Demo 1b: Modelling of Physical Systems

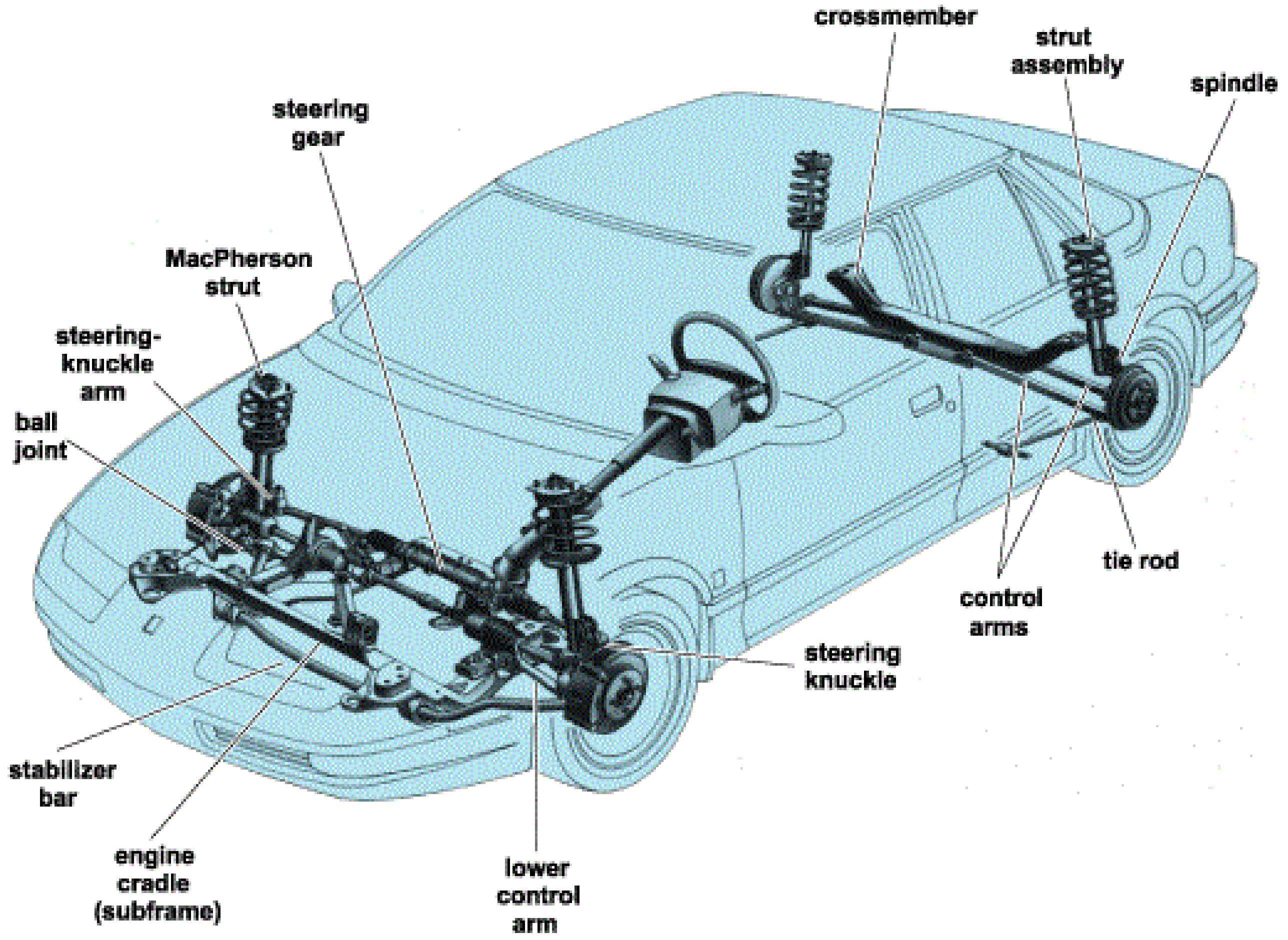
XMUT315 Control Systems Engineering

Topic

- Modelling of a simple control system.
- Modelling of a complex control system.

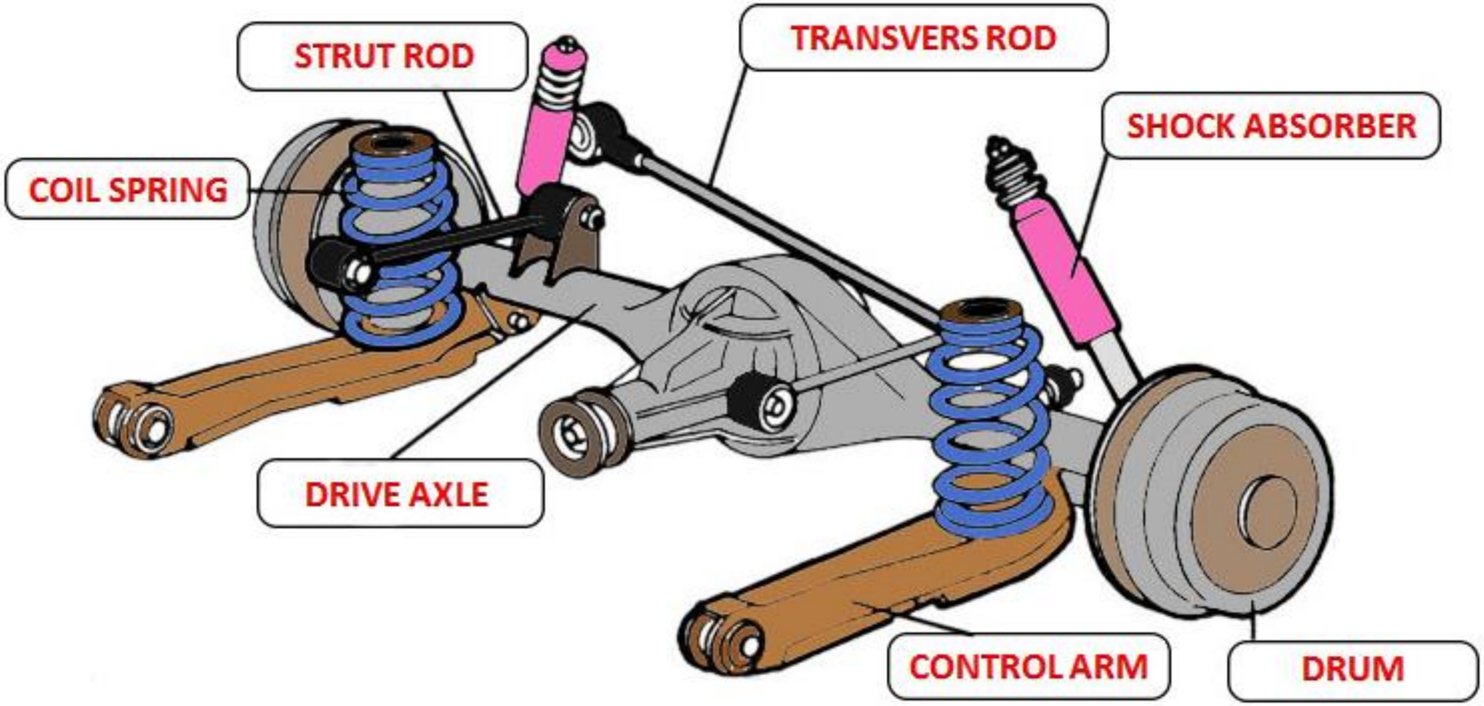
Modelling a Simple Control System

- Car suspension system in the car



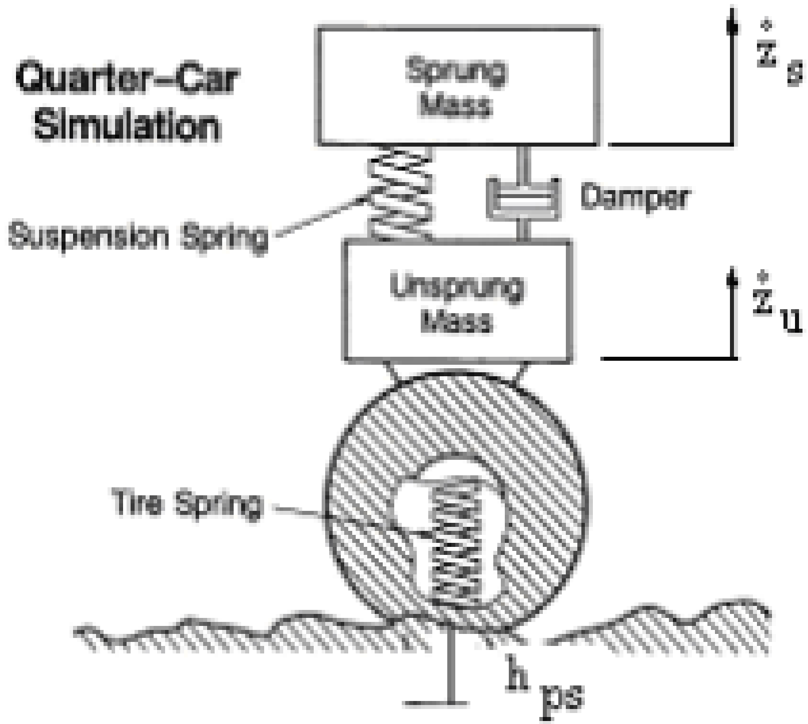
Mechanical System Modelling

- Car suspension system



Mechanical System Modelling

- Explain what happens when a car goes over a bump?



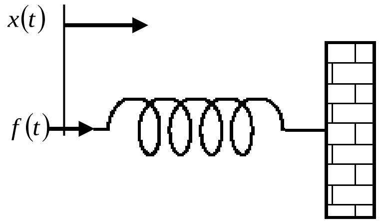
Mechanical System Modelling

- Explain what happens when *a different damper is installed in the system*?
- Simplify to single-input-single-output system.
- Form individual component models.
- Determine their relationships (use physical laws).
- Combine (and simplify if possible).
- This gives us an instantaneous differential equation, but want a time response.

Mechanical System Modelling

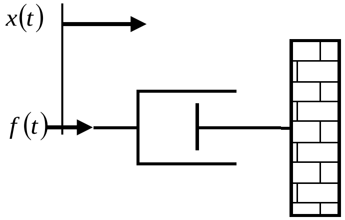
Force - Distance

- Spring



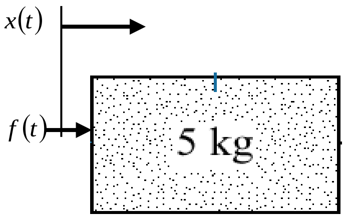
$$f_s(t) = Kx(t)$$

- Damper



$$f_c(t) = C\frac{dx(t)}{dt}$$

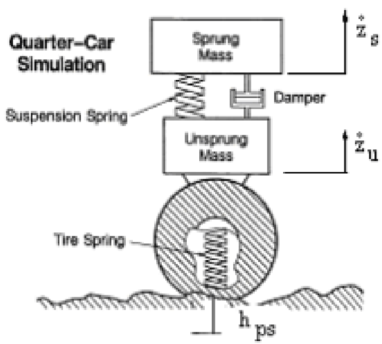
- Mass

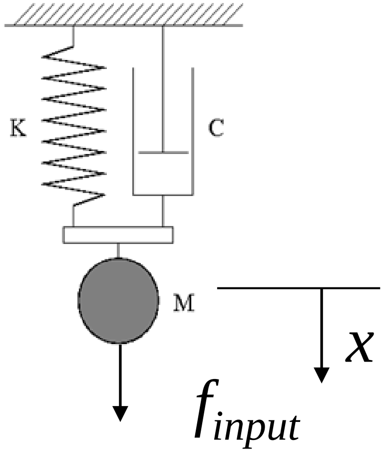


$$f_m(t) = M\frac{d^2x(t)}{dt^2}$$

Mechanical System Modelling

- Physical model and component model of the car suspension system

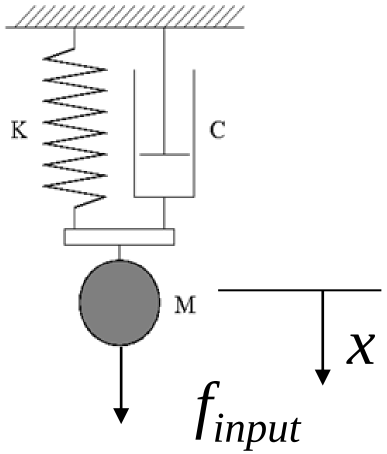




Physical model

Component model

Mechanical System Modelling



Parts of the system:

K = Spring constant
C = Damper coefficient
M = Mass
X = Displacement
F_{input} = Applied force

- Applying Newton second law:

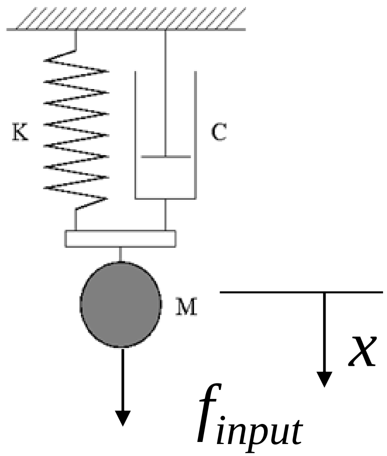
$$F(applied) = F(reaction)$$

- Hence:

$$F(input) = F(mass) + F(spring) + F(damper)$$

Mechanical System Modelling

$$f_{input}(t) = f_s(t) + f_d(t) + f_m(t)$$



- Forces acting in the spring:

$$f_s(t) = Kx(t)$$

- Forces acting in the damper:

$$f_d(t) = C\left(\frac{dx(t)}{dt}\right)$$

- Forces acting in the mass:

$$f_m(t) = M\left(\frac{d^2x(t)}{dt^2}\right)$$

Mechanical System Modelling

- Thus, we have the following equation:

$$f_{input}(t) = Kx(t) + C\left(\frac{dx(t)}{dt}\right) + M\left(\frac{d^2x(t)}{dt^2}\right)$$

- This gives us an instantaneous differential equation.
- But, we want a time response to evaluate the characteristic and behaviour of the system!
- Integrate: numerically, theoretically, or using tables.

Matlab Simulation

MATLAB Code:

```
% Declaration of parameters and assignment of
values into the parameters.

clf; % clear all graphs

K = 10 % Spring constant
C = 3 % Damping constant
m = 1 % Mass (constant)

T = [0: 0.01: 20];% set up the time increments

stept = 1 + 0*T; % graph to show step response
```

Matlab Simulation

```
% Plot and labelling of the graph

plot(t,stept,'m');

xlabel('Time t (s)')

ylabel('Distance x (m)')

hold on % put each graph on top of each other
```

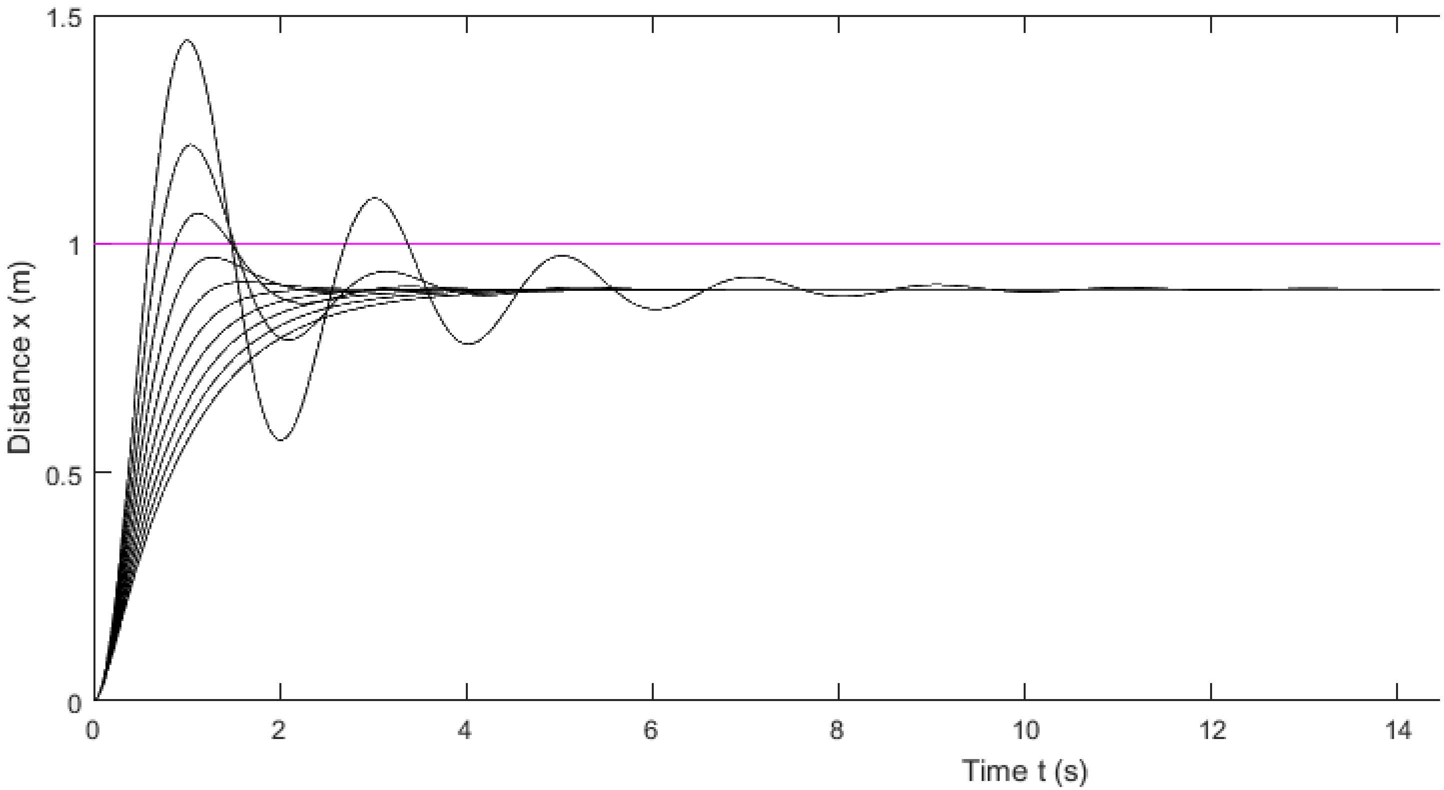
Matlab Simulation

```
% Plot of the graph

for C = 1.0: 1: 10.0
d = tf(9,[m C K])
[y,t]=step(d,T);% step response over 1 second
plot(t,y,'k');
pause(2)
end
```

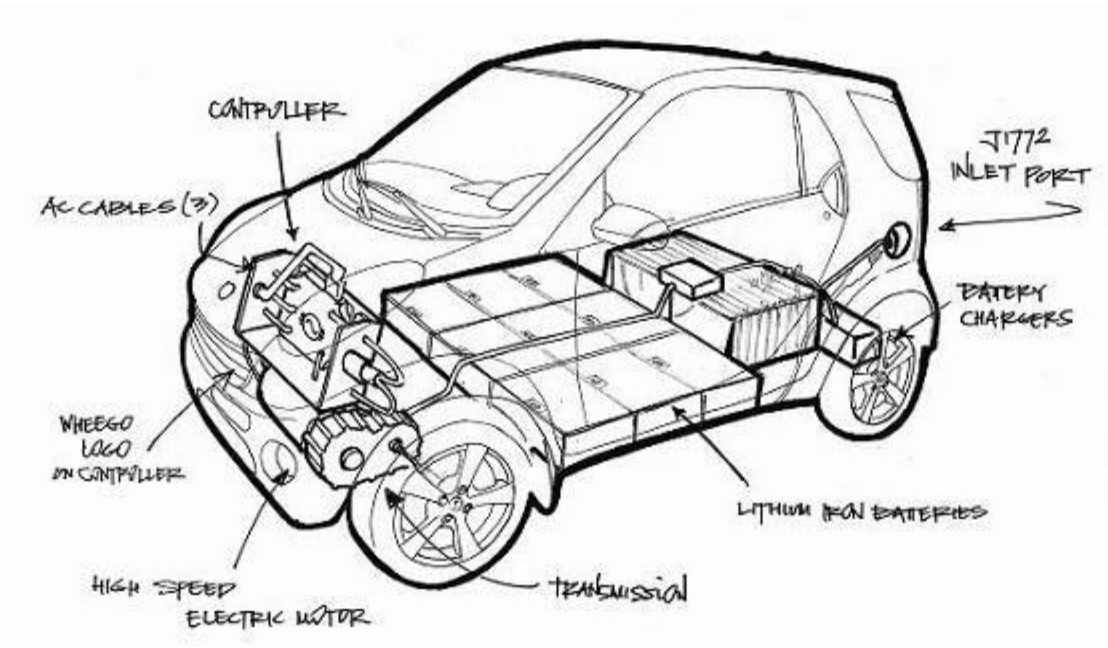
Matlab Simulation

- Graphs as results of the simulation



Modelling a Complex Control System

- Electrical motor in a given electric vehicle car: electromechanical system

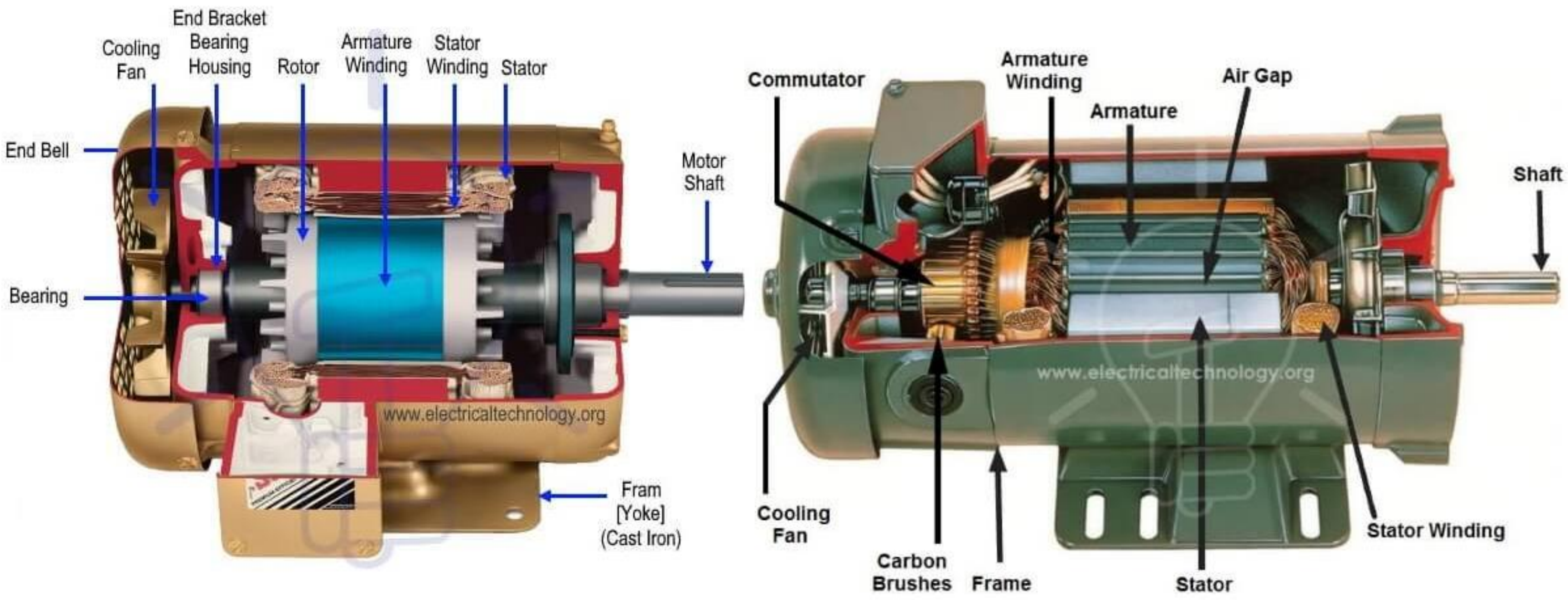


Electromechanical Systems

- Examples of electromechanical systems are DC motor (left) and AC motor (right):

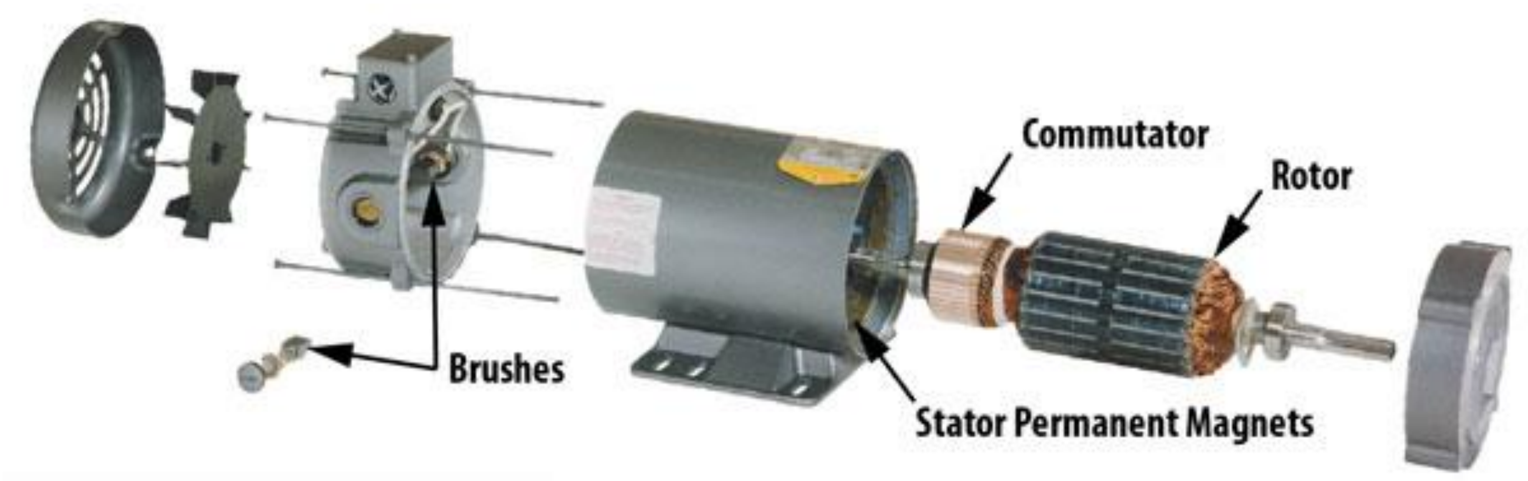
Electromechanical Systems

- Parts of DC motor (left) and AC motor (right):



Electromechanical Systems

- (Loose) parts of the DC motor:



Electromechanical Systems

Steps for modelling the electromechanical system from its physical system:

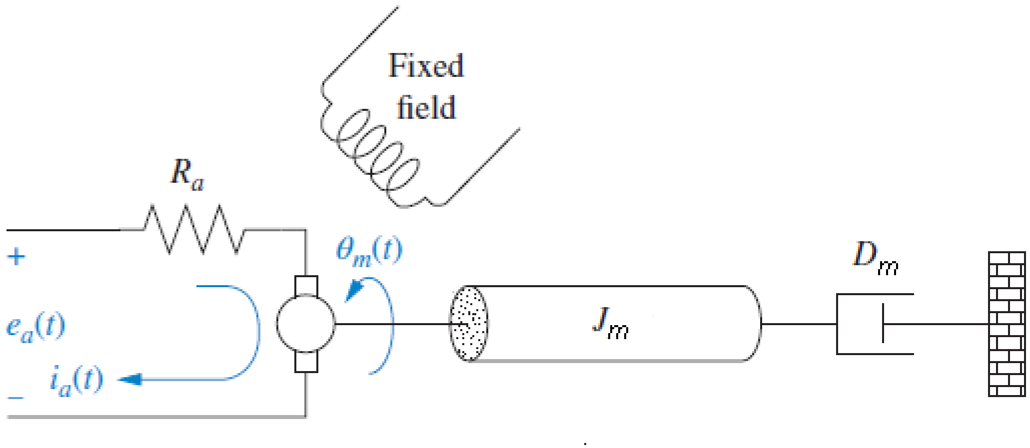
- Explain what happens *when a step input is applied to the system*.
- Start working with the electrical or mechanical system first.
- Simplify to single-input-single-output system.
- Form individual component models.

Electromechanical Systems

- Determine their relationships (use physics/electrical circuit laws).
- Combine (and simplify if possible).
- This gives us an instantaneous differential equation but want a time response.
- Repeat the above steps for the mechanical or electrical system.

Electromechanical Systems

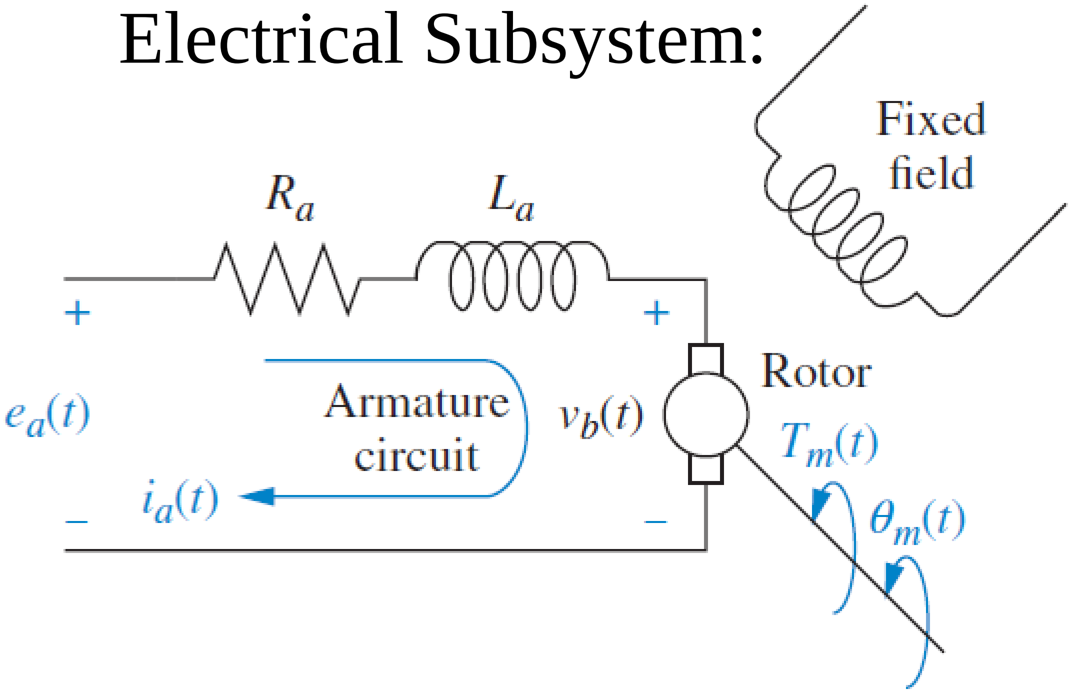
- For a given example of electromechanical system i.e. a DC motor system.

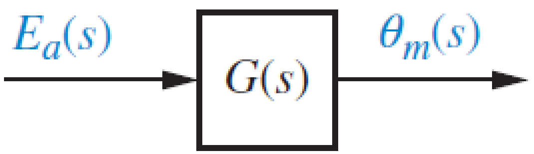


- We evaluate the system in terms of its electrical and mechanical subsystems.

Electromechanical Systems

Electrical Subsystem:





R_a = Armature resistance

L_a = Armature inductance

$V_b(t)$ = Back EMF

$E_a(t)$ = Voltage supply

Electromechanical Systems

- Electrical subsystem:

$$R_a I_a(t) + L_a \frac{dI_a(t)}{dt} + V_b(t) = E_a(t) \qquad (1)$$

- Back EMF coupling:

$$V_b = K_b \frac{d\theta_m(t)}{dt} \qquad (2)$$

Electromechanical Systems

- Substituting $V_b(t)$ in equation (1) with equation (2) and applying Laplace transform, this gives:

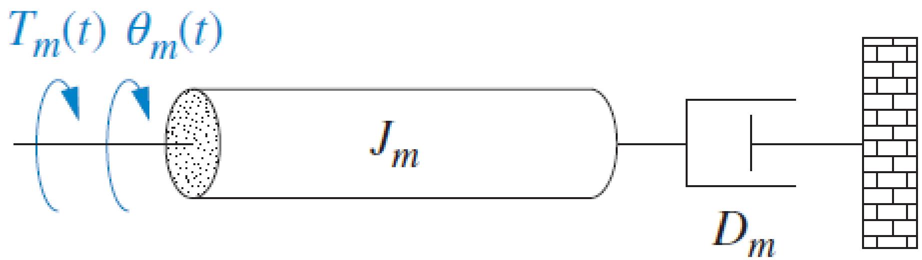
$$(R_a + L_a s)I_a(s) = E_a(s) - K_b s\theta_m(s)$$

- Or

$$I_a(s) = \frac{E_a(s) - K_b s\theta_m(s)}{(R_a + L_a s)} \qquad (3)$$

Electromechanical Systems

Mechanical Subsystem:



J_m = Inertia of motor shaft

D_m = Damping of motor shaft

$T_m(t)$ = Applied torque

$\theta_m(t)$ = Angular displacement

Note: $\omega_m(t) = d\theta_m(t)/dt$ = Angular speed

Electromechanical Systems

- Mechanical subsystem:

$$T_m(t) = J_m \frac{d^2\theta_m(t)}{dt^2} + D_m \frac{d\theta_m(t)}{dt} \qquad (4)$$

- Torque coupling:

$$T_m(t) = K_t I_a(t) \qquad (5)$$

- Substituting T_m in equation (4) with equation (5) and apply Laplace transform, this gives:

$$s(J_m s + D_m)\theta_m(s) = K_t I_a(s) \qquad (6)$$

Electromechanical Systems

Overall Electromechanical System:

- By considering both equations (3) and (6), we can eliminate $I_a(s)$.

$$s(J_m s + D_m)\theta_m(s) = K_t\left[\frac{E_a(s) - K_b s\theta_m(s)}{(R_a + L_a s)}\right]$$

- Rearrange the equation above:

$$(J_m s + D_m)(R_a + L_a s)s\theta_m(s) + K_t K_b s\theta_m(s) = K_t E_a(s)$$

Electromechanical Systems

- As a result, we can obtain the following open-loop transfer function where the rotational speed is the output and the voltage at the armature is the input:

$$\frac{\omega_m(s)}{E_a(s)} = \frac{K_t}{\big(J_m s + D_m\ (s)\big)(R_a + L_a s) + K_b K_t}$$

Note: $\omega_m(s) = s\theta_m(s)$

Electromechanical Systems

- For a DC motor circuits with the following specification: R_a = 2 Ω, L_a = 0.5 H, J_m = 0.02 kg-m^2, D_m = 0.2 N-m s/rad, K_t = 0.1 N-m-A, and K_b = 0.1 V-s/rad.

- Find the relationship $\omega_m(t)/E_a(t)$ by simulating the DC motor system in MATLAB.

$$\frac{\omega_m(s)}{E_a(s)} = \frac{0.1}{(0.02s + 0.2)(0.5s + 2) + (0.1)^2}$$
$$= \frac{0.1}{0.01s^2 + 0.14s + 0.41}$$

Matlab Simulation

MATLAB code:

```
s=tf('s');

% defining of parameters of simulation

Ra=2;
La=0.5;
Jm=0.02;
Dm=0.2;
Kt=0.1;
Kb=0.1;
```

Matlab Simulation

```
% transfer function of the system

G=Kt/[(Jm*s+Dm)*(Ra+La*s)+Kt*Kb];

% apply step function to the system
step(G);

title('Step Response for the Open Loop System');
```

Matlab Simulation

Graph of simulation results: