Analysis and Design with Nyquist Diagram

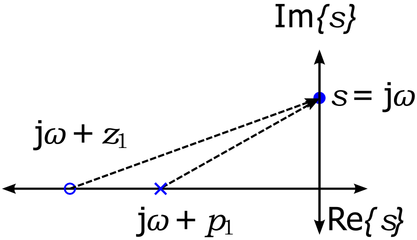
XMUT315 Control System Engineering

Topics

- Construction of Nyquist diagram.
- Poles on the complex axis.
- Gain and phase margins in Nyquist diagram.
- Stability in Nyquist diagram.
- Design with compensators in Nyquist diagram.
- Analysis with Nichols chart.
- Design procedures of control systems with Nyquist diagram and Nichols chart.

Construction of Nyquist Diagram

- We can draw a Nyquist diagram directly, without needing to draw a Bode plot first.
- We just consider the system response as we move around the required contour.



- Recall the magnitude and phase of frequency response.
- Magnitude:

$$|G(s)| = \frac{|K||s+z_1||s+z_2|\ldots|s+z_k|}{|s+p_1||s+p_2|\ldots|s+p_k|}$$

- Phase:

$$\angle G(s) = \angle(s+z_1) + \angle(s+z_2)\ldots + \angle(s+z_k)$$
$$-\angle(s+p_1) - \angle(s+p_2)\ldots - \angle(s+p_k)$$

Construction of Nyquist Diagram

- For example, consider the contribution of a single LHP pole at pole location of $-a$.

$$G(s) = \frac{1}{(s+a)}$$

- As we move from zero to infinite frequency, the phase will move from zero to −90°.

$$\angle G(j\omega) = \tan^{-1}\left(\frac{j\omega}{a}\right) = \theta°$$

- At the same time, the gain will drop.

$$|G(j\omega)| = \frac{1}{\sqrt{(j\omega)^2 + (a)^2}}$$

- We can therefore sketch the Nyquist diagram those results.

Example of Construction of Nyquist Diagram

Consider a first-order system with the transfer function:

$$G(s) = \frac{1}{s + 0.2}$$

a. Determine the equations for calculating magnitude and phase of the frequency response of the system.

[4 marks]

b. Calculate the magnitude and phase of the frequency response of the system for ω = 0, 0.2, and 1 rad/s.

[6 marks]

c. Sketch the Nyquist diagram based on the results obtained in part (b). [4 marks]

Example of Construction of Nyquist Diagram

- For the given first-order system with the transfer function:

$$G(s) = \frac{1}{s\ +\ 0.2}$$

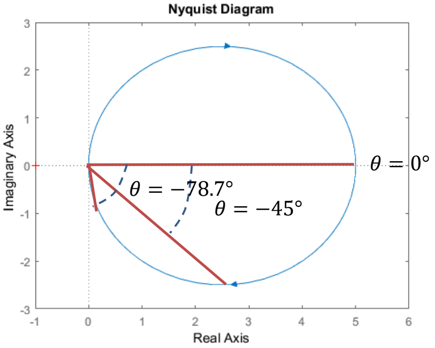
- Magnitude: $|G(j\omega)| = 1/\sqrt{(j\omega)^2 + (0.2)^2}$
- Phase: $\angle G(j\omega) = -\tan^{-1}(j\omega/0.2)$

$\boldsymbol{\omega}$	$\lvert \boldsymbol{G(s)} \rvert$	$\angle \boldsymbol{G(s)}$
0	$\frac{1}{\sqrt{(0)^2 + (0.2)^2}} = 5$	$-\tan^{-1}\left(\frac{0}{0.2}\right) = 0°$
0.2	$\frac{1}{\sqrt{(0.2)^2 + (0.2)^2}} = 3.5$	$-\tan^{-1}\left(\frac{0.2}{0.2}\right) = -45°$
1	$\frac{1}{\sqrt{(1)^2 + (0.2)^2}} = 0.98$	$-\tan^{-1}\left(\frac{1}{0.2}\right) = -78.7°$

Example of Construction of Nyquist Diagram

- The Nyquist diagram of the first-order system with transfer function:

$$G(s) = \frac{1}{s + 0.2}$$



Example of Construction of Nyquist Diagram

See the following second order system with transfer function:

$$G(s) = \frac{1}{(s + j + 2)(s - j + 2)}$$

a. Determine the equations for calculating magnitude and phase of the frequency response of the system.

[4 marks]

b. Calculate the magnitude and phase of the frequency response of the system for ω = 0, 1, and 10 rad/s.

[6 marks]

c. Sketch the Nyquist diagram based on the results obtained in part (b). [4 marks]

Example of Construction of Nyquist Diagram

- For the given second order system:

$$G(s) = \frac{1}{(s+j+2)(s-j+2)} = \frac{1}{s^2+4s+5}$$

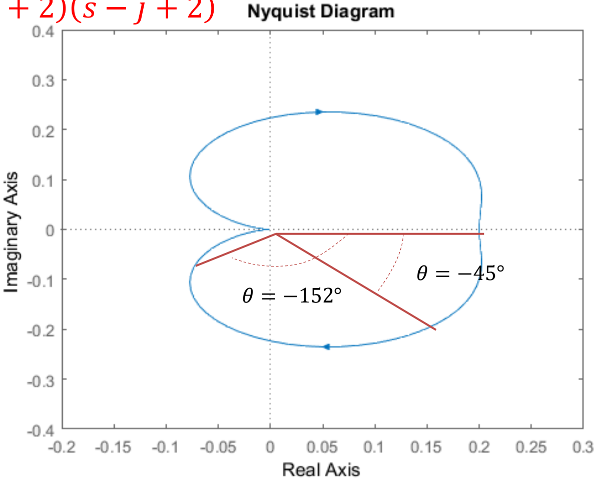
- Magnitude: $|G(j\omega)| = 1/\sqrt{(5-(j\omega)^2)^2+(4j\omega)^2}$
- Phase: $\angle G(j\omega) = -\tan^{-1}[4j\omega/(5-(j\omega)^2)]$

$\boldsymbol{\omega}$	$\lvert \boldsymbol{G(s)} \rvert$	$\angle \boldsymbol{G(s)}$
0	$\frac{1}{\sqrt{(5-0^2)^2+(0)^2}} = 0.2$	$-\tan^{-1}\left(\frac{0}{5}\right) = 0°$
1	$\frac{1}{\sqrt{(5-1^2)^2+(4)^2}} = 0.17$	$-\tan^{-1}\left(\frac{4}{4}\right) = -45°$
10	$\frac{1}{\sqrt{(5-10^2)^2+(40)^2}} = 0.097$	$-\tan^{-1}\left(\frac{40}{75}\right) = -152°$

Example of Construction of Nyquist Diagram

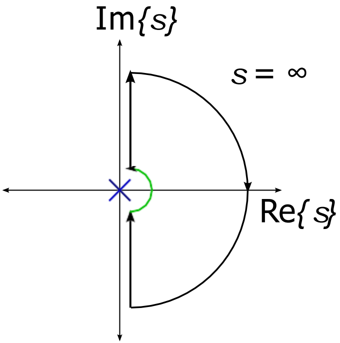
- The Nyquist diagram of the system with transfer function:

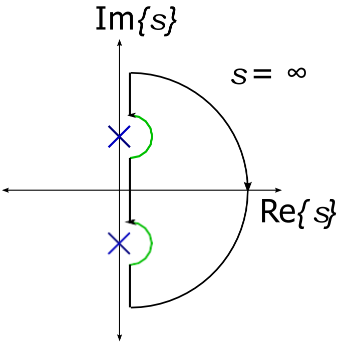
$$G(s) = \frac{1}{(s+j+2)(s-j+2)}$$



Poles on the Complex Axis

- Following the contour we discussed before does not work if we have poles (or zeros) that would lie on the contour.
- Thus, if we have poles of zeros on the imaginary axis, we modify the contour so that it takes an infinitesimally small “detour” around the imaginary root. We then proceed as normal.





Poles on the Complex Axis - Example

For example, consider a second-order system with transfer function:

$$G(s) = \frac{4}{s(s+1)}$$

a. Simulate the Nyquist diagram of the system in MATLAB.

[4 marks]

b. Determine the stability of the system by evaluating the encirclement at the test point (-1, 0).

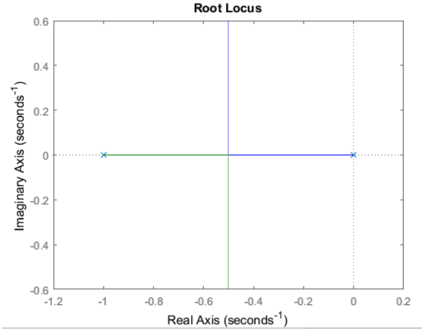
[4 marks]

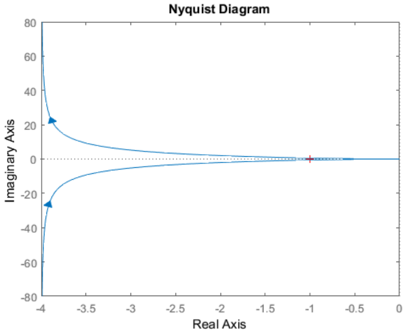
Poles on the Complex Axis - Example

Root locus and Nyquist diagrams of system with transfer function:

$$G(s) = \frac{4}{s(s+1)}$$

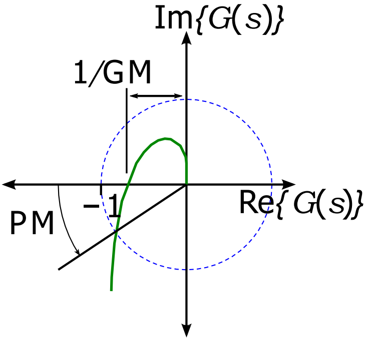
The Nyquist diagram of the above given system is shown in the figure below.





Margins on the Nyquist Diagram

- We can examine the Nyquist diagram to determine the gain and phase margins.
- The phase margin can simply be read as the difference between the phase = −180° line and the point where the curve crosses the unit circle.
- The gain margin is the inverse of the distance to the point where the curve crosses the negative real axis.



Note: that we can again have multiple gain and phase margins if the curve crosses the negative x-axis multiple times.

Stability from the Nyquist Diagram

- Nyquist showed mathematically that the number of poles in the right-half plane of the closed-loop transfer function can be determined by examining the Nyquist diagram of the open-loop transfer function.
- Number of closed-loop poles in right-half of s-plane:

  = Number of open-loop poles in right-half of s-plane

  + Number of clockwise encirclements of $-1 + 0j$.
- Remember that for the system to be stable we must have no closed-loop poles.
- If we have the transfer function of the system, we can easily determine the number of open-loop poles. So, if we use the Nyquist diagram to count the clockwise encirclements, we will be able to determine the closed-loop stability.

Counting Encirclements

- To count the clockwise encirclements:
  1. Draw a straight line from $-1 + 0j$ to ∞ in any direction.
  2. Count how many times the Nyquist diagram crosses from left-to-right over your chosen line. For each such crossing, add one to the your count of the number of encirclements.
  3. Count how many times the Nyquist diagram crosses from right-to-left over your chosen line. For each such crossing, subtract one from your count of total encirclements.
- If you have drawn a correctly constructed Nyquist diagram, then the final number that you get will be the same, regardless of which orientation you chose for your original line (so choose a direction that makes counting easy).

Example of Third-order System

Consider a third-order system:

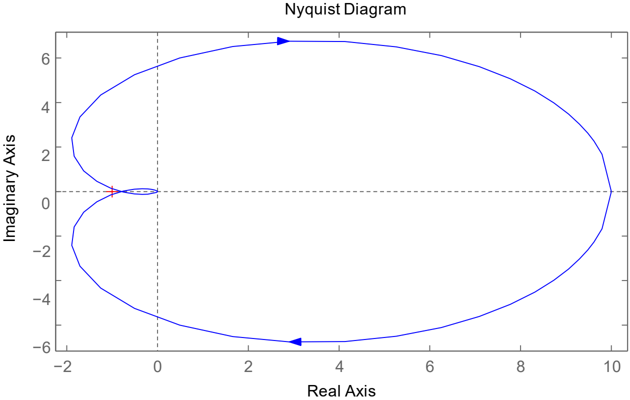
$$G(s) = \frac{100}{(s+1)(s+2)(s+5)}$$

a. Simulate the Nyquist diagram of the system in MATLAB. [4 marks]

b. Determine the stability of the system. [2 marks]

Example of Third-order System

- The Nyquist diagram is as shown in the figure below.



- This system will be closed-loop stable, but if we were to increase the gain, then it would eventually encircle $s = -1$ and become unstable.

Example of Conditionally Stable System

Consider a complex system given as the following transfer function equation:

$$G(s) = \frac{s^2 + 2s + 4}{s(s+4)(s+6)(s^2 + 1.4s + 1)}$$

a. Simulate the Bode plots of the system in MATLAB.

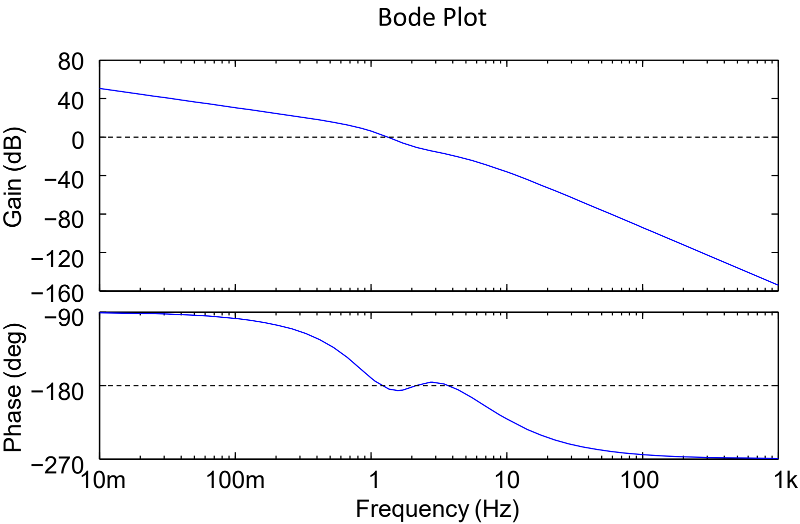
[4 marks]

b. Determine the stability of the system from results obtained in part (a). [2 marks]

c. Simulate the Nyquist diagram of the system in MATLAB. Comment on the difference using this method compared with Bode plots. [6 marks]

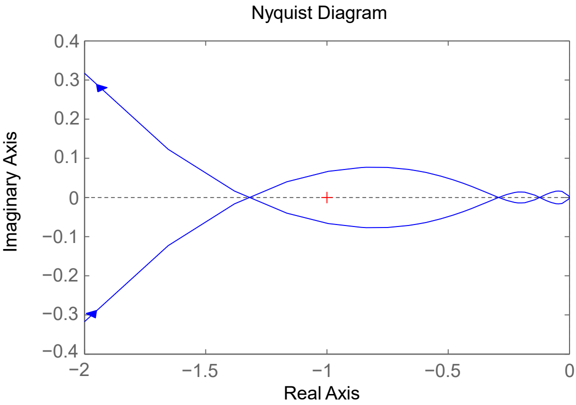
Example of Conditionally Stable System

- The Bode plots are as shown in the figure below. There are multiple crosses for determining the gain and phase margins.



Example of Conditionally Stable System

- Nyquist diagram of system $G(s) = \frac{s^2+2s+4}{s(s+4)(s+6)(s^2+1.4s+1)}$



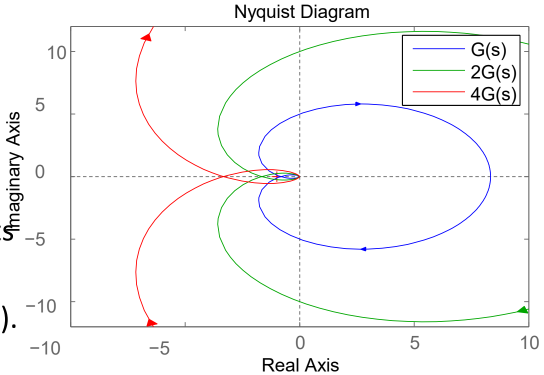
- For some gains, this will be stable, for others unstable. You cannot easily determine this from a Bode plot.

Proportional Compensators

- A proportional compensator simply scales the Nyquist contour.
- Consider a third-order system:

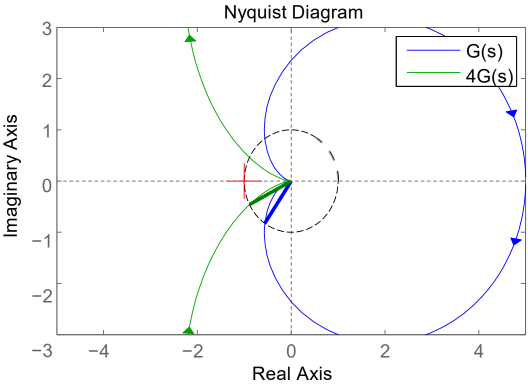
$$G(s) = \frac{50}{(s+1)(s+2)(s+3)}$$

- Notice that a too much proportional gain makes this system closed loop unstable.
- Increasing the gain results in the contour of Nyquist diagram to encircle (-1, 0).



Proportional Compensators

- Even if the system does not become unstable, the phase margin will typically still be reduced by increasing the gain.



- This example shows a second-order system with its phase margin is reduced when a proportional compensator with a gain of 4 is used.

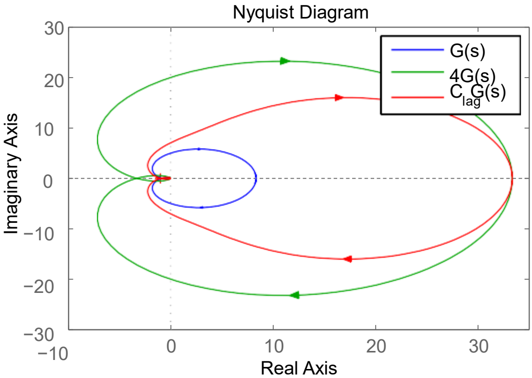
Lag Compensators

- A lag compensated system starts with a higher gain than the uncompensated system but approaches the Nyquist contour of the uncompensated system before it is near −1 + j0.
- Consider a lag compensator with transfer function:

$$C(s) = \frac{s + 0.4}{s + 0.1}$$

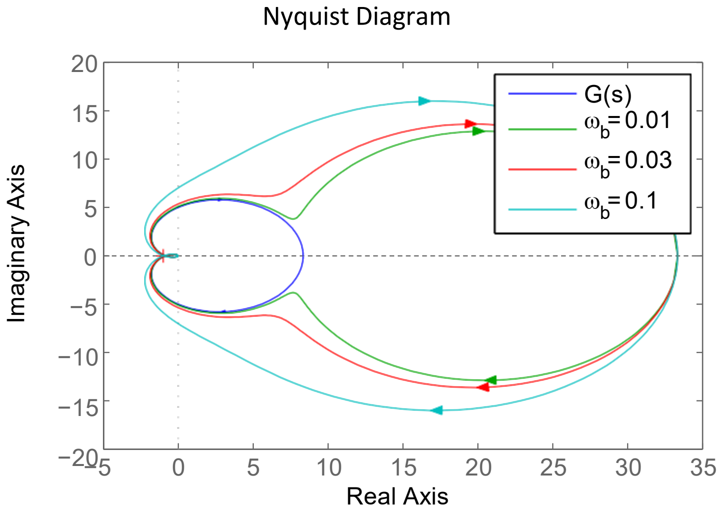
Where:

$\omega_b = 0.1$ and $\alpha = 4$



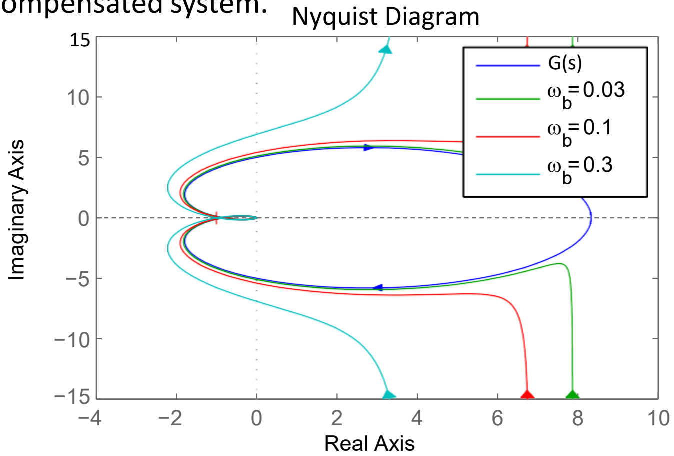
Lag Compensators - Effect of ω_b

- If ω_b is reduced, then the compensated system approaches the uncompensated more "quickly".



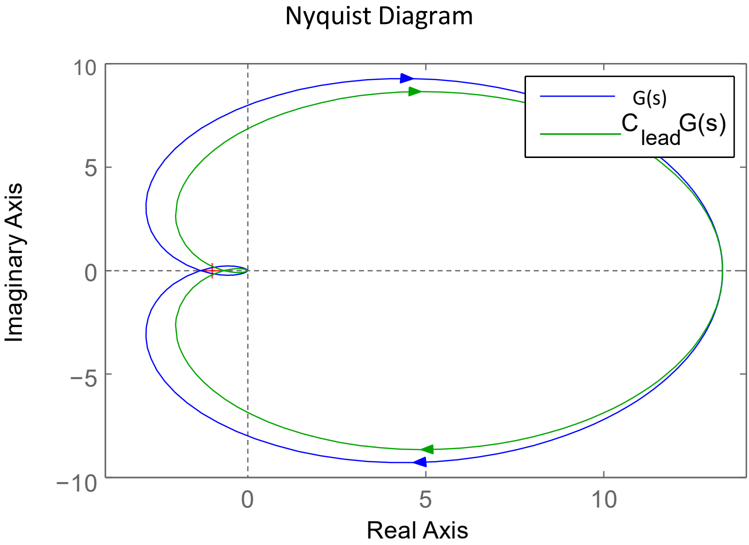
PI Controllers - Effect of ω_b

- A PI controller changes the shape of the Nyquist diagram at low frequencies.
- Again, ω_b determines how quickly the system approaches the uncompensated system.

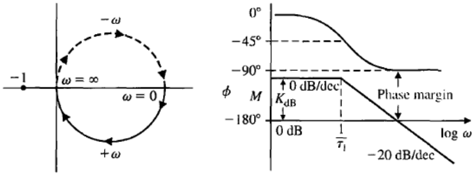


Lead Compensators

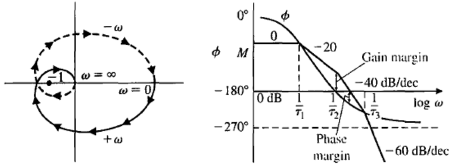
- The lead compensator "rotates" the Nyquist contour away from −1.



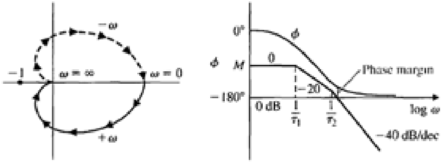
Other Examples of Nyquist Diagram



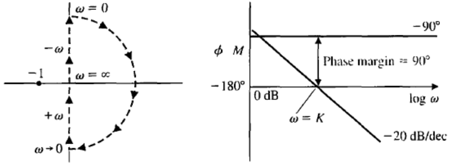
$$G1(s) = \frac{K}{s\tau_1 + 1}$$



$$G3(s) = \frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)}$$

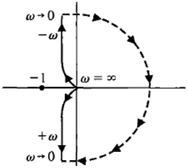


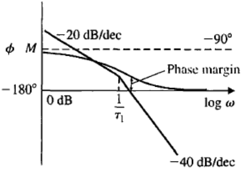
$$G2(s) = \frac{K}{(s\tau_1 + 1)(s\tau_2 + 1)}$$

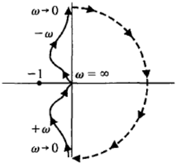


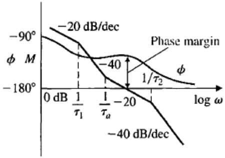
$$G4(s) = \frac{K}{s}$$

Other Examples of Nyquist Diagram



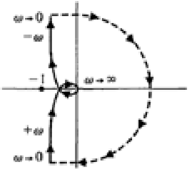


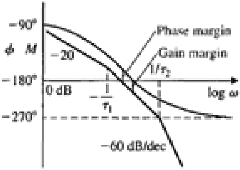


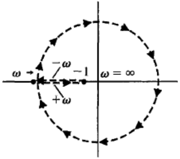


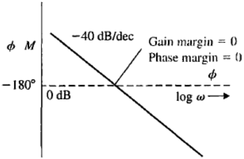
$$G5(s) = \frac{K}{s(s\tau_1 + 1)}$$

$$G7(s) = \frac{K(s\tau_a + 1)}{(s\tau_1 + 1)(s\tau_2 + 1)}$$





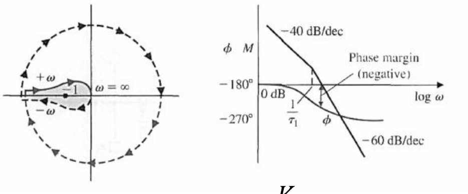




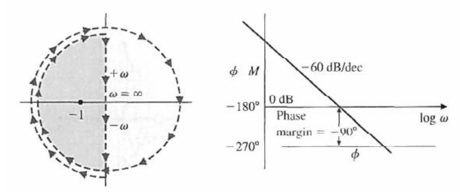
$$G6(s) = \frac{K}{s(s\tau_1 + 1)(s\tau_2 + 1)}$$

$$G8(s) = \frac{K}{s^2}$$

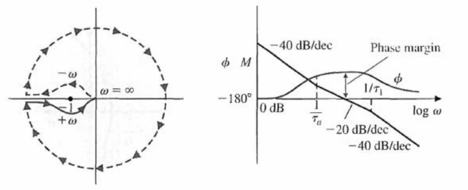
Other Examples of Nyquist Diagram



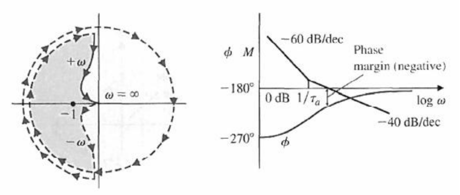
$$G9(s) = \frac{K}{s^2(s\tau_1 + 1)}$$



$$G11(s) = \frac{K}{s^3}$$



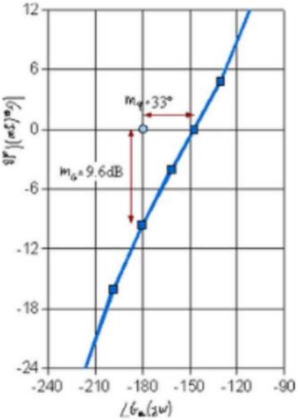
$$G10(s) = \frac{K(s\tau_a + 1)}{s^2(s\tau_1 + 1)} \quad (\tau_a > \tau_1)$$



$$G12(s) = \frac{K(s\tau_a + 1)}{s^3}$$

Analysis with Nichols Chart

- Like Nyquist diagram, it is used to analyse stability of control systems.
- Its graph is magnitude (in dB) vs. phase (in degrees) of the system's frequency response.
- Stability analysis of the system can be determined in terms of these parameters.
- Either evaluating the curve at test point (-180°, 0) or through determining the gain and phase margins of the system.



Example of Construction of Nichols Chart

For a system with the transfer function given below,

$$G(s) = \frac{1}{(s+2)}$$

a. Determine the equations for determining magnitude and phase shift of the frequency response. [4 marks]

b. Determine the magnitude and phase shift of the system for ω = 0, 1, 2, 5 and 10 rad/s. [10 marks]

c. Sketch the Nichols chart from the results obtained in part (b). [4 marks]

d. Simulate the Nichols chart in MATLAB. [5 marks]

e. Evaluate the frequency response of the system based on the results in part (d). [3 marks]

Example of Construction of Nichols Chart

a. For the given system with the transfer function given below

$$G(s) = \frac{1}{(s+2)}$$

Magnitude:

$$|G(j\omega)| = \frac{1}{\sqrt{(2)^2 + \omega^2}}$$

Phase shift:

$$\angle\theta = -\tan^{-1}\left(\frac{j\omega}{2}\right)$$

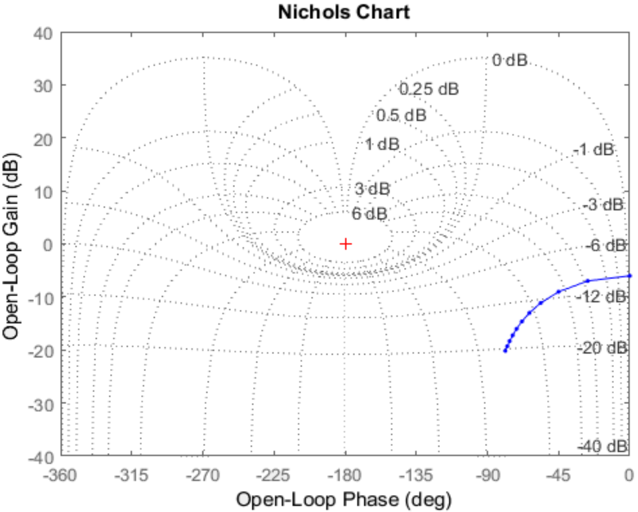
Example of Construction of Nichols Chart

b. The frequency range of the points to be plotted are selected from some frequencies between 0 to 10 rad/s.

Frequency (rad/s)	Magnitude (dB)	Phase Shift (degree)
0	$\frac{1}{\sqrt{(2)^2-(0)^2}} = 0.5 = -6\text{ dB}$	$-\tan^{-1}\left(\frac{0}{2}\right) = 0°$
1	$\frac{1}{\sqrt{(2)^2+(1)^2}} = 0.447 = -6.99\text{ dB}$	$-\tan^{-1}\left(\frac{1}{2}\right) = -26.56°$
2	$\frac{1}{\sqrt{(2)^2+(2)^2}} = 0.353 = -9.04\text{ dB}$	$-\tan^{-1}\left(\frac{2}{2}\right) = -45°$
5	$\frac{1}{\sqrt{(2)^2+(5)^2}} = 0.186 = -14.61\text{ dB}$	$-\tan^{-1}\left(\frac{5}{2}\right) = -68.2°$
10	$\frac{1}{\sqrt{(2)^2+(10)^2}} = 0.098 = -19.82\text{ dB}$	$-\tan^{-1}\left(\frac{10}{2}\right) = -78.69°$

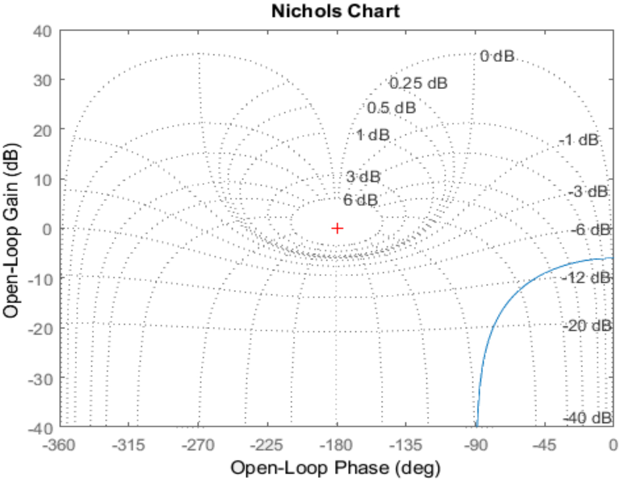
Example of Construction of Nichols Chart

c. The sketch of the Nichols chart is shown in the figure below.



Example of Construction of Nichols Chart

d. The result of the Nichols chart simulation as shown in the figure below.



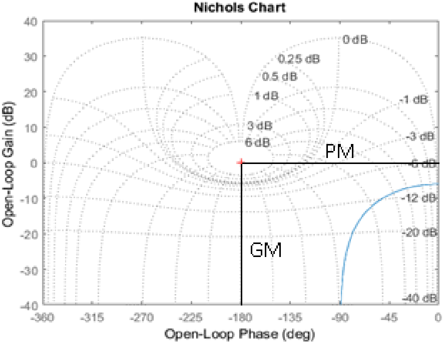
Example of Analysis of Nichols Chart

e. The frequency response of the system based on the results in part (d):

- At low frequency, the magnitude of the system are around less than -6 dB and phase shift from 0 up to -90°.
- At high frequency, the magnitude and phase of the system are settling at negative infinity gain and -90° phase shift. The contour is underneath test point (-180, 0), thus system is stable.
- The gain and phase shift is nowhere near the -180° mark, the magnitude and phase margins of the system are both positive values.

Example of Analysis of Nichols Chart

- From the resulting graph of the Nichols chart, we could analyse the characteristics of the system.



- Gain margin: ∞ dB
- Phase margin:∞ degree
- Stability of the system:

  Since both margins are positive, then the given system is stable.

Design with Nyquist Diagram

- Nyquist diagram is primarily used for analysing and designing the stability of the control systems.
- Procedure for designing of control systems using Nyquist diagram.

  i. Construct the Nyquist diagram for the given control system.

  ii. Determine the stability of the system by evaluating the position of the curve relative to the critical-test point of Nyquist diagram (-1,0).

  iii. Determine the gain and phase margins of the system.

Design with Nyquist Diagram

iv. Increase or decrease the gain of the system to meet the required steady-state condition and transient response of the system.

v. If previous step is not successful, introduce compensator or controller to meet the required steady-state condition and transient response of the system.

vi. Readjust, if necessary, the gain of the system to meet the desired design specification.

Design with Nichols Chart

- Like Nyquist diagram, Nichols chart could provide alternative for analysing and designing the stability of control systems.
- Procedure for designing of control systems using Nichols chart:

i. Construct the Nichols chart for the given control system.

ii. Determine the stability of the system by evaluating the position of the curve relative to the critical-test point of Nichols chart (-180°,0).

Design with Nichols Chart

iii. Determine the gain and phase margins of the system.

iv. Increase or decrease the gain of the system to meet the required steady-state condition and transient response of the system.

v. If previous step is not successful, introduce compensator or controller to meet the required steady-state condition and transient response of the system.

vi. Readjust, if necessary, the gain of the system to meet the desired design specification.