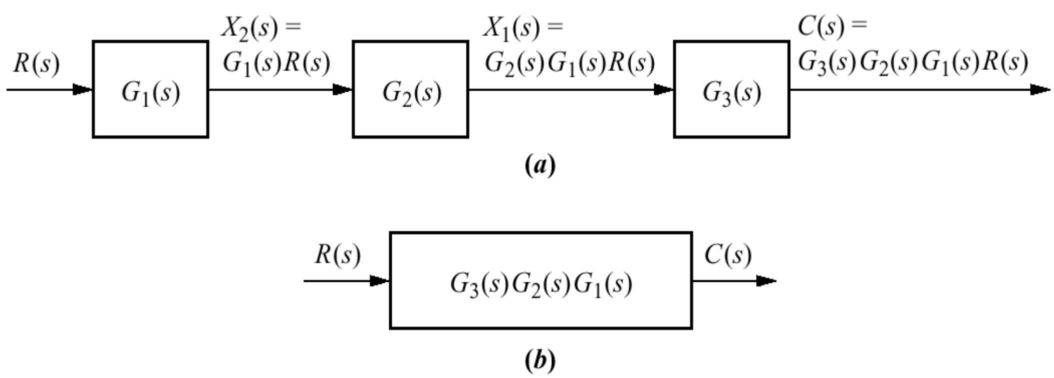


Figure 8: Combining blocks in series.

3.2. Combining blocks in parallel

In the block diagram modelling, blocks in parallel can be combined to form a bigger block. As shown in the figure below, three blocks (G_1, G_2, and G_3) in parallel are combined to form a single block.

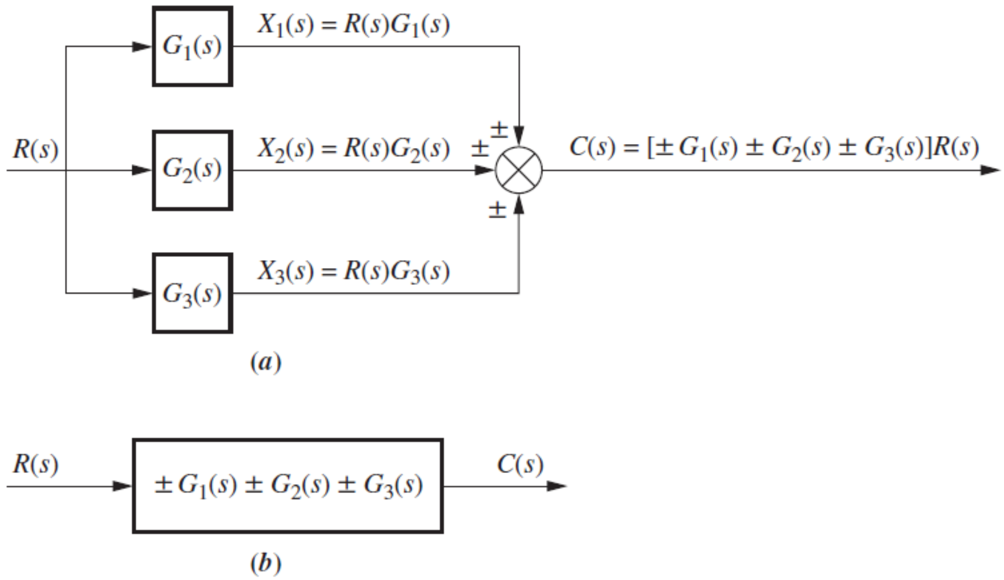


Figure 9: Combining blocks in parallel.

3.3. Changing the flow direction of a block

Direction of the flow of a block can be changed in the block diagram modelling and it requires inversion of the content of the block. The following figures illustrates the reversion of the flow of the signal. As shown in the diagram that we require the inversion of the transfer function inside the block.

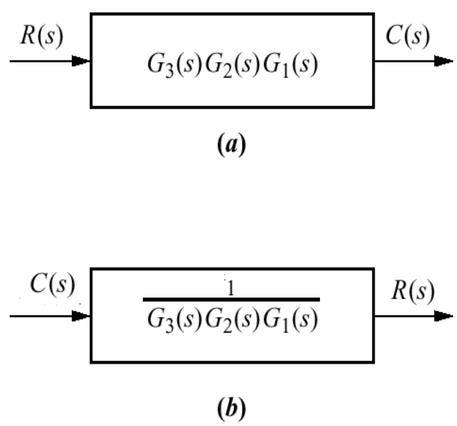


Figure 10: Changing the flow direction of a block.

3.4. Summing Junction Manipulation

In the block diagram modelling, we could move a block to **before** a summing junction. As illustrated in the figure below, it also affects the flows of other relevant signals related to the summing junction.

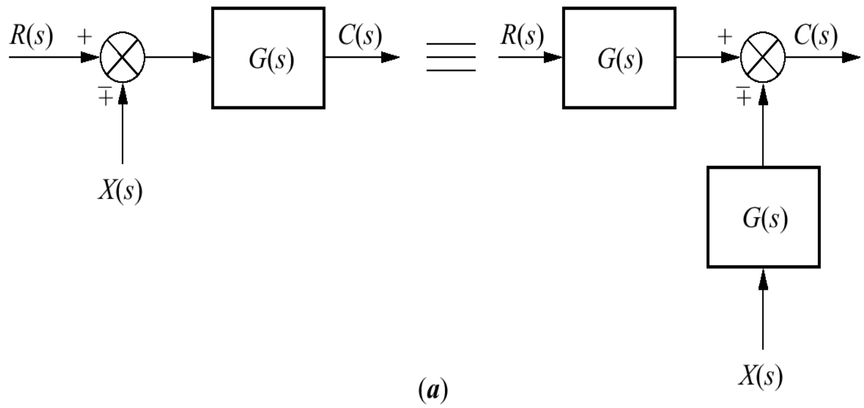


Figure 11: Move a block to before a summing junction.

Respectively, we could also move a block to **after** a summing junction. Also notice the effect of moving a block to another relevant signal related to the summing junction.

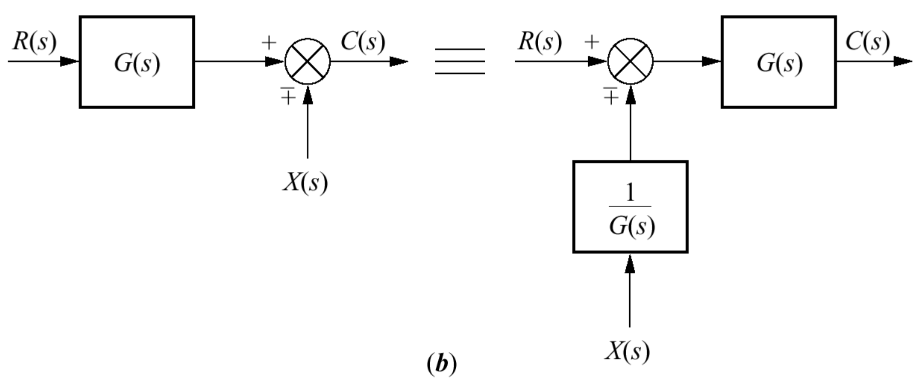
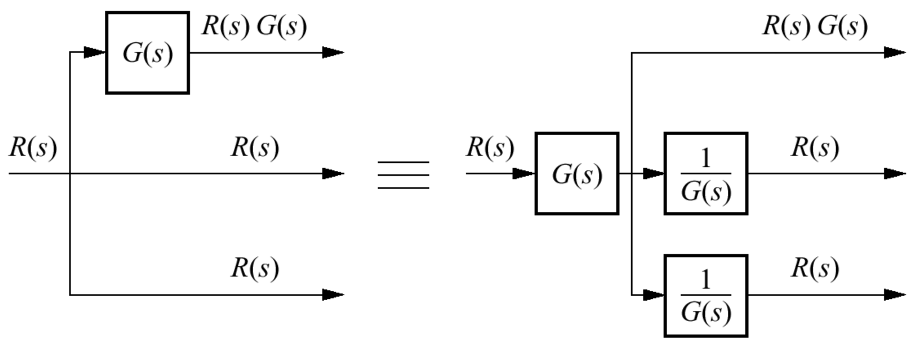


Figure 12: Move a block to after a summing junction.

3.5. Take-off Point Manipulation

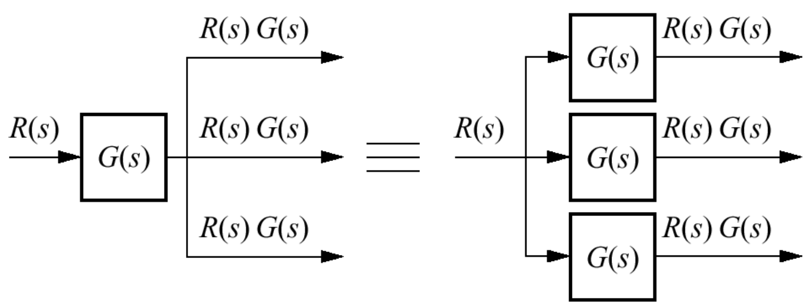
We could also move a block to **before** a take-off point in the block diagram. As illustrated, that moving around the take-off point requires you to modify the other relevant signal related to the take-off point.



(*a*)

Figure 13: Move a block to before a take-off point.

Respectively, we could also move a block to **after** a take-off point. As shown in the figure given below, we need to consider the modification of other relevant signals related to the take-off point.



(*b*)

Figure 14: Move a block to after a take-off point.

3.6. Feedback Structure Manipulation

For a feedback structure as shown in the figure below, the feedback system can be simplified into the following equation. The simplification of the given feedback structure into a single block is shown in the figure.

$$F(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Using block diagram manipulation, a given feedback structure as given in the figure (a) below can be simplified into a structure as shown in (b) and then subsequently to a single block (c).

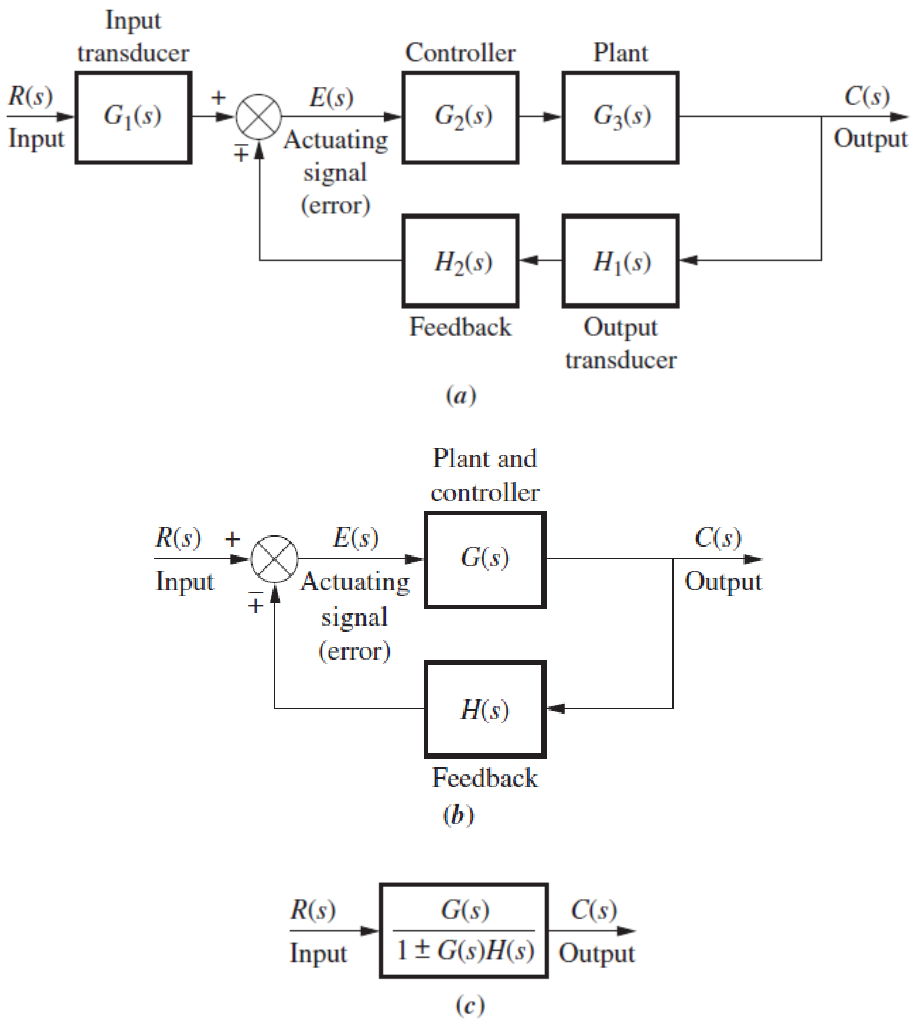


Figure 15: Feedback structure simplification with block manipulation: a. initial structure; b. simplified structure; and c. a single block.

Notice the $\pm$ sign in the denominator. For positive feedback the sign is (–) negative, and for negative feedback, the sign is (+) positive.

4. Block Diagram Reduction

There is a heuristic guideline that can be used to perform simplification of the block diagram. The steps for solving block diagram reduction problems are:

- Rule 1 – Check for the blocks connected in series and simplify.
- Rule 2 – Check for the blocks connected in parallel and simplify.
- Rule 3 – Check for the blocks connected in feedback loop and simplify.

- Rule 4 – If there is difficulty with take-off point while simplifying, shift it towards right.
- Rule 5 – If there is difficulty with summing point while simplifying, shift it towards left.
- Rule 6 – Repeat the above steps till you get the simplified form, i.e., single block.

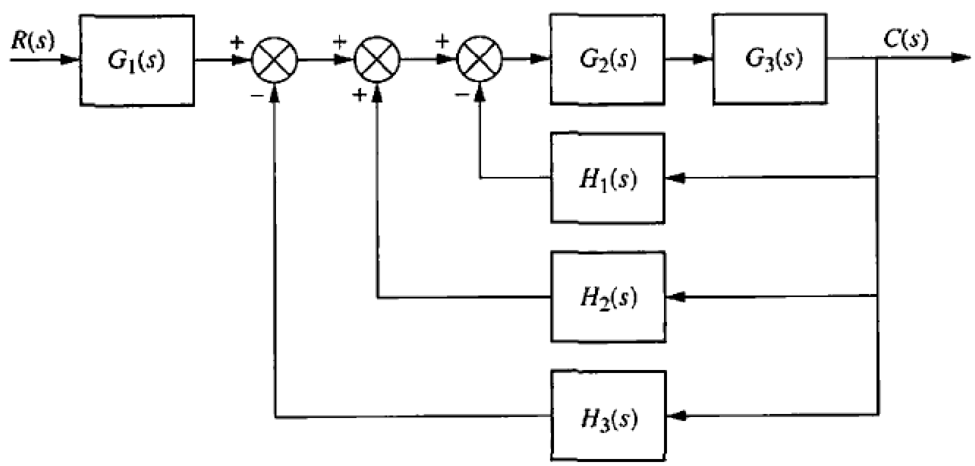
4.1 Reduction via Familiar Forms

Some of manipulation and reduction via familiar forms in the block diagram are:

- Blocks connected in series.
- Block connected in parallel.
- Block connected in feedback loop.

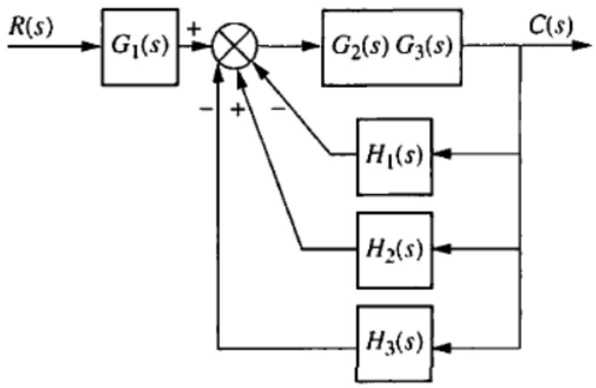
Example for Tutorial 1: Familiar Form Reduction of Block Diagram

By reduction via familiar forms approach, we can reduce the block diagram given below into a simpler form. [8 marks]



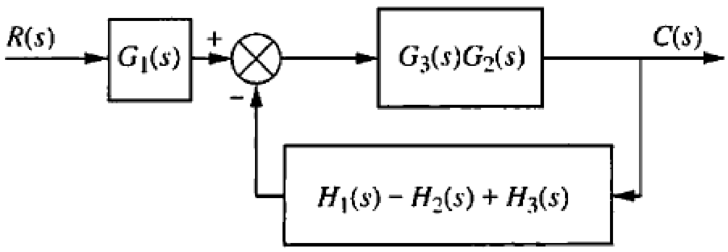
Answer

Combine G_2 and G_3 blocks in the forward path as shown in the figure below.

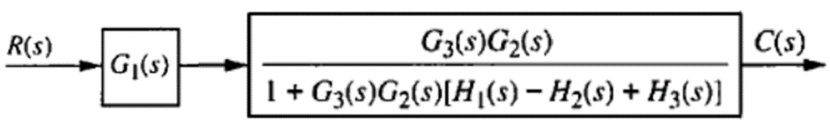


From the block diagram given above, combine the feedback path blocks (H_1, H_2, and H_3) into a single summing junction as they are connected to the same summing junction as shown in the figure below.

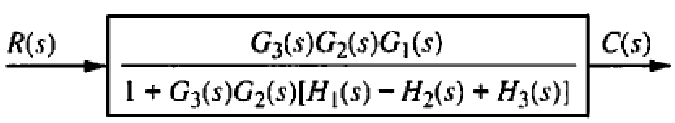
Then, we could solve all parallel feedback paths blocks (H_1, H_2, and H_3).



Furthermore, we can solve feedback path block ($H_1 - H_2 + H_3$) with forward path block (G_3G_2).



Finally, we can combine the result of feedback path block and forward path block with G_1 block in series.



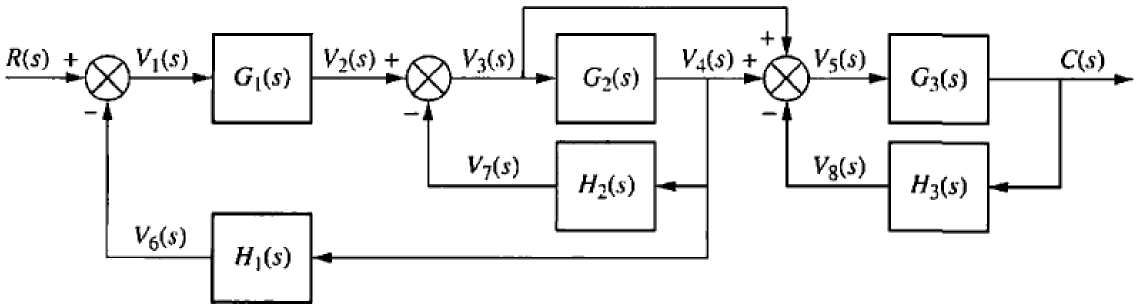
4.2. Reduction by Moving Blocks

Two most common reduction by moving blocks in the block diagram are:

- If there is difficulty with take-off point while simplifying, shift it towards right.
- If there is difficulty with summing point while simplifying, shift it towards left.

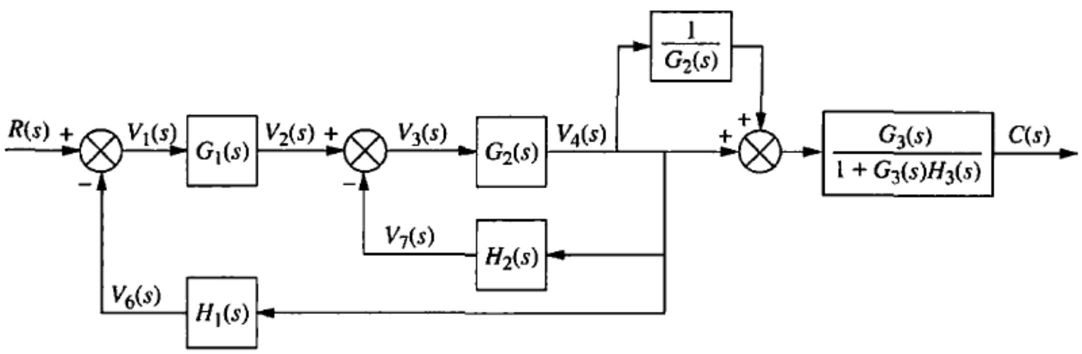
Example for Tutorial 2: Moving Block Reduction of Block Diagram

Referring to the block diagram given below, we can reduce the block diagram below into a simpler form. [10 marks]

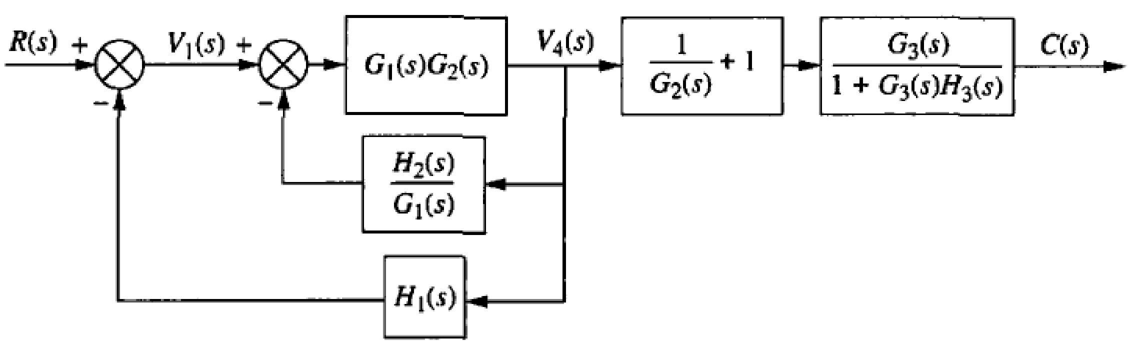


Answer

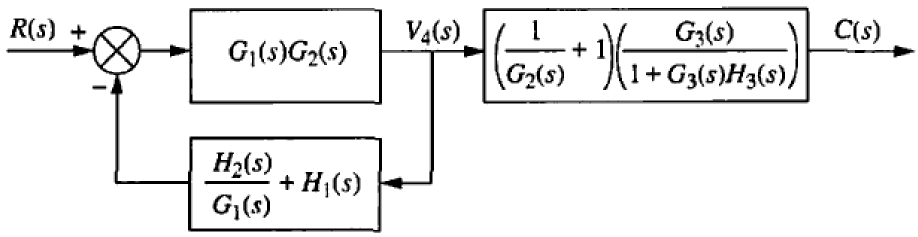
This time we reduce the block diagram by moving the blocks approach. First, we move the take-off point from the left-hand side of G_2 block to its right-hand side. Furthermore, solve the feedback structure that has G_3 and H_3 too.



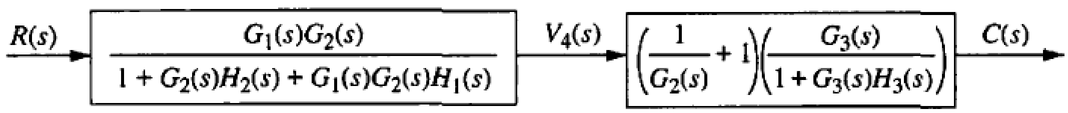
As shown in the figure below, we then sum up the moved block ($1/G_2$). Then, move ($V_2 - V_3$) summing junction from right of G_1 block to its left and combine blocks G_1 with G_2 which are in series.



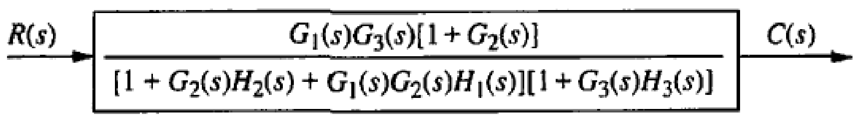
Furthermore, we sum up feedback path (H_2/G_1) and block H_1 into a single block and combine the two blocks on the right-hand side part of the forward path as shown below.



As illustrated in the figure below, we solve the feedback loop on the left-hand side of the block diagram.



In the end, we combine the resulting two blocks in the forward path that are in series as shown in the figure below.



5. Block Diagram and Physical Modelling of Systems

You can combine block diagram with physical modelling of the systems from their entities.

The block diagram is perceived as one of de facto standards for modelling of the systems in control system engineering.

As a result, their combinations would result in a much efficient and standardised process of modelling of the systems.

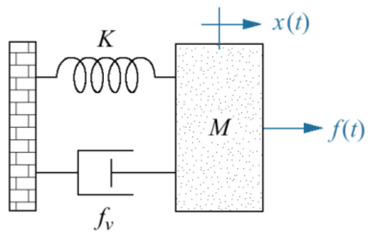
Steps for using the block diagram with the physical modelling of the system are:

1. Develop physical model of the system.
2. Derive mathematical solution.
3. Form transfer function.
4. Convert to block diagram.
5. Combine block diagram.
6. Reduce block diagram.

The following section covers some case study examples combining the block diagram with physical modelling of the systems.

Example for Tutorial 3: Block Diagram of Mechanical System

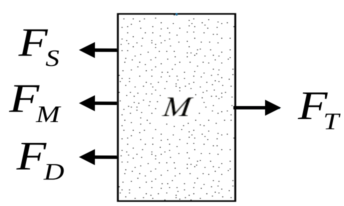
For a mechanical system as shown in the figure below, how do we determine the block diagram for each component? [20 marks]



Answer

Develop Physical Model of the System:

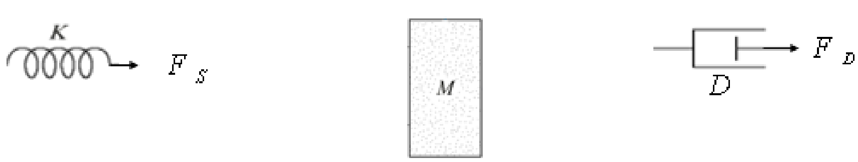
The following figure shows a physical model of the example mechanical system given above.



The physical model of the given example of mechanical system shows the interactions between the forces in the system as illustrated in the diagram above. Notice that all the components in the system e.g. spring, mass, and damper exerted forces (e.g. these are F_S, F_M and F_D respectively) opposing the force applied to the system (F_T).

Derive Mathematical Solution:

Given a physical system, we could then form mathematical solution for the mass, spring, and damper system. As illustrated in the figure below, each of the components of physical system has its relevant mathematical formulae.



$$f(t) = Kx(t) \qquad f(t) = M\frac{d^2x(t)}{dt^2} \qquad f(t) = D\frac{dx(t)}{dt}$$

Notice that the differential equation for determining the force in the spring subsystem is:

$$f_S(t) = kx(t)$$

Where: k is spring constant and $x(t)$ is the displacement.

For the mass system, the following differential equation is used to represent the force acting in this system:

$$f_M(t) = Ma(t)$$

Or

$$f_M(t) = M\left[\frac{d^2x(t)}{dt^2}\right]$$

Where: M is mass of the system and $d^2x(t)/dt^2$ is the acceleration of the second derivative of displacement.

The differential equation for the force in the damper system is given as follows:

$$f_D(t) = Dv(t)$$

Or

$$f_D(t) = D\left[\frac{dx(t)}{dt}\right]$$

Where: D is spring constant and $dx(t)/dt$ is the velocity or the first derivative of the displacement.

Then, the mathematical expressions given below form relationships between parts (from model):

$$f(t) = f_S(t) + f_D(t) + f_M(t)$$

Finally, substituting the differential equation, the equation above becomes:

$$f(t) = kx(t) + D\left[\frac{dx(t)}{dt}\right] + M\left[\frac{d^2x(t)}{dt^2}\right]$$

Form Transfer Function:

Applying the Laplace transform to the differential equation of the given mechanical system.

$$F(s) = kX(s) + DsX(s) + Ms^2X(s)$$

Rearranging the equation given above, we determine transfer function of the system:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Ds + k}$$

Think how the system responds:

$$X(s) = \frac{F(s)}{Ms^2 + Ds + k}$$

For the above given system, referring to the transfer function equation given above, we can determine the relationships among the parameters of the system:

- Larger force results in larger distance.
- Larger mass, spring stiffness or damping results in smaller distance.

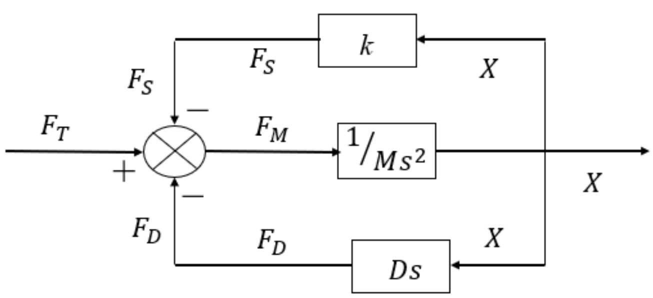
Convert to Block Diagram:

For realising the block diagram for the given mechanical system, we combine and rearrange components together:

$$F_M(s) = F_T(s) - F_S(s) - F_D(s)$$

Where: $F_S(s) = kX(s)$, $F_M(s) = Ms^2X(s)$, $F_D(s) = DsX(s)$, and $F_T(s)$.

As a result, the block diagram of the given mechanical system is as shown in the figure below.



Combine Block Diagram:

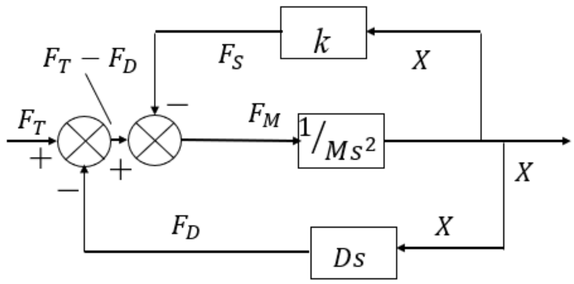
Then, we gather components together to form system, the equation of the system becomes:

$$Ms^2X(s) = F_T(s) - kX(s) - DsX(s)$$

Reduce Paths of Block Diagram:

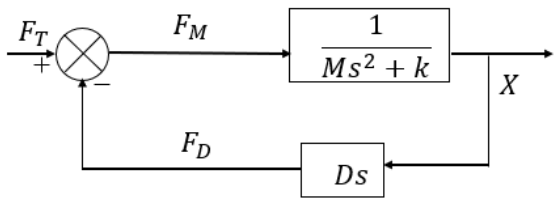
Furthermore, we can simplify paths of the block diagram. The equations of the system become:

- $F_M(s) = Ms^2X(s)$ and $F_S(s) = kX(s)$ in top feedback system.
- $F_T(s)$, $F_M(s) = Ms^2X(s)$ and $F_D(s) = DsX(s)$ in bottom feedback system.

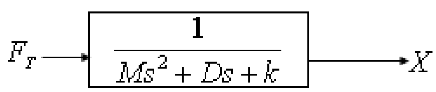


Reduce Block Diagram Further:

Reducing the block diagram further, we can simplify more the two parts that make up the block diagram (e.g. solve $F_M(s)$ which is formed from $F_M(s)$ and $F_S(s)$ in feedback):

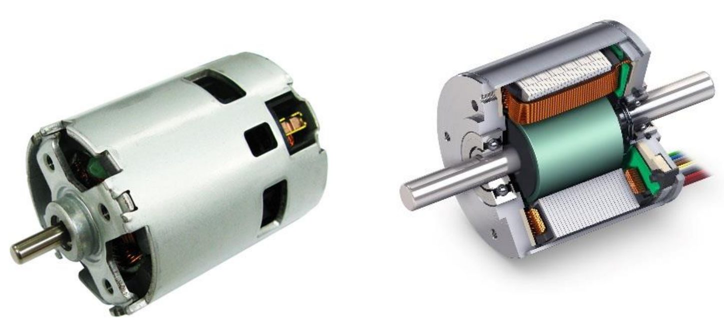


The simplest block diagram (e.g. solve from $F_M(s)$ and $F_D(s)$ in feedback):



Example for Tutorial 4: Block Diagram of Electromechanical System

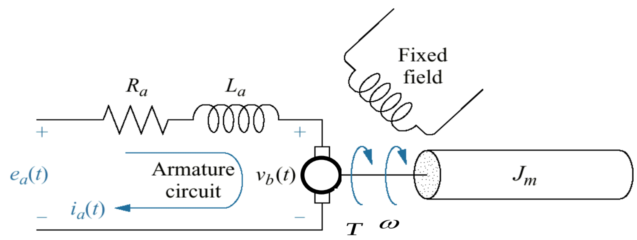
For a given electromechanical system given as a brushless direct current (DC) motor as shown below, derive a model the system. [40 marks]



Answer

Develop Physical Model of the Systems:

Given in the following figure is a schematic diagram of a brushless DC motor that outlines the typical components that make up electrical and mechanical components of the motor.



For the electrical components, there are armature circuit and fixed field components. The armature circuit consists of armature resistance (R_a) that is in series with armature inductance (L_a) and back EMF of the motor ($v_b(t)$).

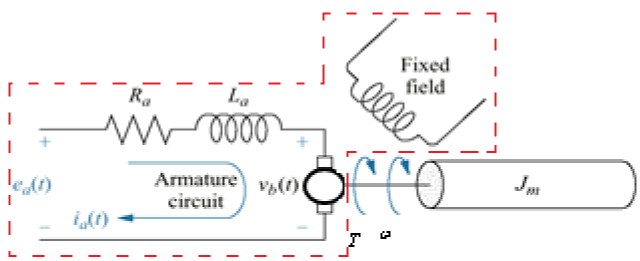
The winding in the fixed field circuit produces electromagnetic flux that will interact with flux generated by the at the armature that is energised by armature voltage ($e_a(t)$) producing torque (T). In most model of the DC motor, we tend not to include the fixed field winding circuit in the modelling, except if it is for analysis of power loss or torque calculation.

For mechanical components, the torque generated (T) is used to turn the motor shaft (i.e. overcoming its inertia (J_m) and also load if it is connected) at specified angular velocity ($\omega(t)$).

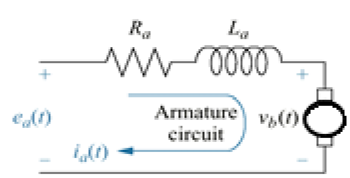
As a result, we need to form a relationship between input voltage (e.g. $e_a(t)$) and output velocity ($\omega(t)$).

Determine Electrical Components:

The following figure shows the main electrical components of a brushless DC motor.



Notice the three elements in the armature circuit e.g. armature inductance, armature resistor and back EMF of the motor.



Now, include armature inductance:

$$v_L(t) = L_a\left[\frac{di_a(t)}{dt}\right]$$

Armature resistor:

$$v_R(t) = i_a(t)R_a$$

Back EMF of motor:

$$v_b(t) = K_e\omega(t)$$

Derive Equation of Electrical System:

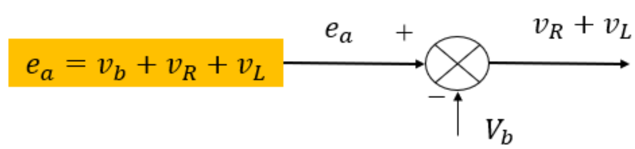
Apply KVL to the armature circuit:

$$e_a(t) = v_R(t) + v_L(t) + v_b(t)$$

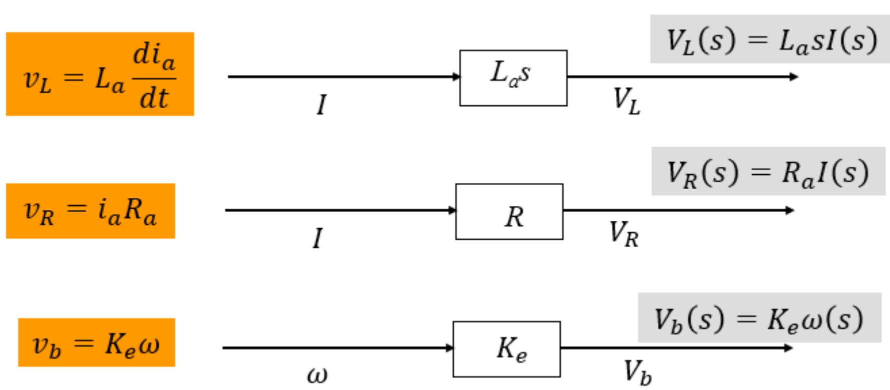
Arrange the equation above:

$$e_a(t) - v_b(t) = v_R(t) + v_L(t)$$

The following figure shows the block diagram of the armature circuit components of brushless DC motor.



Apply Laplace transform and gather electrical components to block diagram. The following figure outlines main electrical components of brushless DC motor in block diagram.

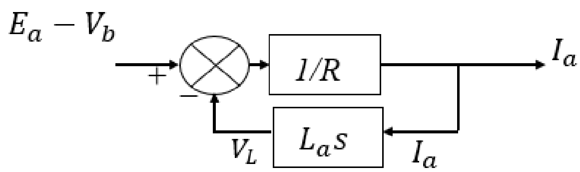


Model Electrical System in Block Diagram:

Represent the electrical component as feedback (e.g. $V_L(s)$ is in feedback loop):

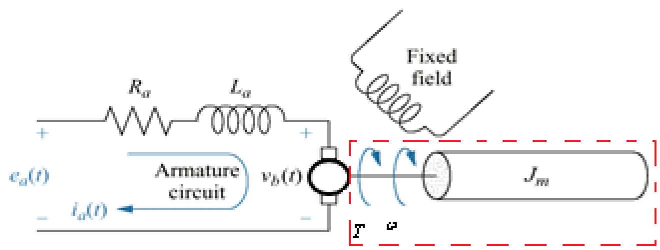
$$E_a(s) - V_b(s) - V_L(s) = V_R(s)$$

The figure given below show the final block diagram of the main electrical components in the brushless DC motor.

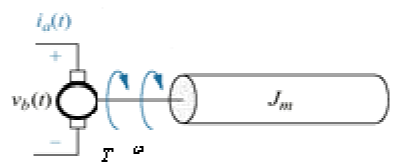


Determine Mechanical Components:

The diagram given below shows the main mechanical components of the DC motor system.



Notice how the mechanical components of the motor that is developed torque in the armature and opposing torque due to inertia (J_m) and the load of motor (not shown in the diagram).



Torque proportional to armature current:

$$T(t) = K_T i_a(t)$$

Torque is opposed by the inertia torque:

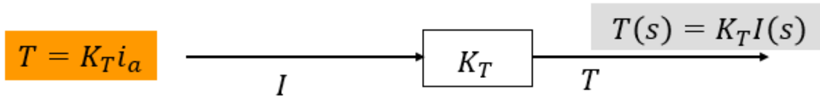
$$T(t) = J\left[\frac{d\omega(t)}{dt}\right]$$

Derive Equation of Mechanical System:

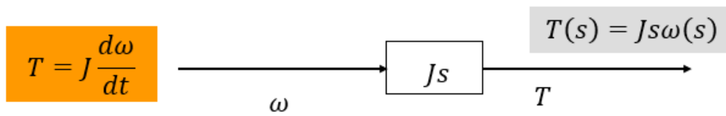
Using the energy conservation law, the amount of generated electrical energy is equal to the expended mechanical energy. The equation for the mechanical system is as derived as follows.

$$K_T i_a(t) = J\left[\frac{d\omega(t)}{dt}\right]$$

Apply Laplace transform and gather mechanical components to block diagram. The block diagram of the electromechanical system becomes as the figure given below. The figure below shows the block diagram of the torque and armature current in brushless DC motor.

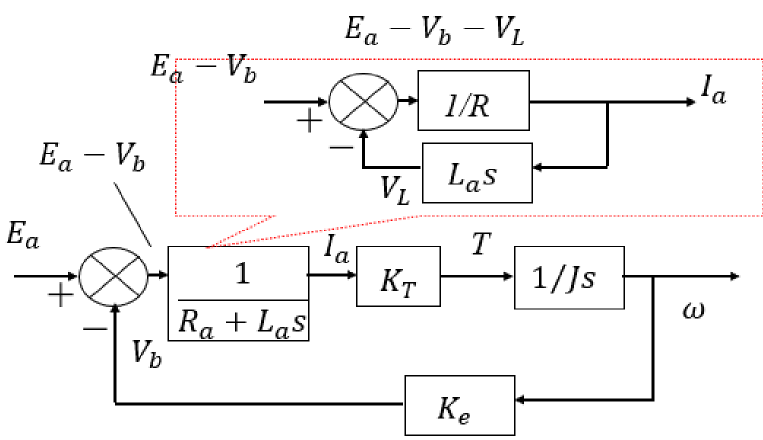


The following figure shows the block diagram of the torque and inertia in brushless DC motor.



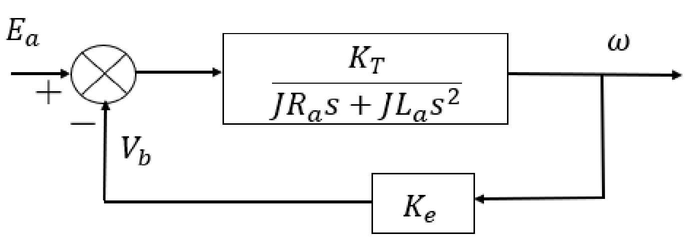
Add Mechanical System to Block Diagram:

Add variables of the mechanical system with the variables of the electrical system in the block diagram. The block diagram ends up becoming as shown in the figure below. It shows the block diagram of both the mechanical system and the electrical system in brushless DC motor.



Reduce Block Diagram:

For the given block diagram of overall brushless DC motor system as shown below, simplify the block diagram.



The figure above shows the overall block diagram of brushless DC motor system.

Reduce the block diagram of the given electromechanical system into a single block diagram of brushless DC motor system.

$$E_a \longrightarrow \boxed{\dfrac{1/K_e}{\dfrac{L_a J}{K_e K_t} s^2 + \dfrac{R_a J}{K_e K_t} s + 1}} \longrightarrow \omega$$