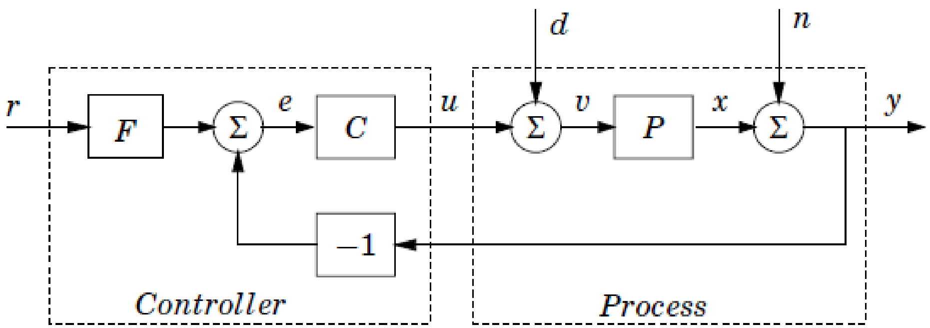
Feedback Control Systems

XMUT315 Control Systems Engineering

Topics

- Introduction to feedback control systems.
- Mechanisms in feedback control systems.
- Effect of feedback on gain, stability, noise and sensitivity.
- Input functions for evaluation and testing.
- Feedforward systems.
- Controller or compensator in feedback system.
- Examples of controller or compensator in feedback system.

Introduction to Feedback Control



For a block diagram of feedback system:

- P is process
- C is controller
- F is feedforward

Signals in the feedback control:

- x is the real physical variable to control
- u is the control variable
- r is reference signal
- e is error signal
- n is measurement noise
- d is load disturbance
- v is control variable distorted by disturbance
- y is process variable distorted by noise

Introduction to Feedback Control

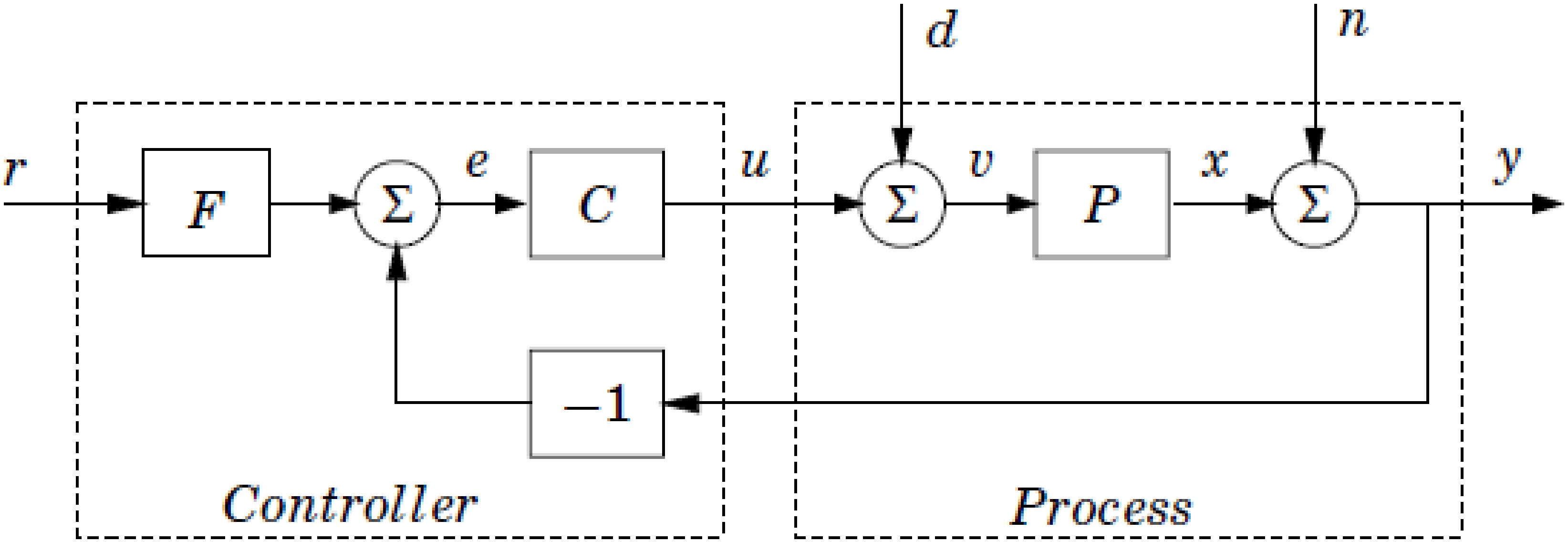
Many issues must be considered in analysis and design of control systems. Some of the basic requirements are:

- Stability of the systems - Avoiding instability is a primary goal.
- Ability to follow reference signals - It is also desirable that the process variable follows the reference signal faithfully.
- Reduction of effects of load disturbances - The system can reduce the effect of load disturbances.
- Reduction of effects of measurement noise - it is essential that not too much noise is injected.
- Reduction of effects of model uncertainties - models might be inaccurate and properties of the process may also change. The control system should be able to cope with moderate changes.

Introduction to Feedback Control

- The following relations are obtained from the block diagram in feedback control systems.

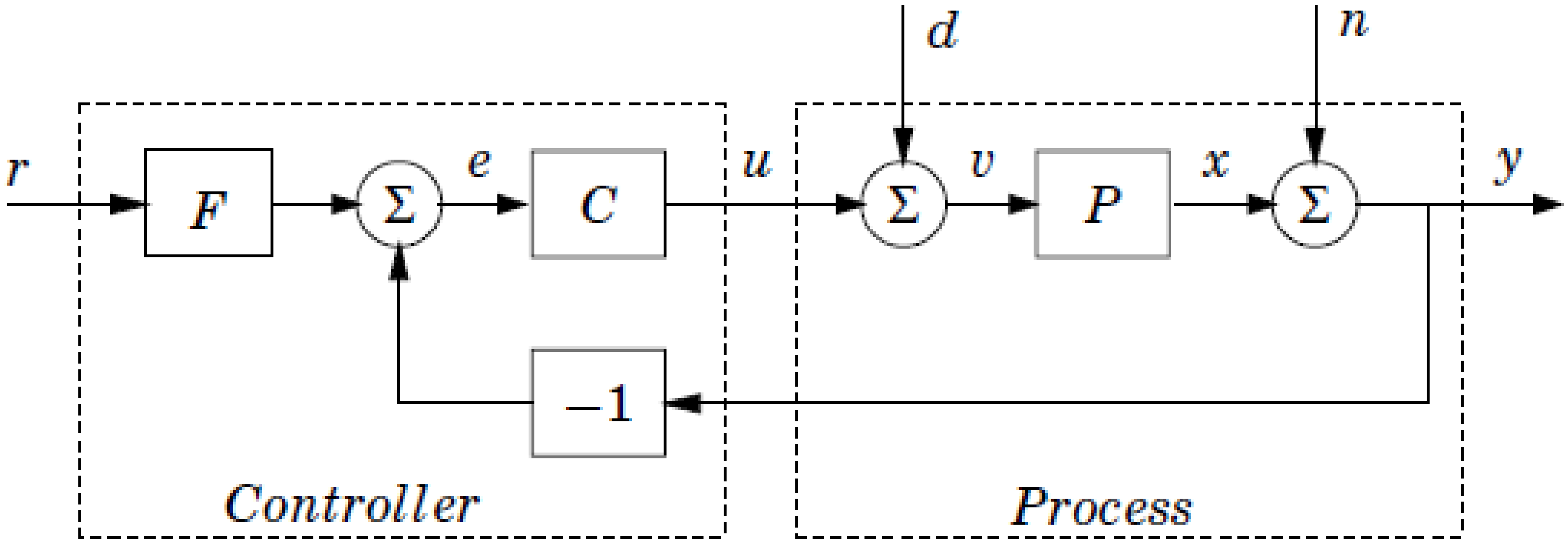
$$X(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right] D(s) - \left[\frac{P(s)C(s)}{1 + P(s)C(s)}\right] N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right] R(s)$$



Introduction to Feedback Control

- The following relations are obtained from the block diagram in feedback control systems.

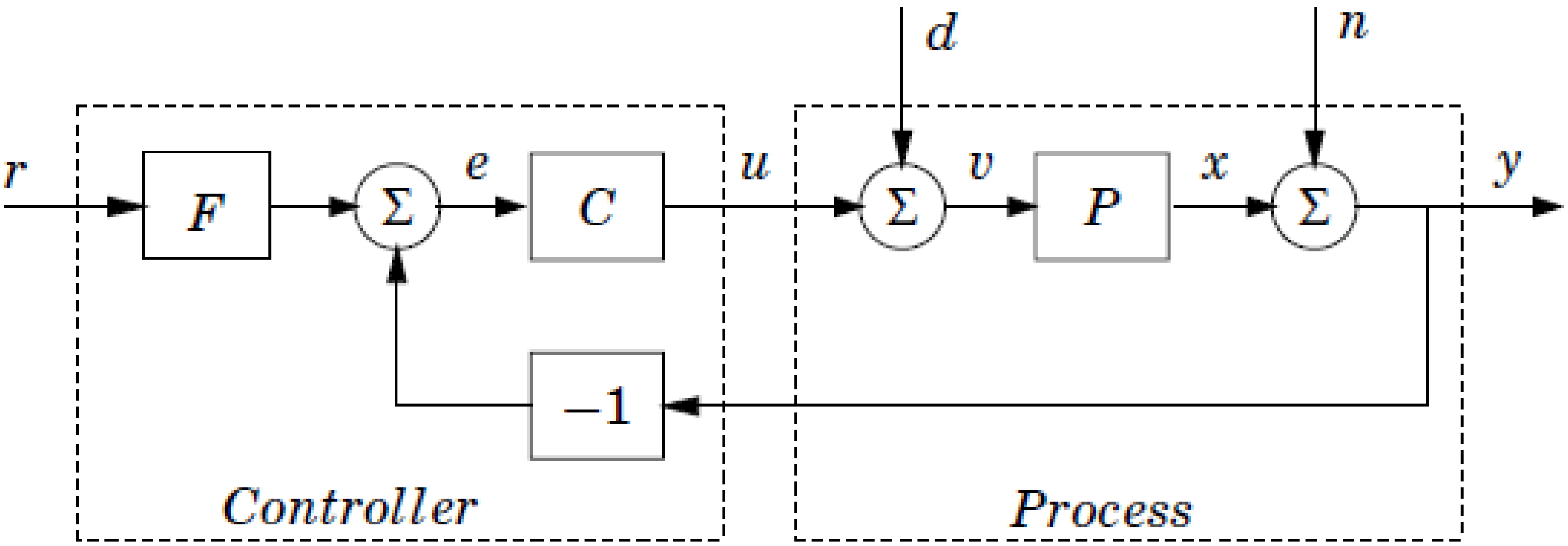
$$Y(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right] D(s) + \left[\frac{1}{1 + P(s)C(s)}\right] N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right] R(s)$$



Introduction to Feedback Control

- The following relations are obtained from the block diagram in feedback control systems.

$$U(s) = -\left[\frac{P(s)C(s)}{1+P(s)C(s)}\right]D(s) - \left[\frac{C(s)}{1+P(s)C(s)}\right]N(s) + \left[\frac{C(s)F(s)}{1+P(s)C(s)}\right]R(s)$$

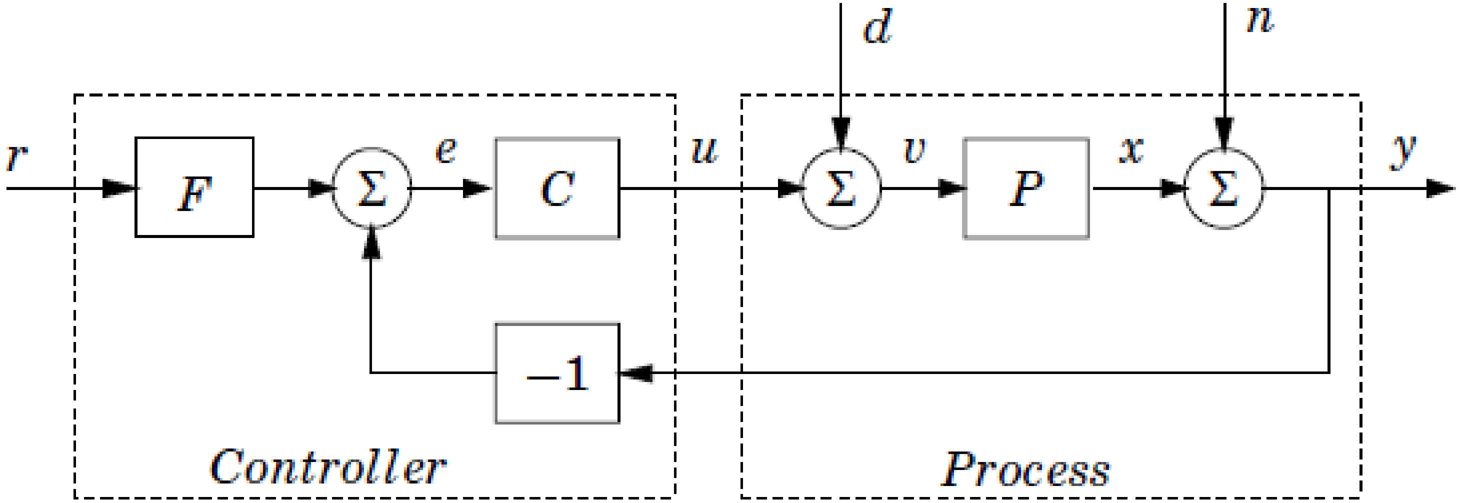


Introduction to Feedback Control

- Several transfer functions are the same and that all relations are given by the following set of six transfer functions.

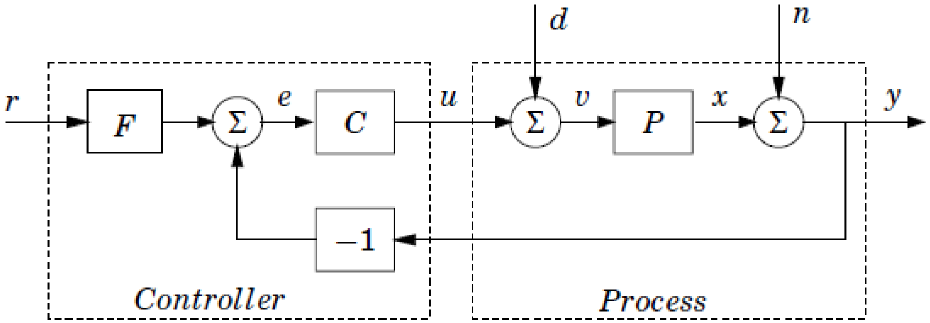
$$\frac{PCF}{1+PC} \qquad \frac{PC}{1+PC} \qquad \frac{P}{1+PC}$$

$$\frac{CF}{1+PC} \qquad \frac{C}{1+PC} \qquad \frac{1}{1+PC}$$



Introduction to Feedback Control

- When $F = 1$ i.e. a system with (pure) error feedback.



$$\frac{PC}{1+PC}$$ Complementary sensitivity function

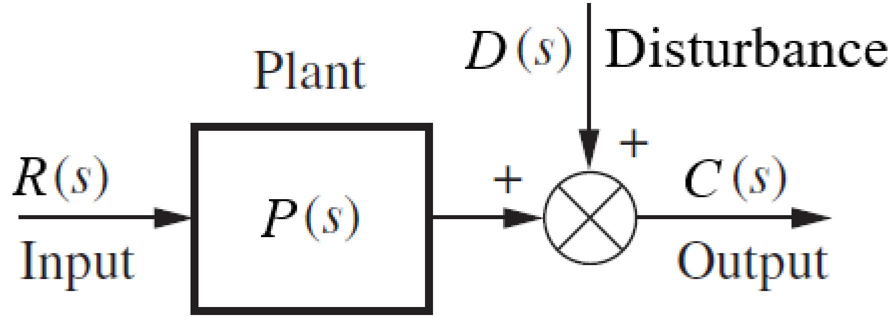
$$\frac{P}{1+PC}$$ Load disturbance sensitivity function

$$\frac{C}{1+PC}$$ Load disturbance sensitivity function

$$\frac{1}{1+PC}$$ Sensitivity function

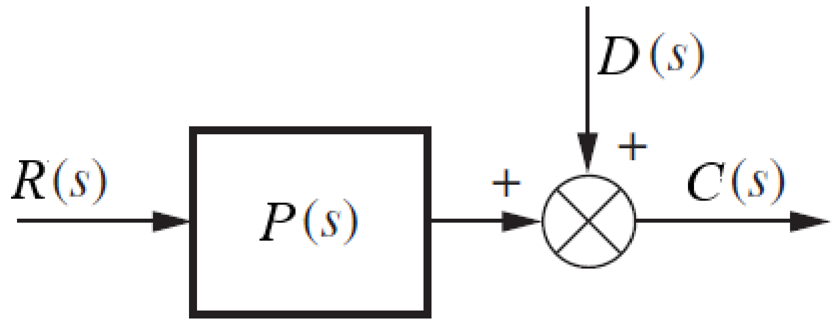
Mechanisms in Feedback Control

- We wish for the given system a controlled output i.e. output = input, despite disturbances.
- This is achieved by adding feedback system.
- Open-loop systems (i.e. without feedback).
- Process with transfer function *P*, perturbed by a disturbance *D*.
- Depending on its magnitude, the disturbance might affect the system significantly.



Example of Open-Loop Control

For an open-loop system as shown below, attempt the following tasks.



a. Suppose that the P is 10 and disturbance D is 0, determine the value of R so, the output of the system (C) becomes 1.

[2 marks]

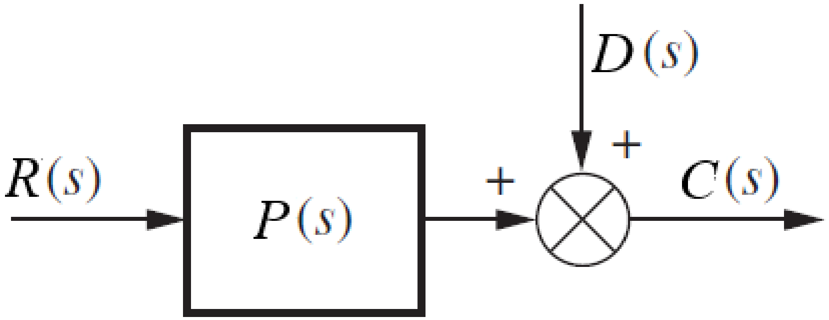
b. When P changes by 10% to 11, determine the value of output (C). Describe what would happen if disturbance were changed by 0.1. [2 marks]

Example of Open-Loop Control

a. If the output C is to be 1, just make input $R = 0.1$.

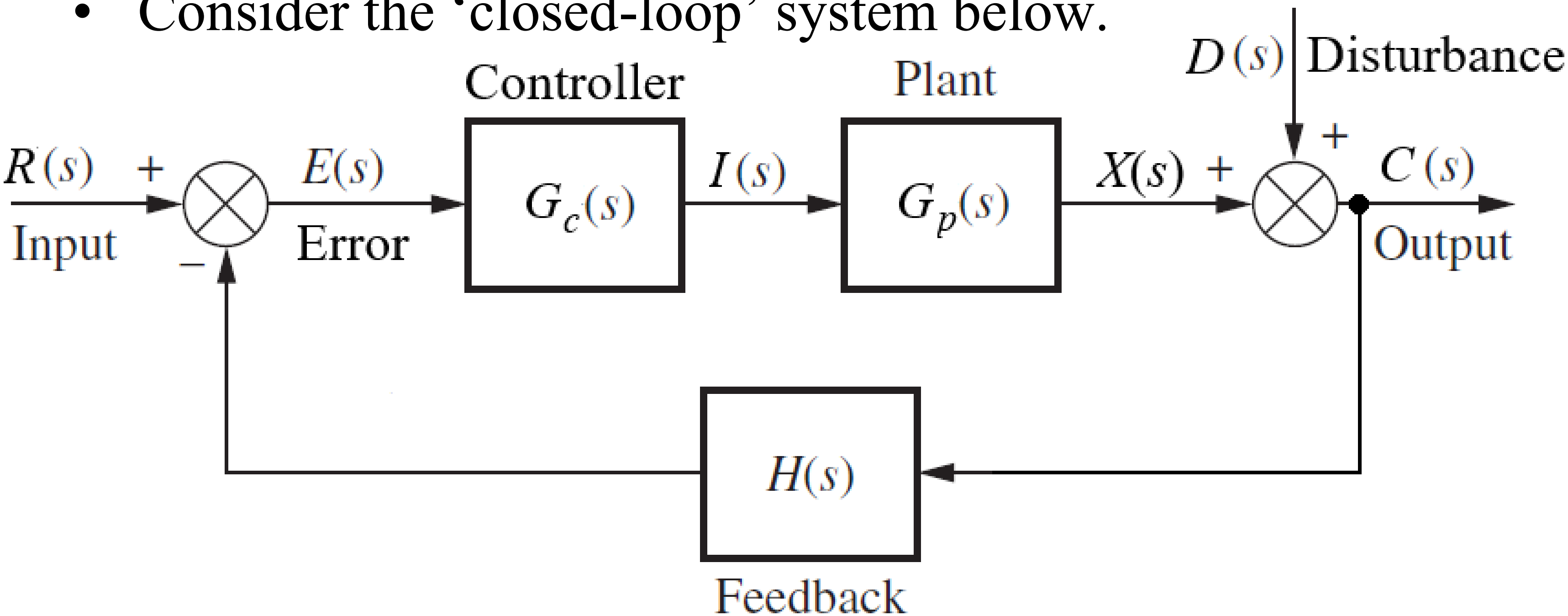
b. But, if P changes by 10% to 11 then C changes by 10% to 1.1.

If disturbance D is 0.1, then C will also change by 0.1.



Mechanisms in Feedback Control

- Now, we have a closed-loop system (i.e. feedback added).
- Consider the 'closed-loop' system below.



- G_P represents the system or device being controlled and G_C is the controller. Thus:

$$C = \left(\frac{G_C G_P}{1 + G_C G_P}\right) R + \left(\frac{1}{1 + G_C G_P}\right) D$$

Mechanisms in Feedback Control

- Considering unity feedback, $H(s) = 1$, error signal (E) is:

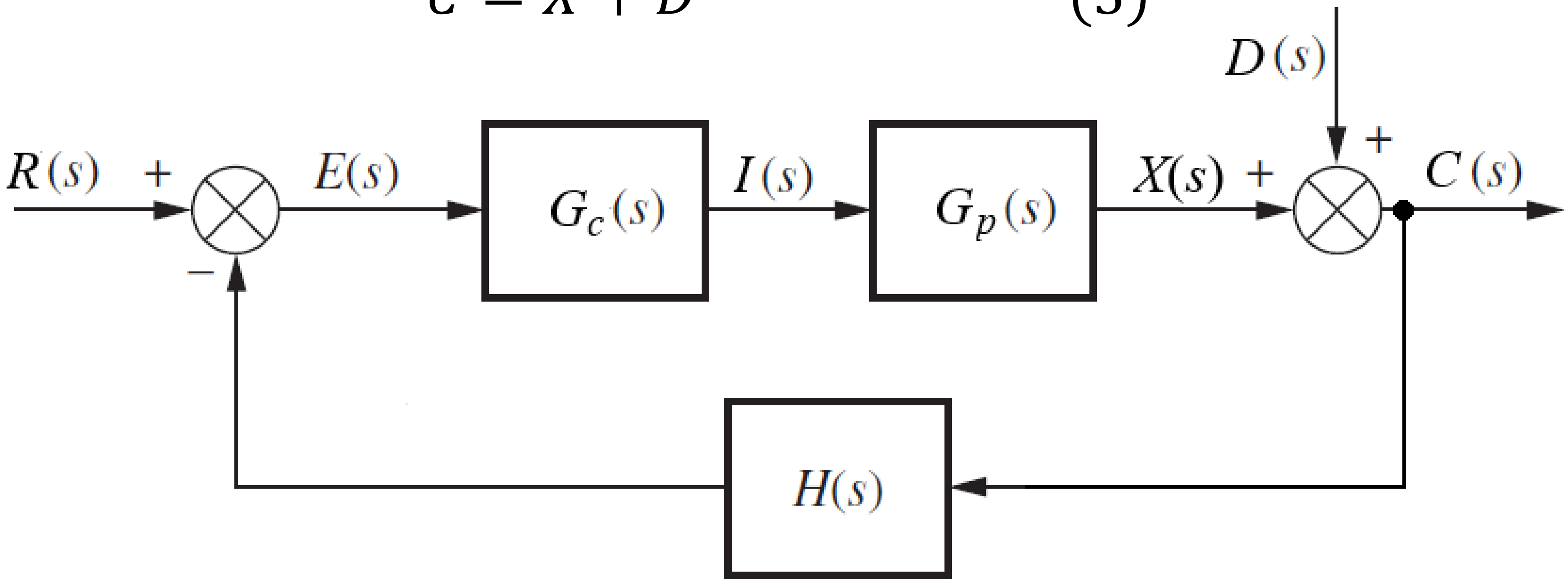
$$E = R - C \quad (1)$$

- Plant output (X) is:

$$X = E(G_C)(G_P) \quad (2)$$

- Output of feedback system with disturbance (C):

$$C = X + D \quad (3)$$



Mechanisms in Feedback Control

- From (3) knowing $X = C - D$, substitute X in (2):

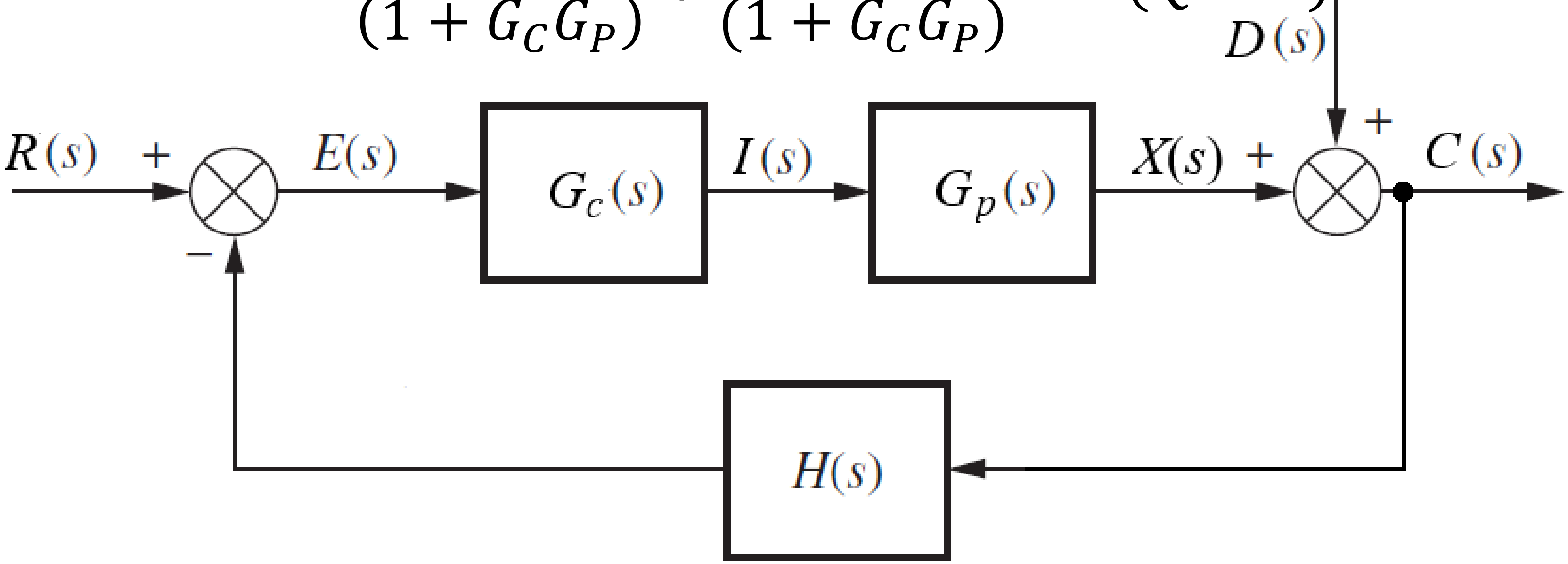
$$C - D = E(G_C)(G_P) \qquad (4)$$

- Substituting E in (4) with (1):

$$C - D = (R - R)(G_C)(G_P)$$

- Rearranging the equation above:

$$C = \frac{(G_C G_P)R}{(1 + G_C G_P)} + \frac{D}{(1 + G_C G_P)} \qquad \text{(Q.E.D)}$$



Example of Feedback Control

For a unity feedback system equation as shown below, ignoring disturbances, perform the following tasks:

$$C = \left(\frac{G_C G_P}{1 + G_C G_P}\right) R + \left(\frac{1}{1 + G_C G_P}\right) D$$

a. If G_P is 10 and G_C and R are 1 and 10 respectively, determine the value of output (C). [2 marks]

b. Now, change G_P to 11, but G_C and R are the same, determine the value of output (C). [2 marks]

c. Comment on the results obtained in part (a) and (b). [2 marks]

Example of Feedback Control

a. Ignoring disturbances ($D = 0$), by applying feedback equation:

$$C = \left(\frac{G_C G_P}{1 + G_C G_P}\right) R$$

Let $G_P = 10$, as before, and $G_C = 10$;

$$C = \left[\frac{(10)(10)}{1 + (10)(10)}\right] R = \left(\frac{100}{101}\right) R = (0.99) R$$

If R is 1, then C is 0.99 (i.e. it is within 1% of being 1).

b. If G_P changes to 11, $C = R$ (110/111) = 0.99. Thus, unlike open-loop, if R is 11, C is still about 0.99.

Example of Feedback Control

c. Negative feedback:
   - reduces effects on output of parameter changes.
   - makes output almost the same as input.

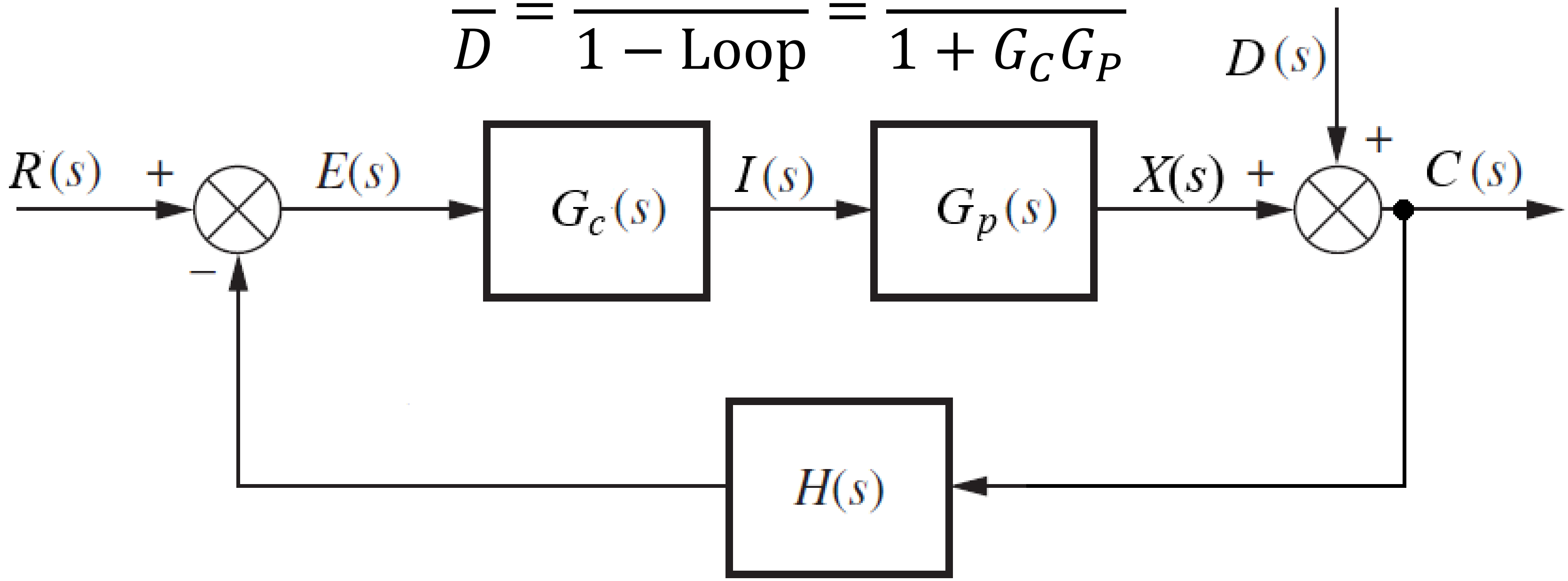
Disturbances Control

- For a unity feedback system ($H(s) = 1$) with disturbance, its equation is:

$$C = \left(\frac{G_C G_P}{1 + G_C G_P}\right) R + \left(\frac{1}{1 + G_C G_P}\right) D$$

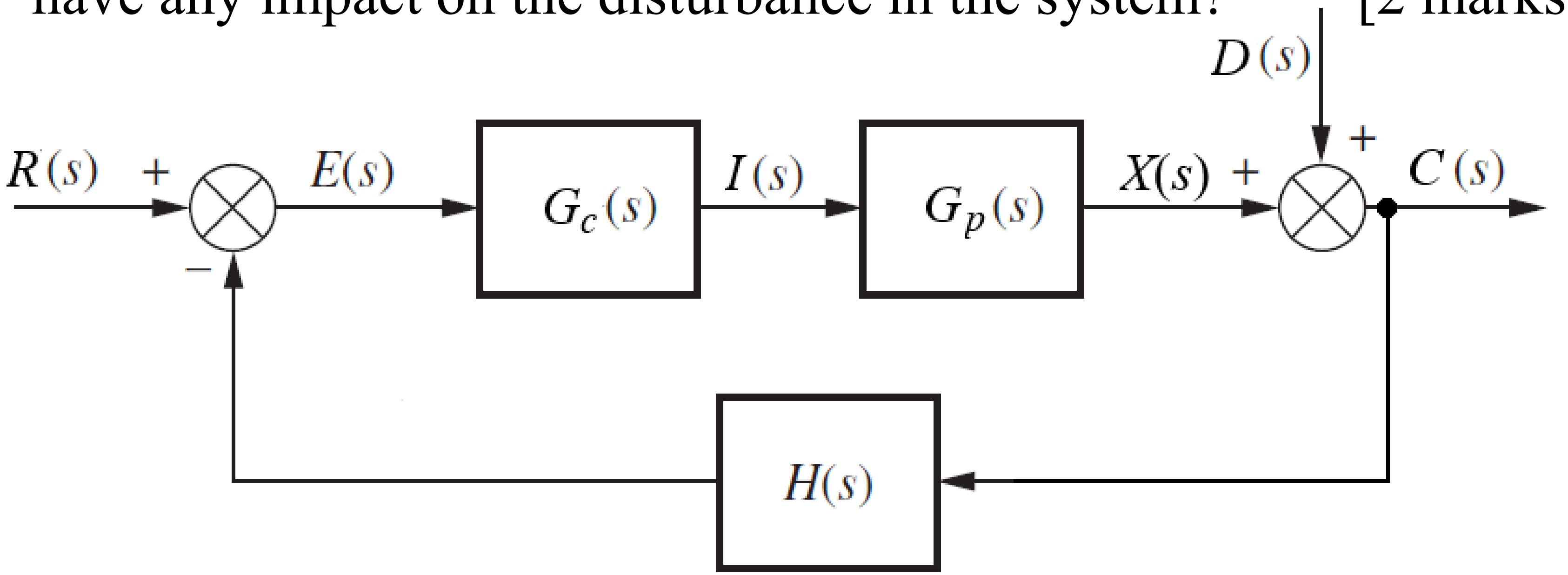
- To see the effect of disturbances, assume R is 0, then:

$$\frac{C}{D} = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{1}{1 + G_C G_P}$$



Example of Disturbances Control

For a unity feedback system with disturbance, determine the output of the system (C) if $G_C = 10$, $G_P = 10$ and $D = 0.1$. Does feedback have any impact on the disturbance in the system? [2 marks]

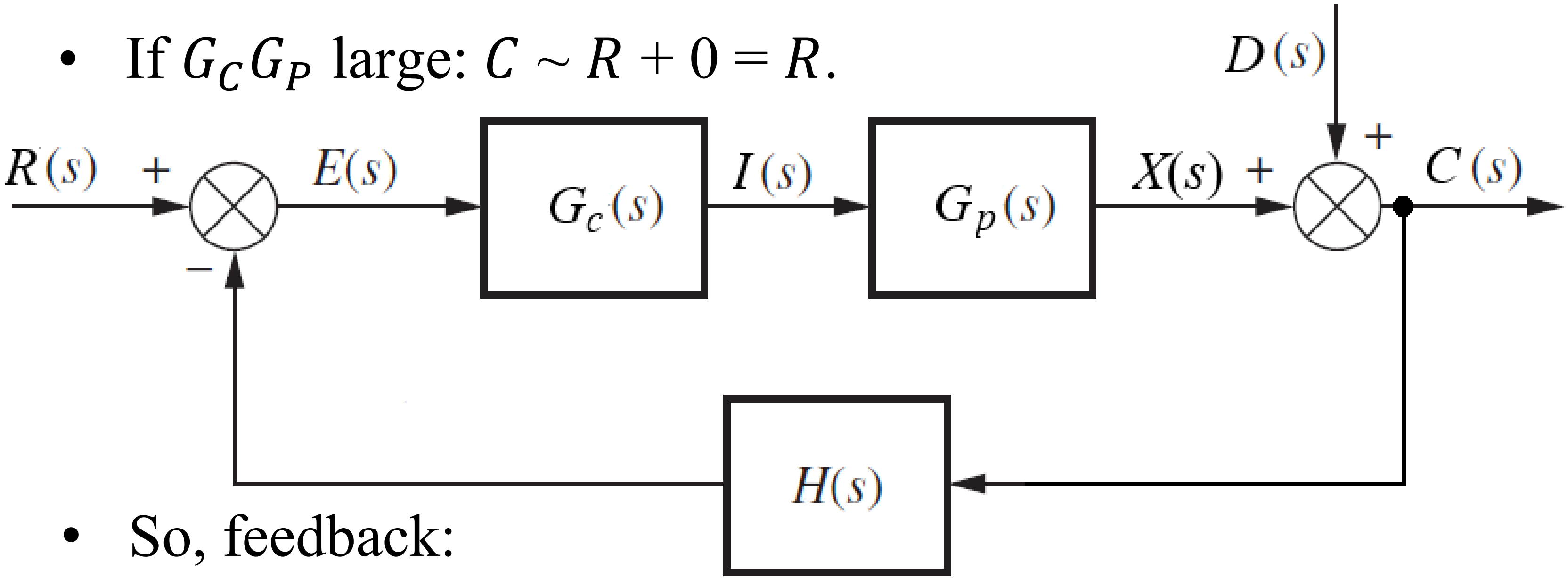


For $H(s) = 1$, $G_C = 10$, $G_P = 10$ and $D = 0.1$, output due to disturbance:

$$C = \frac{\text{Forward}}{1 - \text{Loop}} = \frac{1}{1 + G_C G_P} = \left[\frac{1}{1 + (10)(10)}\right] 0.1 = 0.00099$$

Principle of Superposition

- If $G_C G_P$ large: $C \sim R + 0 = R$.

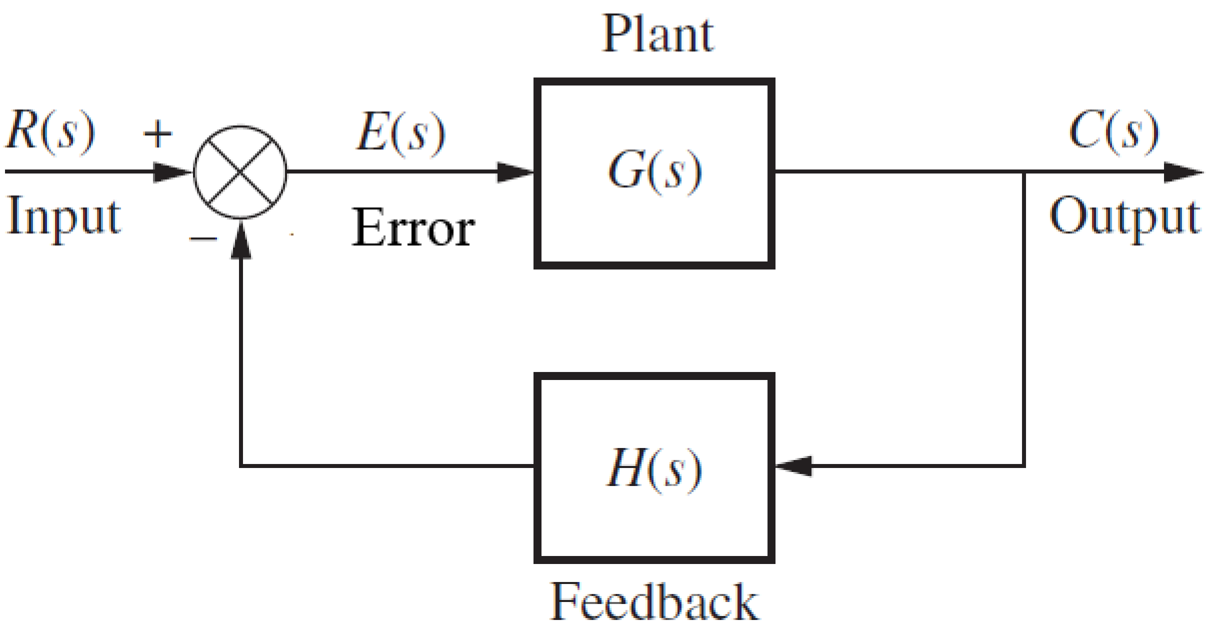


- So, feedback:
  - makes output almost the same as input.
  - reduces the effect of change in the device.
  - Minimises the effects of disturbances.
- This is true because the ‘loop gain’, $G_C G_P$, is high.

$$C = \left(\frac{G_C G_P}{1 + G_C G_P}\right) R + \left(\frac{1}{1 + G_C G_P}\right) D$$

Effect of Feedback on Gain

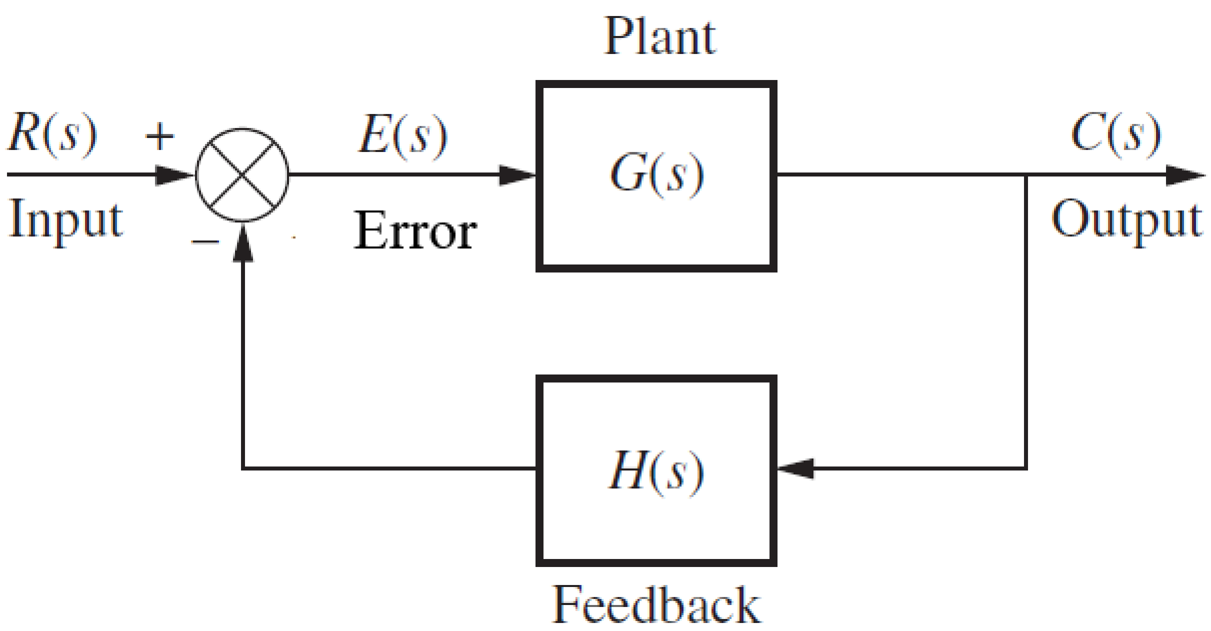
- Note: We can't just keep increasing the 'gain' of $G(s)$. We need to consider the dynamics of the blocks, e.g. there could be a block in the feedback path which would be affected.
- Negative feedback reduces the error between the reference input, $R(s)$ and system output ($C(s)$).



Effect of Feedback on Gain

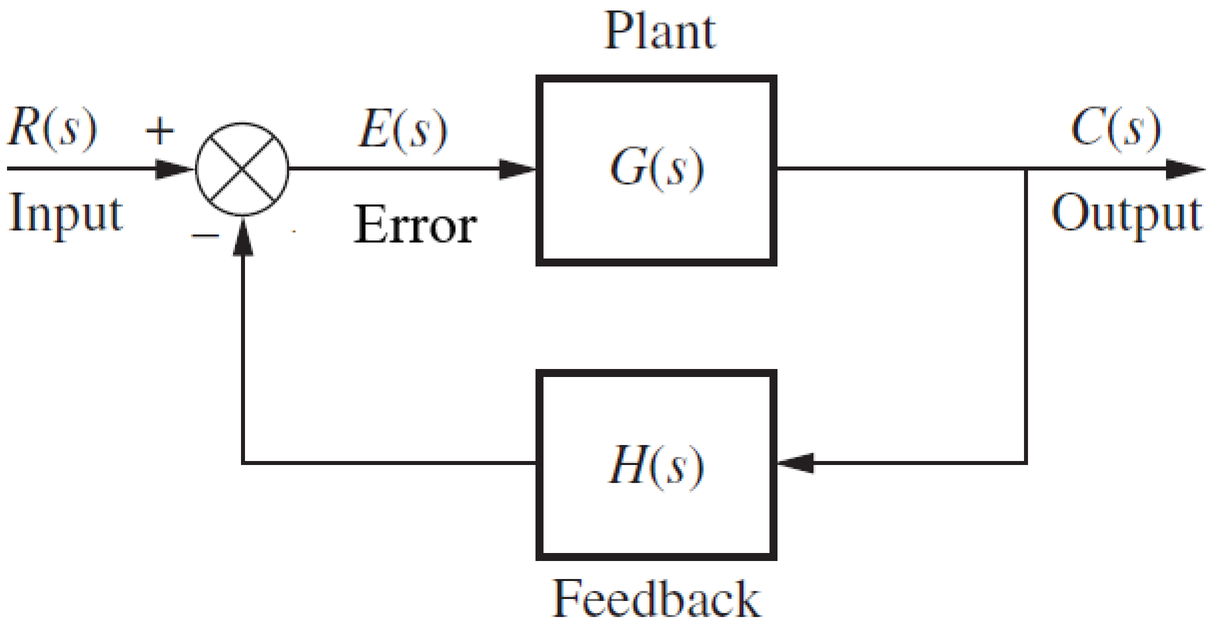
- From the equation below, the overall gain of closed-loop control system with negative feedback is the ratio of 'G' and $(1 + GH)$. Thus:

$$\text{Gain} = \left|\frac{G}{1 + GH}\right| \qquad \text{and} \qquad T = \frac{G}{1 + GH}$$



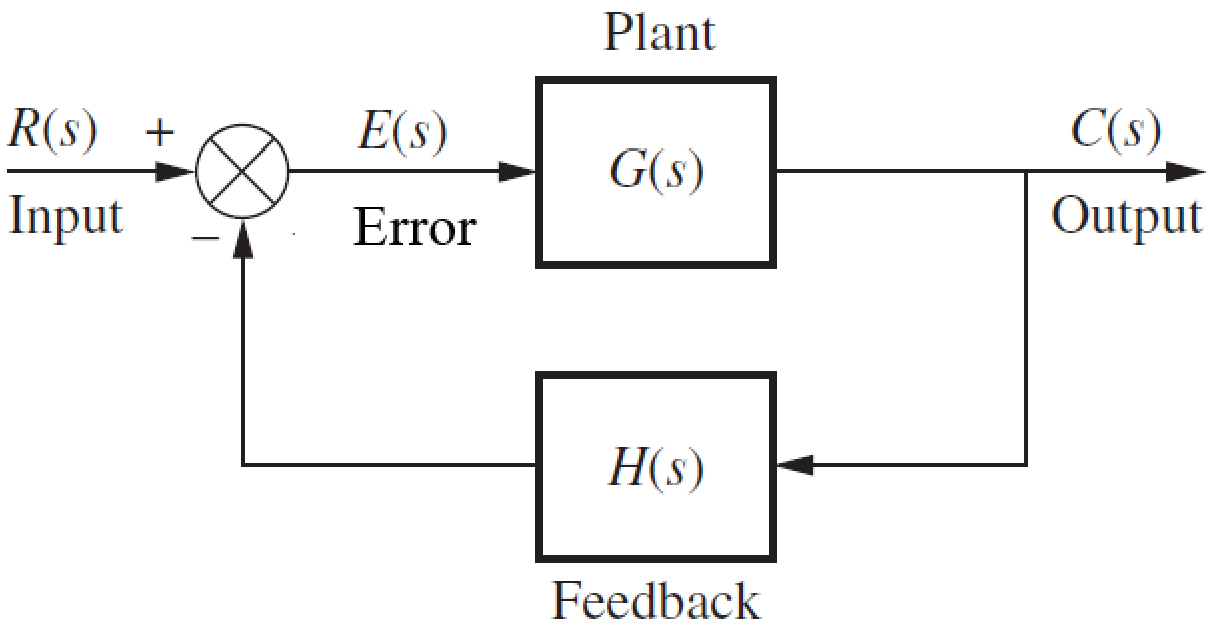
Effect of Feedback on Gain

- So, the overall gain may increase or decrease depending on the value of $(1 + GH)$.
  - If $(1 + GH) < 1$, the overall gain increases -> 'GH' < 0 (gain of the feedback path is negative).
  - If $(1 + GH) > 1$, the overall gain decreases -> 'GH' > 0 (gain of the feedback path is positive).



Effect of Feedback on Stability

- In general, 'G' and 'H' are functions of frequency.
- The feedback will increase the overall gain of the system in one frequency range and decrease in the other frequency range.



Effect of Feedback on Stability

- A system is said to be stable if its output is under control. Otherwise, it is considered unstable.

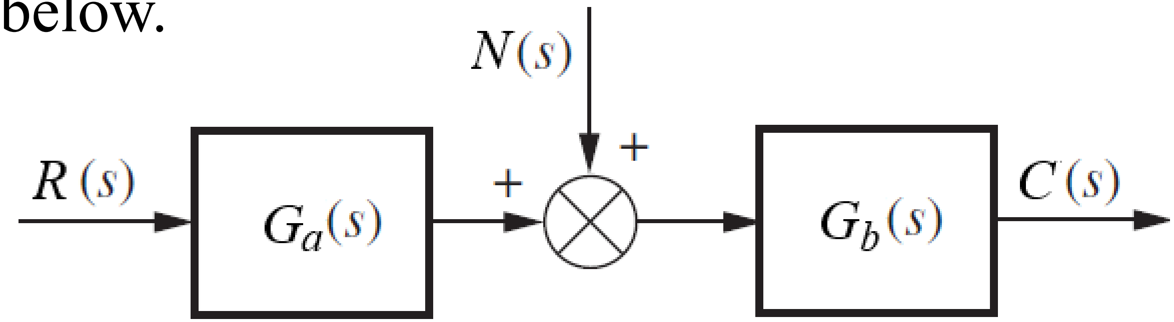
$$T = \frac{G}{1 + GH}$$

(for example, $1 + GH$ is a characteristic equation)

- In the equation above, if the denominator value is zero (i.e. $GH = -1$), then the output of the control system will be infinite. So, the control system becomes unstable.
- We must properly choose the feedback to make the control system stable.
- We will look more closely the stability of control system in the subsequent topic in this course.

Effect of Feedback on Noise

- To know the effect of feedback on noise, let us compare the transfer function relations with and without feedback due to noise signal alone.
- Consider an open-loop control system with noise signal as shown below.

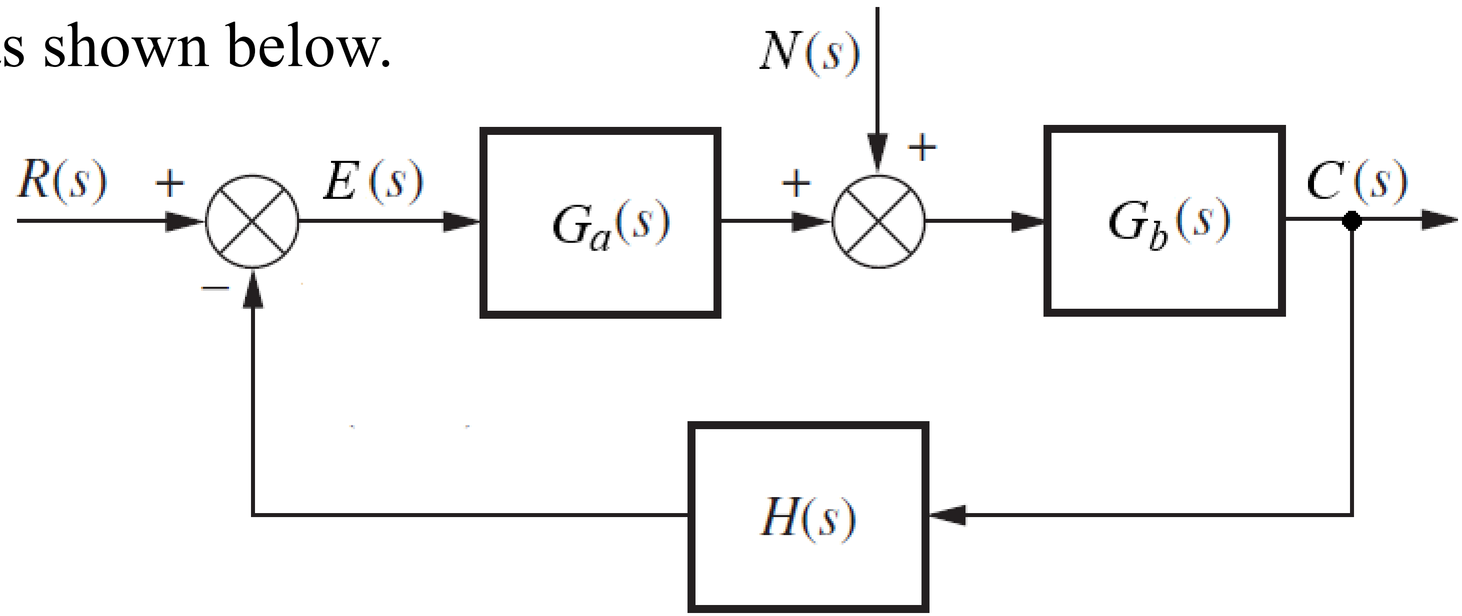


- By making the other input $R(s)$ equal to zero, the open-loop transfer function due to noise signal alone is:

$$\frac{C(s)}{N(s)} = G_b$$

Effect of Feedback on Noise

- Consider a closed-loop control system with noise signal ($N(s)$) as shown below.



- By making the other input $R(s)$ equal to zero, the closed-loop transfer function equation due to noise signal alone is:

$$\frac{C(s)}{N(s)} = \frac{G_b}{(1 + G_a G_b H)}$$

Effect of Feedback on Noise

- When we compare the open-loop transfer function equation due to noise with the closed-loop transfer function equation due to noise, feedback reduce the noise in the system
- In the closed-loop control system, the gain due to noise signal is decreased by a factor of $(1 + G_a G_b H)$ provided that the term $(1 + G_a G_b H)$ is greater than one.
- Feedback reduces impact of the noise on the system.

Sensitivity of System Parameters

- The changes in system parameters affect the behaviour of a system.
- Ideally, parameter changes due to heat, or other causes should not appreciably affect a system's performance.
- The degree to which changes in system parameters affect system transfer functions, and hence performance, is called sensitivity.
- A system with zero sensitivity (that is, changes in the system parameters have no effect on the transfer function) is ideal.
- The greater the sensitivity, the less desirable the effect of a parameter change.

Sensitivity of System Parameters

- Let us formalise a definition of sensitivity:

  Sensitivity is the ratio of the fractional change in the function to the fractional change in the parameter as the fractional change of the parameter approaches zero.

- That is,

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\text{Fractional change in the function}, F}{\text{Fractional change in the parameters}, P}$$

$$= \lim_{\Delta P \to 0} \frac{\Delta F/F}{\Delta P/P} = \lim_{\Delta P \to 0} \frac{P\Delta F}{F\Delta P}$$

- which reduces to:

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{P\delta F}{F\delta P}$$

Effect of Feedback on Sensitivity

- Sensitivity of the overall gain of negative feedback closed-loop control system (T) to the variation in open-loop gain (G) is defined as:

$$S_G^T = \frac{\frac{\partial T}{T}}{\frac{\partial G}{G}} = \frac{\text{Percentage change in } T}{\text{Percentage change in } G}$$

Where: ∂T is the incremental change in T due to incremental change in G.

- We can rewrite the equation above as:

$$S_G^T = \frac{\partial T}{\partial G}\left(\frac{G}{T}\right)$$

Effect of Feedback on Sensitivity

- Perform partial differentiation with respect to G on both sides of basic equation for feedback system.

$$\frac{\partial T}{\partial G} = \frac{\partial\left(\frac{G}{1+GH}\right)}{\partial G}$$

- Since the partial differentiation is in the form of $u'/v' = (u'v - uv')/v^2$, thus:

$$\frac{\partial T}{\partial G} = \frac{(1)(1+GH) - (GH)}{(1+GH)^2} = \frac{1}{(1+GH)^2}$$

- Then, from basic equation for feedback system, you will get:

$$T = \frac{G}{1+GH} \qquad \text{thus} \qquad \frac{G}{T} = 1 + GH$$

Effect of Feedback on Sensitivity

- Substitute the above given equations as below.

$$S_G^T = \frac{\partial T}{\partial G}\left(\frac{G}{T}\right) = \left[\frac{1}{(1+GH)^2}\right](1+GH) = \frac{1}{(1+GH)}$$

- We got the sensitivity of the overall gain of closed-loop control system as the reciprocal of $(1+GH)$.
- Sensitivity may increase or decrease depending on the value of $(1+GH)$.
  - If $(1+GH) < 1$, sensitivity increases -> 'GH' value is negative because the gain of feedback path is negative.
  - If $(1+GH) > 1$, sensitivity decreases -> 'GH' value is positive because the gain of feedback path is positive.

Effect of Feedback on Sensitivity

- In general, 'G' and 'H' are functions of frequency.
- Feedback increase the sensitivity of the system gain in one frequency range and decrease in the other frequency range.
- We have to choose the values of 'GH' in such a way that the system is insensitive or less sensitive to parameter variations.

Example of Sensitivity of System Parameters

For example, assume a system described as a transfer function equation as shown below:

$$F = \frac{K}{K + a}$$

a. Determine the value of F when $K = 10$ and $a = 100$.

[2 marks]

b. If you triple the value of a, determine the value of F now.

[2 marks]

c. Comment on the results obtained in parts (a) and (b).

[2 marks]

d. Describe advantage of feedback based on your comment given in part (c). [2 marks]

Example of Sensitivity of System Parameters

a. If $K = 10$ and $a = 100$, then $F = 0.091$.

b. If parameter a triples to 300, then $F = 0.032$.

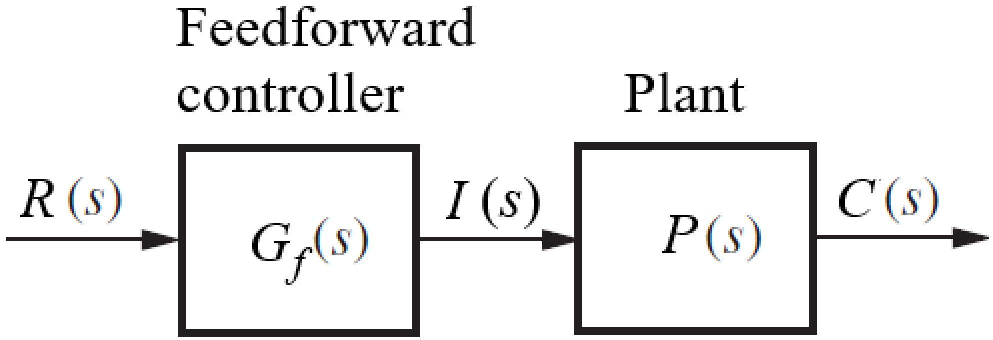
c. We see that a fractional change in parameter a of $(300 - 100)/100 = 2$ (a 200% change) yields a change in the function F of $(0.032 - 0.091)/0.091 = 0.65$ (65% change).

   Thus, the function F has reduced sensitivity to changes in parameter a.

d. As we proceed, we will see that another advantage of feedback is that in general it affords reduced sensitivity to parameter changes.

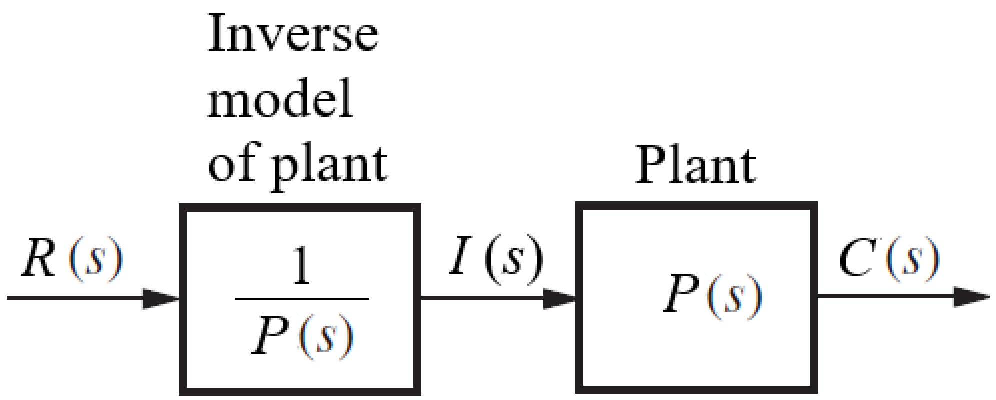
Feedforward Control System

- Control element responds to change in command or measured disturbance in a pre-defined way.
- Based on prediction of plant behavior (requires model).
- Can react before error actually occurs:
  - Overcome sluggish dynamics and delays.
  - Does not jeopardise stability.



Example of Feedforward Control System

- One of the feedforward control implementations, e.g. model-based prediction of input.
- Ideally, it consists of an exact inverse model of the plant.
- Can compensate for known plant dynamics, delays (before you get errors).
- No sensors needed.
- System response must be predictable.



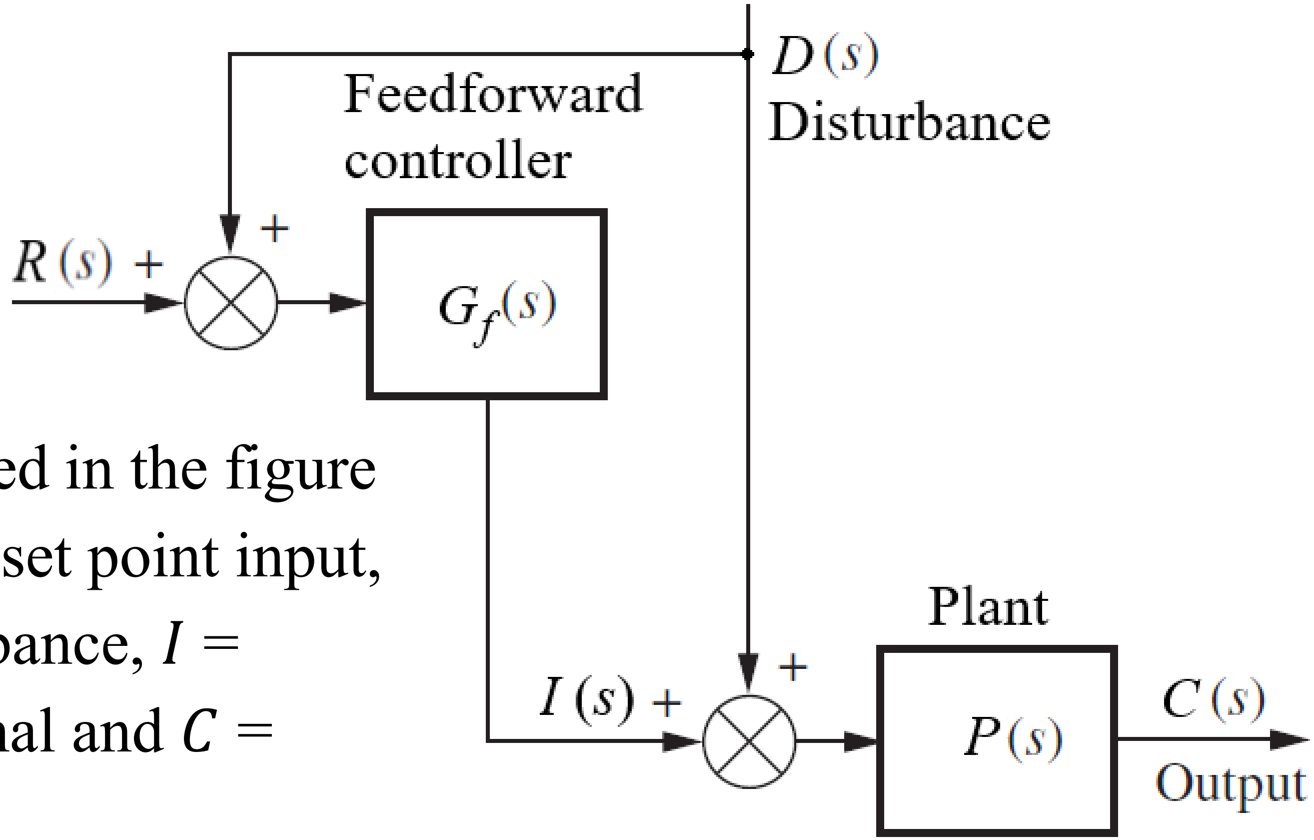
Limitations of Feedforward Control

- The disturbance variables must be measured online. In many applications, this is not feasible.
- To make effective use of feedforward control, at least an approximate process model should be available.
- In particular, we need to know how the controlled variable responds to changes in both the disturbance and manipulated variables. The quality of feedforward control depends on the accuracy of the process model.
- Ideal feedforward controllers that are theoretically capable of achieving perfect control may not be physically realisable. Fortunately, practical approximations of these ideal controllers often provide very effective control.

Difference of Feedback vs. Feedforward

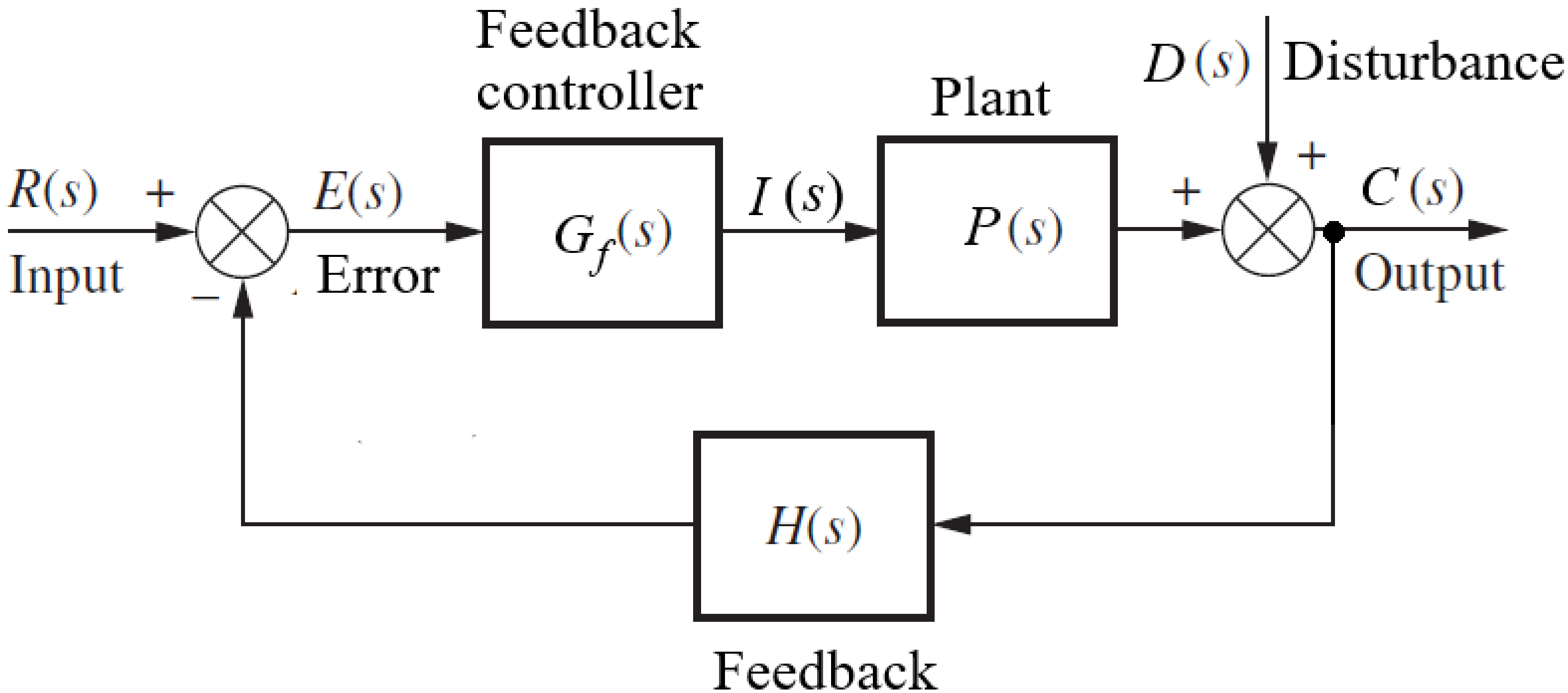
- There are several differences between a feedforward system and a feedback system.

- As illustrated in the figure below, R = set point input, D = Disturbance, I = control signal and C = output.



Difference of Feedback vs. Feedforward

Feedforward and feedback systems are distinguished by how they handle their signals, disturbances, loops, focus, and variables used.



Difference of Feedback vs. Feedforward

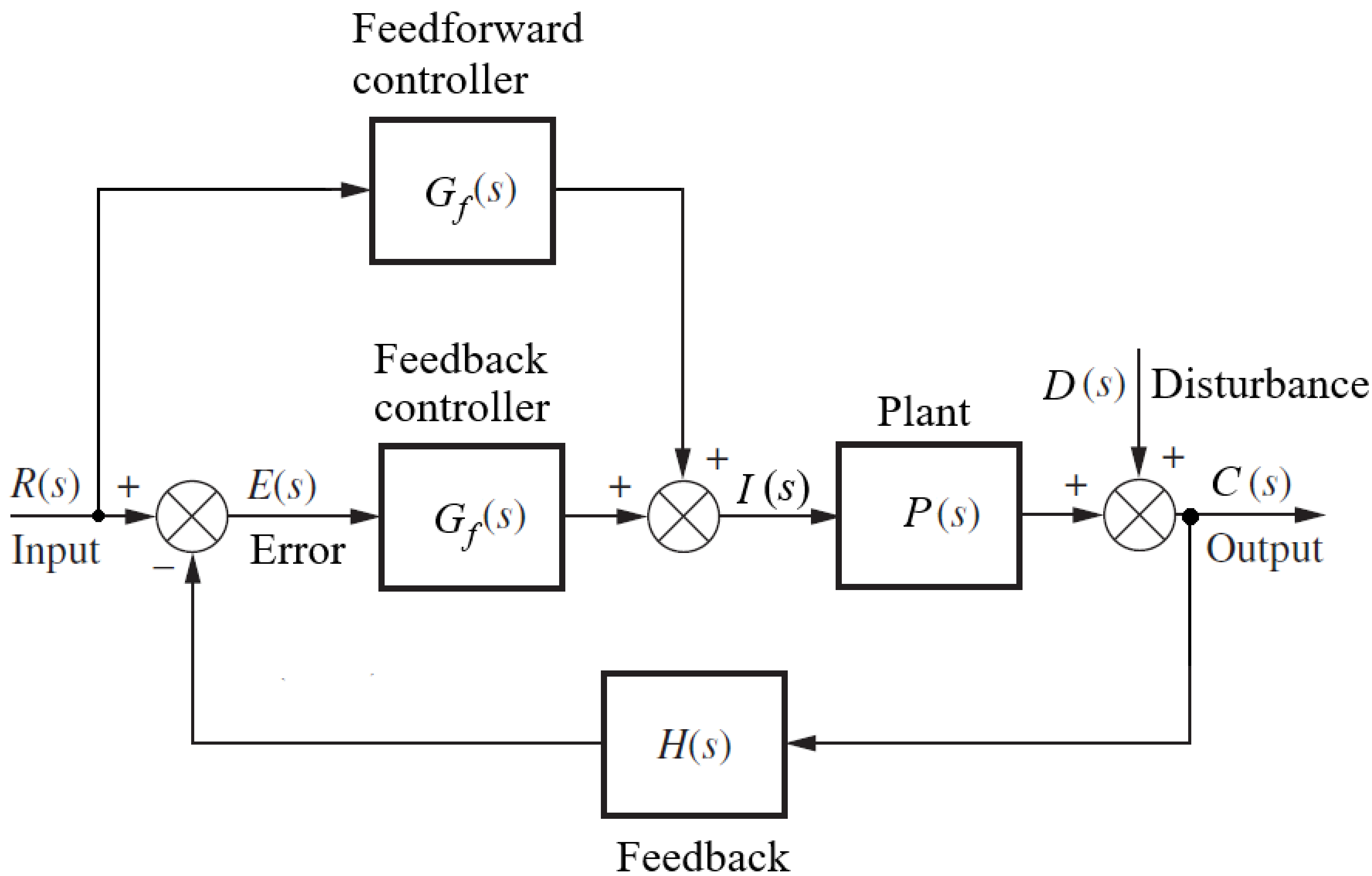
Category	**Feedback System**	**Feedforward System**
Signal	Output depends on the generated feedback signal.	The signal is passed to some external load.
Measure of disturbance in the system	Not needed by feedback system	Needed by feedfoward system.
Disturbances	Detected in feedback system	Not detected in feedforward system
Loop	Closed loop	Open loop
Focus	Output of the system	Input of the system
Variable	Adjusted on the basis of errors	Adjusted on the basis of knowledge (model)

Combining Feedback and Feedforward

Feedforward and feedback are often used together:

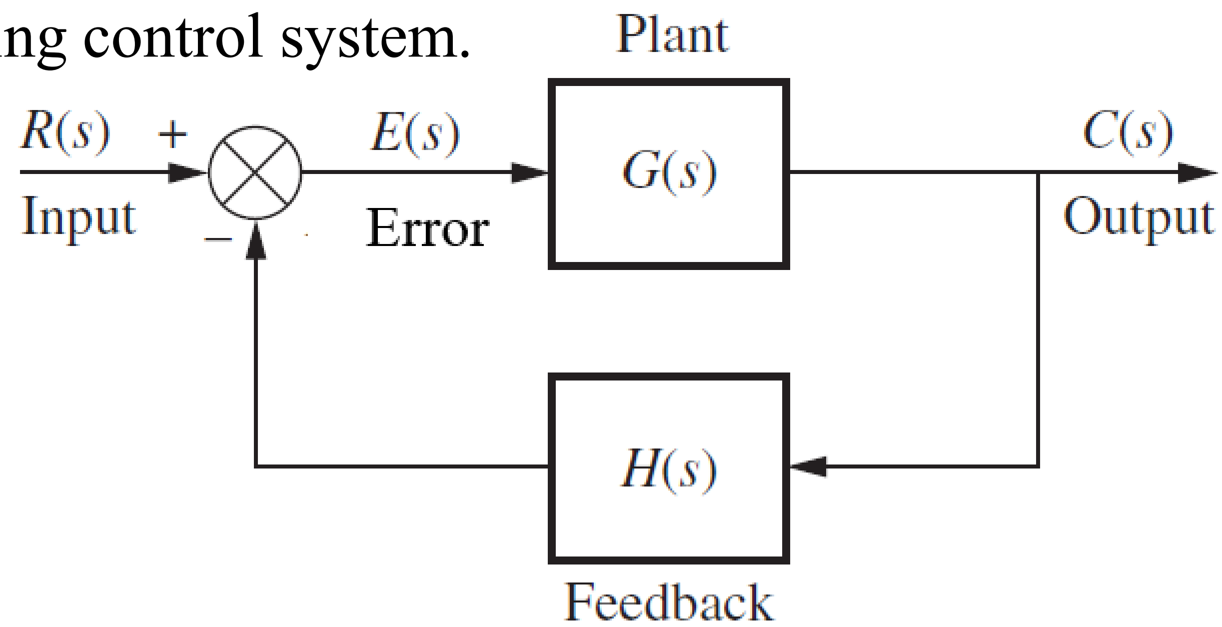
- Feedforward component provides rapid response.
- Feedback component fills in the rest of the response accurately, compensating for errors in the model.

Combining Feedback and Feedforward



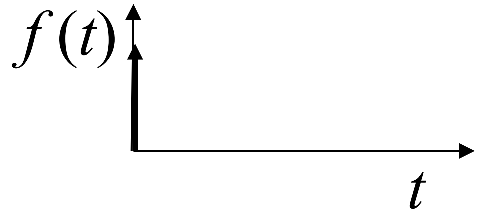
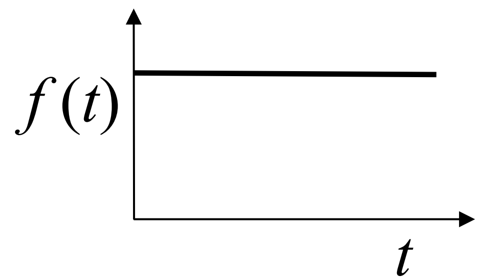
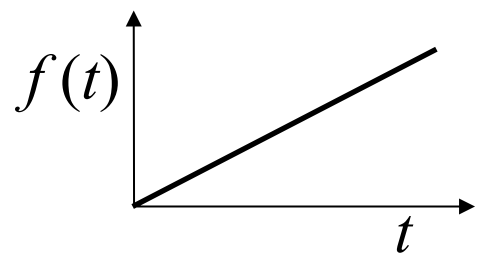
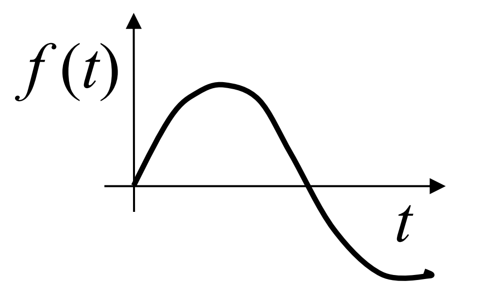
Input Signals

- There are various types of input, $R(s)$ for evaluating and testing control system.



- These are needed to cope with different characteristics and behaviours of practical control systems that exist out there.
- Inputs used in the course: impulse, step, ramp and sinusoid.
- Other types of input in practice: parabolic, square wave, triangle wave, PWM, etc.

Input Signals

<table>
<tr><th>Input</th><th>Function</th><th>Waveforms</th></tr>
<tr><td>Impulse</td><td>$\delta(t)$</td><td></td></tr>
<tr><td>Step</td><td>$u(t)$</td><td></td></tr>
<tr><td>Ramp</td><td>$tu(t)$</td><td></td></tr>
<tr><td>Sinusoid</td><td>$\sin \omega t$</td><td></td></tr>
</table>

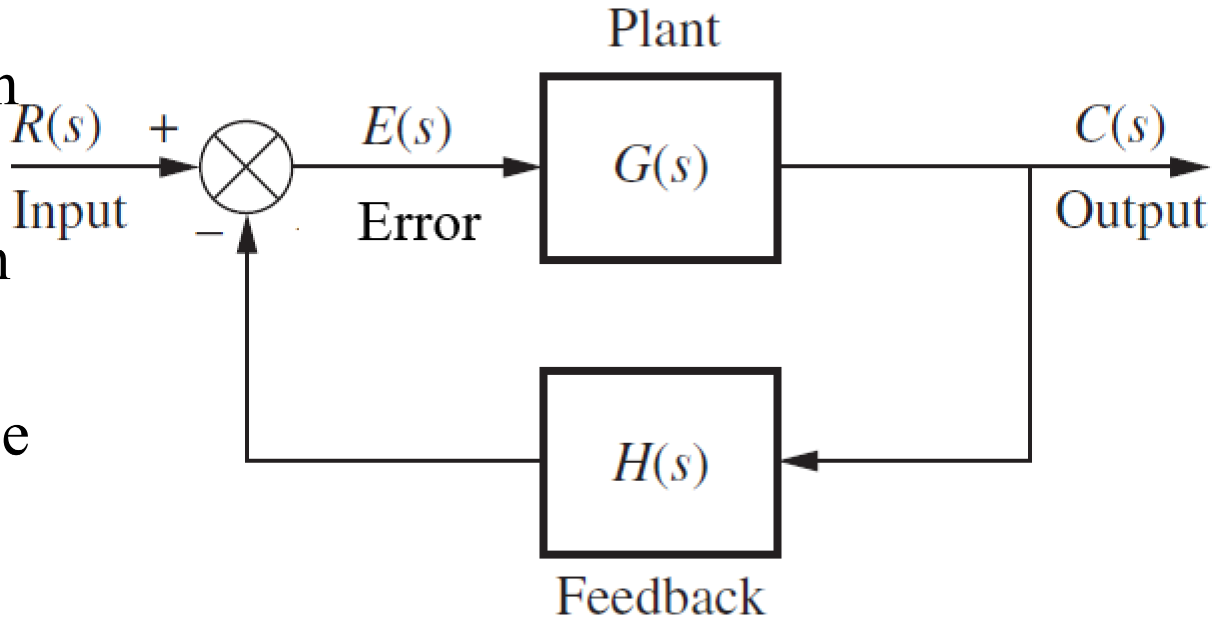
Input Signals

- An impulse input is a very high amplitude pulse applied to a system over a very short time (i.e., it is not maintained for long period of time). That is, the magnitude of the input approaches infinity while the time approaches zero.
- A step input is instantaneously applied at some time (typically taken as zero) and thereafter held at a constant level.
- A ramp input increases linearly with time. However, in practice, there is a physical limit, or the dynamic problem ends before the input gets too large.
- A sinusoid input is used for applying a sinusoid signal into the system.

Controller in Feedback System

- A controller is one of the most important components of the control system. It is designed to be responsible for the management and control of the performance of the system.

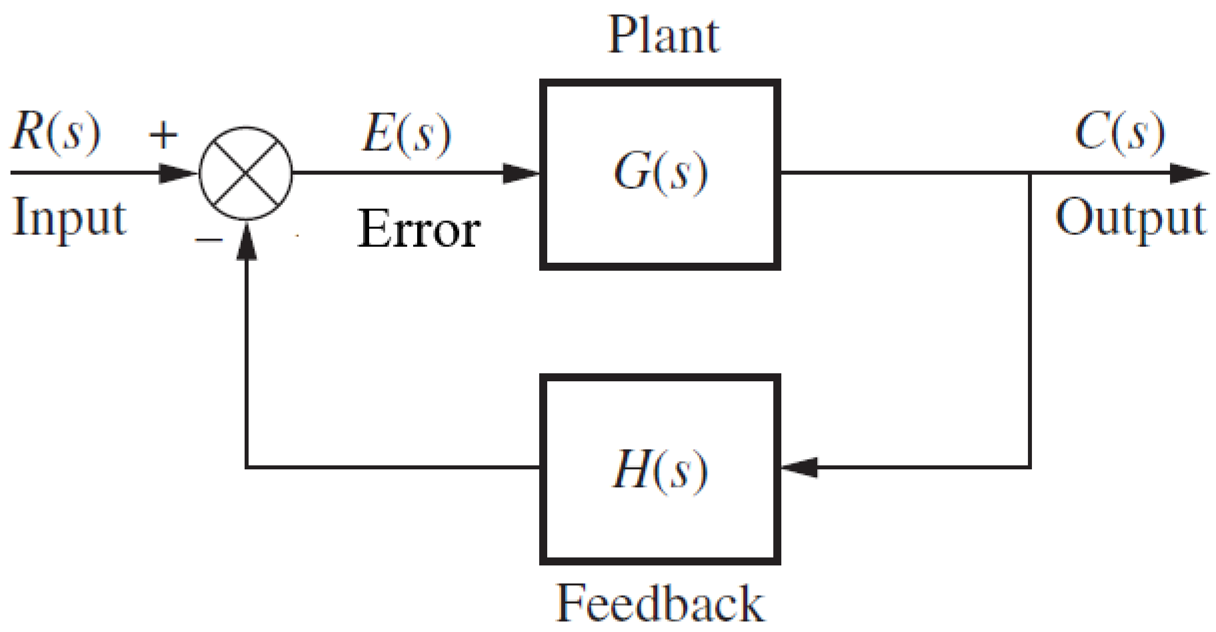
- It is a device or an algorithm that works to maintain the value of the controlled variable at set point.



- A control system can control its output(s) to a particular value or perform a sequence of events or perform an event if the specified conditions are satisfied based on the input(s) given.

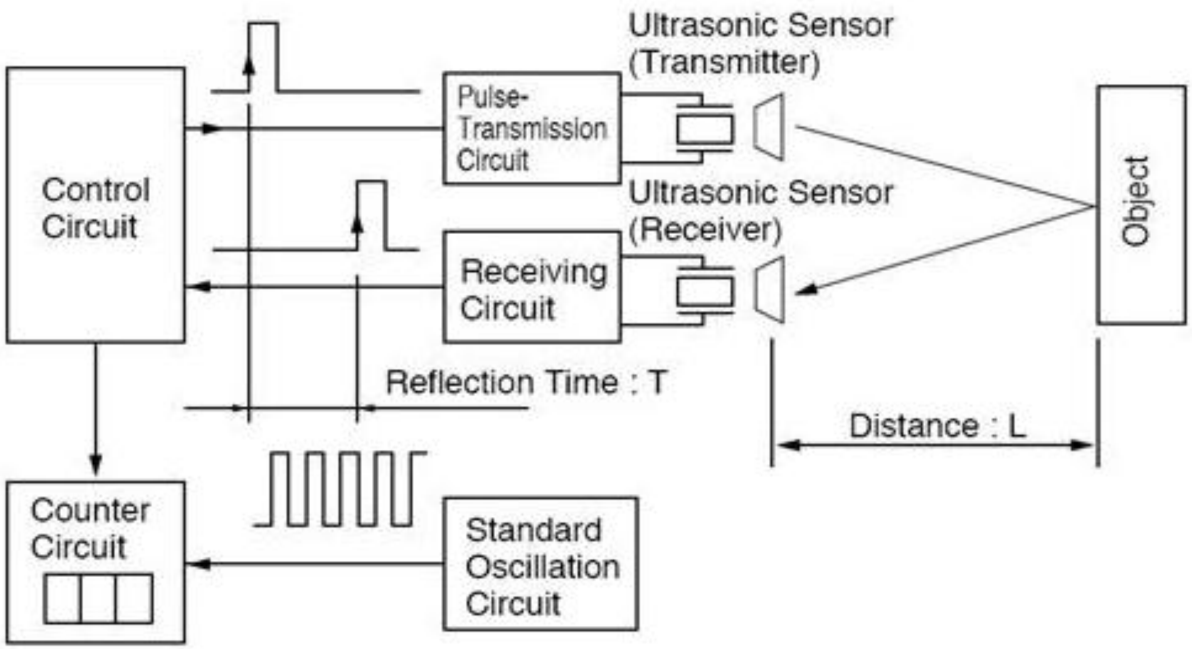
Controller in Feedback System

- The controller receives the difference between the reference set point and the measured output (e.g. known as the error) and generates a control action to make the error to zero.
- The generated control action manipulates the process variable closer to the set point.



Measurements in Controller

- The measurement system consists of a sensor and transmitter.
- The sensor measures the process variable and produces mechanical, electrical or other related phenomena.
- The transmitter converts the phenomenon into a signal that is suitable for transmission.



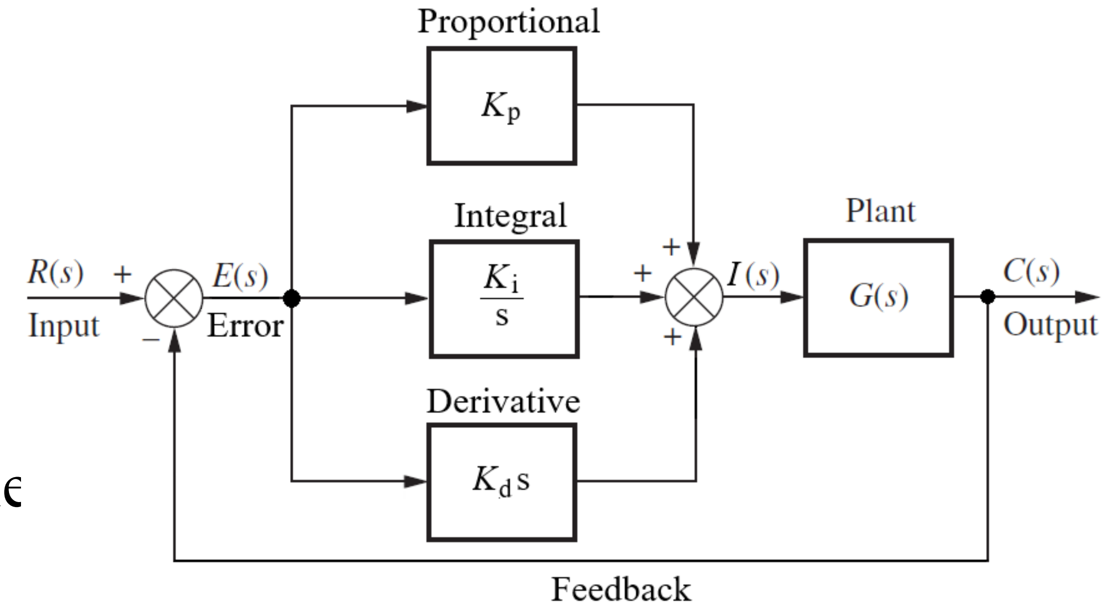
Types of Controller

Types of controller in control systems engineering:

- Proportional controller (P)
- Integral controller (I)
- Derivative controller (D)
- Any combination of above e.g. PD, PI, PID, etc.

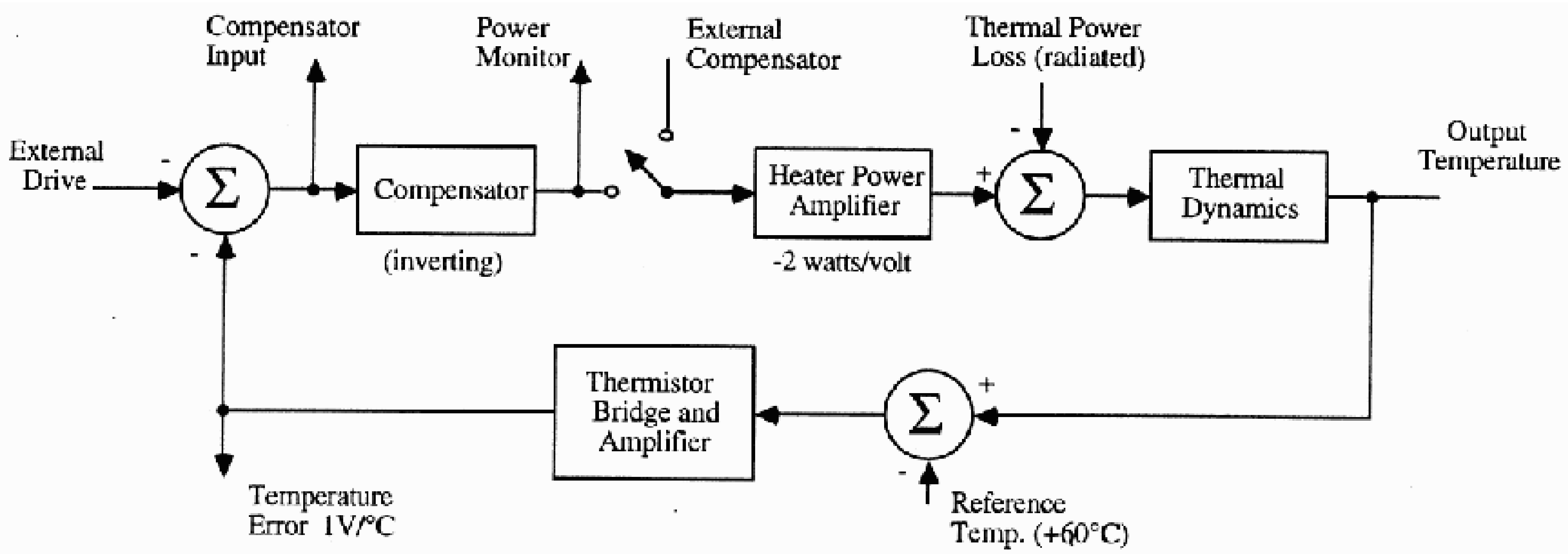
Other types:

- Fuzzy logic controller.
- Model based controller.
- Neural network controlle



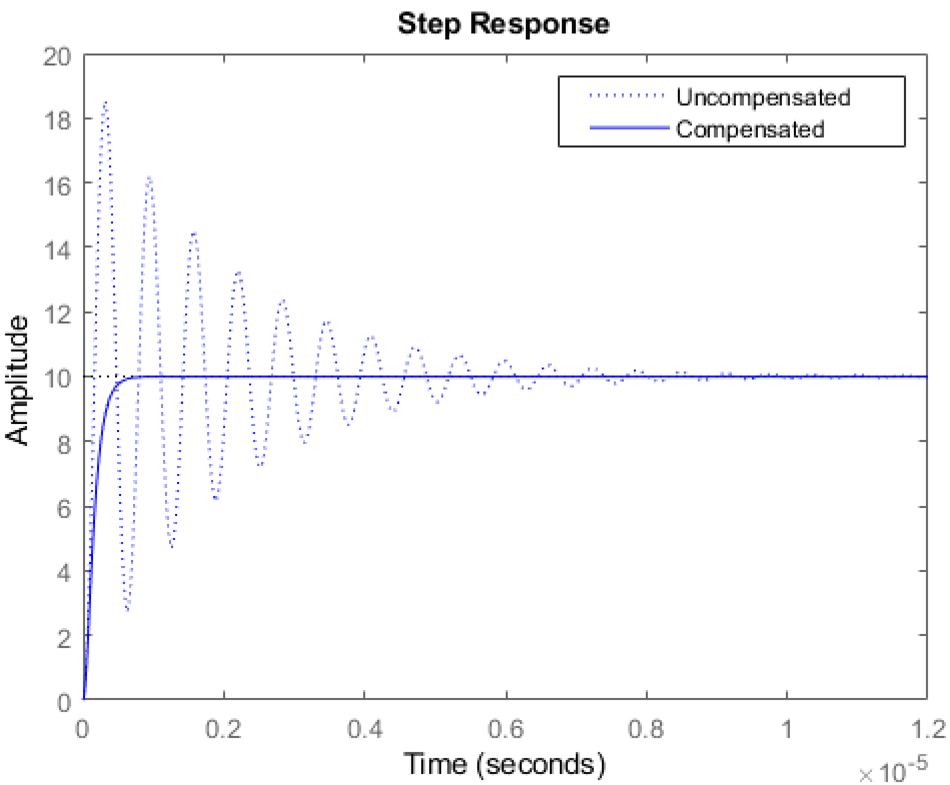
Compensator

- A slightly different form of component than the controller.
- Like the controller, a compensator is used for changing the characteristics and behaviour of a given system.
- It consists typically of passive components, not active components, like in the controller's case.



Methods of Compensation

- Compensator (is a type of controller which) makes some adjustments in order to make up for deficiencies in the system.
- Compensating devices are may be in the form of electrical, mechanical, hydraulic etc.
- Most electrical compensator are RC filter.



Why Do We Need Compensator?

Reasons/Rationales:

- Compensate a unstable system to make it stable.
- A compensating network is used to minimise overshoot.
- Compensators could increase the steady-state accuracy of the system. An important point to be noted here is that the increase in the steady-state accuracy could bring instability to the system.
- Compensator could also introduce poles and zeros in the system, thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system could potentially change.

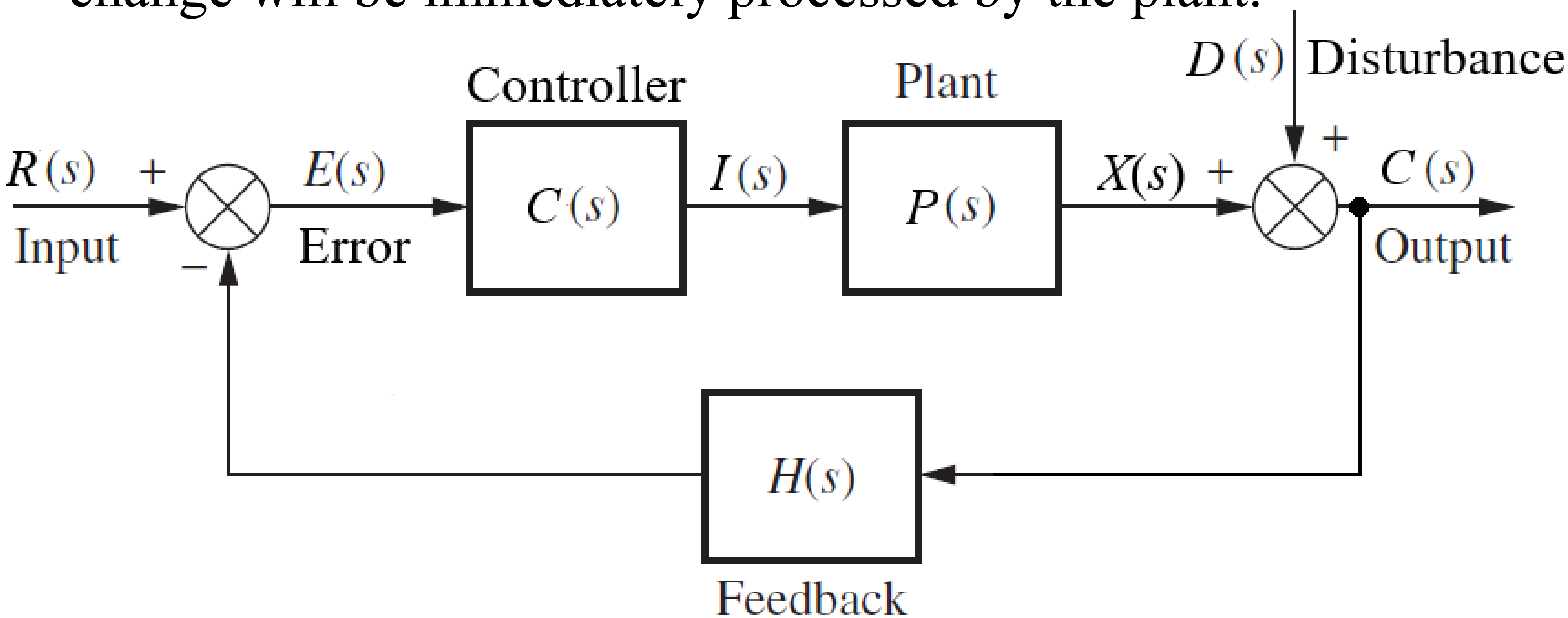
Types of Compensator

- Methods of compensation:
  - Series compensator
  - Feedback compensator
  - Load compensator

- The simplest compensating network used for compensators in control systems engineering is known as:
  - Lead compensators.
  - Lag compensators.
  - Lead-lag compensators.

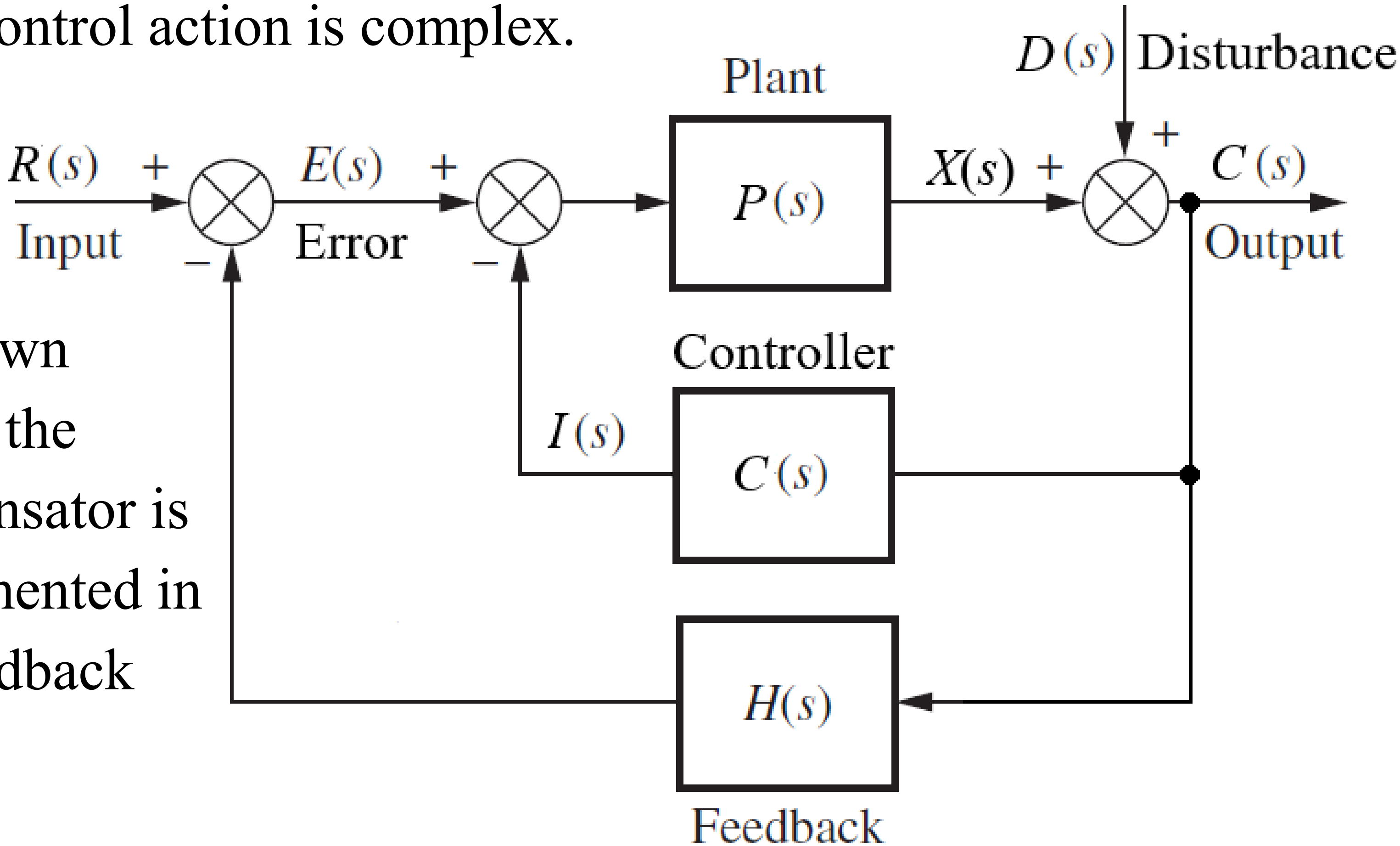
Series Compensator

- Series compensator: connecting a compensating circuit between the error detector and plants, known as series compensation.
- Series compensator is implemented if the plant ($G(s)$) is a small-scale system.
- Connected in series with the plant means faster response, as any change will be immediately processed by the plant.



Feedback Compensator

- Feedback compensator: When a compensator is used in a feedback manner called feedback compensation.
- This arrangement is considered when plant ($G(s)$) is large in scale, or the control action is complex.



- As shown below, the compensator is implemented in the feedback loop.

Load Compensator

- Load compensator: a combination of series and feedback compensator is called load compensation.
- This is often implemented to accommodate both speed (open loop) and accuracy/complexity (feedback loop).