VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

Blocks Diagram Modelling

XMUT315 Control Systems Engineering

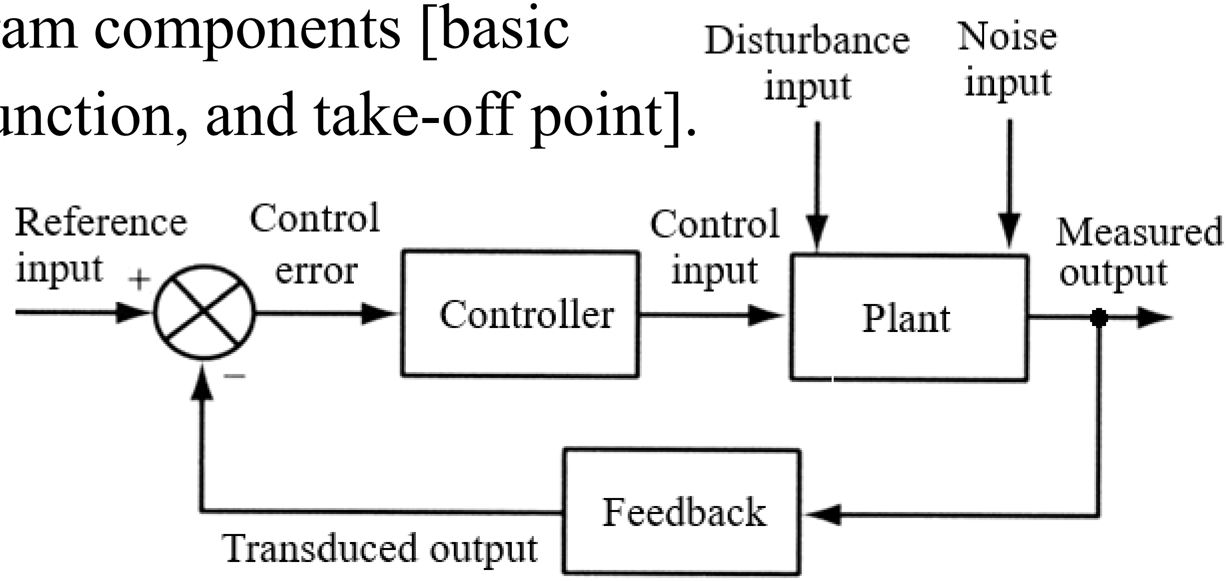
Topics

- Introduction to block diagram modelling.
- Feedback systems in block diagram.
- Block diagram manipulation.
- Block diagram reduction.
- Block diagram and physical modelling.

Introduction to Block Diagram Modelling

In order to analyse a system:

- We identify an input signal [a variable].
- Using block-diagram components [basic block, summing junction, and take-off point].
- We combine internal signals [modified variables].



- To produce the output signal [another variable].

The input-output relationship may then be determined.

Components

Signals:

- It is used to combine several signals in the system.
- + and/or – the system signals.
- Up to three inputs and only one output.

Signals

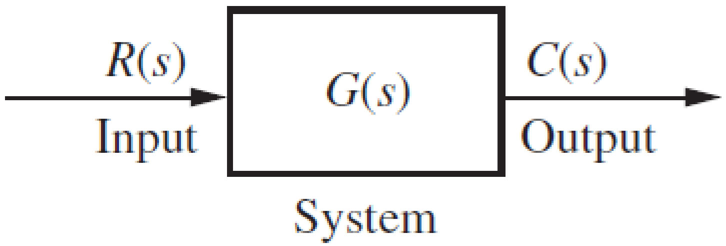
Signals in electrical system

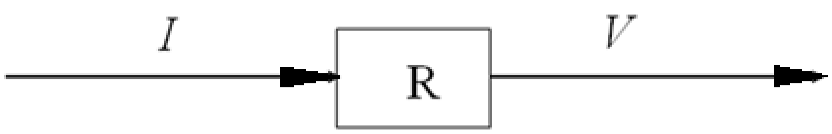
Signals in mechanical system

Components

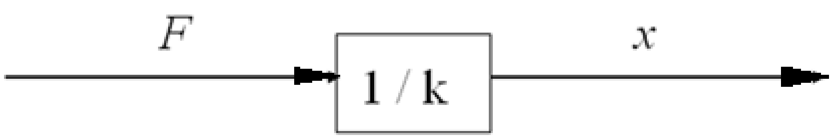
Block:

- It is used to house a function or feature in the system.
- System or function that acts on the system signal.





Resistance as function in electrical system

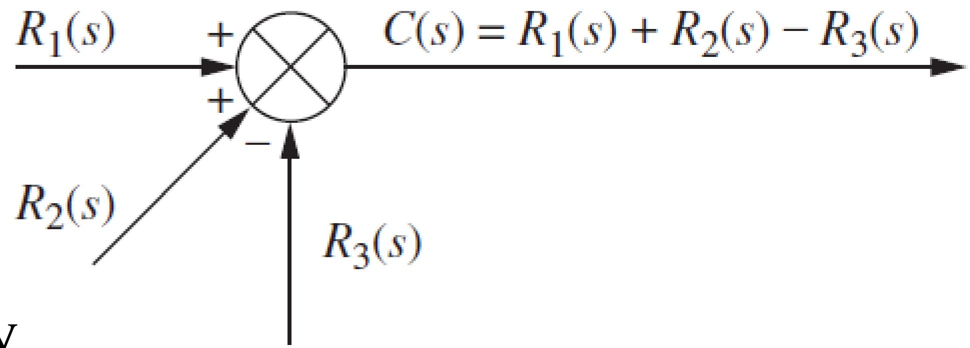


Spring constant as function in spring system

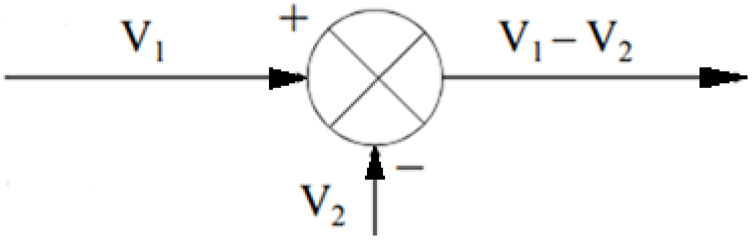
Components

Summing Junction:

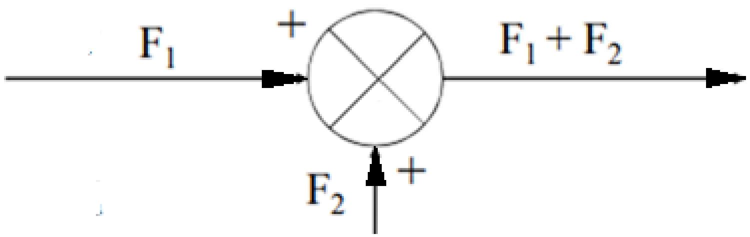
- It is used to combine several signals in the system.
- + and/or – the system signals.
- Up to three inputs and only one output.



Summing junction



Voltages in electrical system

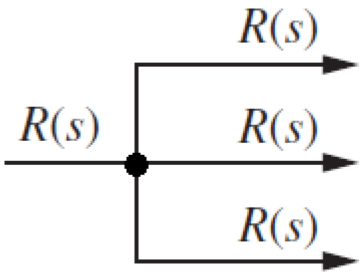


Flows in fluid system

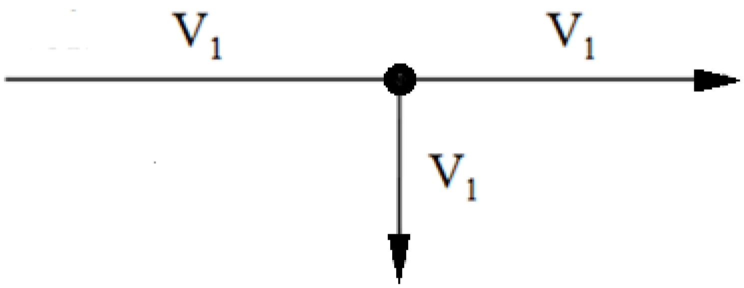
Components

Take-off point:

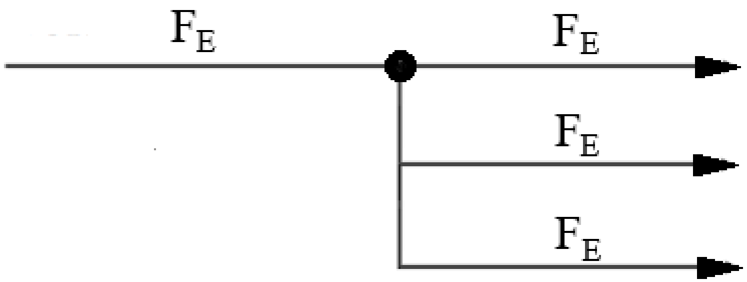
- It is used to split a signal into a number of it.
- The system signal can be used elsewhere, but is not affected by the split.
- Only one input and many outputs.



Take-off point



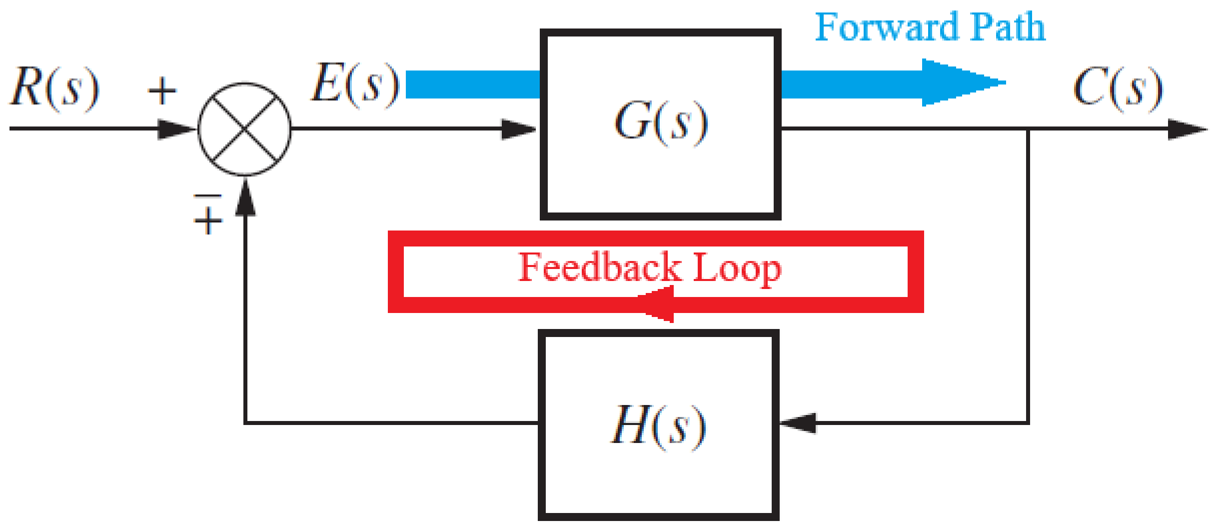
Voltages in electrical system



Forces in mechanical system

Feedback System in Block Diagram

- Feedback system is realised as negative in the feedback loop



- Create transfer function from the variables (input and output) and constants (bits inside the boxes).

$$\frac{\text{Output}}{\text{Input}} = \frac{\text{Forward}}{1 \pm \text{Loop}}$$

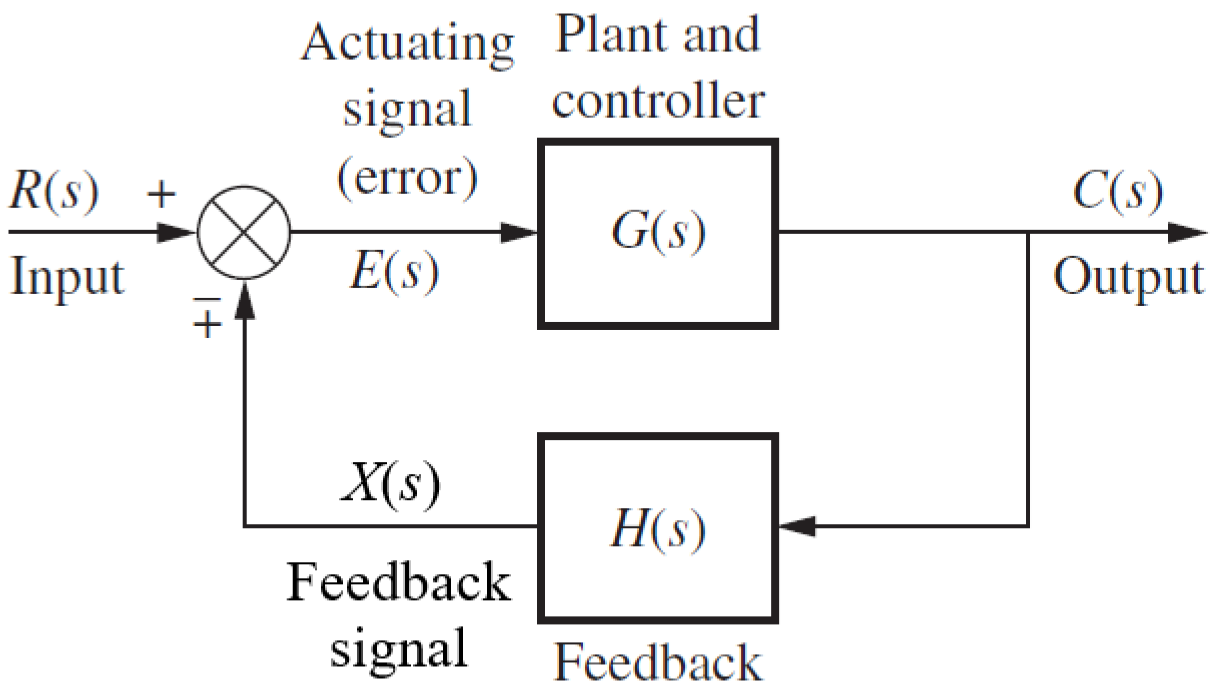
Feedback System in Block Diagram

- Error of the feedback system:

$$\text{Error} = \text{Input} \pm \text{Feedback}$$

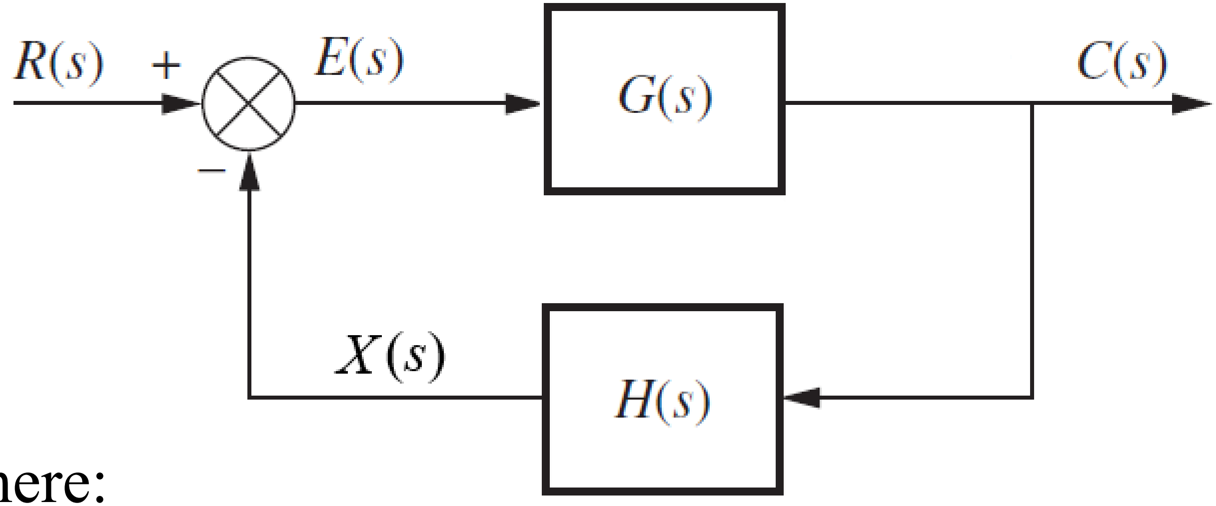
- Output of the feedback system:

$$\text{Output} = \text{Error} \times \text{Plant}$$



Feedback System in Block Diagram

- Feedback system can be expressed as functions in respect to the changing variable:



Where:

$E(s)$ = Error signal

$C(s)$ = Output signal

$X(s)$ = Feedback signal

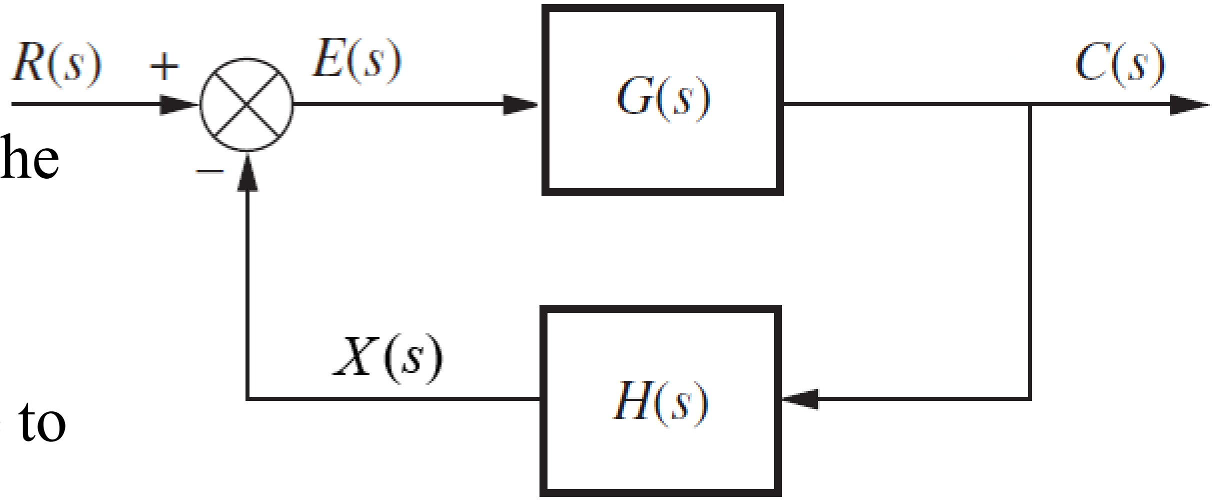
$R(s)$ = Input (Reference) signal

$G(s)$ = Plant function

$H(s)$ = Feedback function

Feedback System in Block Diagram

- Note: this derivation example is for negative feedback.
- For the positive feedback, change the sign in front of parameter $X(s)$ in equation (1) above to (+) positive.



- We can form three equations:

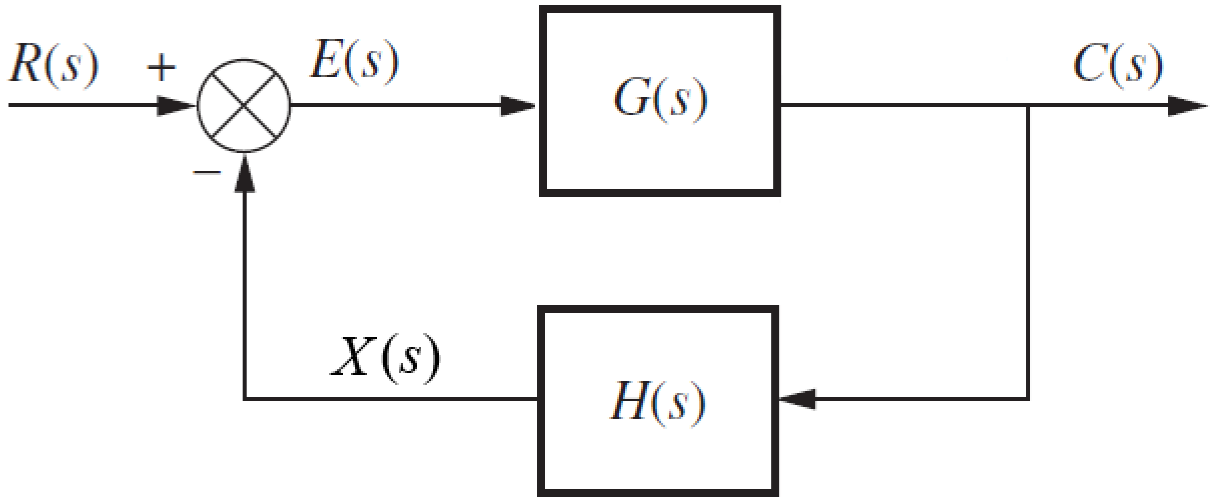
$$E(s) = R(s) - X(s) \qquad (1)$$

$$C(s) = G(s)E(s) \qquad (2)$$

$$X(s) = H(s)C(s) \qquad (3)$$

Feedback System in Block Diagram

- We need to form a relationship between input and output by removing the intermediate variables:



- Combine equation (1) + (3):

$$E(s) = R(s) - H(s)C(s) \qquad (4)$$

- Combine equation (2) + (4):

$$C(s) = G(s)[R(s) - H(s)]$$

Feedback System in Block Diagram

- We need to form a relationship between input and output by removing the intermediate variables.
- Multiply out the bracket:

$$C(s) = G(s)R(s) - G(s)H(s)C(s)$$

- Collect output terms:

$$C(s) + G(s)H(s)C(s) = G(s)R(s)$$

- Rearrange the equation:

$$C(s)[a + G(s)H(s)] = G(s)R(s)$$

- As a result (e.g. positive feedback: loop is (+), negative feedback: loop is (-)):

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = \frac{\text{Forward}}{1 - \text{Loop}}$$

Block Manipulation

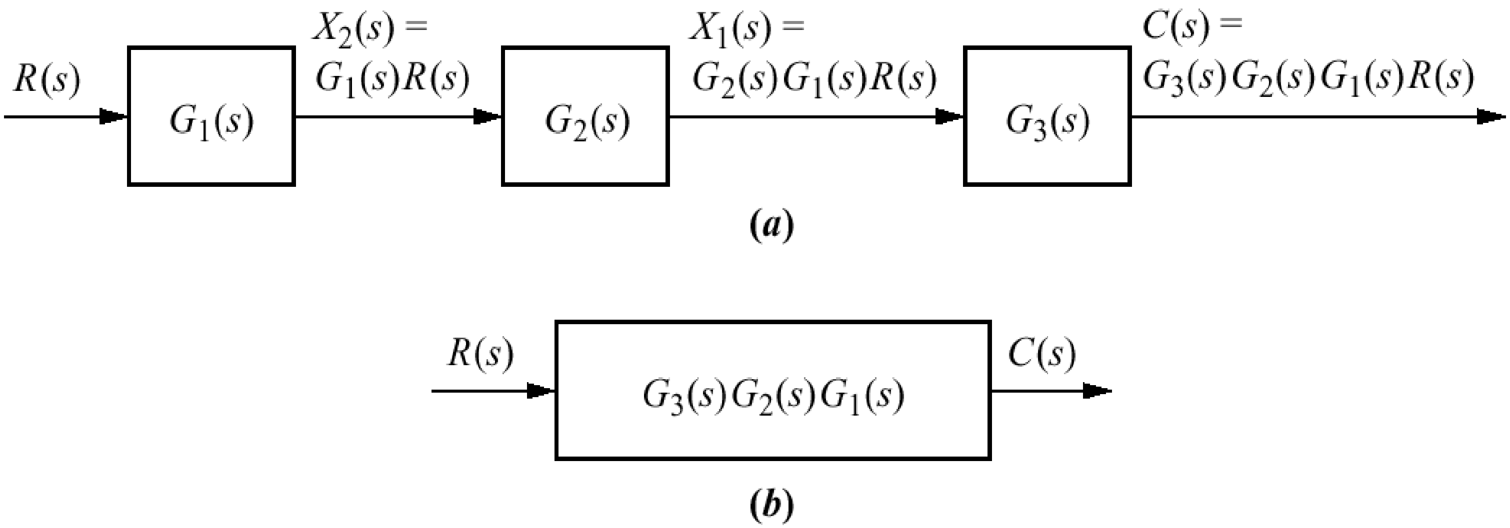
Block manipulations:

- Combining blocks in series.
- Combining blocks in parallel.
- Changing the flow direction of a block
- Summing junction manipulation.
- Take-off point manipulation.
- Feedback structure manipulation.

Block Manipulation

Combining blocks in series:

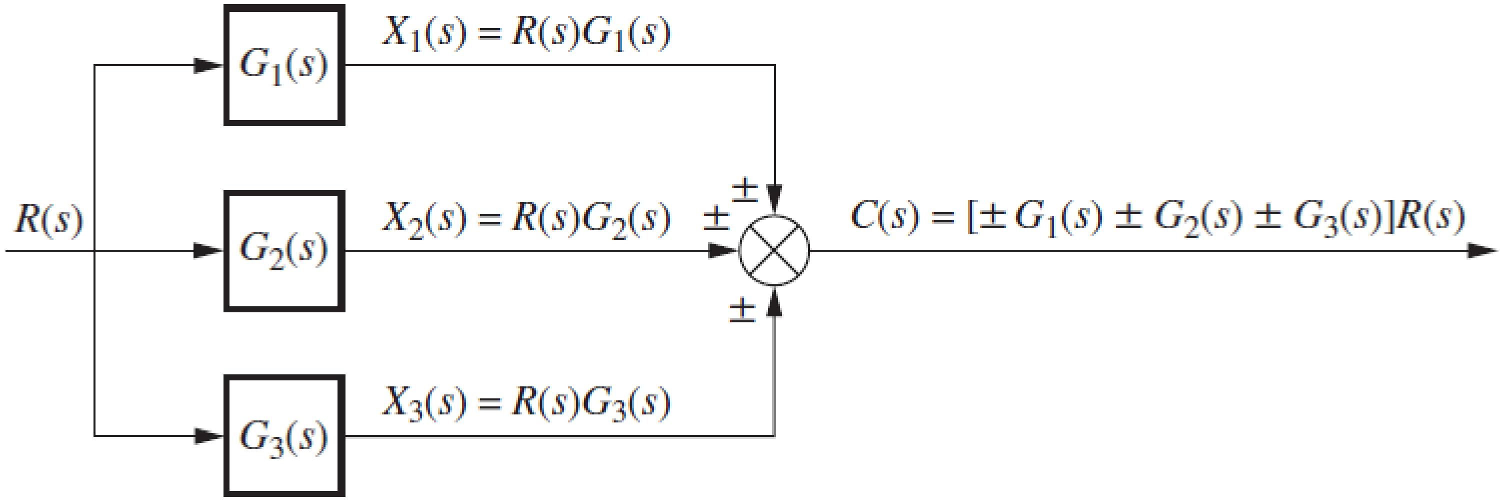
- Blocks in series can be combined to form a bigger block.



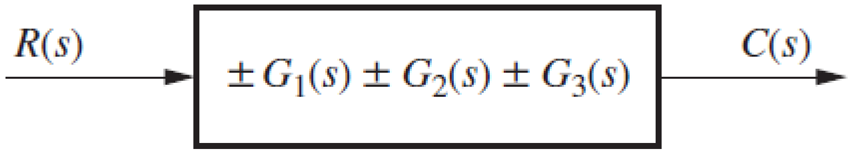
Block Manipulation

Combining blocks in parallel:

- Blocks in parallel can be combined to form a bigger block.



(a)

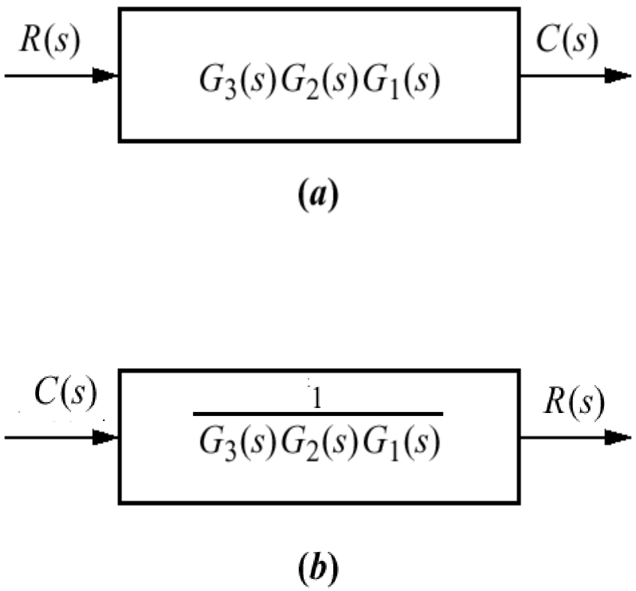


(b)

Block Manipulation

Changing the flow direction of a block:

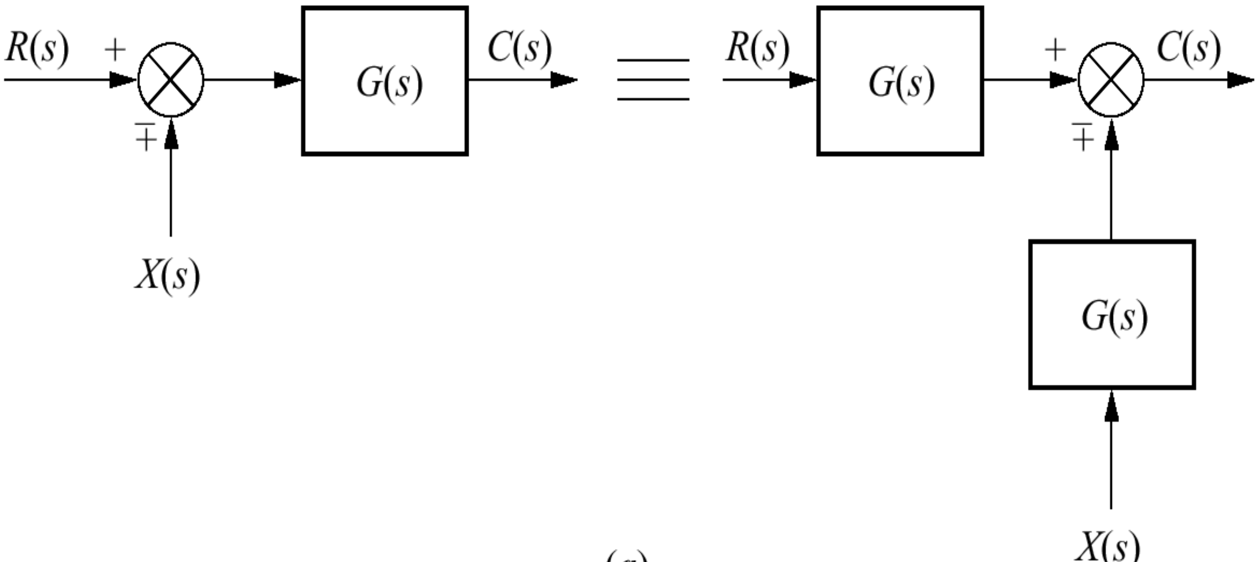
- The direction of the flow of a block can be changed, and it requires inversion of the content of the block.



Block Manipulation

Summing Junction manipulation:

- Move a block to **before** a summing junction.

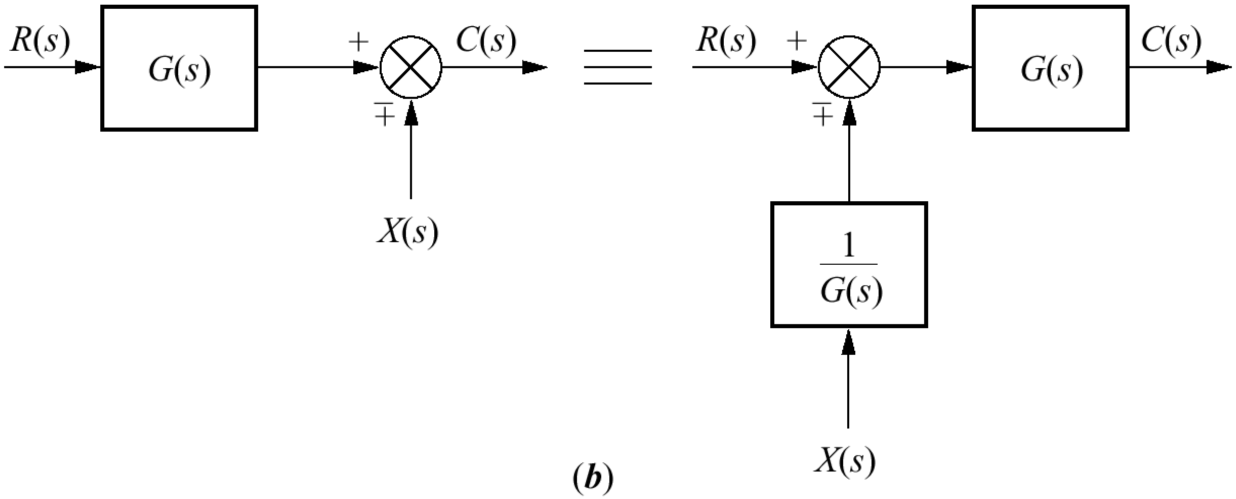


(a)

Block Manipulation

Summing Junction manipulation:

- Move a block to **after** a summing junction.

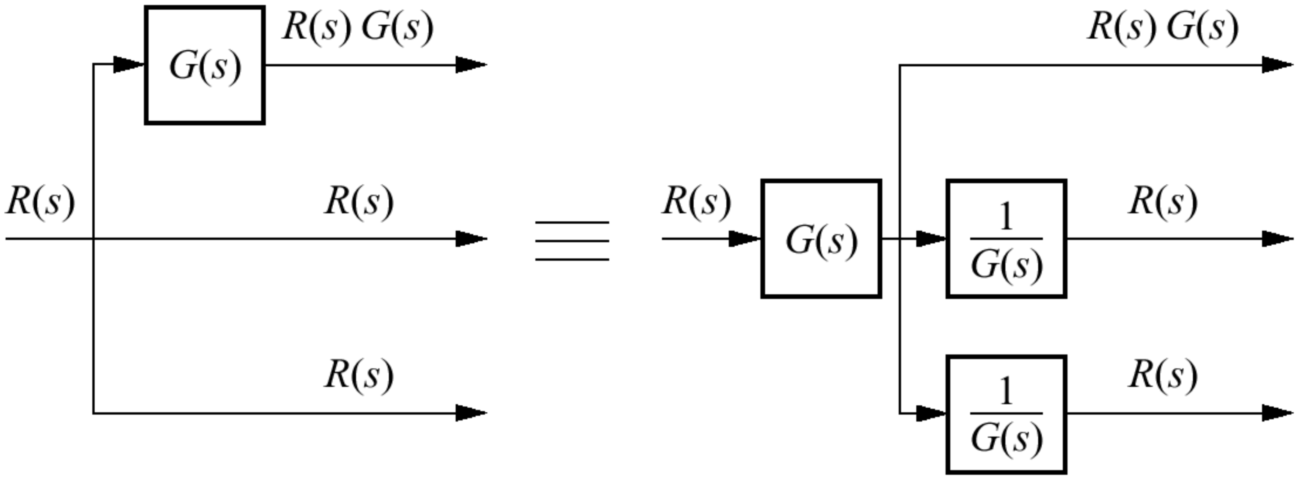


(*b*)

Block Manipulation

Take-off Point manipulation:

- Move a block to **before** a take-off point:

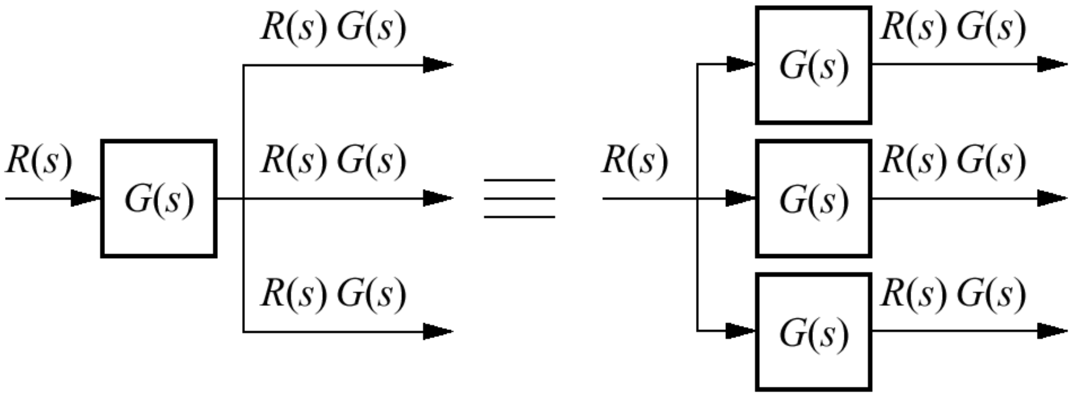


(a)

Block Manipulation

Take-off point manipulation:

- Move a block to **after** a take-off point:



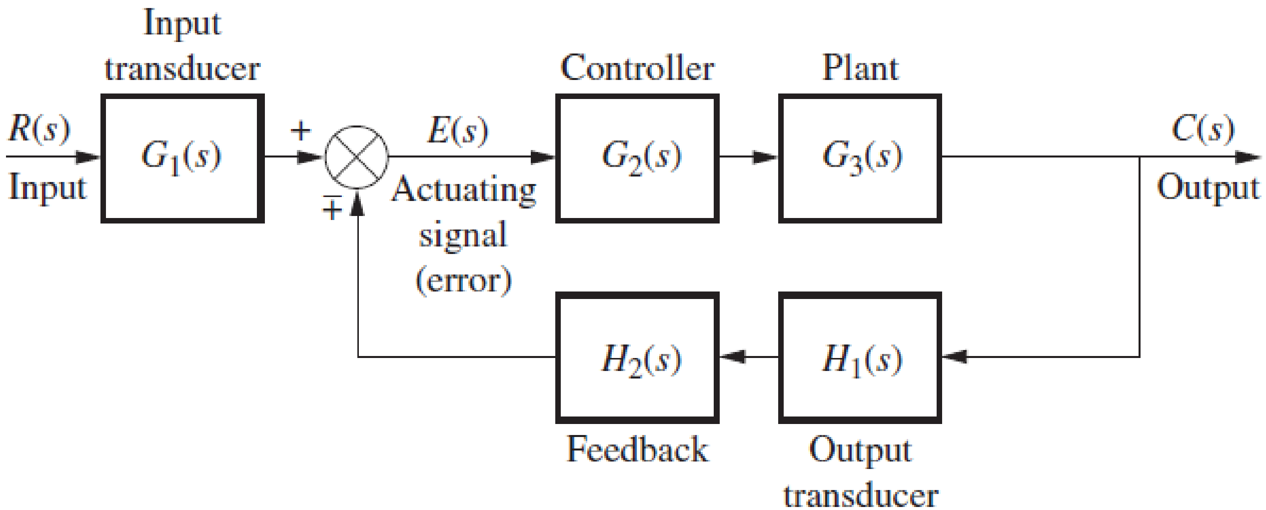
(b)

Block Manipulation

Feedback structure manipulation:

- For a feedback structure shown below, using block diagram manipulation, its simplification as is shown below.

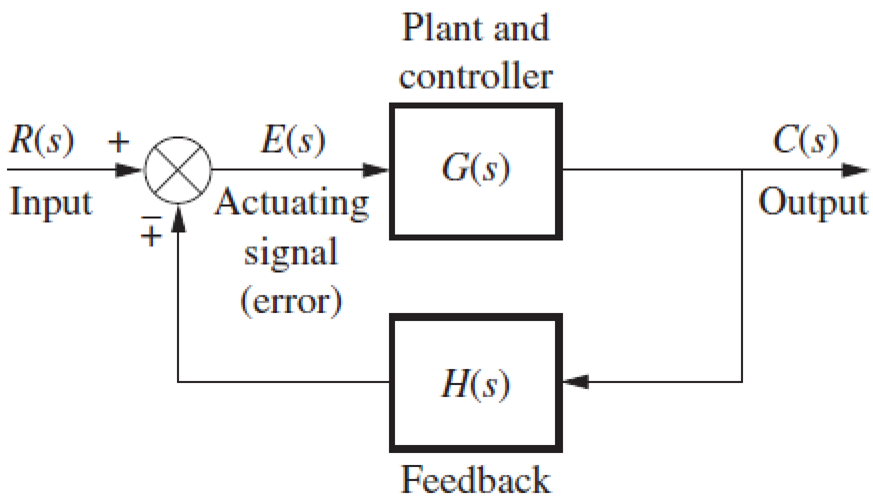
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 \pm G(s)H(s)}$$



(a)

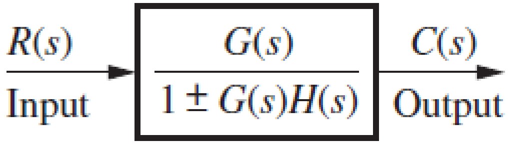
Block Manipulation

- Simplified structure.



(b)

- A single block (notice the sign $\pm$ in the denominator: for positive feedback – sign is (-), and for negative feedback – sign is (+)).



(c)

Block Diagram Reduction

Steps for solving block diagram reduction problems:

- Rule 1 − Check for the blocks connected in series and simplify.
- Rule 2 − Check for the blocks connected in parallel and simplify.
- Rule 3 − Check for the blocks connected in the feedback loop and simplify.
- Rule 4 − If there is difficulty with the take-off point while simplifying, shift it towards the right.
- Rule 5 − If there is difficulty with the summing points while simplifying, shift it towards the left.
- Rule 6 − Repeat the above steps till you get the simplified form, i.e., single block.

Reduction via Familiar Forms

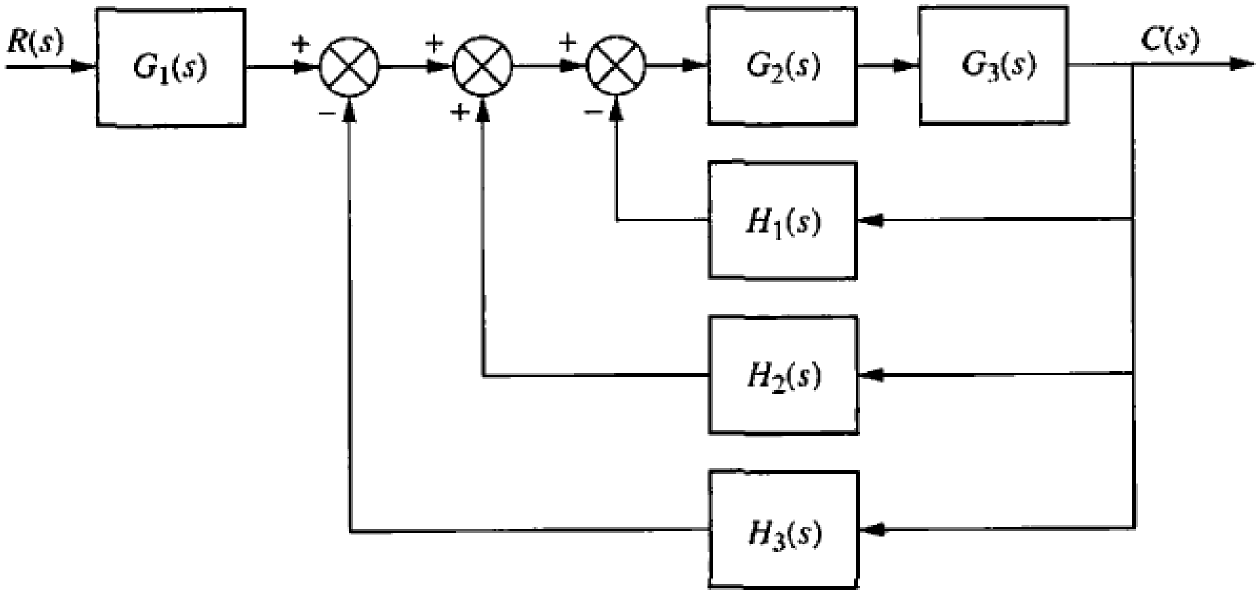
Some of manipulation and reduction via familiar forms in the block diagram are:

- Blocks connected in series.
- Block connected in parallel.
- Block connected in feedback loop.

Block Diagram Reduction Example 1

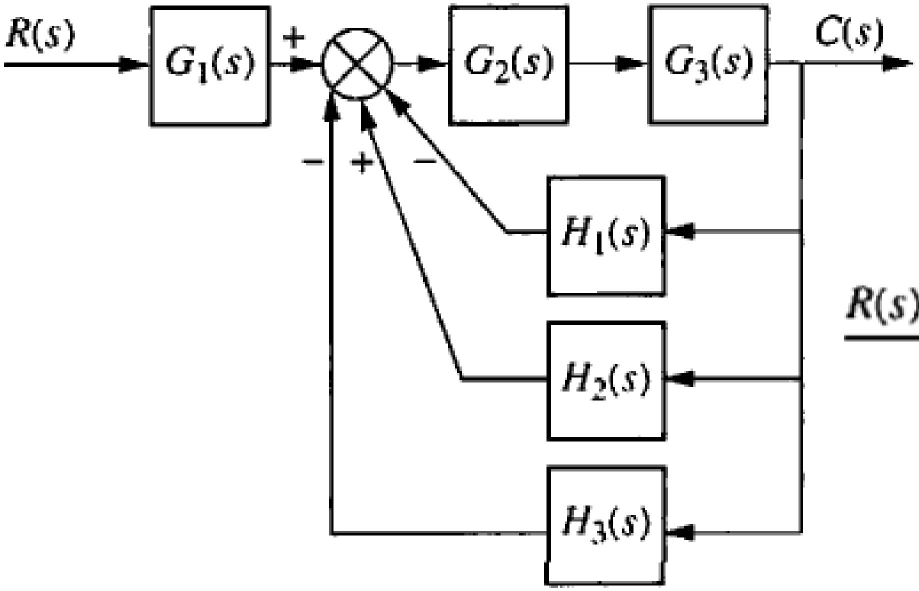
- Reduce the block diagram below into a simpler form:

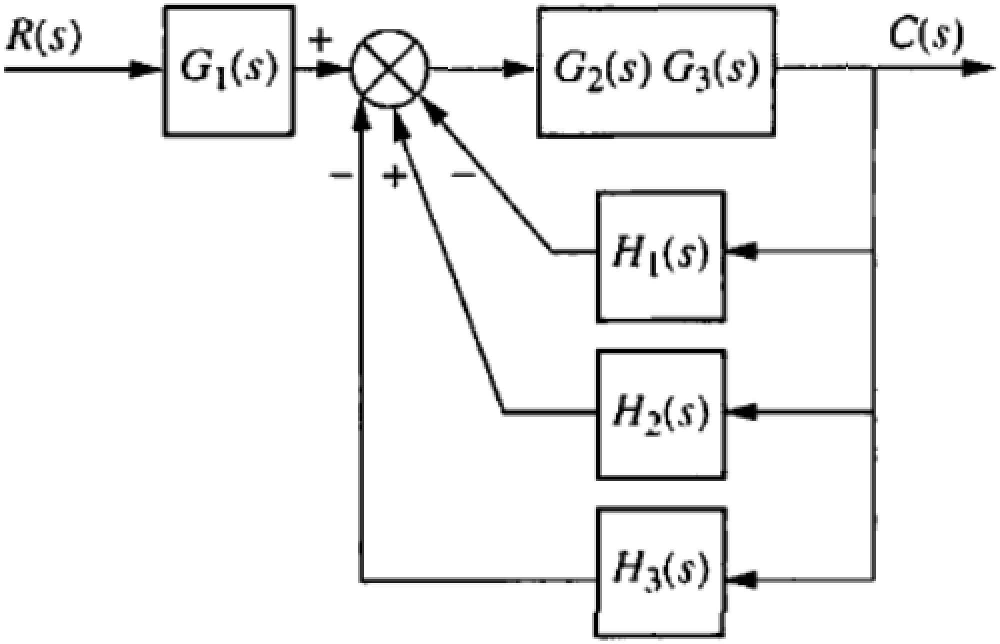
[8 marks]



Reduction via Familiar Forms

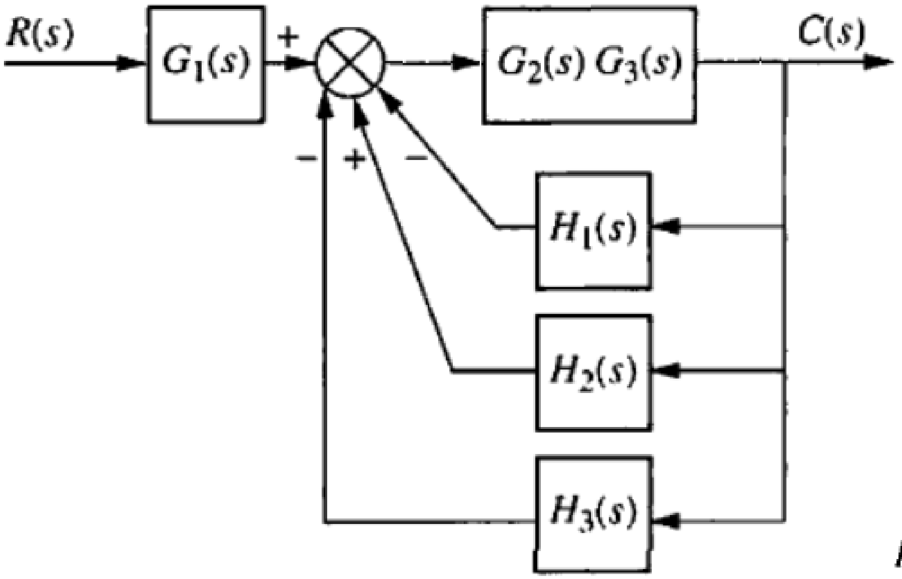
- Combine G_2 and G_3 blocks in the forward path.

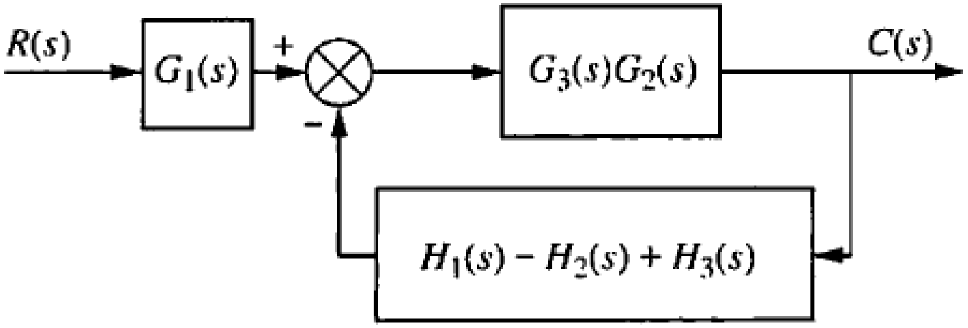




Reduction via Familiar Forms

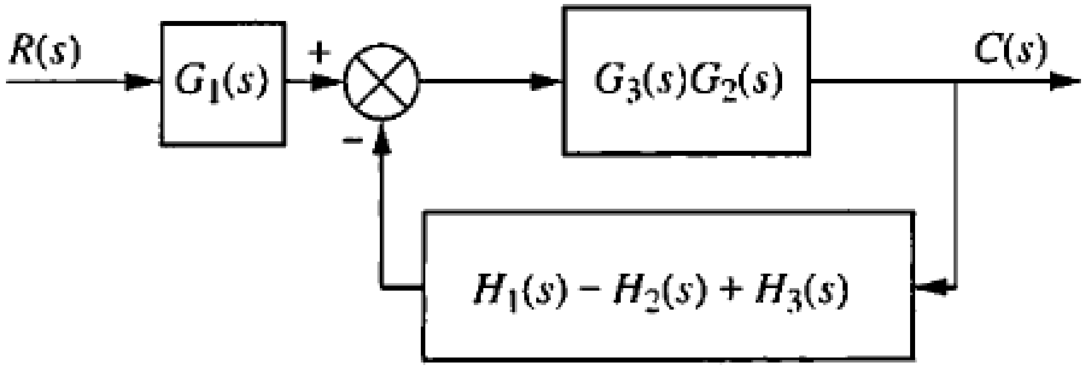
- Solve all parallel feedback paths blocks (H_1, H_2 and H_3)

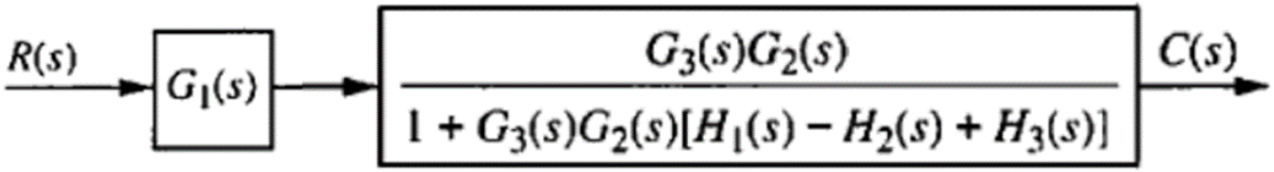




Reduction via Familiar Forms

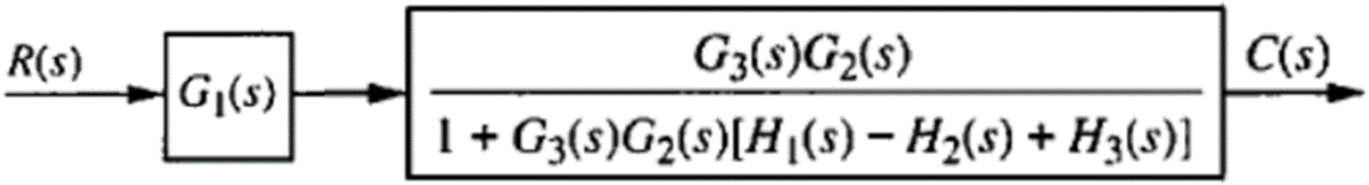
- Solve feedback path block ($H_1 - H_2 + H_3$) with forward path block ($G_3 G_2$).

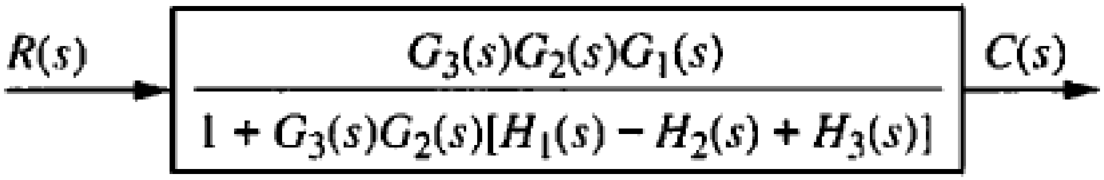




Reduction via Familiar Forms

- Combine the result of feedback path block and forward path block with G_1 block in series.





Reduction via Moving Blocks

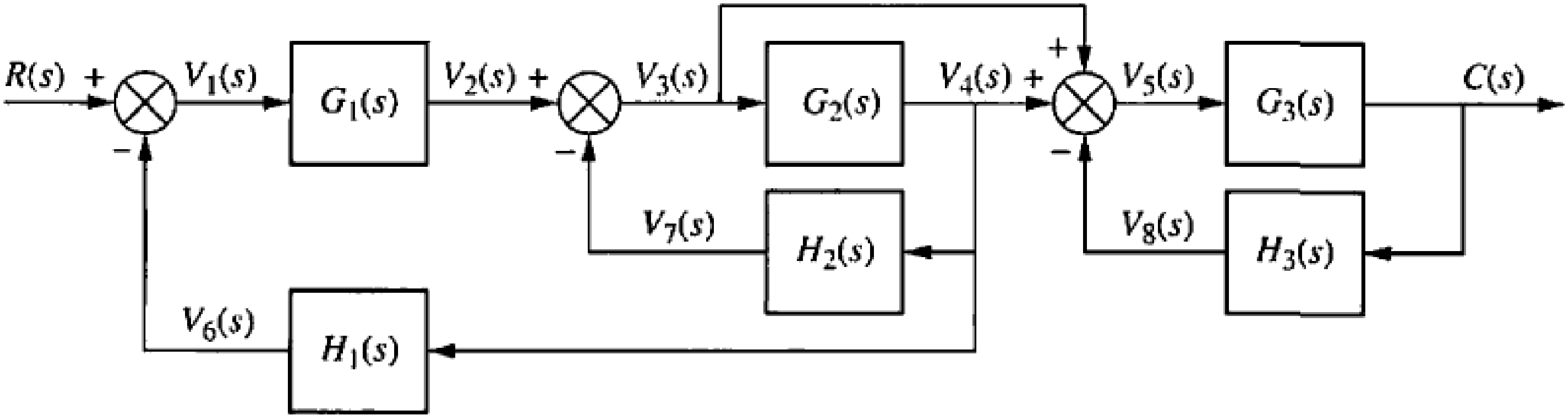
Two most common reduction by moving blocks in the block diagram are:

- If there is difficulty with take-off point while simplifying, shift it towards right.
- If there is difficulty with summing point while simplifying, shift it towards left.

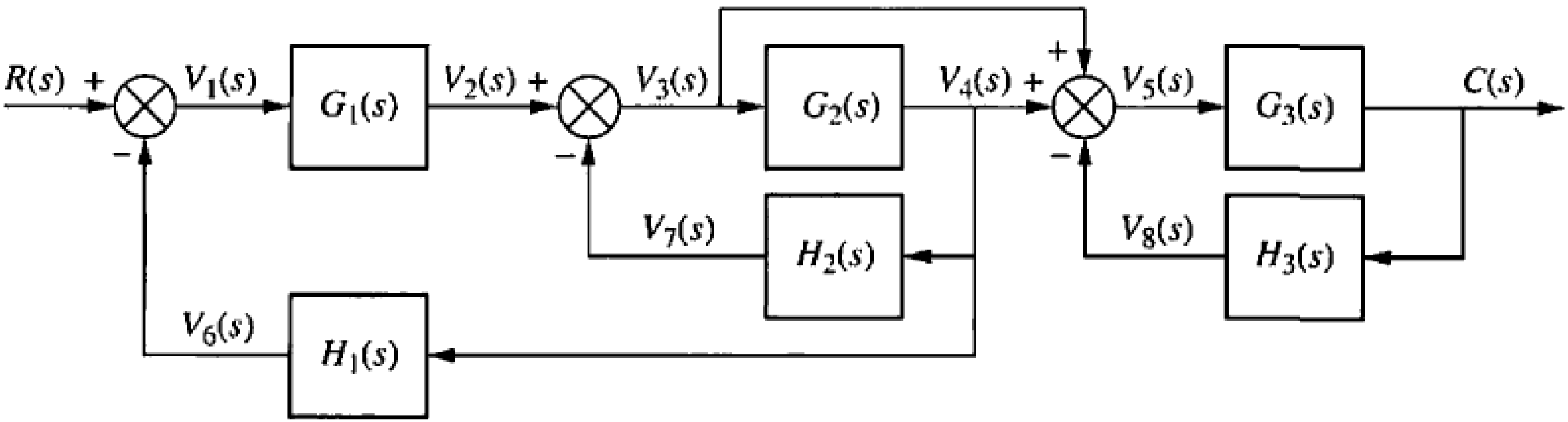
Block Diagram Reduction Example 2

- Reduce the block diagram below into a simpler form:

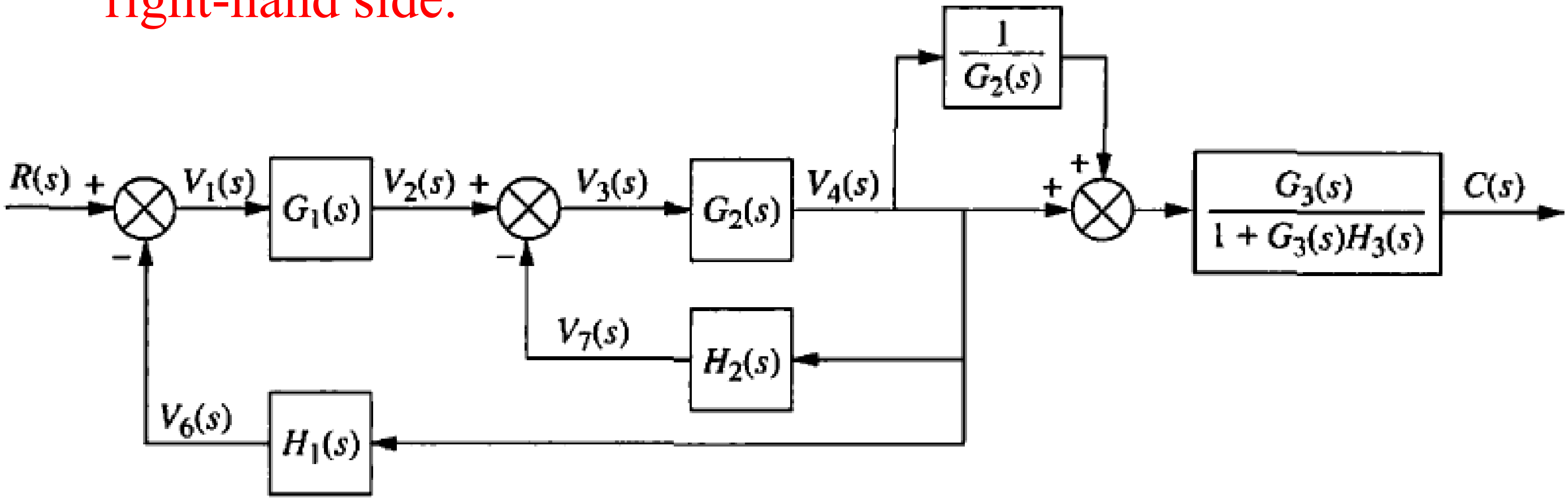
[10 marks]



Reduction by Moving Blocks

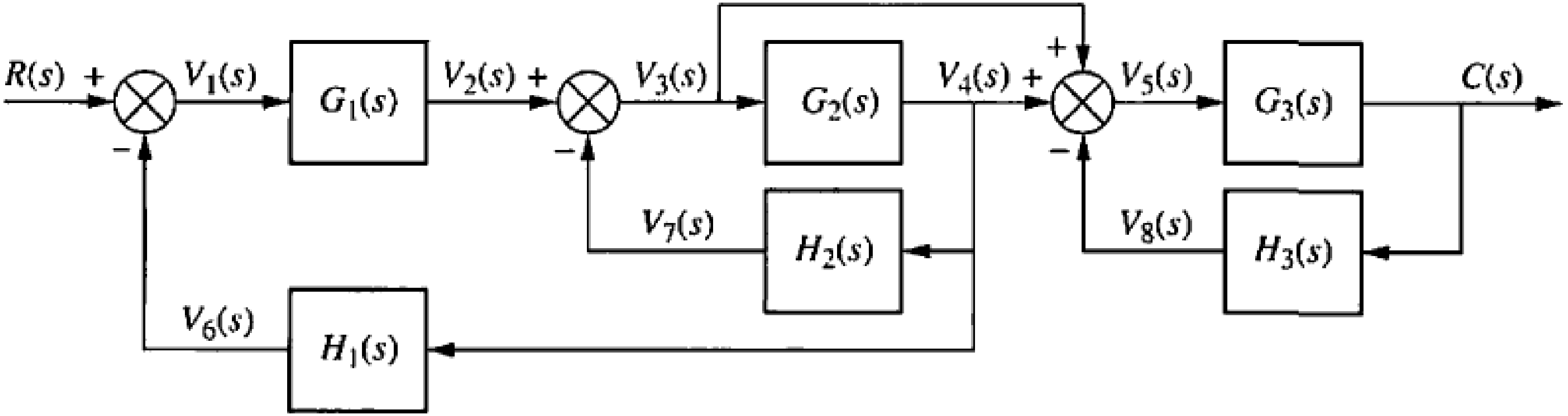


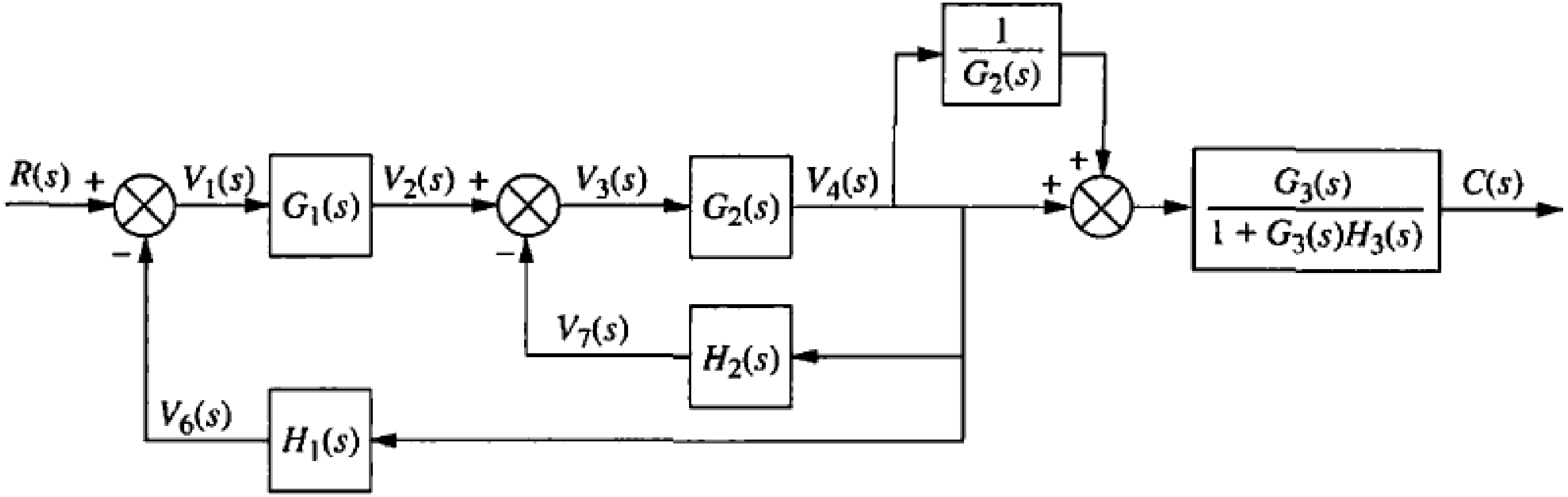
- Move take off point from left-hand side of G_2 block to its right-hand side.



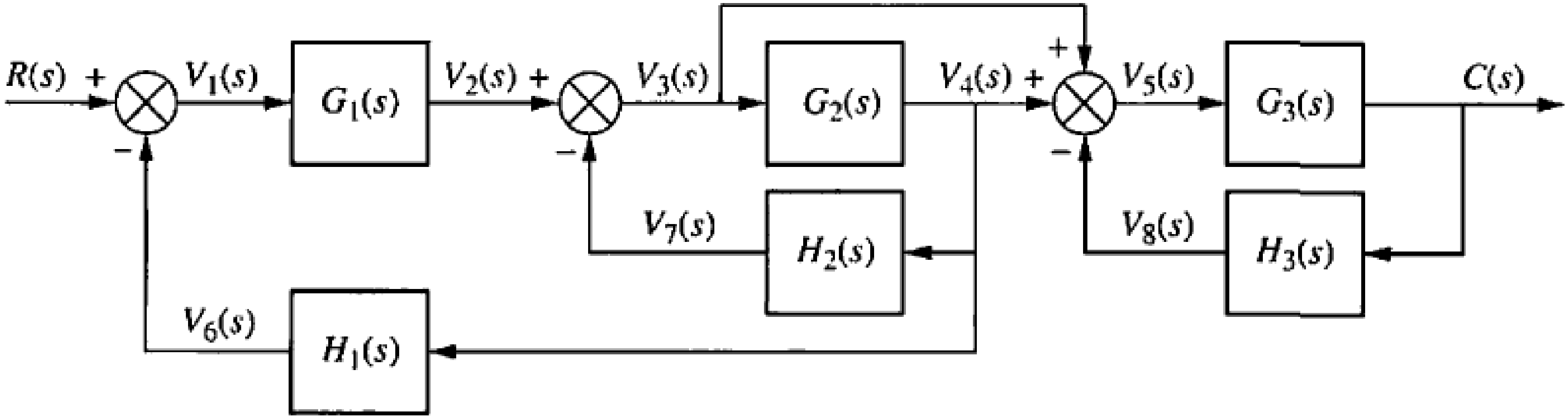
Reduction by Moving Blocks

- Solve feedback part i.e. blocks with G_3 and H_3.

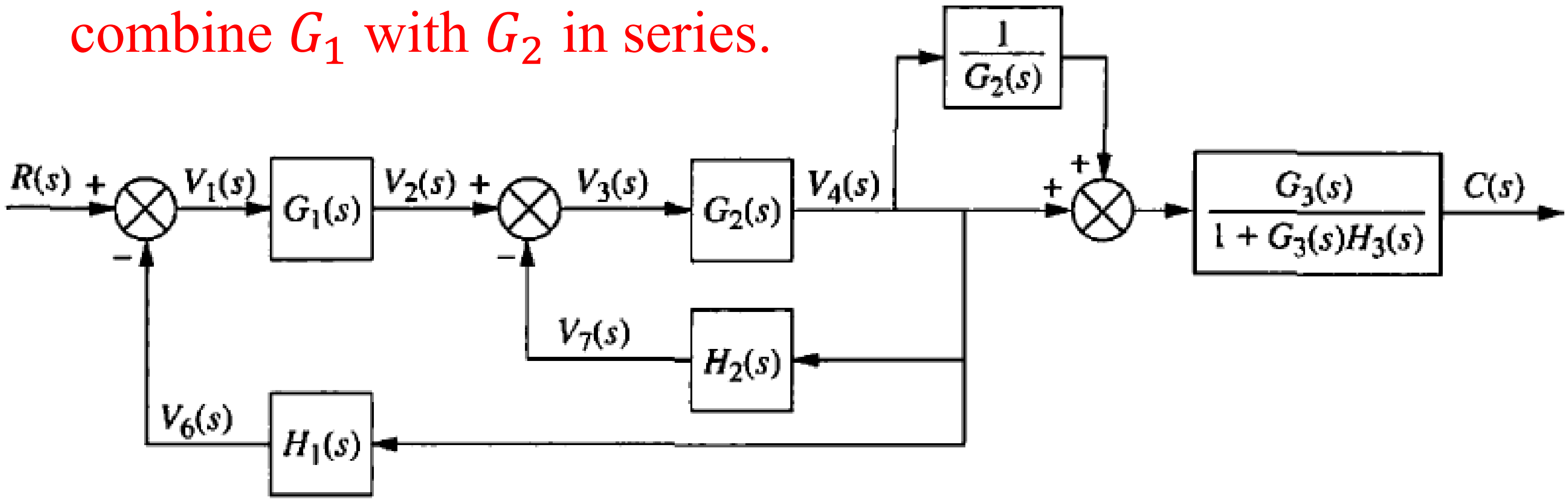




Reduction by Moving Blocks

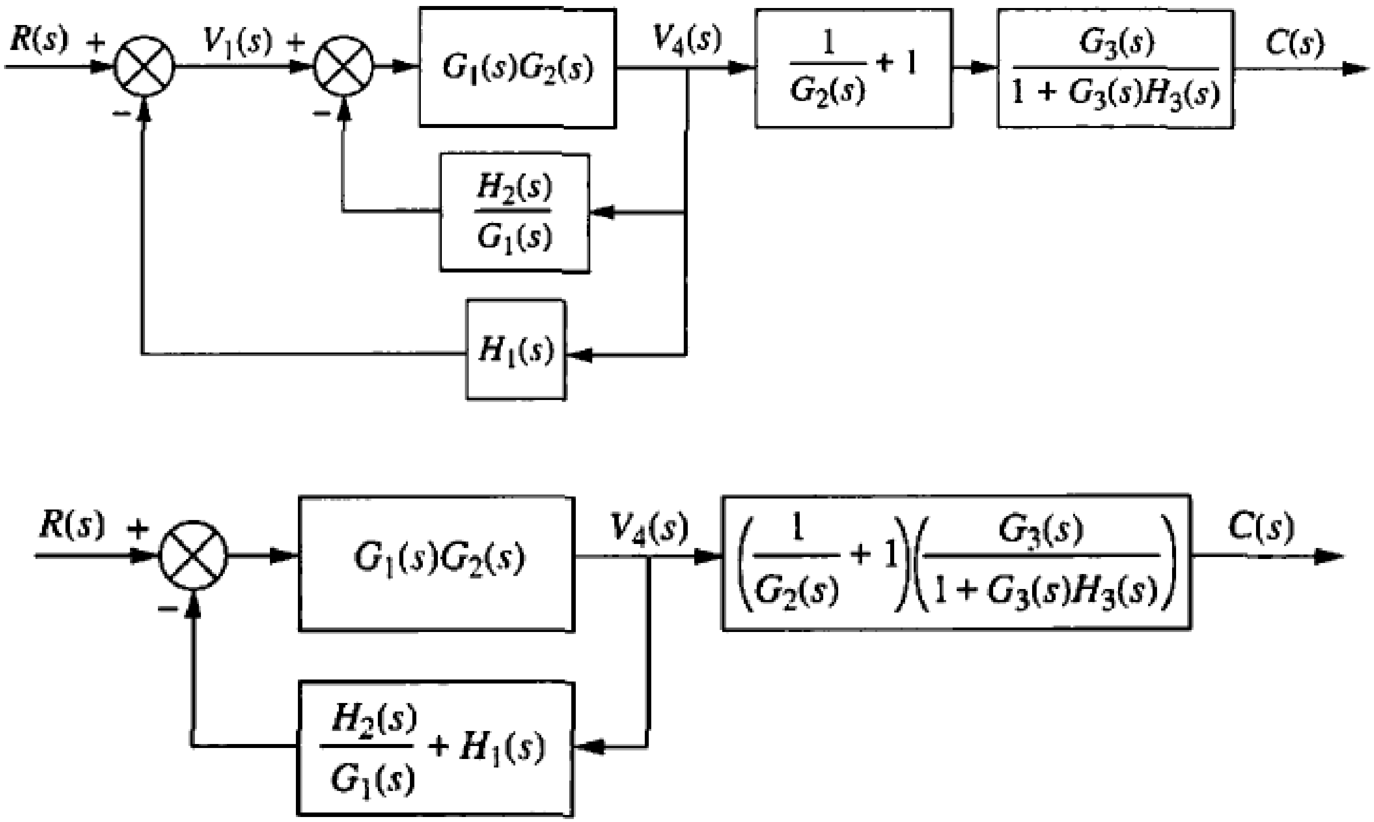


- Sum up the moved block ($1/G_2$). Then, move ($V_2 - V_3$) summing junction from right of G_1 block to its left and combine G_1 with G_2 in series.



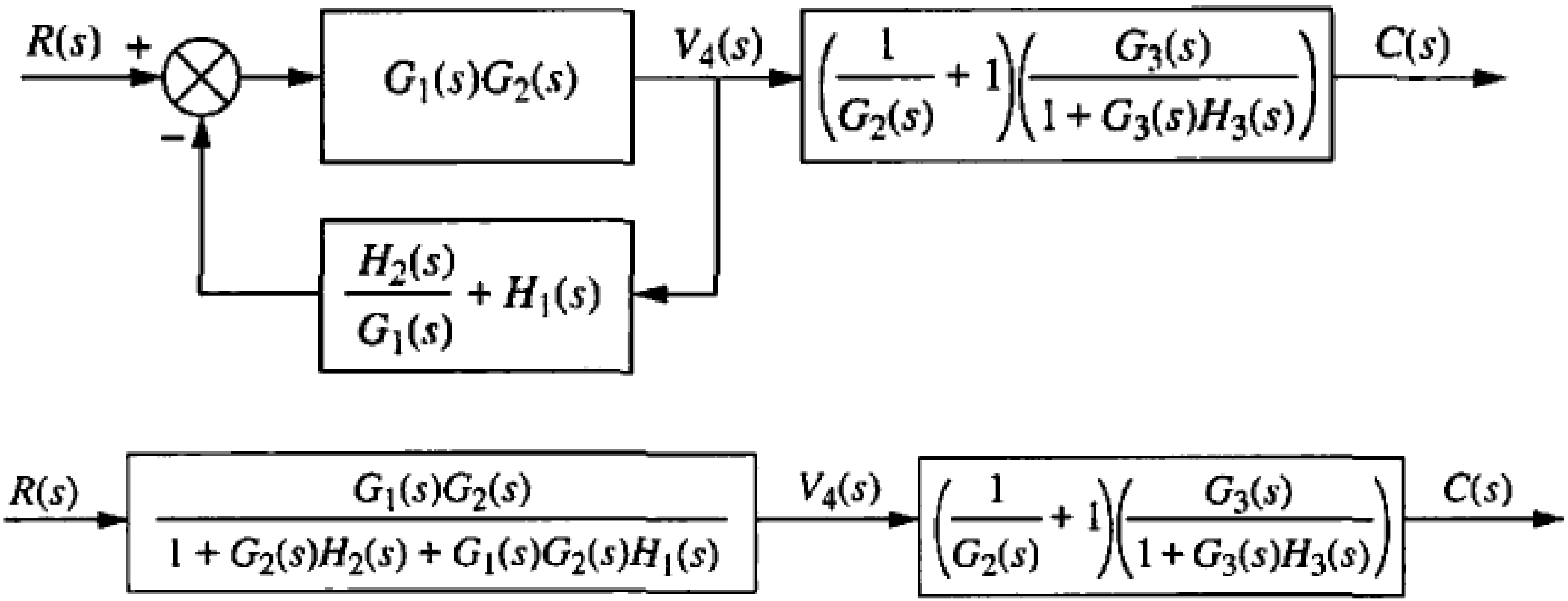
Reduction by Moving Blocks

- Sum up feedback path (H_2/G_1) and H_1 into a single block and combine the two blocks on the right-hand side part of the forward path.



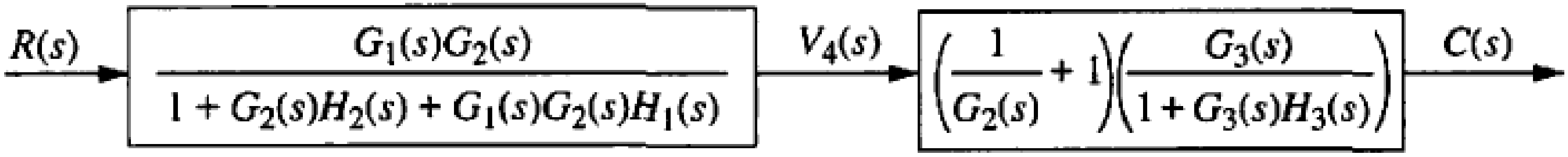
Reduction by Moving Blocks

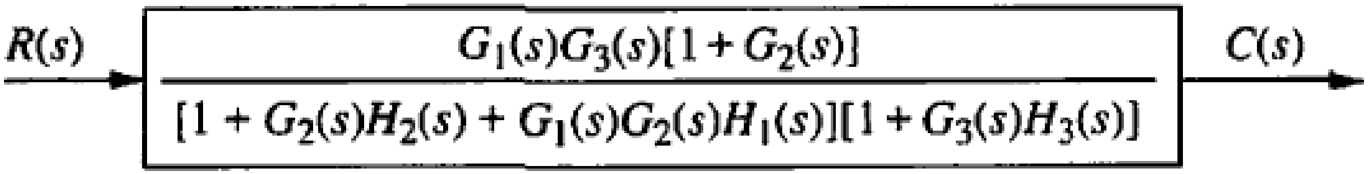
- Solve the feedback loop on the left-hand side of the block diagram.



Reduction by Moving Blocks

- Combine the resulting two blocks in the forward path which are in series.





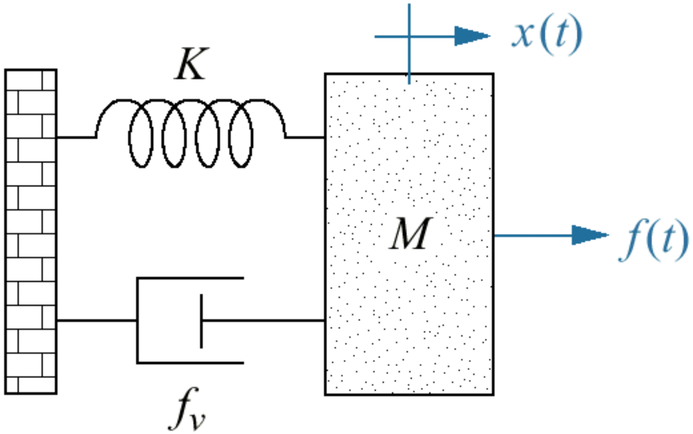
Block Diagram and Physical Model

- Combinations of block diagram with physical model would result in a much efficient and standardised process of modelling of the systems.
- Steps for using the block diagram with the physical modelling of the system are:
  1. Develop physical model of the system.
  2. Derive mathematical solution.
  3. Form transfer function.
  4. Convert to block diagram.
  5. Combine block diagram.
  6. Reduce block diagram.

Example 1: Mass, Spring & Damper System

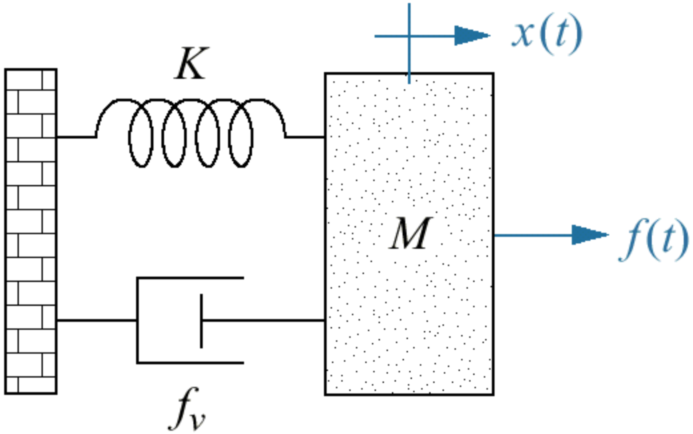
- For a mechanical system as shown in the figure below, how do we determine the block diagram for each component?

[20 marks]

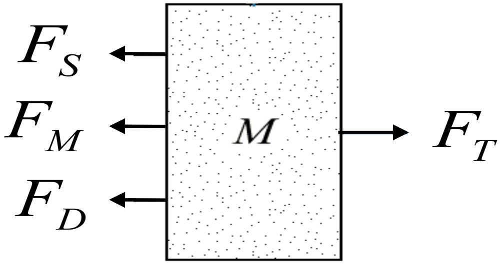


Develop Physical Model of System

- Create a physical model.
- Determine the block diagram for each component of the system.



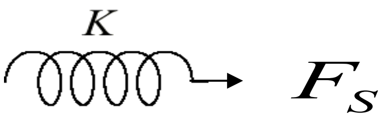
Physical System

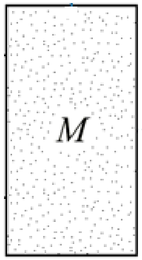


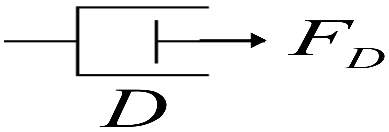
Physical Model

Derive Mathematical Solution

- Each of the components of physical system has its relevant mathematical formulae.







$$f(t) = Kx(t) \qquad f(t) = M\frac{d^2x(t)}{dt^2} \qquad f(t) = D\frac{dx(t)}{dt}$$

- Notice that the differential equation for determining the force in the spring subsystem is:

$$f_S(t) = kx(t)$$

- Where: k is spring constant and $x(t)$ is the displacement.

Derive Mathematical Solution

- For the mass system, the following differential equation is used to represent the force acting in this system:

$$f_M(t) = Ma(t)$$

- Or

$$f_M(t) = M\left(\frac{d^2x(t)}{dt^2}\right)$$

Where: M is mass of the system and $d^2x(t)/dt^2$ is the acceleration of the second derivative of displacement.

- The differential equation for the force in the damper system is given as follows:

$$f_D(t) = Dv(t)$$

Derive Mathematical Solution

- Or

$$f_D(t) = D\left[\frac{dx(t)}{dt}\right]$$

- Where: D is spring constant and $dx(t)/dt$ is the velocity or the first derivative of the displacement.
- Form relationships between parts (from model):

$$f(t) = f_S(t) + f_D(t) + f_M(t)$$

- Substituting the differential equation:

$$f(t) = kx(t) + D\left[\frac{dx(t)}{dt}\right] + M\left[\frac{d^2x(t)}{dt^2}\right]$$

Form Transfer Function

- Apply Laplace transform to differential equation of the given mechanical system.

$$F(s) = kX(s) + DsX(s) + Ms^2X(s)$$

- Rearrange the equation above, determine transfer function:

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Ds + k}$$

- Think how the system responds:

$$X(s) = \frac{F(s)}{Ms^2 + Ds + k}$$

- For the above given system:
  - Larger force -> larger distance.
  - Larger mass & spring stiffness/damping -> smaller distance.

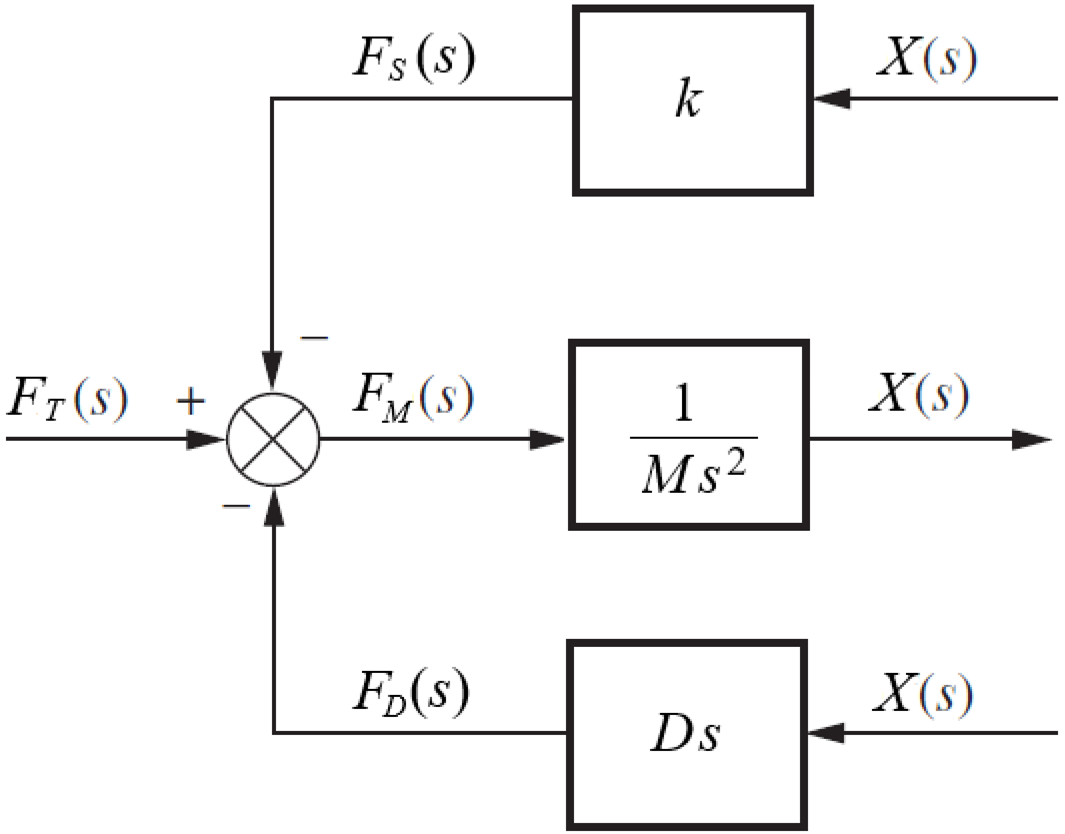
Convert to Block Diagram

- Combine and rearrange components together:

$$F_M(s) = F_T(s) - F_S(s) - F_D(s)$$

Where:

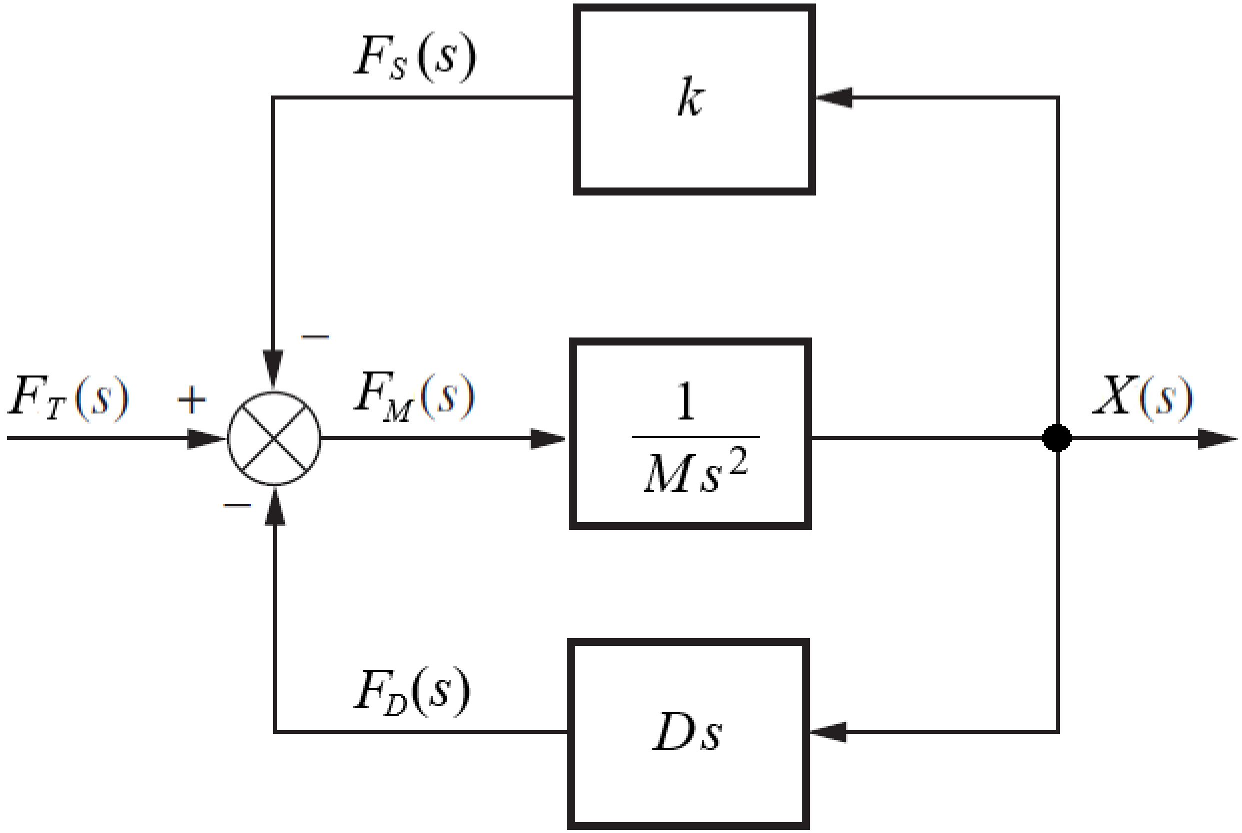
$F_S(s) = kX(s)$,
$F_M(s) = Ms^2X(s)$,
$F_D(s) = DsX(s)$,
and $F_T(s)$.



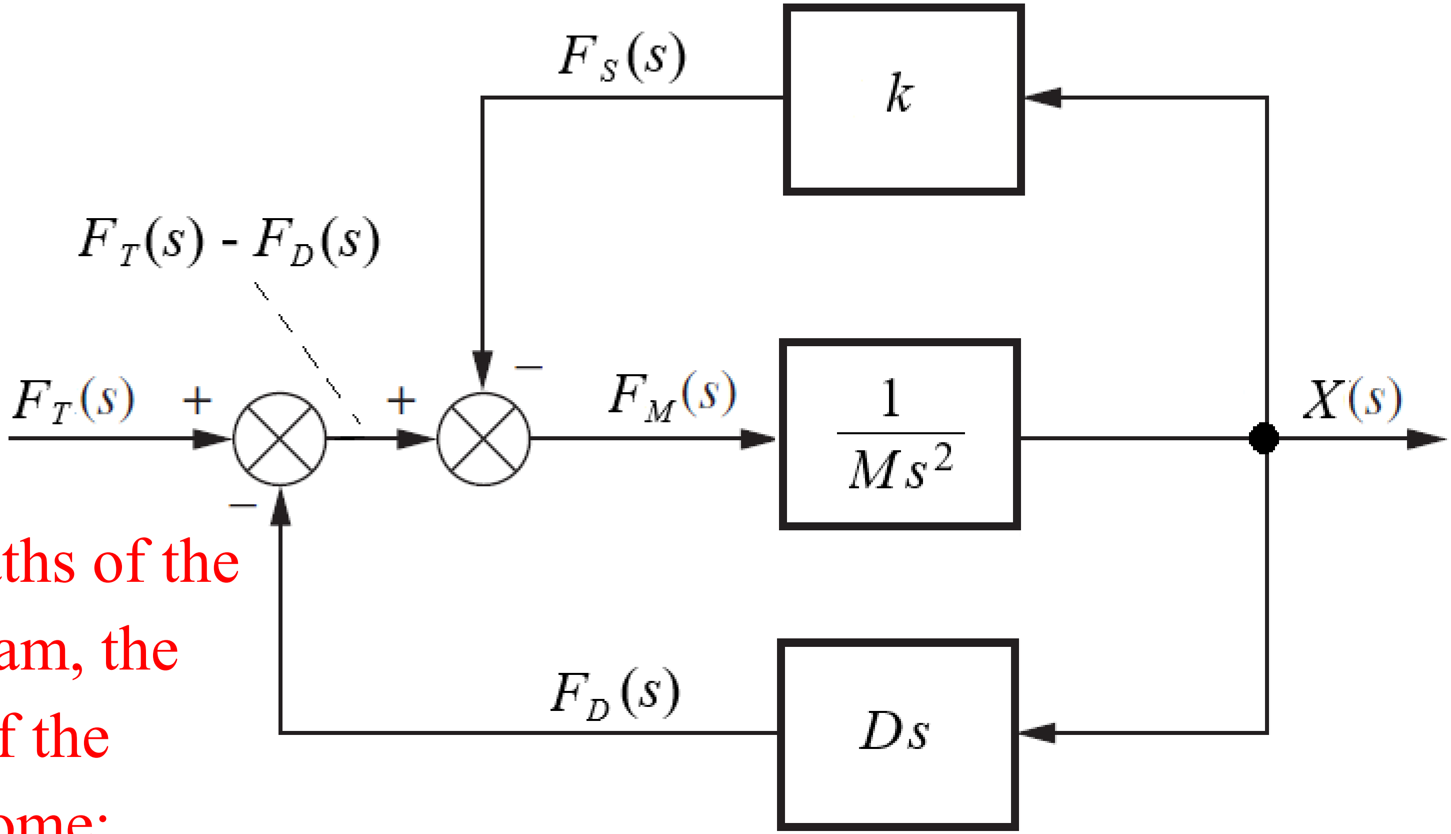
Combine Block Diagram

- Gather components together to form system, the equation of the system becomes:

$$Ms^2X(s) = F_T(s) - KX(s) - DsX(s)$$



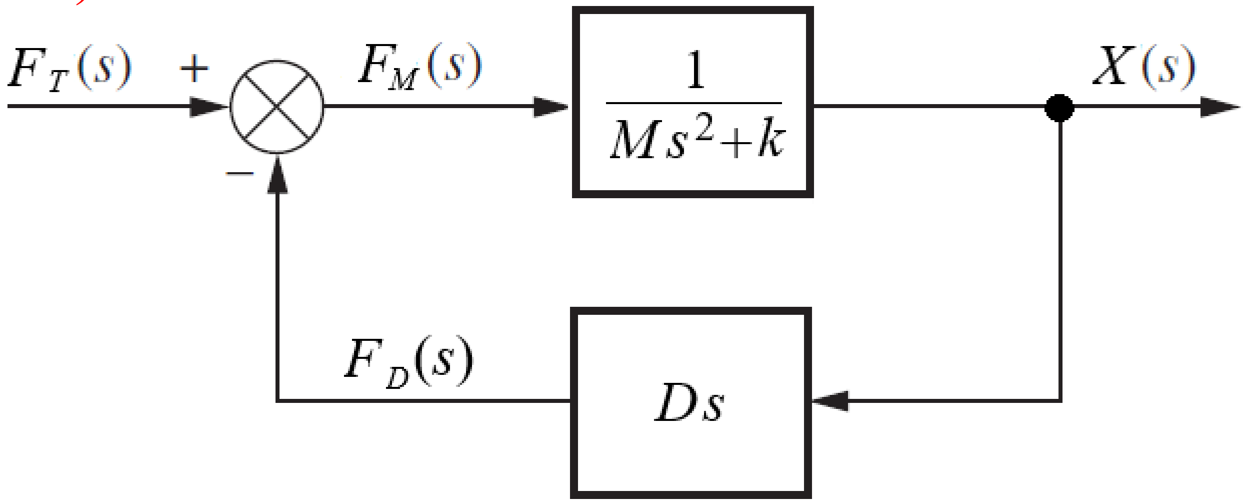
Reduce Paths of Block Diagram



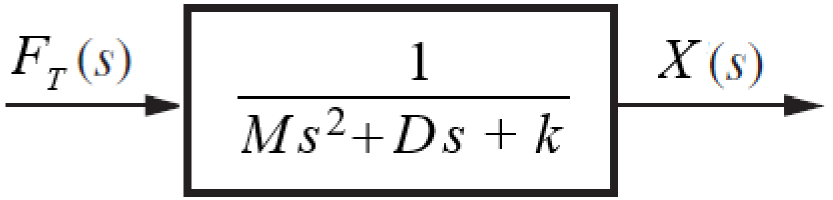
- Simplify paths of the block diagram, the equations of the system become:
  - $F_M(s) = Ms^2X(s)$ and $F_S(s) = kX(s)$ in top feedback system.
  - $F_T(s)$, $F_M(s) = Ms^2X(s)$ and $F_D(s) = DsX(s)$ in bottom feedback system.

Reduce Block Diagram Further

- Simplify the two parts that make up the block diagram (e.g. solve $F_M(s)$ which is formed from $F_M(s)$ and $F_S(s)$ in feedback):

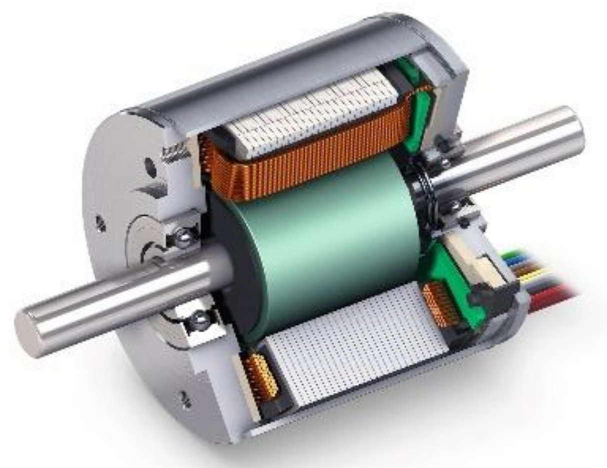


- The simplest block diagram (e.g. solve from $F_M(s)$ and $F_D(s)$ in feedback):



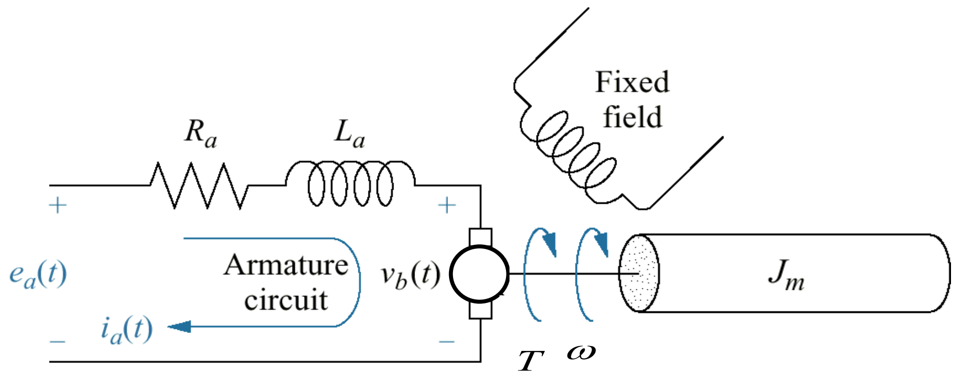
Example 2: Brushless DC Motor System

For a given electromechanical system given as a brushless direct current (DC) motor as shown below, derive a model the system. [40 marks]



Develop Physical Model of System

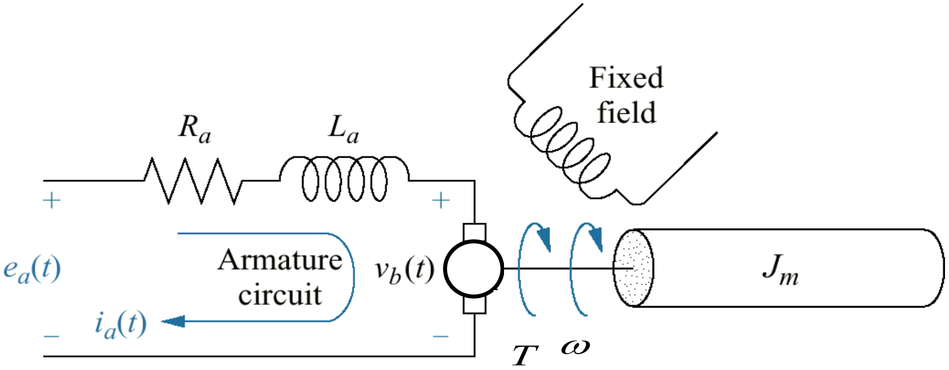
- Given in the following figure is a schematic diagram of a brushless DC motor that outlines the typical components that make up electrical and mechanical components of the motor.



- For the electrical components, there are armature circuit and fixed field circuit. The armature circuit consists of armature resistance (R_a) that is in series with armature inductance (L_a) and back EMF of the motor ($v_b(t)$).

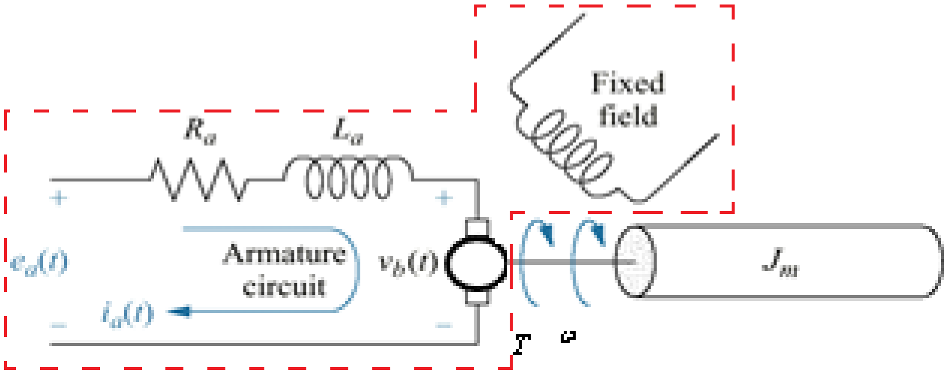
Develop Physical Model of System

- The winding in the fixed field circuit produces electromagnetic flux that interact with flux generated at armature that is energised by armature voltage ($e_a(t)$) producing torque (T).
- In most model of the DC motor, we tend not to include the fixed field winding circuit in the modelling.
- For mechanical part, the torque generated (T) is used to turn the motor shaft (i.e. overcoming its inertia (J_m) and load if it is connected) at specified angular velocity ($\omega(t)$).
- Thus, we need to form a relationship between input voltage (e.g. $e_a(t)$) and output velocity ($\omega(t)$):

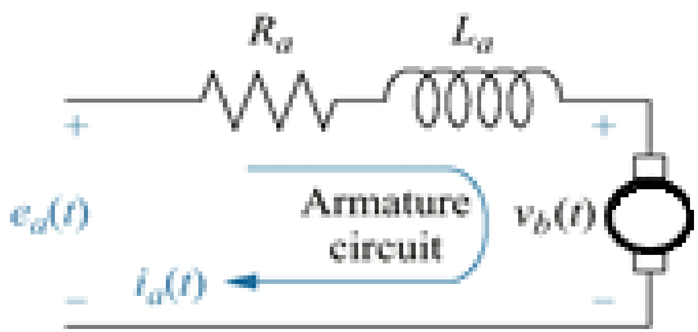


Determine Electrical Components

- The following figure shows the main electrical components of a brushless DC motor.



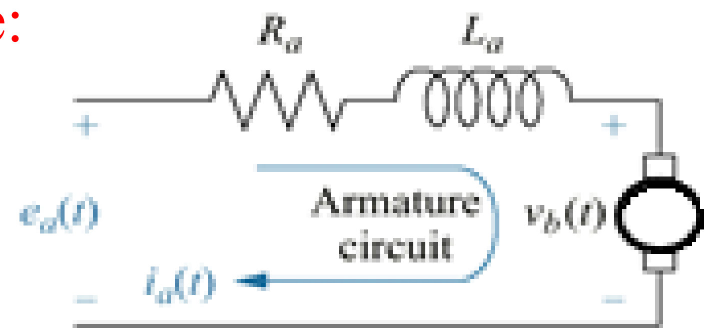
- Notice the three elements in the armature circuit e.g. armature inductance, armature resistor, and back EMF of the motor.



Determine Electrical Components

- Now, include armature inductance:

$$v_L(t) = L_a\left[\frac{di_a(t)}{dt}\right]$$

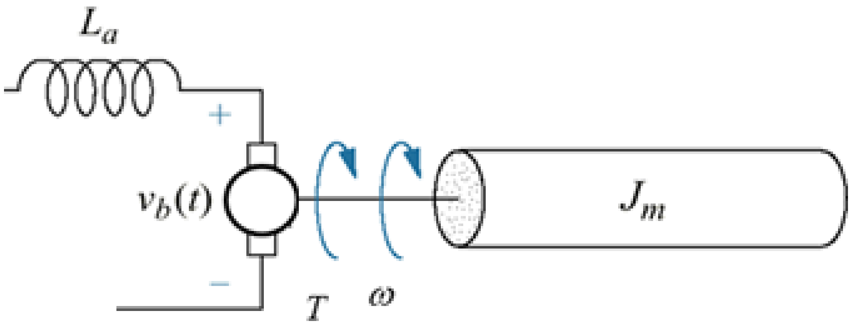


- Armature resistor:

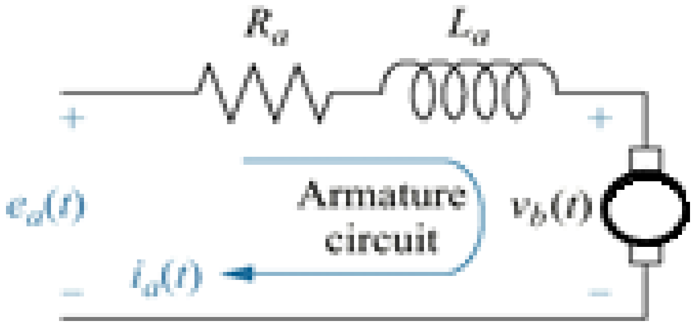
$$v_R(t) = i_a(t)R_a$$

- Back EMF of motor (i.e. connecting electrical system with mechanical system) and its block diagram:

$$v_b(t) = K_e\omega(t)$$



Derive Equation of Electrical System



- Apply KVL to the armature circuit:

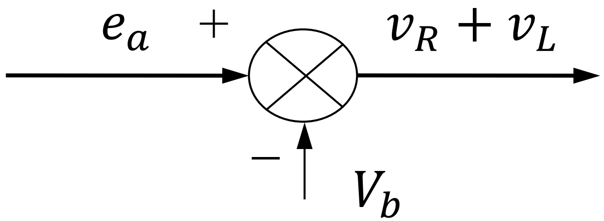
$$e_a(t) = v_R(t) + v_L(t) + v_b(t)$$

- Arrange the equation above:

$$e_a(t) - v_b(t) = v_R(t) + v_L(t)$$

- The following figure shows the block diagram of the armature circuit components of the brushless DC motor.

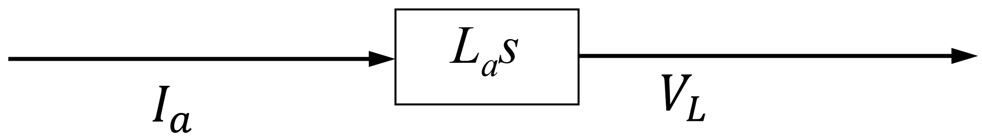
$$e_a - v_b = v_R + v_L$$



Derive Equation of Electrical System

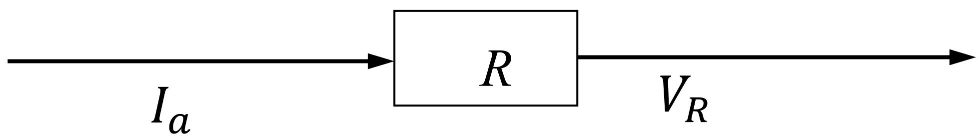
- Apply the Laplace transform and gather electrical components to the block diagram:

$$v_L = L_a \frac{di_a}{dt}$$



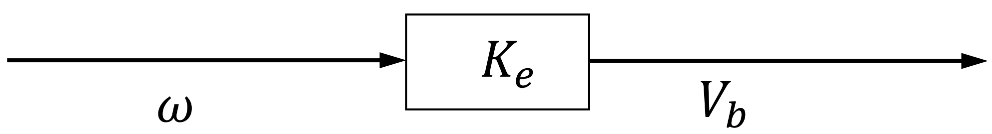
$$V_L(s) = L_a s I_a(s)$$

$$v_R = i_a R_a$$



$$V_R(s) = R_a I_a(s)$$

$$v_b = K_e \omega$$

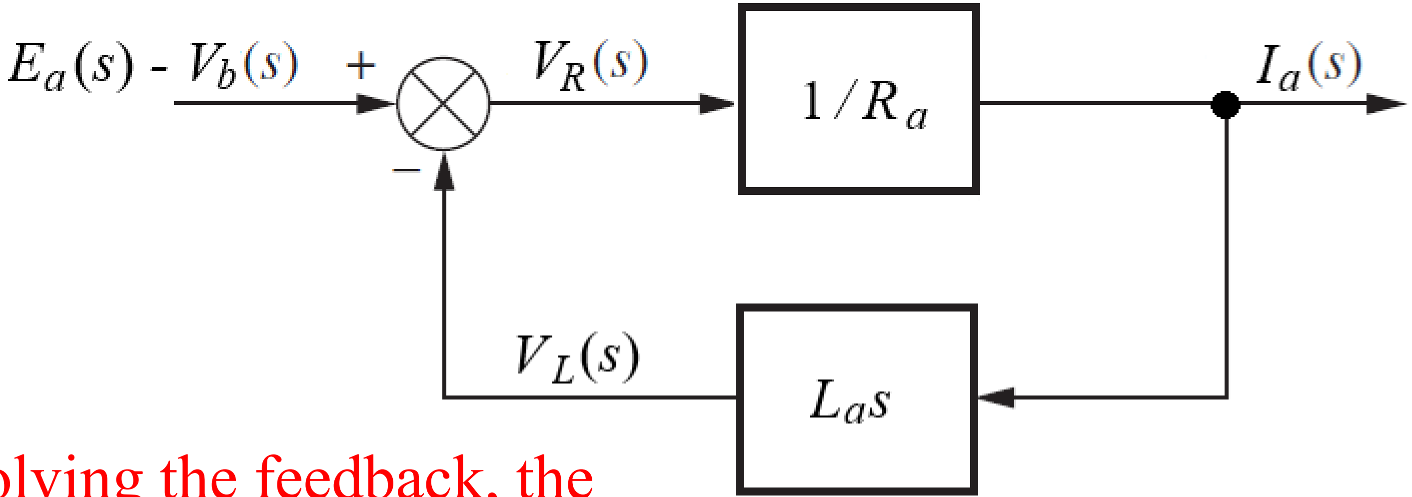


$$V_b(s) = K_e \omega(s)$$

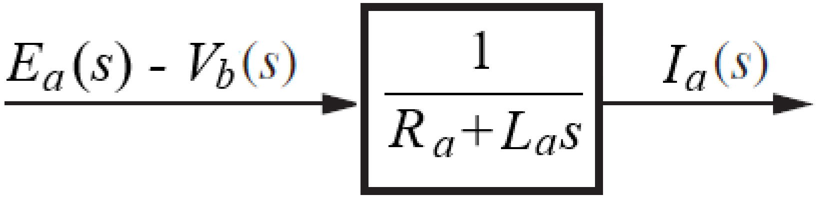
Model Electrical System in Block Diagram

- Represent the electrical component as a feedback (e.g. $V_L(s)$ is in feedback loop):

$$E_a(s) - V_b(s) - V_L(s) = V_R(s)$$

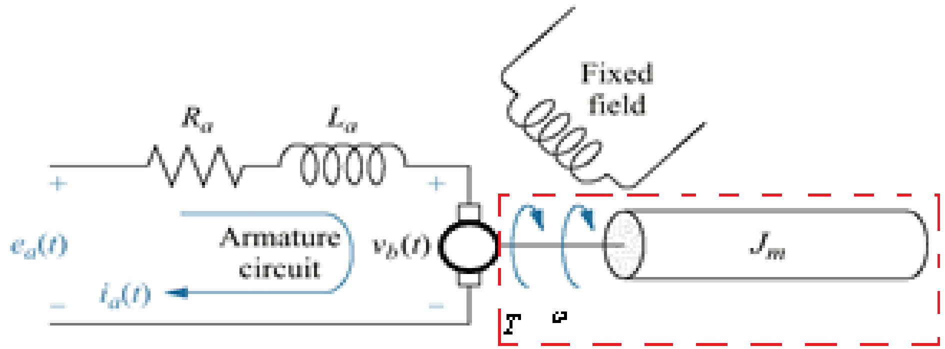


- Solving the feedback, the final block diagram of the main electrical components is shown below.

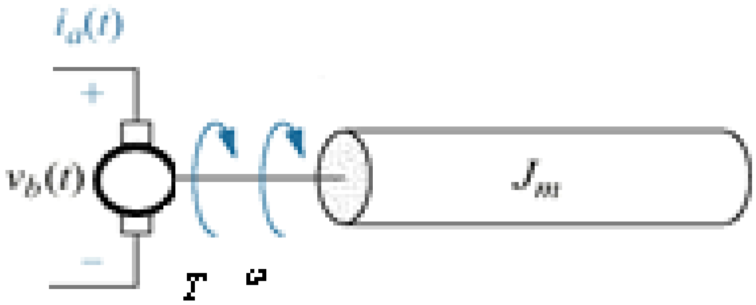


Determine Mechanical Components

- The diagram given below shows the main mechanical components of the DC motor system.



- The developed torque in the armature opposes torque due to inertia (J_m) and the load of motor (not shown in the diagram).



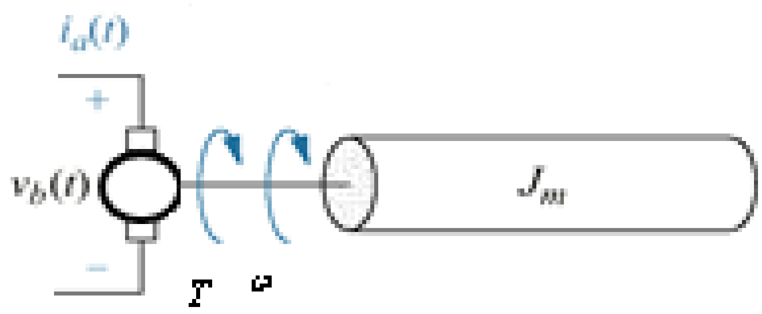
Determine Mechanical Components

- Torque proportional to armature current:

$$T(t) = K_T i_a(t)$$

- Torque is opposed by the inertia torque:

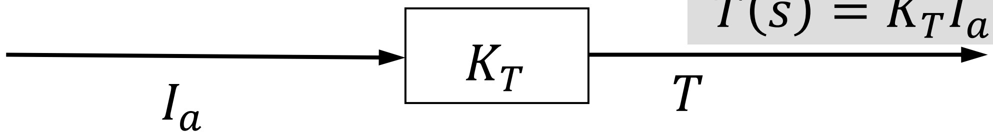
$$T(t) = J\left(\frac{d\omega(t)}{dt}\right)$$



Derive Equation of Mechanical System

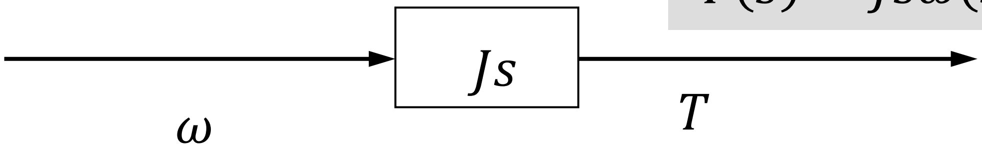
- Apply the Laplace transform and gather mechanical components to the block diagram:

$$T = K_T i_a$$



$$T(s) = K_T I_a(s)$$

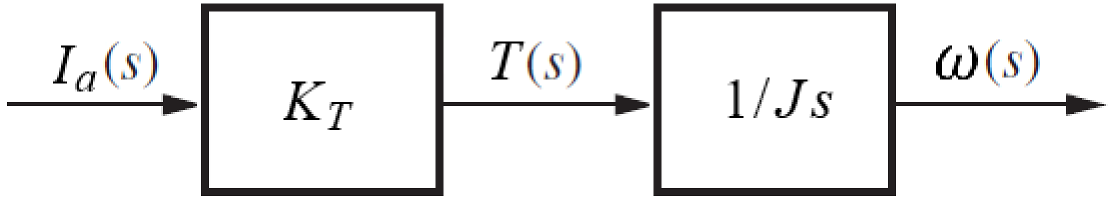
$$T = J\frac{d\omega}{dt}$$



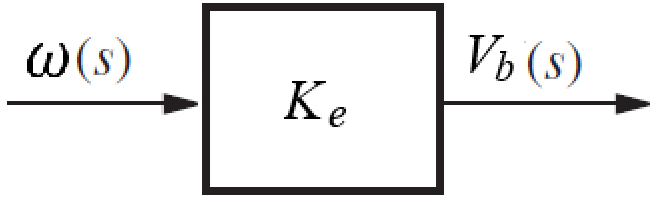
$$T(s) = Js\omega(s)$$

Model Mechanical System to Block Diagram

- Model the mechanical system into the block diagram.
- We need to invert the torque and inertia block diagram to combine both the torque and armature current block and the torque and the inertia block.

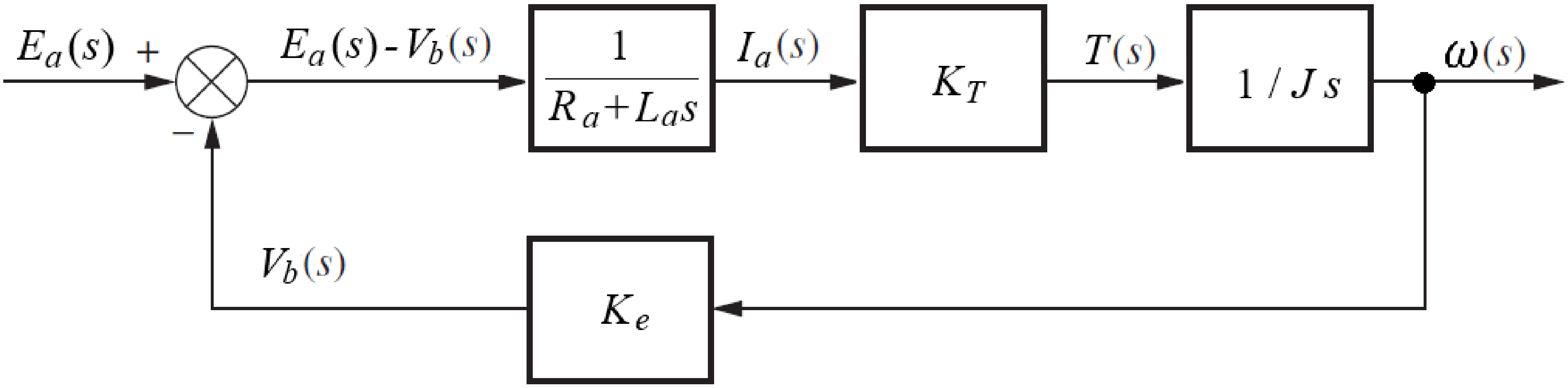


- And, we have the black diagram that connects the mechanical subsystem of the brushless motor with its electrical subsystem.



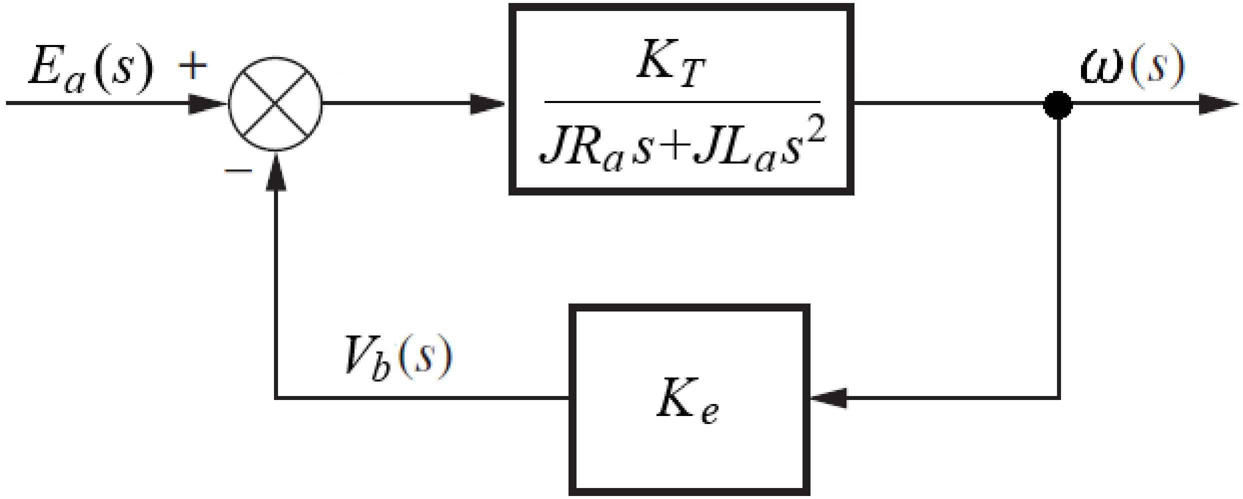
Combine Mechanical and Electrical System

- Put the variables of the mechanical system into the block diagram:



Reduce Block Diagram

- Reduce the block diagram by combining the three blocks in the forward loop:



- The final block diagram of the electromechanical system is shown below.