VICTORIA UNIVERSITY OF WELLINGTON
TE HERENGA WAKA

Introduction to Controllers and Compensators

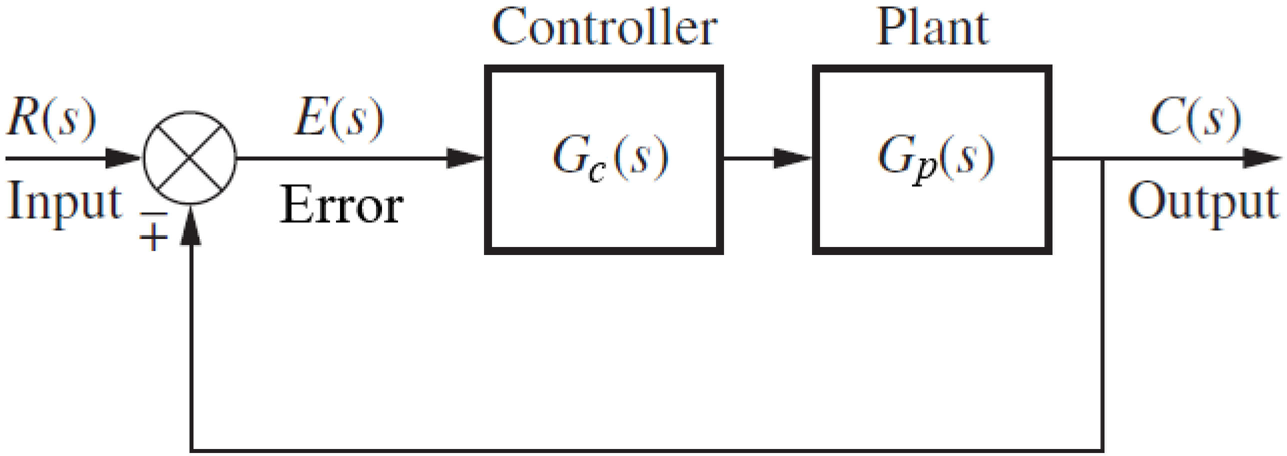
XMUT315 Control Systems Engineering

Topics

- Introduction to Controller and Compensator in the Feedback Control Systems.
- Controllers (e.g. Proportional Controller, Integral Controller, Derivative Controller, Proportional-Integral Controller, Proportional-Derivative Controller, and Proportional-Integral-Derivative Controller).
- Compensators (e.g. Lead Compensator, Lag Compensator, and Lead-Lag Compensator).
- Introduction to Controller/Compensator Design.

Controller and Compensator

- Controller or compensator changes the behaviour of the control system.
  - Controller is an element whose role is to maintain a physical quantity in a desired level.
  - Compensator is an element for modification of system dynamics e.g. to improve characteristics of the open-loop plant so that it can safely be used with feedback control.



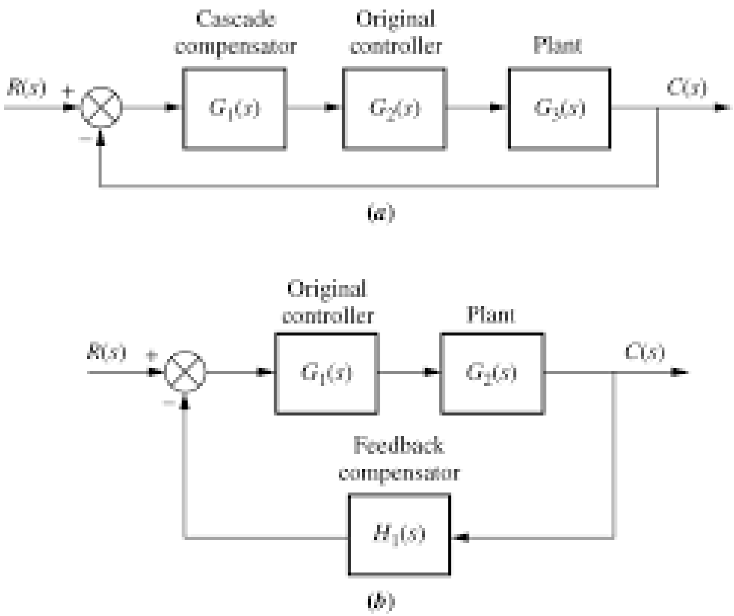
Note: G_c = controller or compensator and G_p = plant.

Controller and Compensator

- Three main types of controller:
  - P (gain or proportional) controller.
  - D (derivative) controller.
  - I (integral) controller.
  - PD (proportional derivative) controller.
  - PI (proportional integral) controller.
  - PID (proportional, integral, and derivative) controller.
- Three main types of compensator:
  - Lag compensator.
  - Lead compensator.
  - Lead-lag compensator.

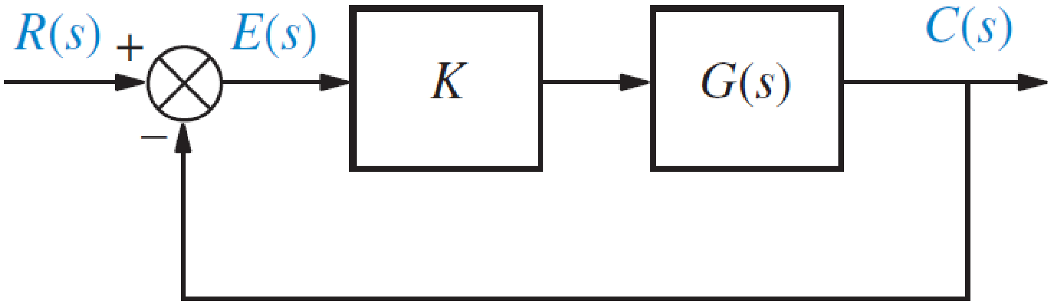
Controller and Compensator

- Controllers or compensators:
  - They change the natural response of the system.
  - They adjust the poles of the system.
  - They help achieve the desired output from a given input.



P Controller

- For a unity feedback system as shown below, a proportional controller ($G_c(s)$) connected in series with the plant ($G(s)$).



- The transfer-function equation of the proportional controller is:

$$G_c(s) = K$$

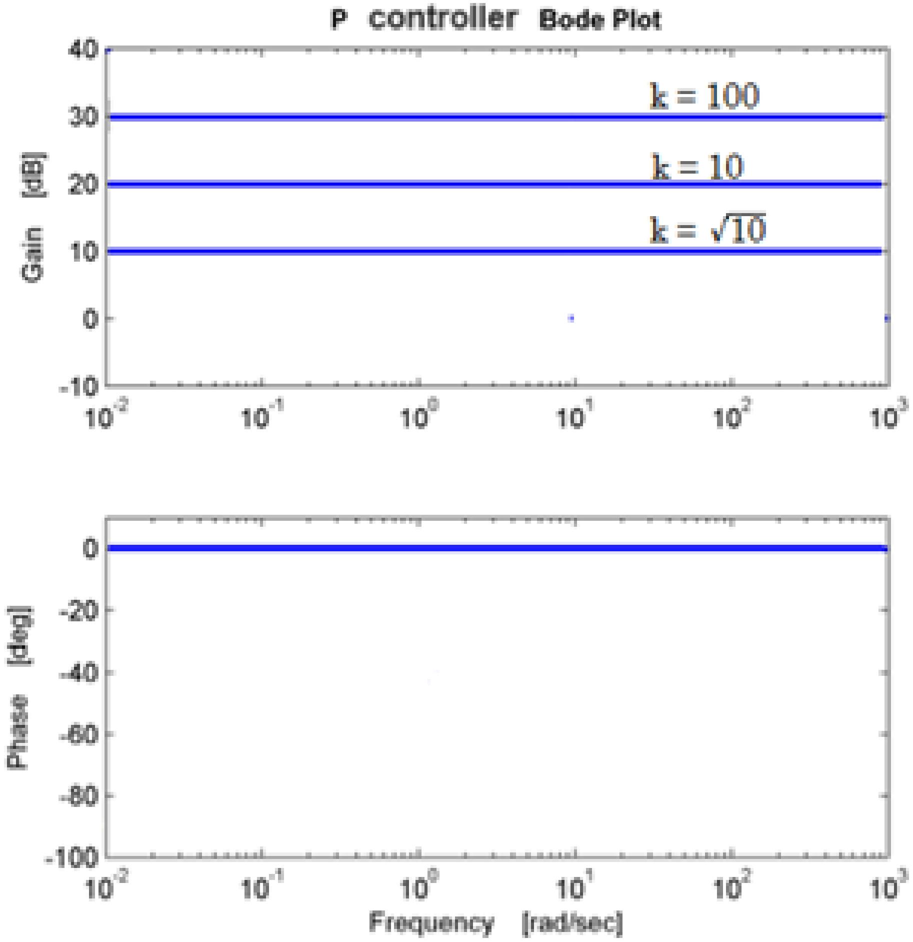
- Thus:

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{KG(s)}{1 + KG(s)}$$

Characteristics of P Controllers

- The frequency response of a P controller is as shown below.

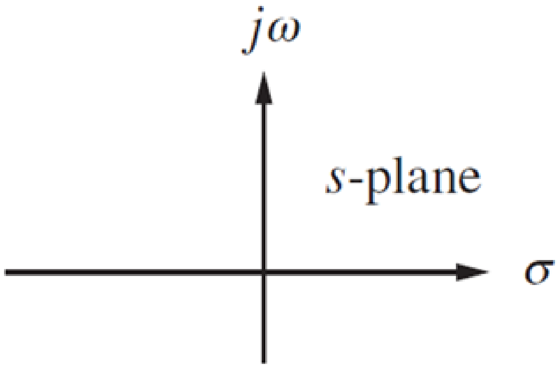
- Magnitude plot:
  - All frequency:
    $20 \log K$
- Phase-shift plot:
  - All frequency: 0°

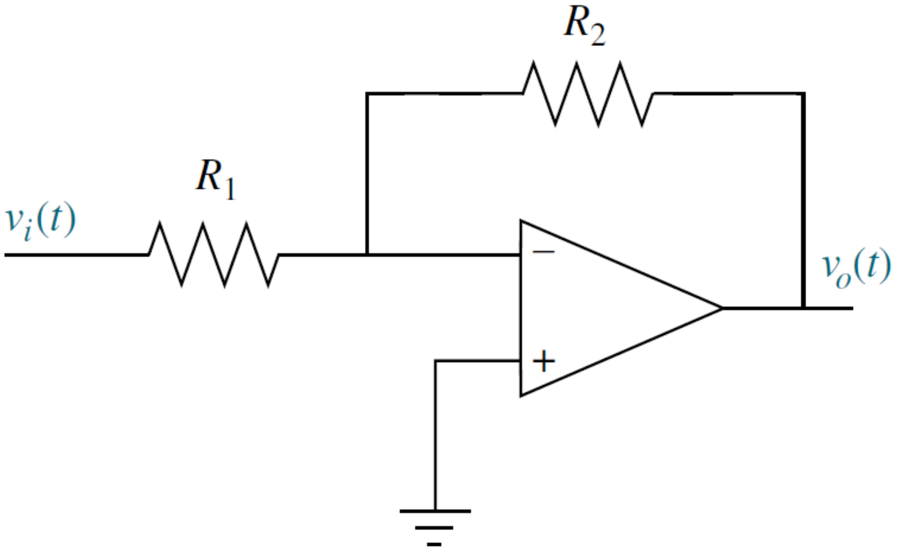


Characteristics of P Controllers

- Features of the P controller:
  - Increases the gain of the system.
  - Improve transient response (up to a point).
  - Often result in a non-zero steady-state error.
  - Relatively easy to implement.

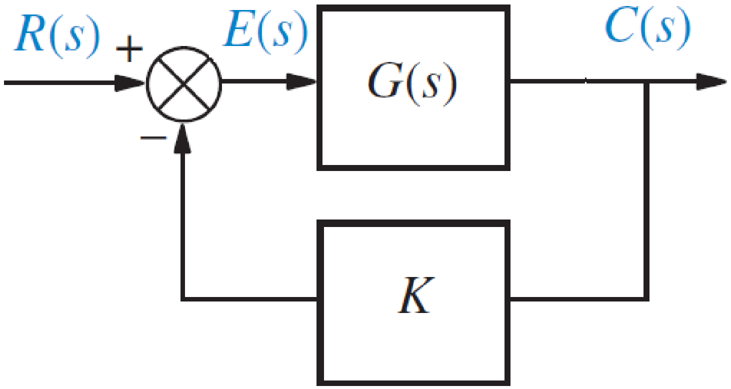
$$G_c(s) = K$$





P Controller

- For a proportional controller ($G_c(s)$) attached in the feedback path, consider a plant in the forward path ($G(s)$).



- The transfer-function equation of the closed-loop feedback system is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 - (-G_c(s))G(s)} = \frac{G(s)}{1 + KG(s)}$$

P Controller

- Unlike the previous setup, notice that the controller is placed in the feedback loop of the given control system.
- Notice the negative sign in the equation for the transfer-function equation of the feedback system.
- If the size of the loop gain is large, that is if:

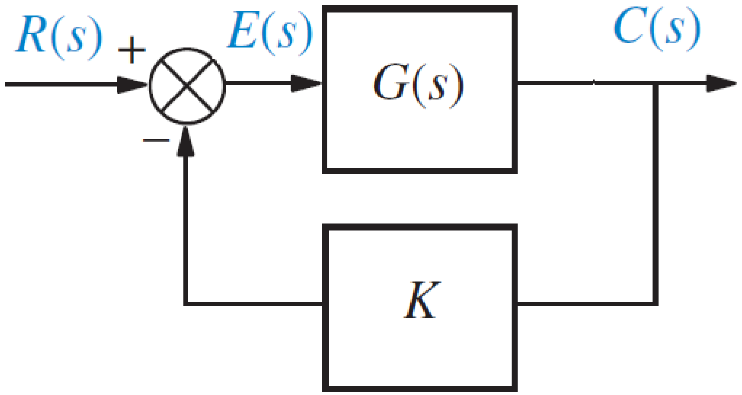
$$|KG(s)| \gg 1$$

- Then, the gain of the closed-loop system approximately is the gain of the controller:

$$T(s) \approx \frac{G(s)}{KG(s)} = \frac{1}{K}$$

P Controller in First-Order System

- For a proportional controller ($G_c(s)$) is attached to the feedback path and the plant in the forward path is a first-order system $G(s)$.



- The transfer-function equations of both the controller and the plant are:

$$G_c(s) = K \qquad \text{and} \qquad G(s) = \left(\frac{A}{1 + sT}\right)$$

P Controller in First-Order System

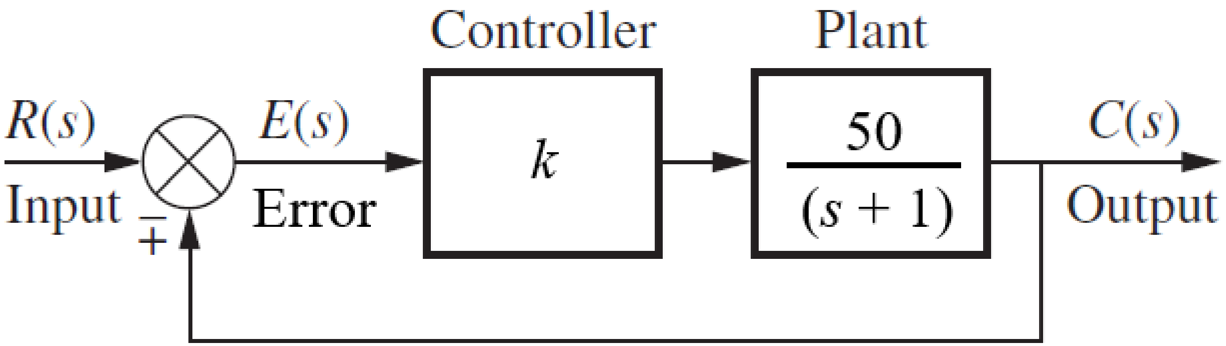
- In term of $T(s) = C(s)/R(s)$, closed-loop transfer-function equation of the system is:

$$T(s) = \frac{G(s)}{1 - \big(-G_c(s)\big)G(s)} = \frac{\left(\frac{A}{1+sT}\right)}{1 - (-K)\left(\frac{A}{1+sT}\right)} = \frac{A}{1 + sT + AK}$$

- Thus, we can see that the time constant of the closed-loop first-order system depends on both gains of the plant A and controller K.
- The gain of the closed-loop system depends on the gain of the plant A.

Example of P Controller in First-Order System

The open-loop transfer-function equation of a first-order system is given below.



a. Determine the time constant of the open-loop system. [2 marks]

b. If a proportional controller connected in series with the system as shown above, determine the gain of proportional controller (K) that will change the time constant (τ) of the closed-loop system to become 0.1 second. [6 marks]

Example of P Controller in First-Order System

a. The time constant of the open-loop first-order system is:

$$G(s) = \frac{50K}{s+a}$$

Thus

$$\tau = \frac{1}{a} = \frac{1}{1} = 1\ \mathrm{s}$$

b. The gain of the proportional controller (K) that will change the time constant of the closed-loop first order system to become 0.1 second is determined as follows.

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{K\left(\frac{50}{s+1}\right)}{1+K\left(\frac{50}{s+1}\right)} = \frac{50K}{s+1+50K}$$

Example of P Controller in First-Order System

Thus, equate the time constant with the part of the transfer-function equation.

$$\tau = \frac{1}{a} = \frac{1}{1 + 50K}$$

For the time constant of 0.1 second, the gain of the proportional controller is calculated from:

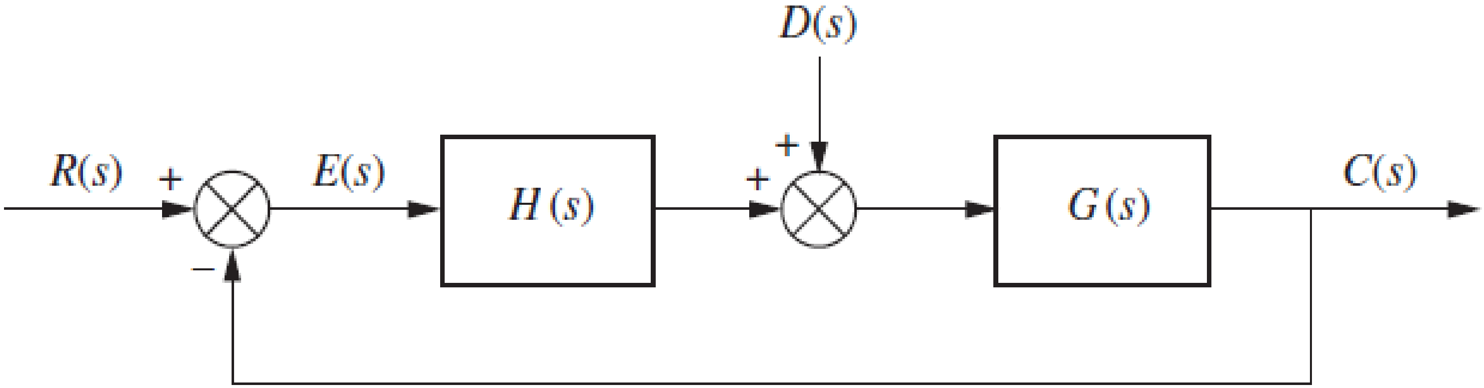
$$0.1 = \frac{1}{1 + 50K}$$

Rearrange the equation above, the value of K is:

$$K = \frac{10 - 1}{50} = 0.18$$

P Controller in Second-Order System

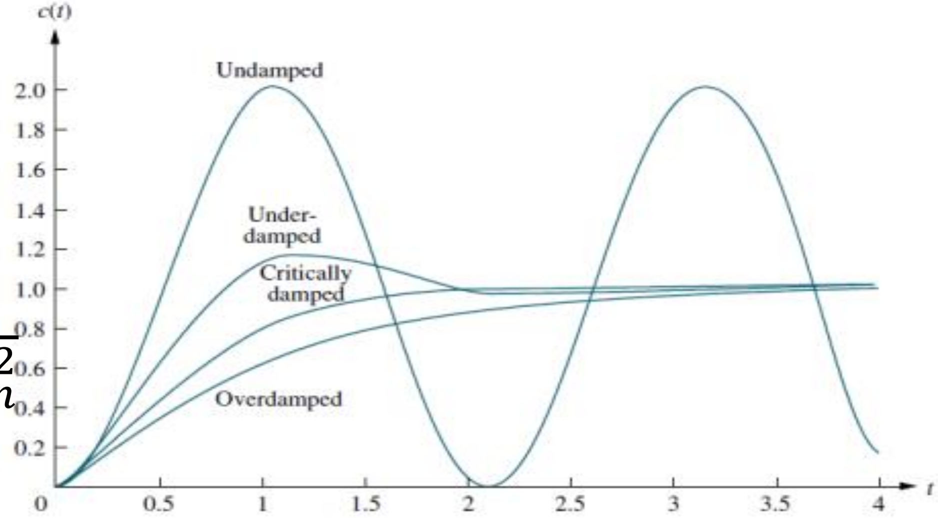
- For a unity feedback second-order system ($G(s)$) with a proportional controller ($H(s)$) added as shown below.



- Unless told otherwise, assume $D(s)$ = 0, if $G(s) = \omega_n^2/(s^2 + 2\omega_n\zeta s + \omega_n^2)$ and $H(s) = K$:

$$T(s) = \frac{H(s)G(s)}{1 + H(s)G(s)}$$

$$= \frac{\omega_n^2 K}{s^2 + 2\omega_n\zeta s + (1+K)\omega_n^2}$$



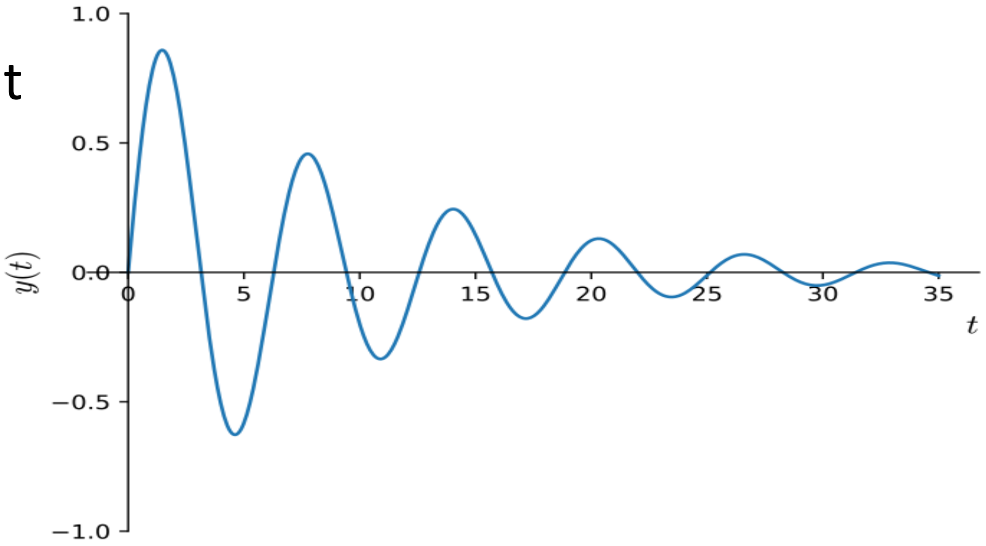
P Controller in Second-Order System

- We can determine values for $1 + K$ to make the system: undamped, underdamped, critically damped, and overdamped.

- As proven for the first-order system before, a higher gain typically yields a faster response, but it is at the expense of a more oscillatory response.

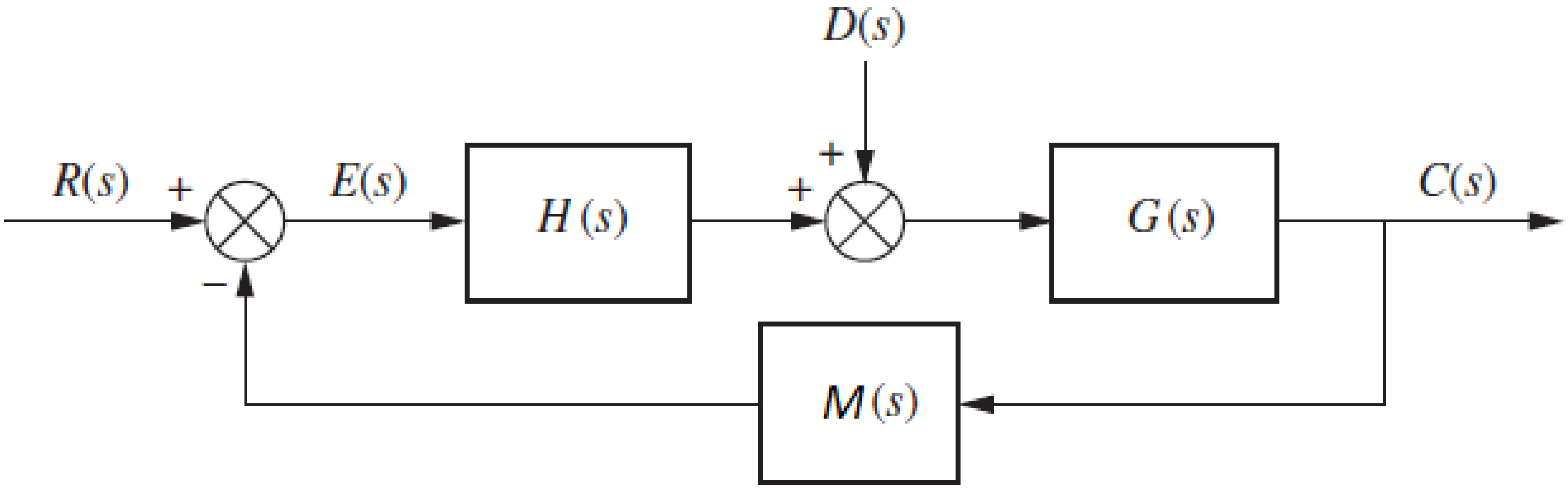
- If the transient response of the system is too oscillatory, it will take time before the system settles to its final value.

- We cannot, therefore, just increase the controller gain.



P Controller in Second-Order System

- For a second-order system with a proportional controller ($H(s)$) and non-unity feedback ($M(s)$) as shown below.



- If $D(s) = 0$, the open-loop gain of the system is:

$$\frac{C(s)}{R(s)} = H(s)G(s)$$

P Controller in Second-Order System

- The transfer-function equation of the closed-loop system is:

$$T(s) = \frac{C(s)}{R(s)} = \frac{H(s)G(s)}{1 + M(s)H(s)G(s)}$$

- Focusing on the characteristic equation in the transfer-function equation, it is:

$$1 + M(s)H(s)G(s)$$

- Notice that $M(s)$ influences the characteristic equation.
- As a result, the variable $M(s)$ affects transient response of the closed-loop system as specified above.

P Controller in Second-Order System

- Applying the final-value theorem, the steady-state equation of the system for a step input is:

$$C(\infty) = \lim_{s\to 0} s\,(R(s))\frac{H(s)G(s)}{1+M(s)H(s)G(s)}$$

$$= \lim_{s\to 0} s\left(\frac{1}{s}\right)\left(\frac{H(s)G(s)}{1+M(s)H(s)G(s)}\right)$$

$$\cong \frac{1}{M(s)}$$

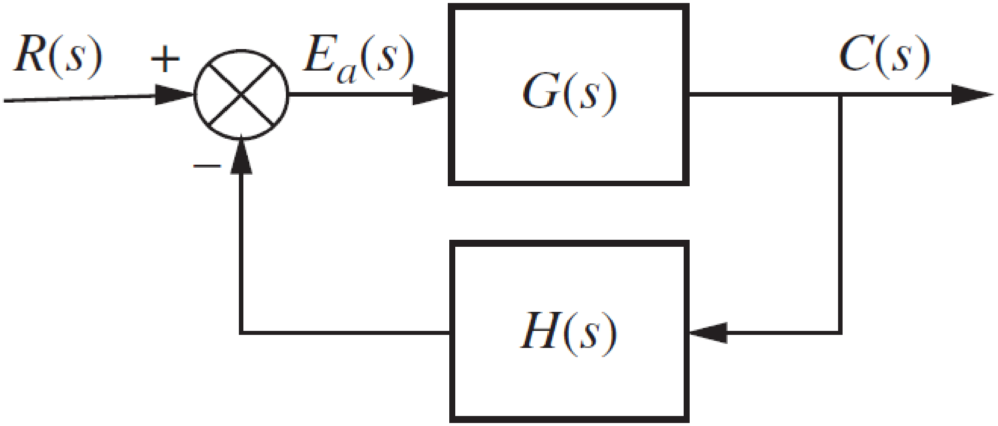
- Thus, the variable $M(s)$ influences also the steady-state response of the closed-loop system for a given step input.

Example of P Controller in Second-Order System

For an open-loop control system described by the transfer-function equation given below, attempt the following tasks:

$$G(s) = \frac{5}{s^2 + 10s + 5}$$

a. Derive the transfer function equation of the closed-loop system with a proportional controller $H(s) = M$ added in the feedback loop as shown below. [4 marks]



Example of P Controller in Second-Order System

b. If M is 9, determine the transient response of the closed-loop system. [6 marks]

c. As part of design specification for the system, for a step input response, determine the feedback gain (M) if we wish the steady-state error condition of the closed-loop system to be 0.6. [8 marks]

Example of P Controller in Second-Order System

a. For the given second-order system with non-unity feedback, the transfer-function equation of the closed-loop system is:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

$$= \frac{\left(\frac{5}{s^2 + 10s + 5}\right)}{1 + \left(\frac{5}{s^2 + 10s + 5}\right)M}$$

Rearrange the equation above, it becomes:

$$T(s) = \frac{5}{s^2 + 10s + 5(1 + M)}$$

Example of P Controller in Second-Order System

b. The transient response of the closed-loop system when M is 9 is:

$$T(s) = \frac{5}{s^2 + 10s + 5(1+9)} = \frac{5}{s^2 + 10s + 50}$$

Evaluating the characteristics equation of the closed-loop system, its roots are:

$$\text{root}_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{(10)^2 - 4(1)(50)}}{2(1)} = -5 \pm j5$$

The roots are complex pair, so the response of the closed-loop system is underdamped.

Example of P Controller in Second-Order System

c. For a step input, the steady-state error of the closed-loop system is:

$$e_{step}(\infty) = \lim_{s\to 0} \frac{s(1/s)}{1 + T(s)}$$

$$= \lim_{s\to 0} \frac{1}{1 + \frac{5}{s^2 + 10s + 5(1+M)}}$$

$$= \lim_{s\to 0} \frac{s^2 + 10s + 5(1+M)}{s^2 + 10s + 5(2+M)}$$

$$= \frac{1+M}{2+M}$$

Example of P Controller in Second-Order System

To achieve a steady-state error of 0.6 for the step response of the system, the gain of proportional controller M is calculated from:

$$e_{step}(\infty) = \frac{1 + M}{2 + M} = 0.6$$

Thus, the gain of the proportional controller that meets the design specification is:

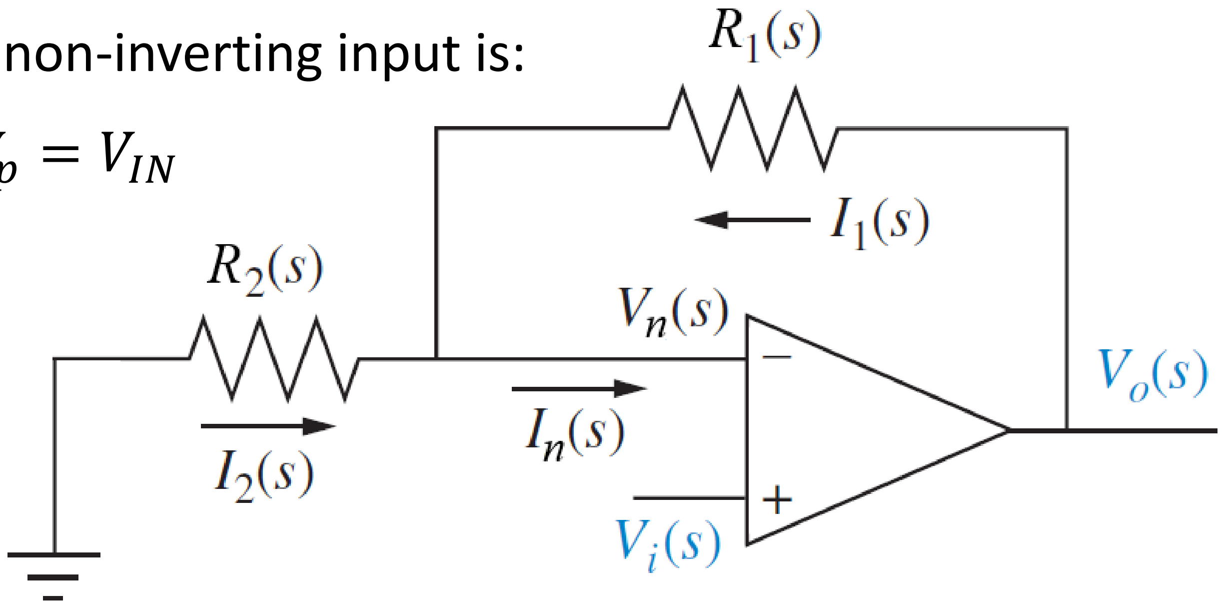
$$M = \frac{1.2 - 1}{1 - 0.6} = 0.5$$

P Controller in Practice

- In practice, the P controller is realised as a non-inverting amplifier with R_1 and R_2 forming the voltage divider part of the circuit.

- Voltage at the non-inverting input is:

$$V_p = V_{IN}$$

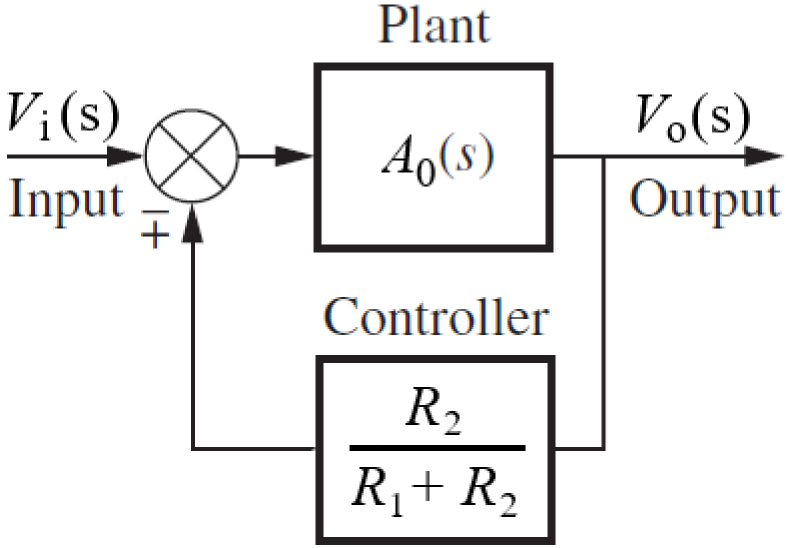


- Due to the potential divider arrangement in the circuit, the voltage at the inverting pin of the op amp is:

$$V_n = V_{OUT}\left(\frac{R_2}{R_1 + R_2}\right)$$

P Controller in Practice

- As an example, given the non-inverting amplifier with open-loop gain of A.



- The transfer-function equation of the amplifier is:

$$\frac{V_o}{V_i} = \frac{A_0(s)}{1 - A_0(s)\left(-\frac{R_2}{R_1 + R_2}\right)} = \frac{A_0(s)(R_1 + R_2)}{R_1 + R_2 + A_0(s)R_2}$$

- If the loop gain $A_0(s)R_2/(R_1 + R_2)$ is large, then:

$$A_0(s)R_2 \gg R_1 + R_2$$

P Controller in Practice

- As a result, the transfer-function equation of the non-inverting amplifier is:

$$\frac{V_o}{V_i} = \frac{A_0(s)(R_1 + R_2)}{A_0(s)R_2} = \frac{R_1 + R_2}{R_2}$$

- In the above equation, the feedback loop path is:

$$\beta = \frac{R_2}{R_1 + R_2}$$

- So, if the loop gain is large:

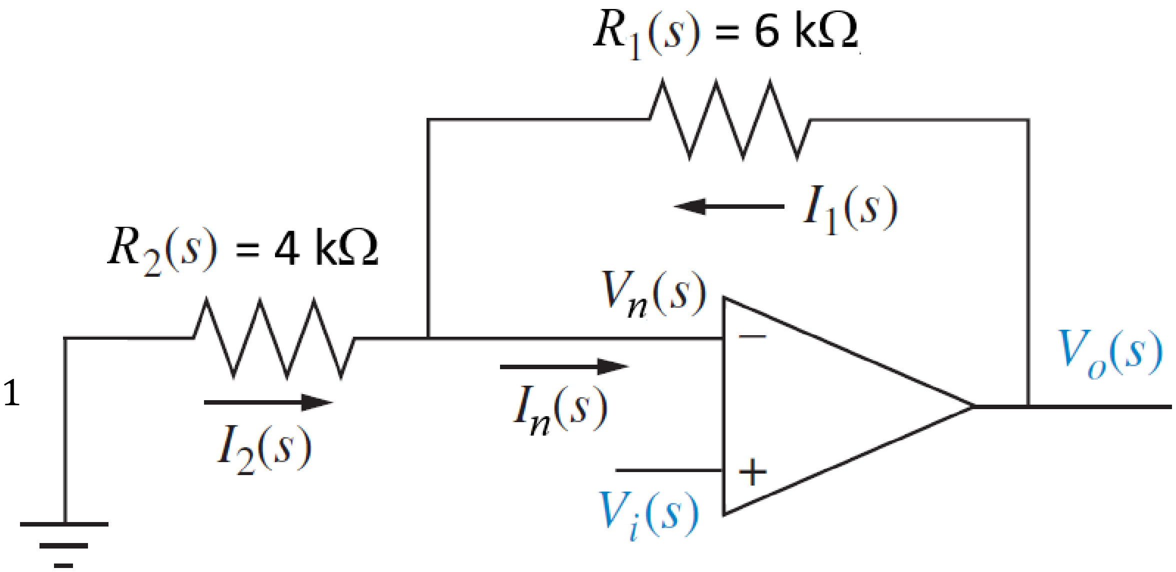
$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_2} = \frac{1}{\beta}$$

Example of P Controller in Practice

For example, given the specification of a non-inverting operational amplifier circuit as shown below, perform the following tasks:

Note:

- Open-loop gain of op amp, $A_0(s)$ = 10^5.
- Feedback resistors: R_1 = 6 kΩ, R_2 = 4 kΩ.



a. Derive the transfer-function equation of the amplifier. [6 marks]

b. Determine whether the forward-loop gain of the amplifier is larger than the feedback loop gain. [4 marks]

c. Calculate the gain of the amplifier. [4 marks]

Example of P Controller in Practice

a. The transfer-function equation of the op amp circuit is derived from:

$$V_p = V_i \qquad (Eq.1)$$

And

$$V_n = V_o\left(\frac{R_2}{R_1 + R_2}\right) = V_o\left(\frac{4\ \mathrm{k\Omega}}{6\ \mathrm{k\Omega} + 4\ \mathrm{k\Omega}}\right) = 0.4V_o \quad (Eq.2)$$

For the non-inverting amplifier with an open-loop gain of A, the output voltage is:

$$V_o = A_0(s)\left(V_p - V_n\right) = A_0(s)(V_i - 0.4V_o) \qquad (Eq.3)$$

Substituting equations (1) and (2) into equation (3), the transfer-function equation of the op amp circuit is:

$$\frac{V_o}{V_i} = \frac{A_0(s)}{(1 + 0.4A_0(s))}$$

Example of P Controller in Practice

b. Consider whether the feedback loop is large, the feedback loop is:

$$A_0(s)R_2 = (10^5)(4 \times 10^3) = 4 \times 10^8$$

And

$$R_1 + R_2 = (6 \times 10^3) + (4 \times 10^3) = 10^4$$

As calculated above, the feedback loop is large (e.g. $4 \times 10^8 \gg 10^4$).

c. As a result, the overall gain of the amplifier is:

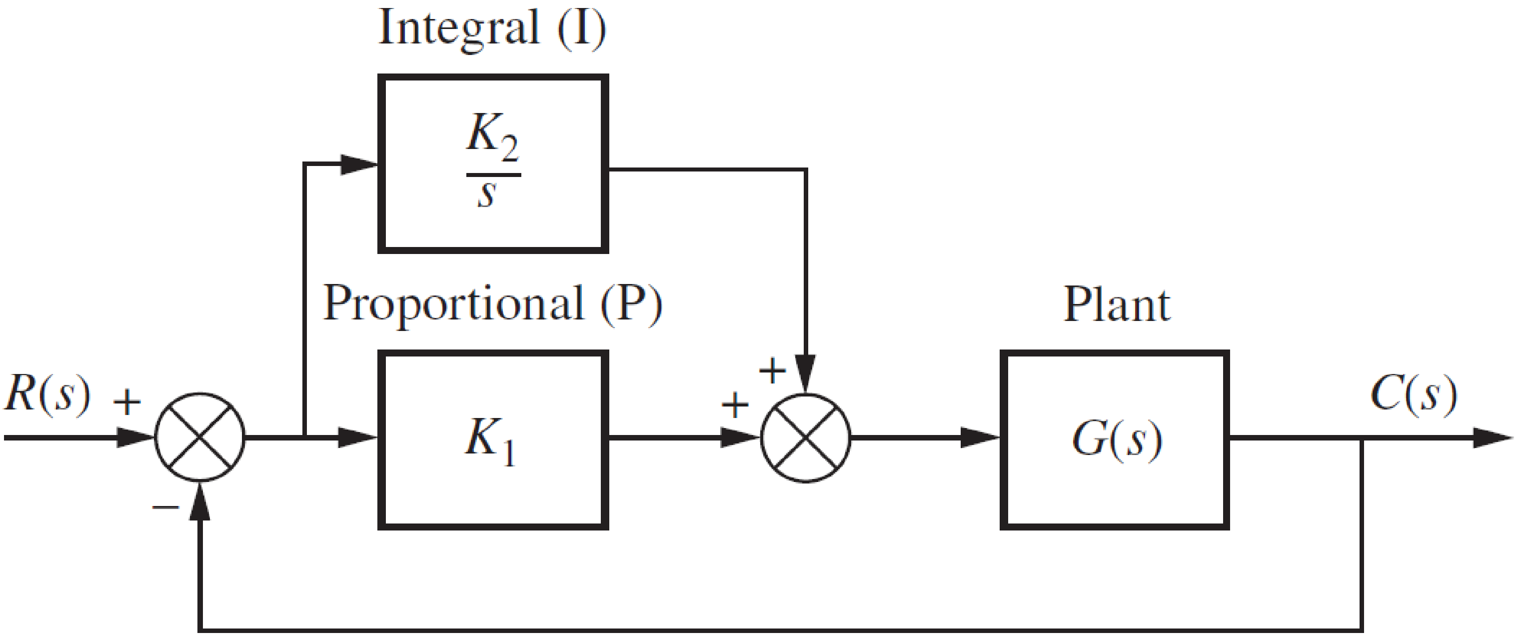
$$\frac{V_o}{V_i} = \frac{R_1 + R_2}{R_2} = \frac{6\ \mathrm{k\Omega} + 4\ \mathrm{k\Omega}}{4\ \mathrm{k\Omega}} = 2.5$$

PI Controllers

- For a PI Controller, its transfer-function equation can be written as:

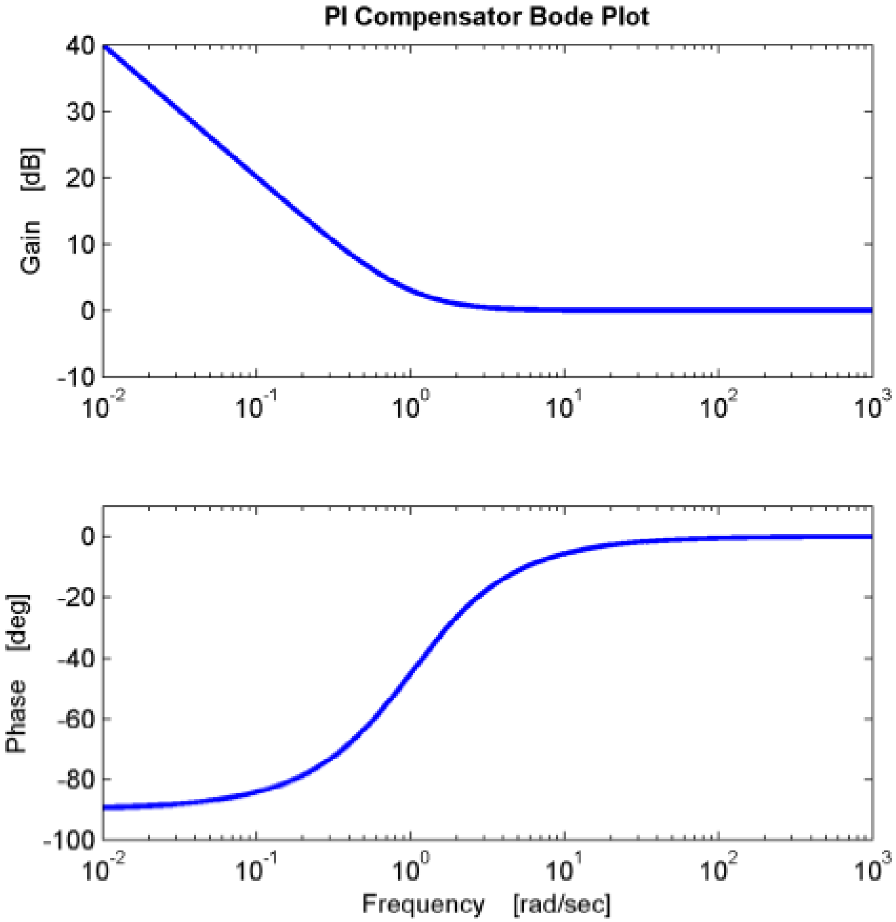
$$G_c(s) = P(s) + I(s) = K_1 + \frac{K_2}{s} = \frac{K_1(s + K_2/K_1)}{s}$$

Where: $P(s) = K_1$ and $I(s) = K_2/s$.



Characteristics of PI Controllers

- The frequency response of a PI controller is as shown below.
- Magnitude plot:
  - Low: -slope gain.
  - Cut-off: half gain.
  - High: zero gain.
- Phase-shift plot:
  - Low: -90°.
  - Cut-off: -45°.
  - High: 0°.

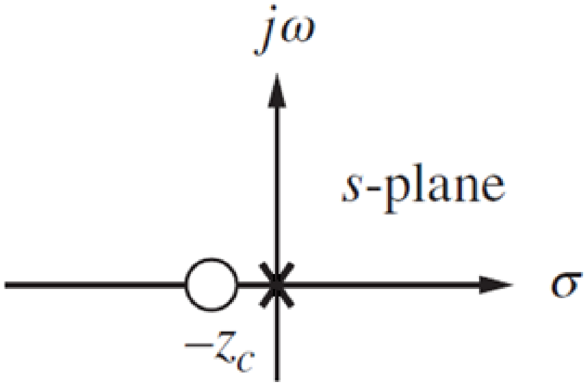


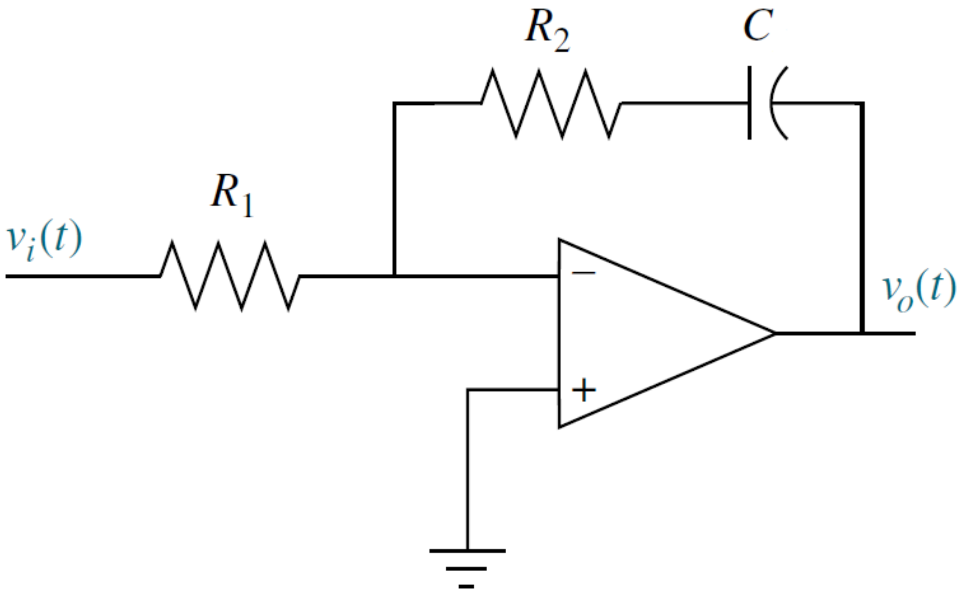
Characteristics of PI Controllers

Features of the PI controller:

- Improve steady-state error.
- Error becomes zero.
- Increases system type.
- Zero at z_c is small and negative.
- Active circuits are required to implement.

$$G_c(s) = \frac{(s + z_c)}{s}$$





Applications of PI Controllers

- The functions $P(s)$ and $I(s)$ can be chosen so the $(s + K_2/K_1)$ term (e.g. controller zero) cancels plant pole.
- Suppose the plant of a second-order system is:

$$G(s) = \frac{1}{(s + T_1)(s + T_2)}$$

- If we apply PI to this plant, and make $K_2/K_1 = T_2$, then

$$\frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{(s + T_2)}{(s + T_1)(s + T_2)} = \frac{1}{s(s + T_1)}$$

- So, the closed-loop transfer-function equation is:

$$T(s) = \frac{O(s)}{I(s)} = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)} = \frac{1}{s^2 + sT_1 + 1}$$

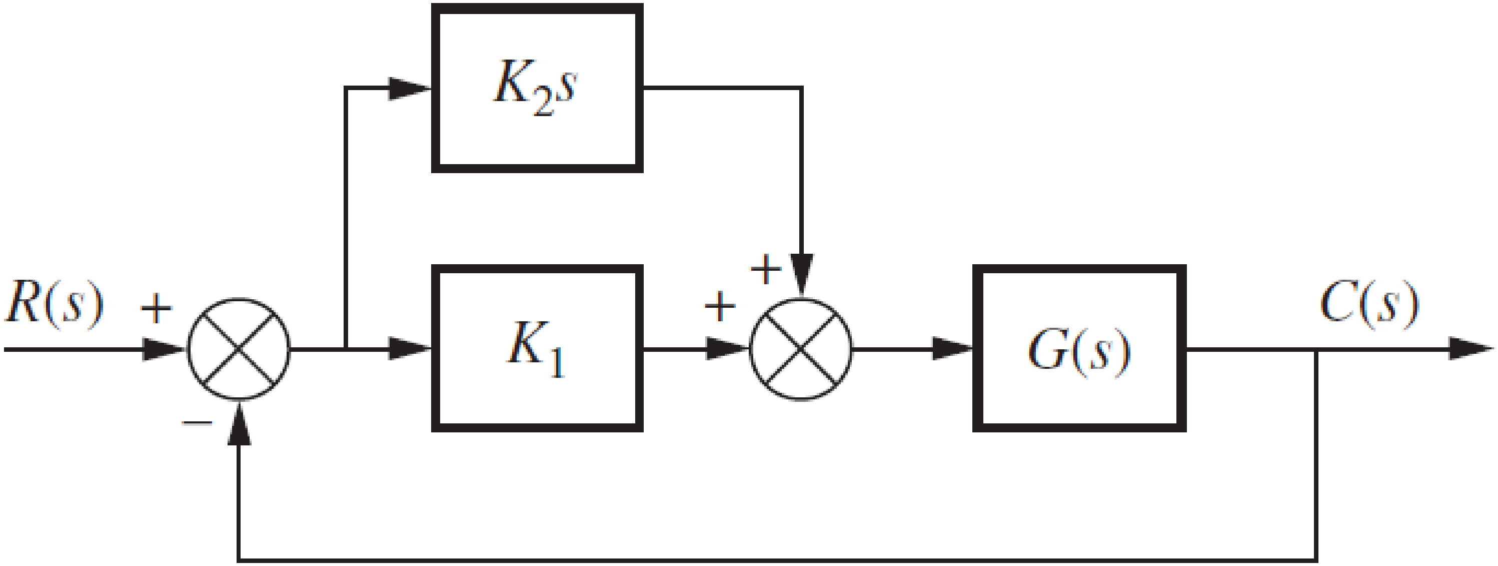
Note, the $I(s)$ term means that the steady-state value is 1.

PD Controllers

- For a PD Controller, its transfer-function equation is:

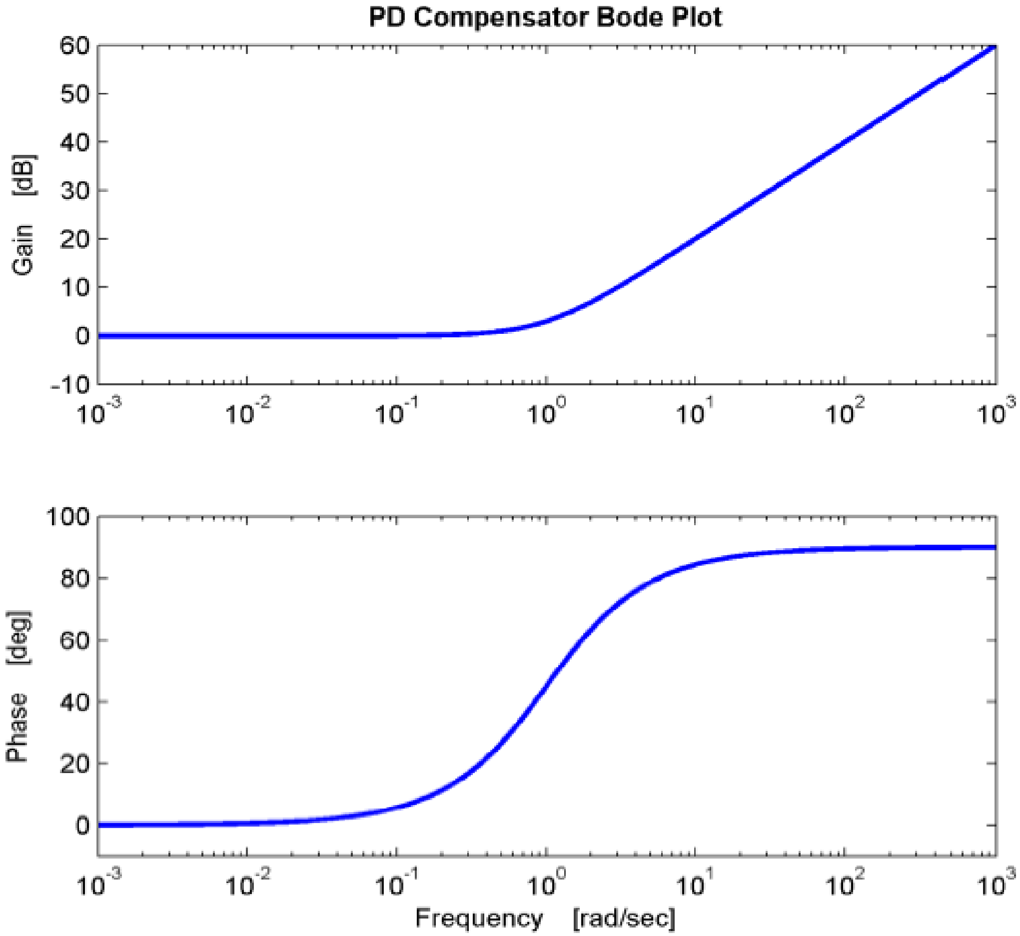
$$G_1(s) = P(s) + D(s) = K_1 + K_2 s = K_2(s + K_1/K_2)$$

Where: $P(s) = K_1$ and $D(s) = K_2 s$.



Characteristics of PD Controllers

- The frequency response of a PD controller is as shown below.
- Magnitude plot:
  - Low: zero gain.
  - Cut-off: half gain.
  - High: +slope gain.
- Phase-shift plot:
  - Low: 0°.
  - Cut-off: +45°.
  - High: +90°.

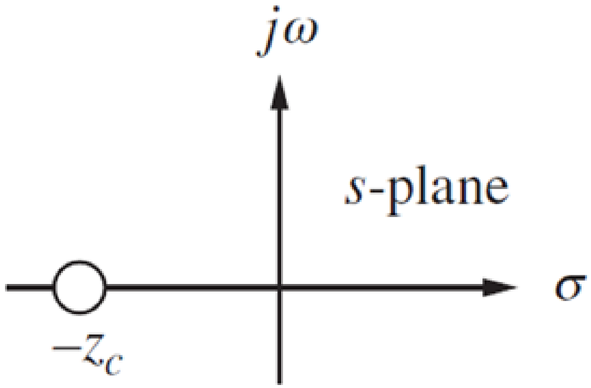


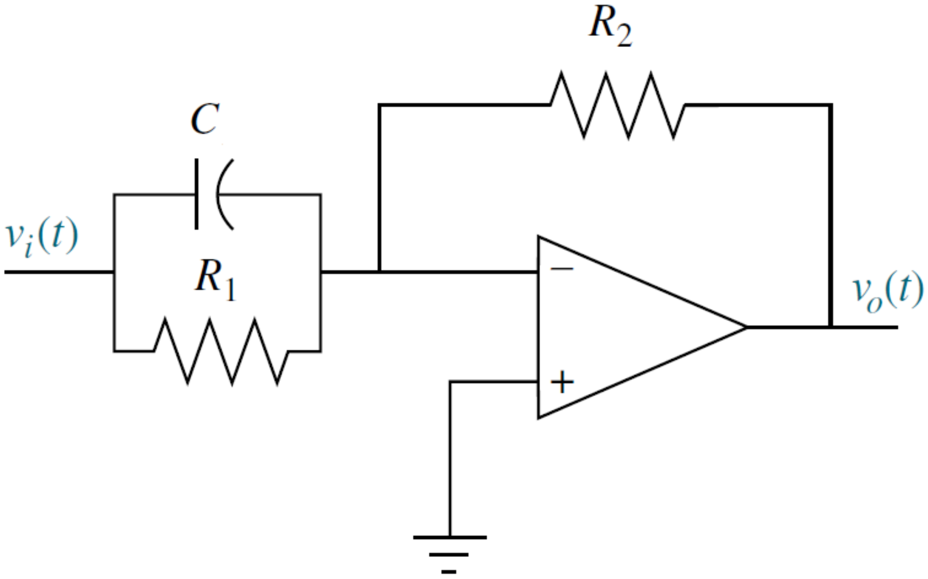
Characteristics of PD Controllers

Features of the PD controller:

- Improve transient response.
- Zero at $-z_c$ is selected to indicate the design point.
- It can cause noise and saturation.
- Implement with rate feedback or with a pole (lead).
- Active circuits are required to implement.

$$G_c(s) = (s + z_c)$$





Applications of PD Controllers

- We could make $s + K_1/K_2$ term to cancel the plant pole.
- If the transfer-function equation of the plant of a second-order system is:

$$G(s) = \frac{1}{s(s+T)}$$

- And PD is applied and assign $K_1/K_2 = T$ then:

$$\frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{s+T}{s(s+T)} = \frac{1}{s}$$

- So, the closed-loop transfer-function equation is:

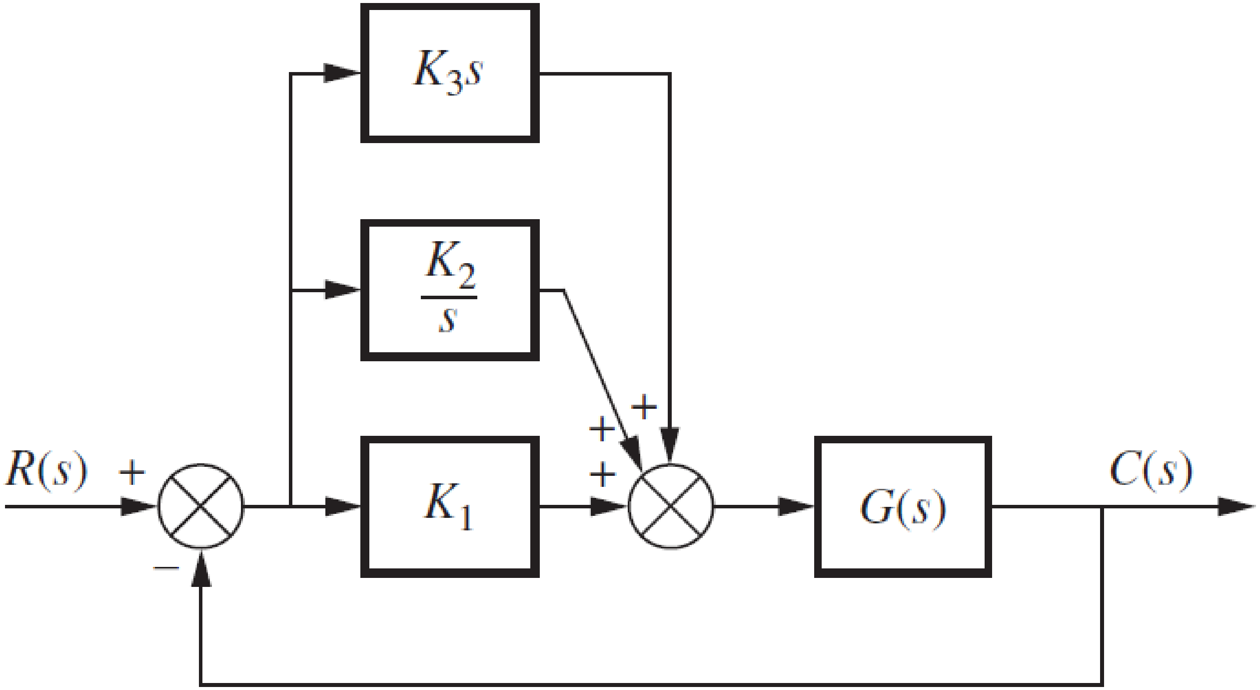
$$T(s) = \frac{O(s)}{I(s)} = \frac{1}{s+1}$$

PID Controllers

- For a PID Controller, its transfer-function equation is:

$$G_c(s) = P(s) + I(s) + D(s) = K_1 + \frac{K_2}{s} + K_3 s$$

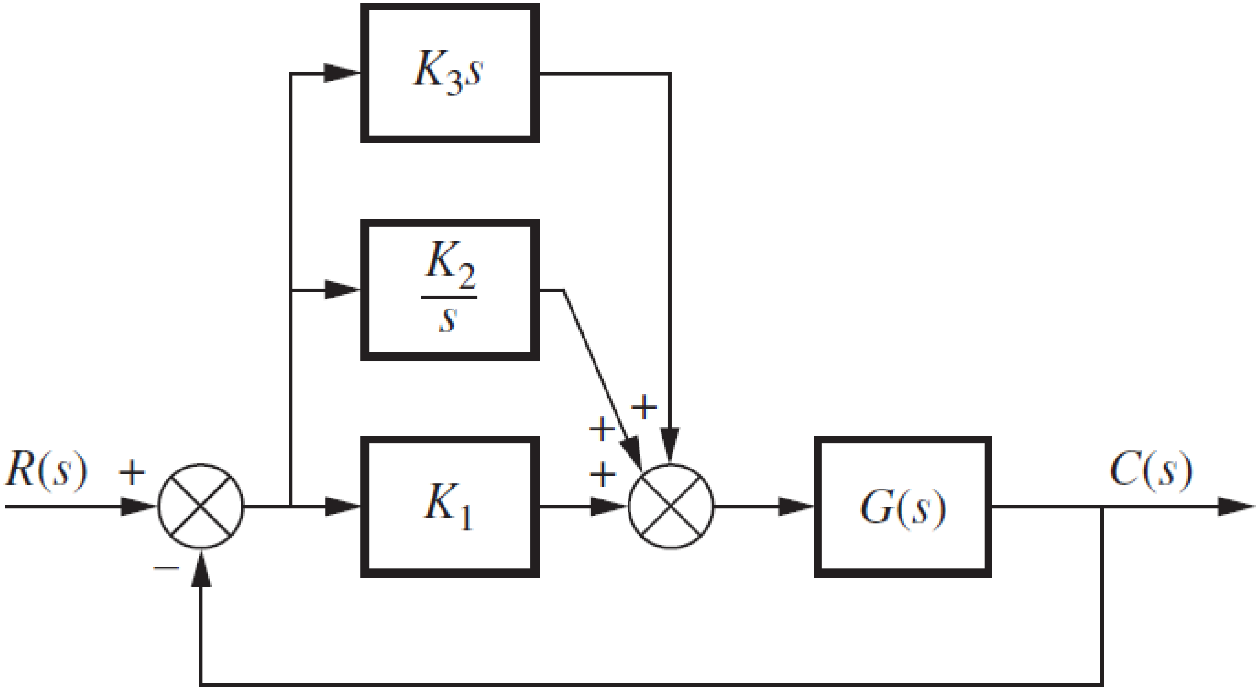
Where: $P(s) = K_1$, $I(s) = K_2/s$, and $D(s) = K_3 s$



PID Controllers

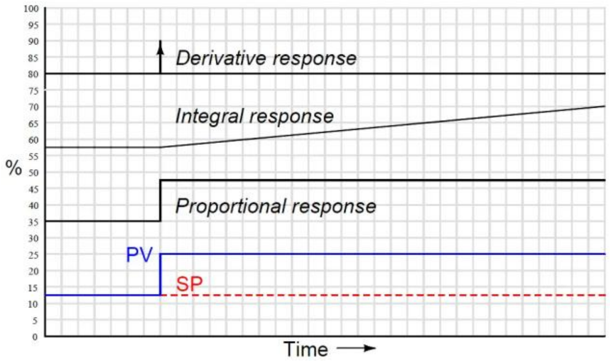
- Alternatively, this gives the transfer-function equation of the PID controller:

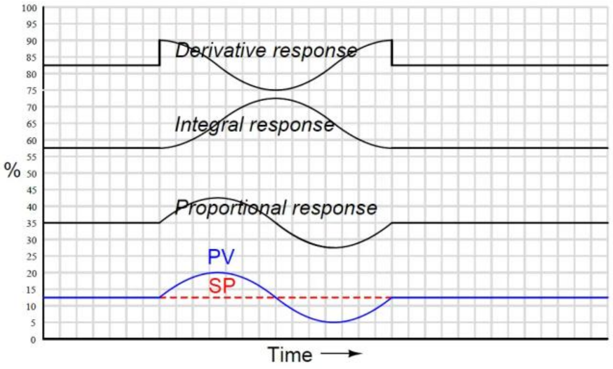
$$G_c(s) = \frac{K_3 s^2 + K_1 s + K_2}{s}$$

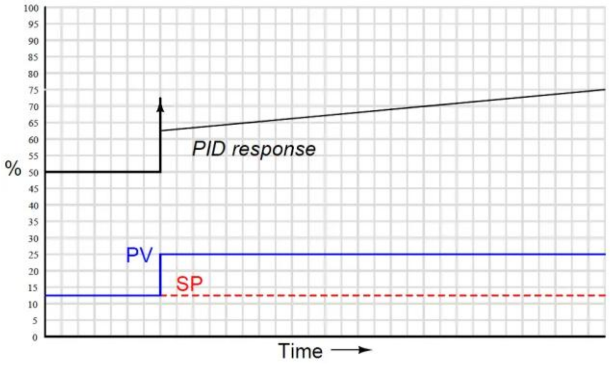


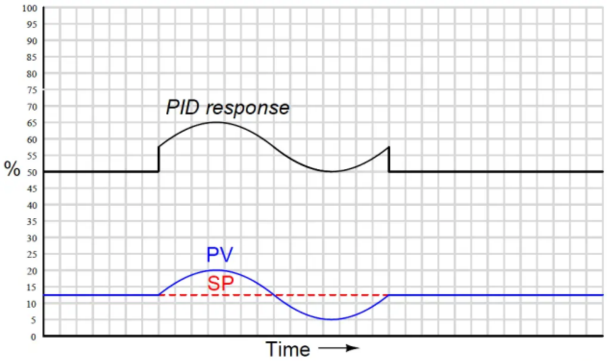
Characteristics of PID Controllers

- The response of a PID controller is as shown below over the step and sinusoidal inputs.







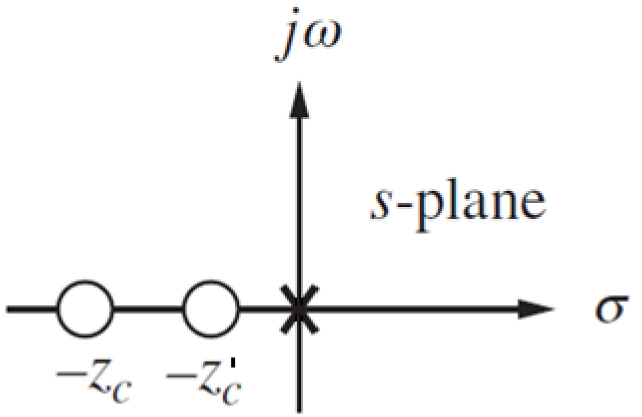


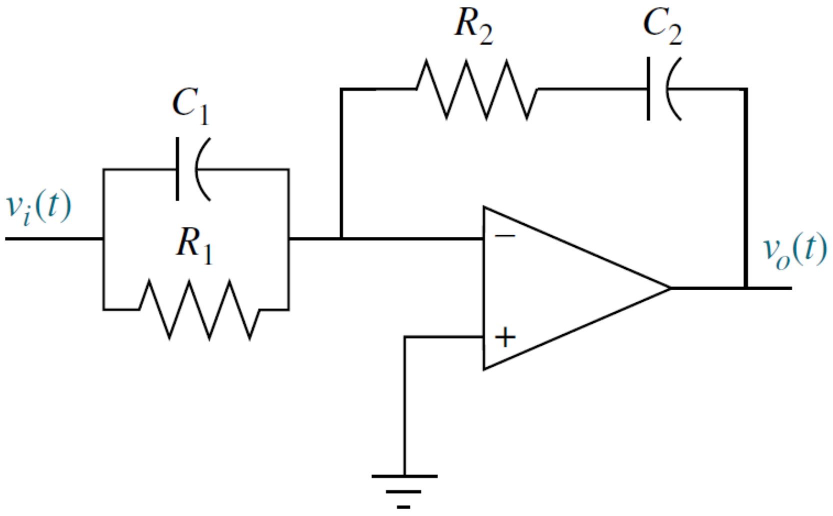
Characteristics of PID Controllers

Features of the PID controller:

- Improve steady-state error and transient response.
- It can cause noise and saturation.
- Implement with rate feedback or with an additional pole.
- Active circuits are required to implement.

$$G_c(s) = \frac{(s + z_c)(s + z_c')}{s}$$



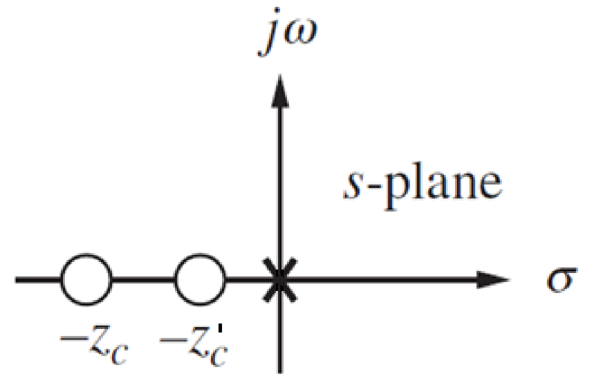


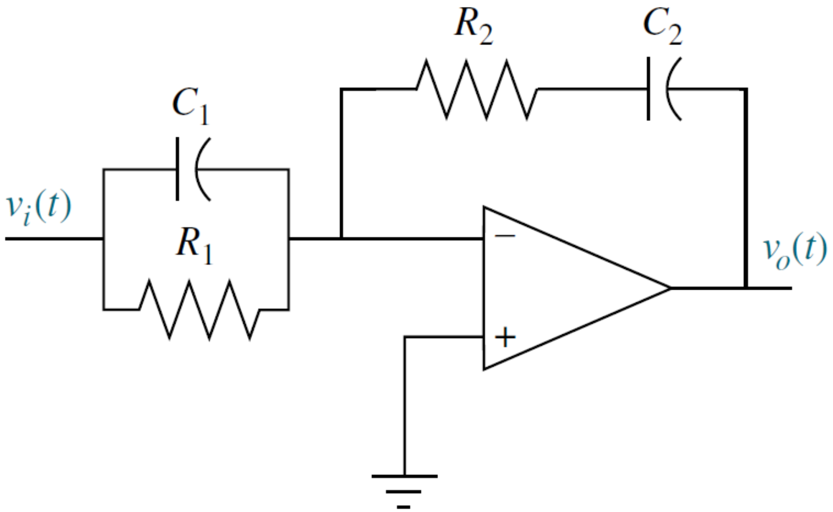
Characteristics of PID Controllers

Features of the PID controller:

- Lag zero at $-z_{lag}$ and pole at the origin improve steady-state error.
- Lag zero at $-z_{lag}$ is close to, and to the left of, the origin.
- Lead zero at $-z_{lead}$ improves transient response.
- Lead zero at $-z_{lead}$ is selected to indicate the design point.

$$G_c(s) = \frac{(s + z_c)(s + z_c')}{s}$$





Applications of PID Controllers

- We could apply a PID controller to a plant of second-order system:

$$G(s) = \frac{1}{1 + bs + as^2}$$

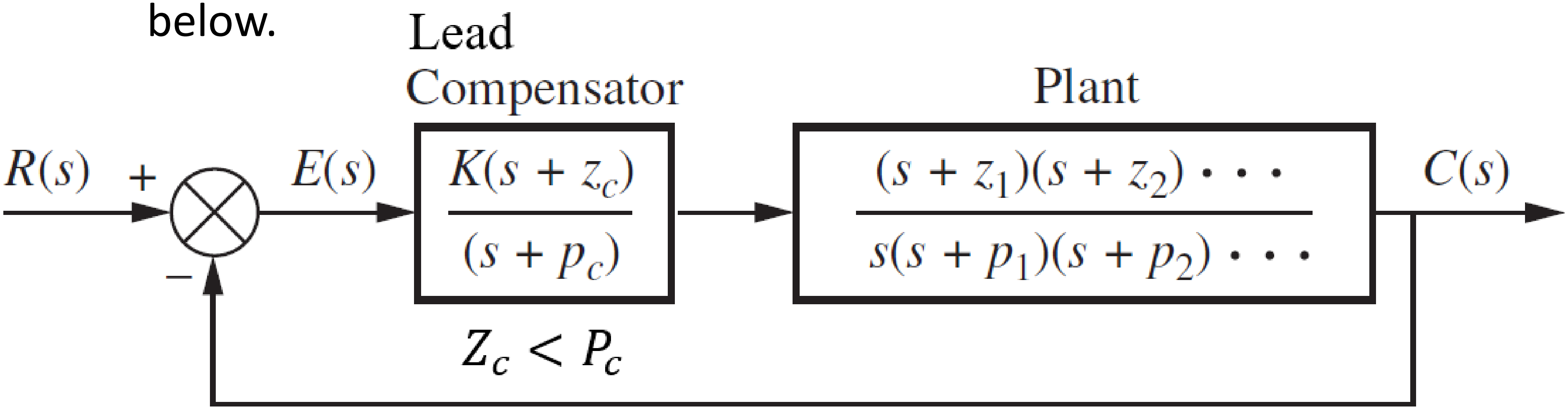
- Thus, set $K_3 = a$, $K_1 = b$, and $K_2 = 1$, the equation is now:

$$T(s) = \frac{O(s)}{E(s)} = G_c(s)G(s) = \frac{\frac{as^2 + bs + 1}{s}}{1 + bs + as^2} = \frac{1}{s}$$

- In all these examples, by careful arrangement, system is either first or second order.
- Cancellation may not give best response, but analysis of systems is easier!

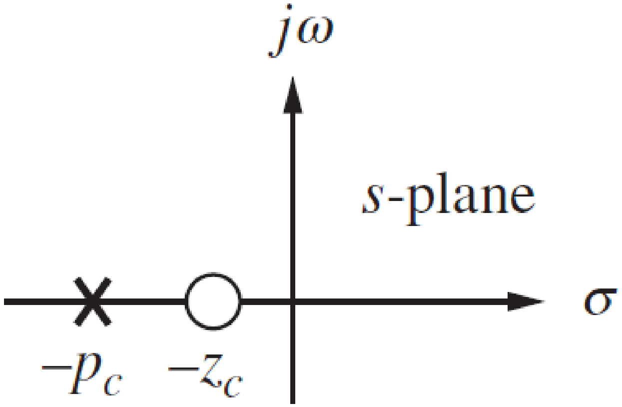
Lead Compensators

- With $P_c > Z_c$, the block diagram of the lead compensator is shown below.



- The pole-zero diagram of a lead compensator is shown below.

$$G_c(s) = \frac{(s+z_c)}{(s+p_c)}$$



Lead Compensators

- The transfer-function equation of a lead compensator is:

$$G_c(s) = \frac{1}{\beta}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}\right) \qquad (\beta < 1)$$

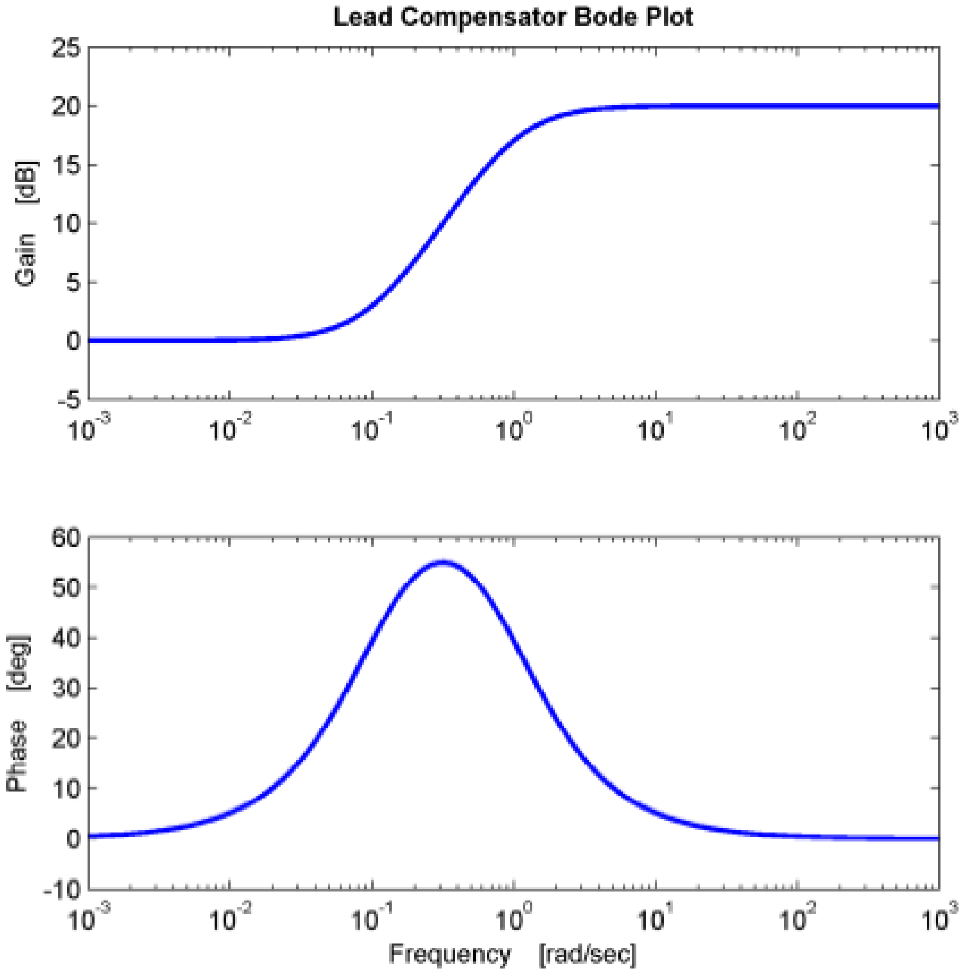
- Or

$$G_{lead}(s) = \frac{s+z_c}{s+p_c} \qquad \text{with} \qquad |p_c| > |z_c|$$

- This compensator consists of 1 pole and 1 zero with |pole| > |zero|.

Characteristics of Lead Compensators

- Frequency response plot of lead compensator.
- Magnitude plot:
    - Low: zero gain.
    - Cut-off: half gain.
    - High: +finite gain.
- Phase-shift plot:
    - Low: 0°.
    - Cut-off: +45°.
    - High: 0°.

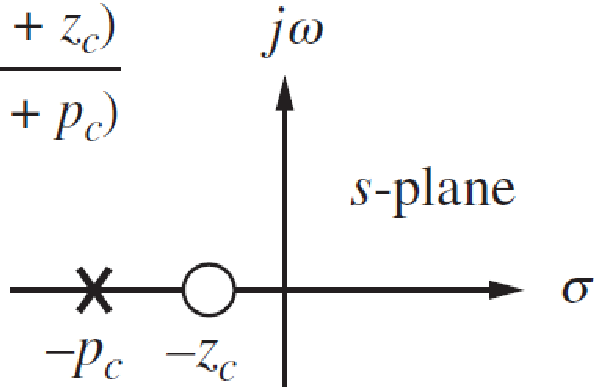


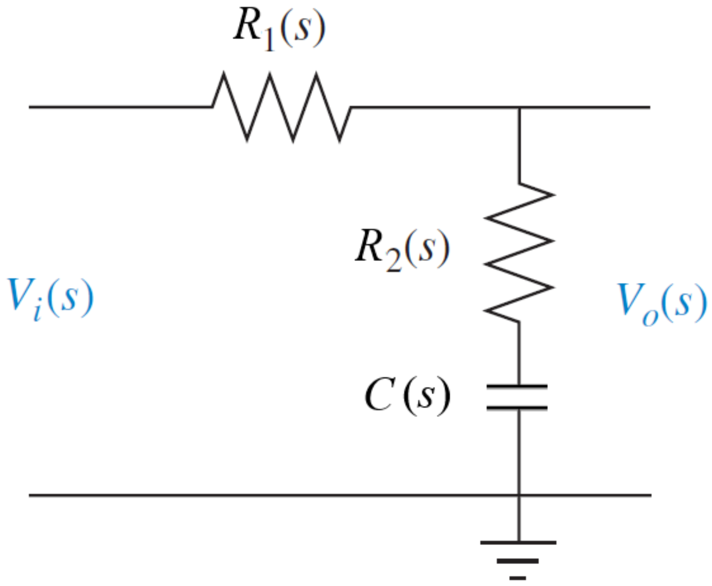
Characteristics of Lead Compensators

Features of the lead compensator:

- Improve transient response.
- Zero at $-z_c$ and pole at $-p_c$ are selected to indicate design point and pole at $-p_c$ is more negative than zero at $-z_c$.
- Active circuits are not required.

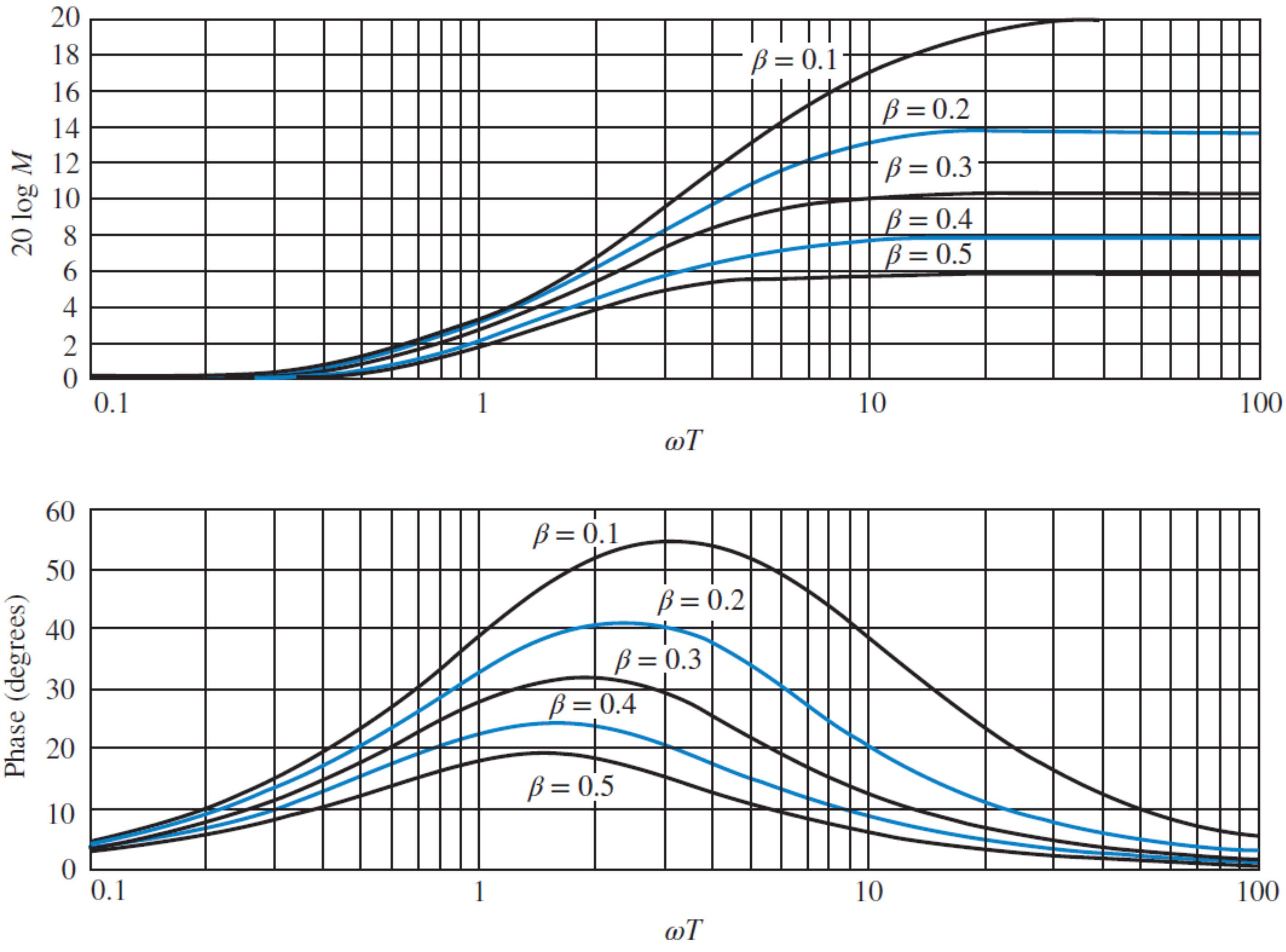
$$G_C(s) = \frac{(s + z_c)}{(s + p_c)}$$





Characteristics of Lead Compensators

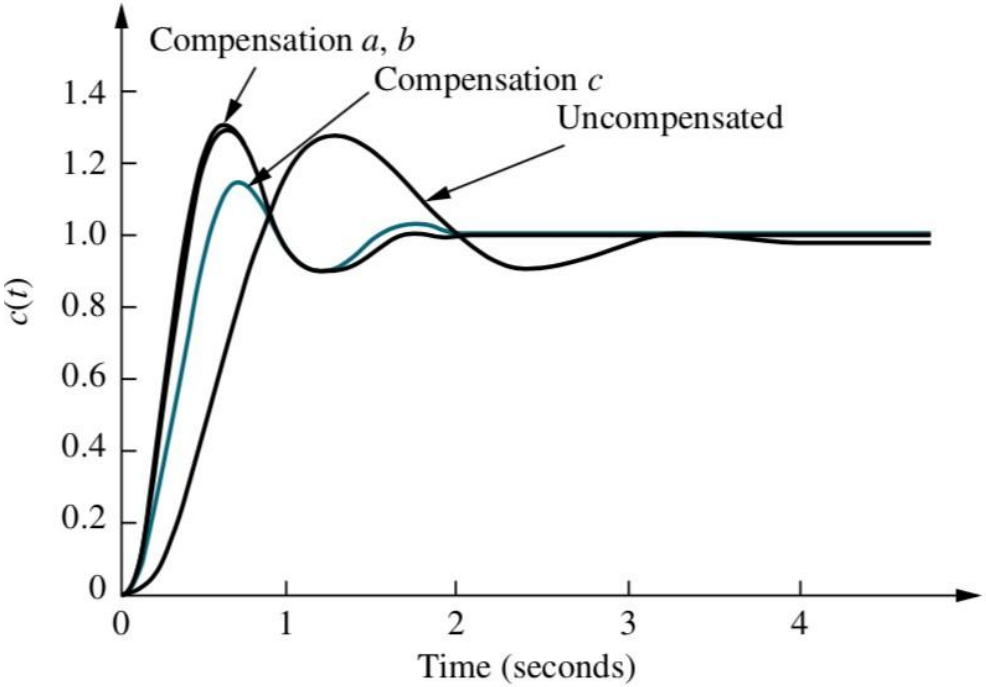
- Frequency response plot of lead compensator with varied β.



Applications of Lead Compensators

- In lead compensator, the zero is closer to the origin than the pole, that is:

$$z_c < p_c$$

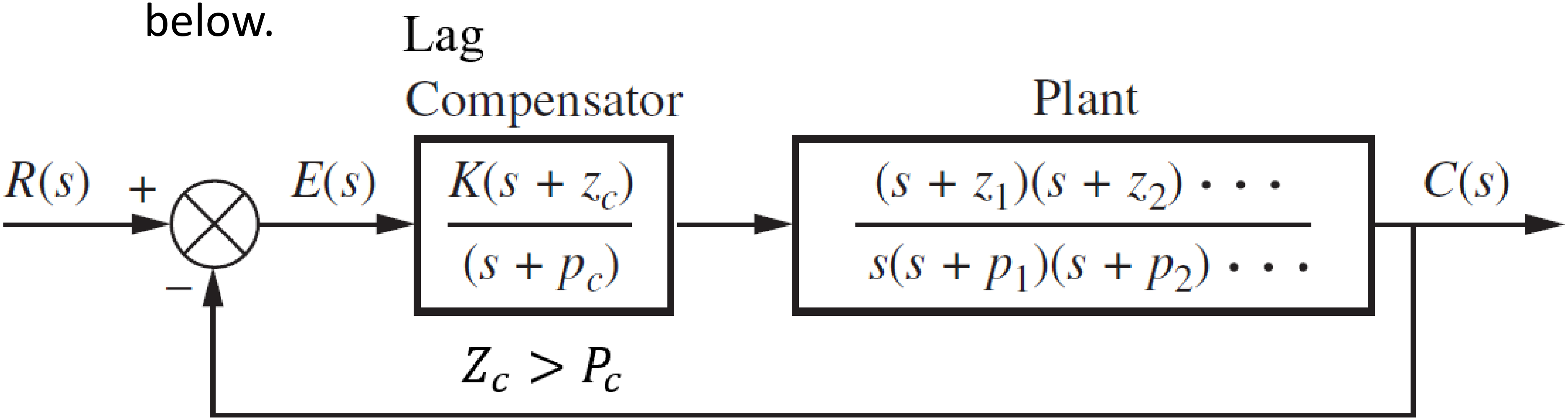


- The lead compensator influences transient response (e.g. percentage overshoot and settling times).

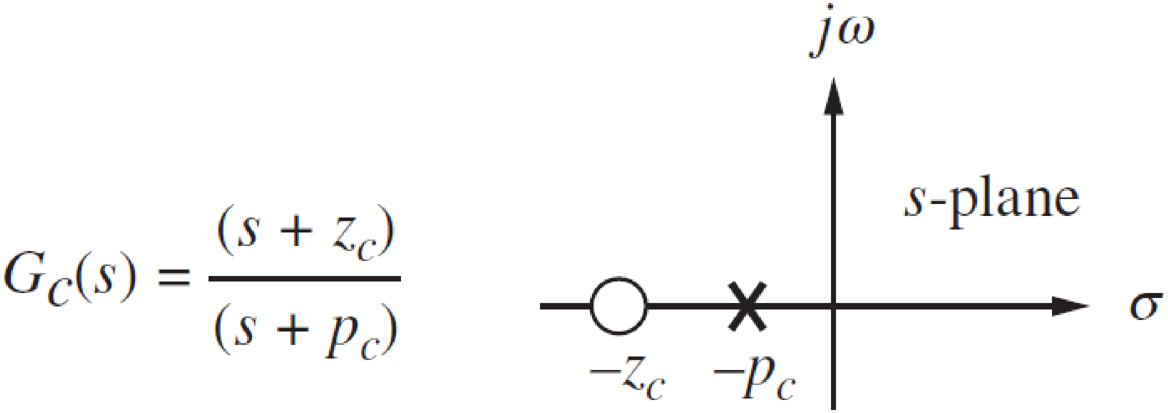
- The uncompensated system is slower compared with the compensated systems (a, b, and c = increasing distance of the poles from origin).

Lag Compensators

- With $Z_c > P_c$, the block diagram of the lag compensator is shown below.



- The pole-zero diagram of a lag compensator is shown below.



$$G_C(s) = \frac{(s+z_c)}{(s+p_c)}$$

Lag Compensators

- The transfer function of the lag compensator is:

$$G_c(s) = \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \qquad (\alpha > 1)$$

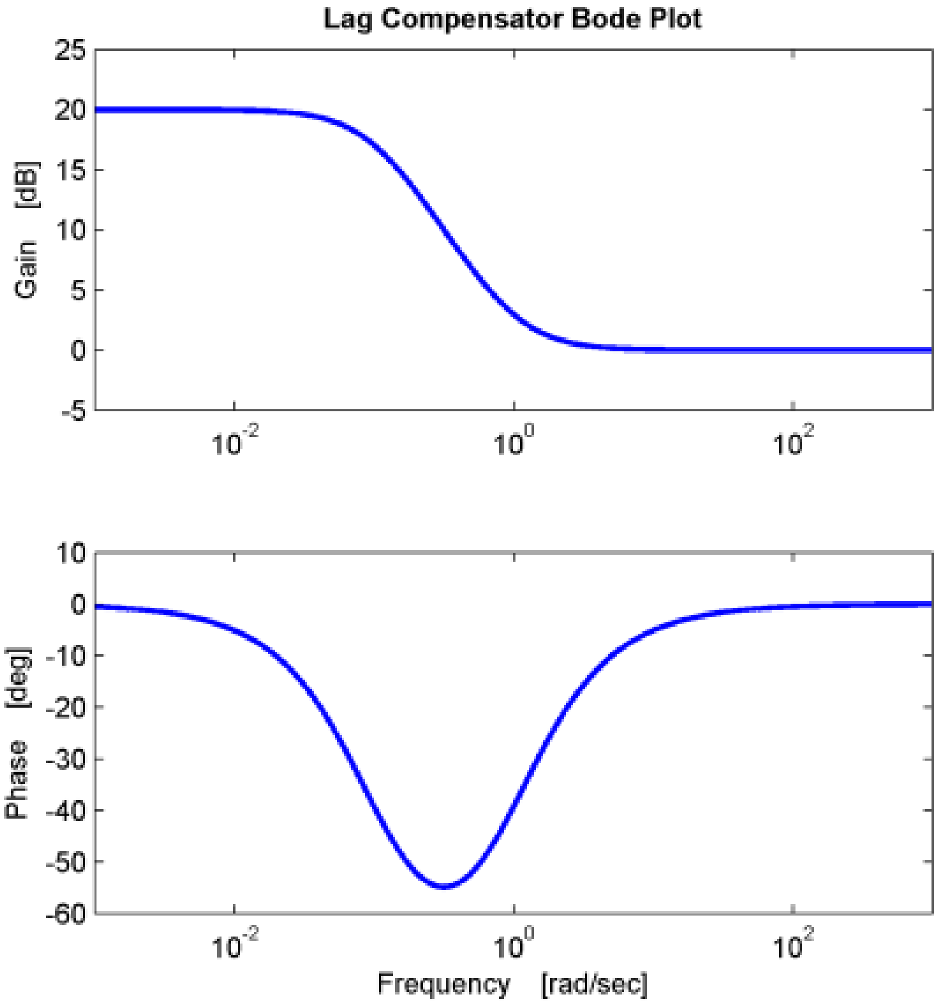
- Or

$$G_{lag}(s) = \frac{s + z_c}{s + p_c} \quad \text{with} \quad |p_c| < |z_c|$$

- This compensator consists of 1 pole and 1 zero with |pole| < |zero|.

Characteristics of Lag Compensators

- Frequency response plot of lag compensator.
- Magnitude plot:
  - Low: +finite gain.
  - Cut-off: half gain.
  - High: zero gain.
- Phase-shift plot:
  - Low: 0°.
  - Cut-off: -45°.
  - High: 0°.

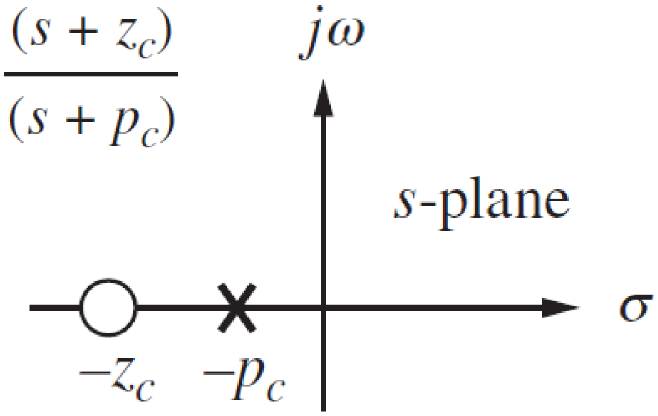


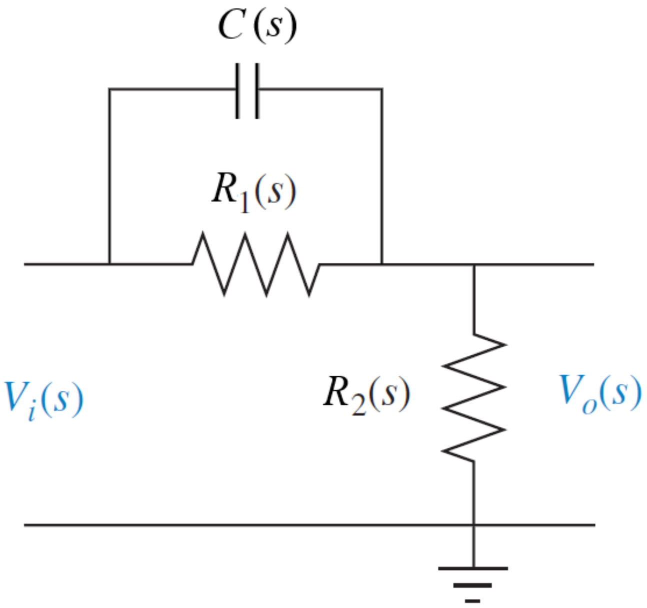
Characteristics of Lag Compensators

Features of the lag compensator:

- Improve steady-state error.
- Error is improved but not driven to zero.
- Pole at $-p_c$ is small and negative.
- Zero at $-z_c$ is close to, and to the left of the pole at $-p_c$.
- Active circuits are not required.

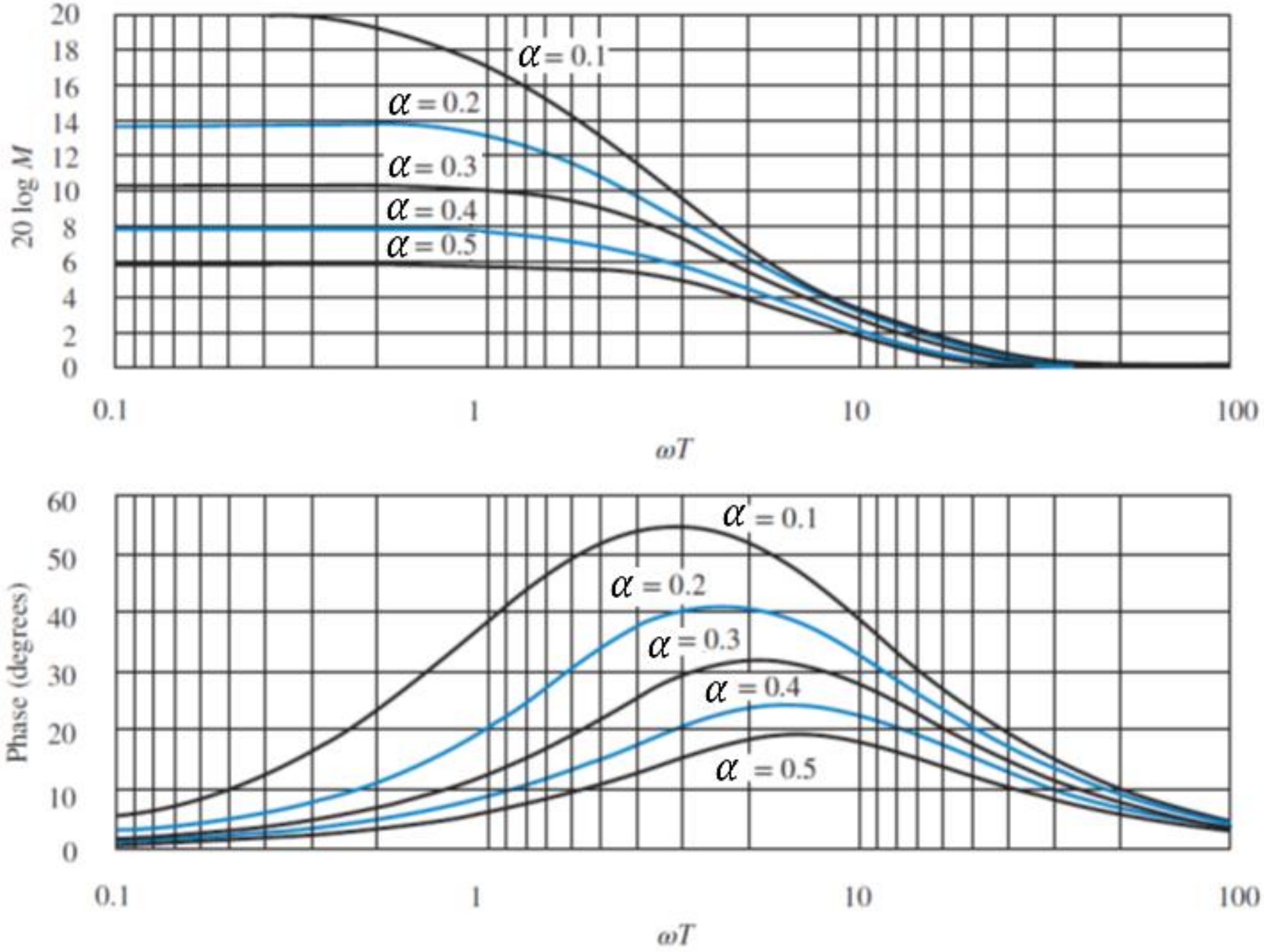
$$G_c(s) = \frac{(s + z_c)}{(s + p_c)}$$





Characteristics of Lag Compensators

- Frequency response plot of lag compensator with varied α.

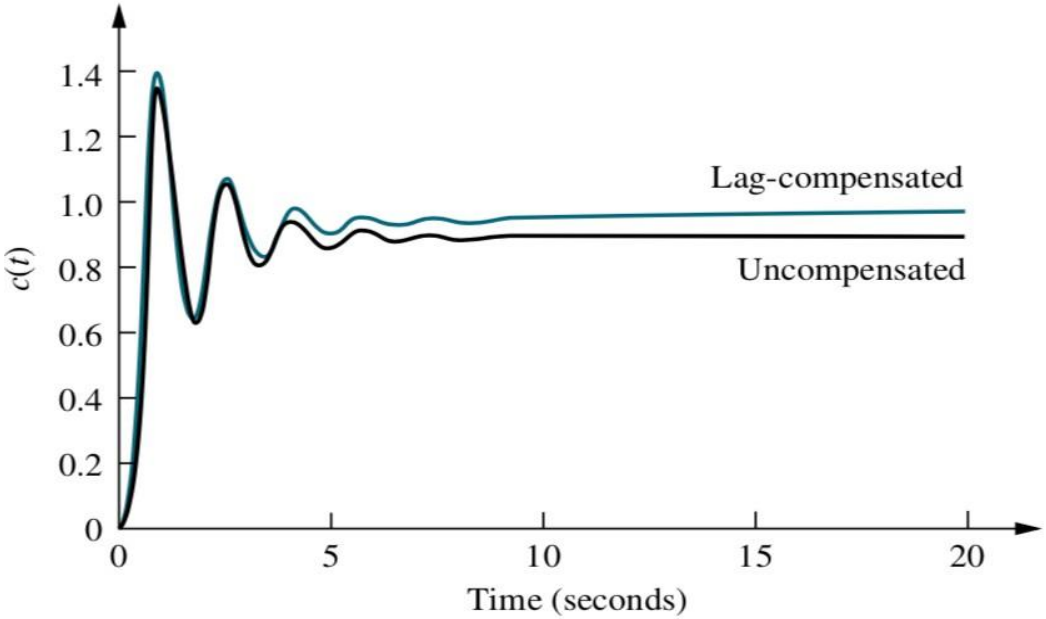


Applications of Lag Compensators

- In a lag compensator, the pole is closer to the origin than the zero, that is:

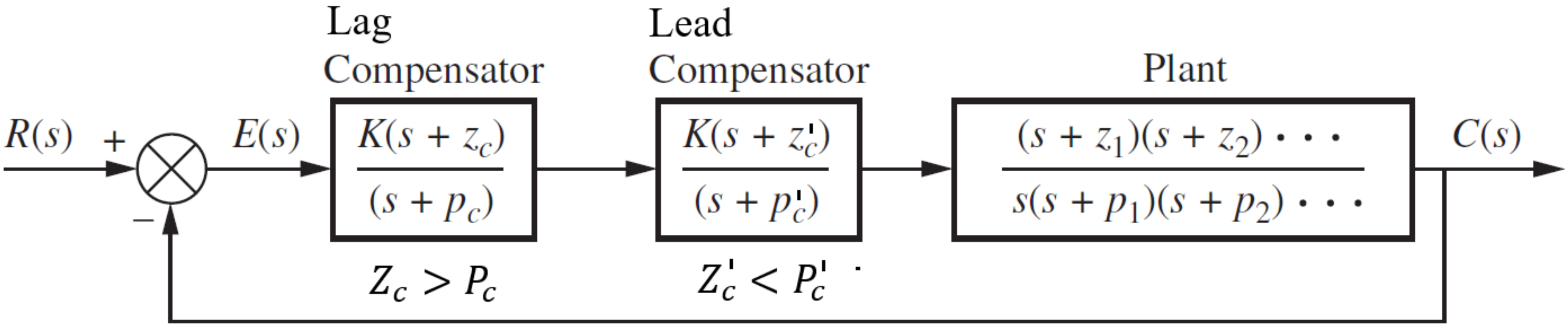
$$z_c > p_c$$

- The lag compensator reduces steady-state error.



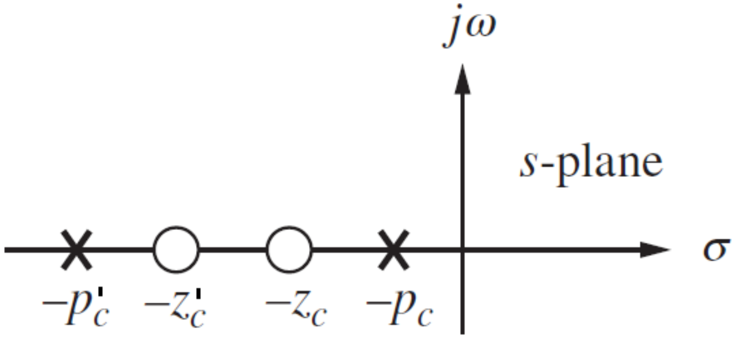
Lead-Lag Compensators

- Block diagram of the lead-lag compensator is shown below.



- The pole-zero diagram of a lead-lag compensator is shown below.

$$G_C(s) = \frac{(s+z_c)}{(s+p_c)}\frac{(s+z'_c)}{(s+p'_c)}$$



Lead-Lag Compensators

- Transfer-function equation of the lead-lag compensator:

$$G_c(s) = \left(\frac{s + \frac{1}{T_1}}{s + \frac{\gamma}{T_1}} \right) \left(\frac{s + \frac{1}{T_2}}{s + \frac{1}{\gamma T_2}} \right) \qquad (\gamma > 1)$$

- Or

$$G_{lead-lag}(s) = G_{lead}(s) G_{lag}(s)$$

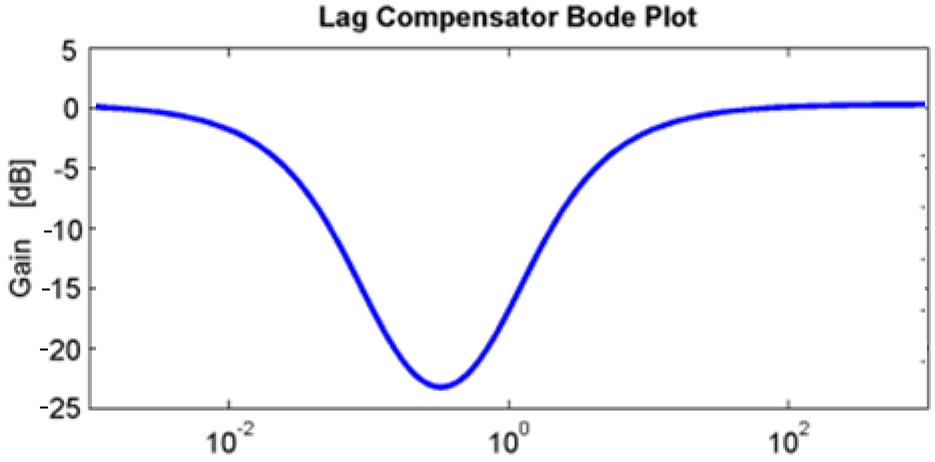
$$= \left(\frac{s + z_{c(lag)}}{s + p_{c(lag)}} \right) \left(\frac{s + z_{c(lead)}}{s + p_{c(lead)}} \right)$$

- The pole and zero of the lag and lead parts of the lead-lag controller are:

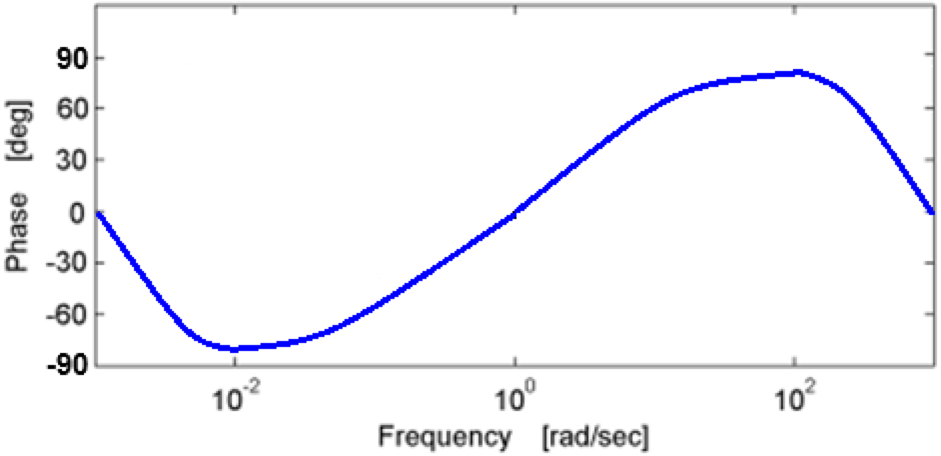
$$\left| p_{c(lag)} \right| < \left| z_{c(lag)} \right| \quad \text{and} \quad \left| z_{c(lead)} \right| < \left| p_{c(lead)} \right|$$

Characteristics of Lead-Lag Compensators

- Frequency response plot of lead-lag compensator.
- Magnitude plot:
  - Low: zero gain.
  - Cut-off 1: half gain.
  - Cut-off 2: half gain.
  - High: zero gain.



- Phase-shift plot:
  - Low: 0°.
  - Cut-off 1: -45°.
  - Cut-off 2: 45°.
  - High: 0°.



Characteristics of Lead-Lag Compensators

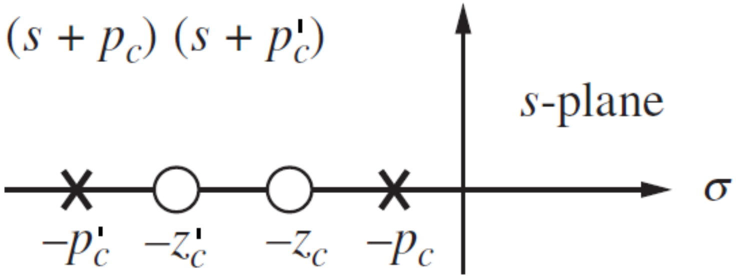
- Lead-lag compensator improves steady-state error and transient response:

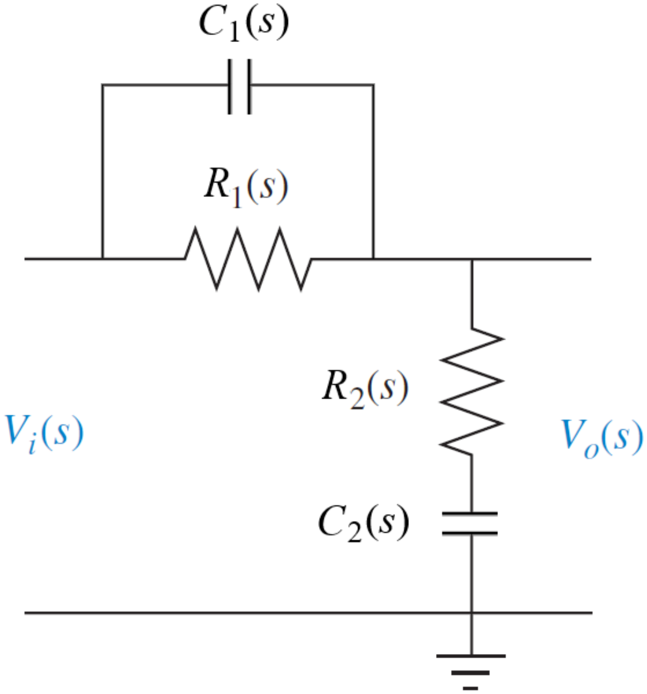
- Lag pole at $-p_{lag}$ and lag zero at $-z_{lag}$ are used to improve steady-state error.

- Lag pole at $-p_{lag}$ is small and negative.

- Lag zero at $-z_{lag}$ is close to, and to the left of, lag pole at $-p_{lag}$

$$G_c(s) = \frac{(s+z_c)}{(s+p_c)}\frac{(s+z_c')}{(s+p_c')}$$

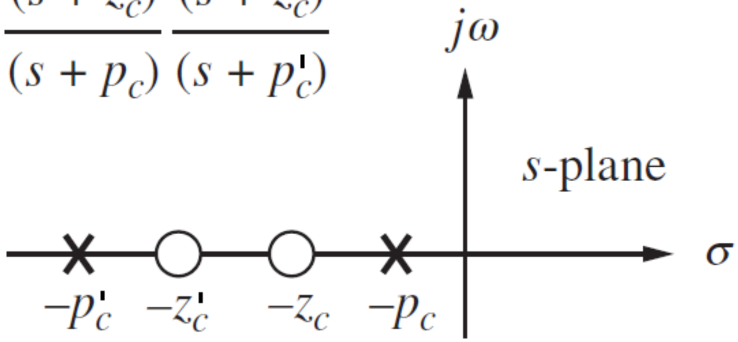


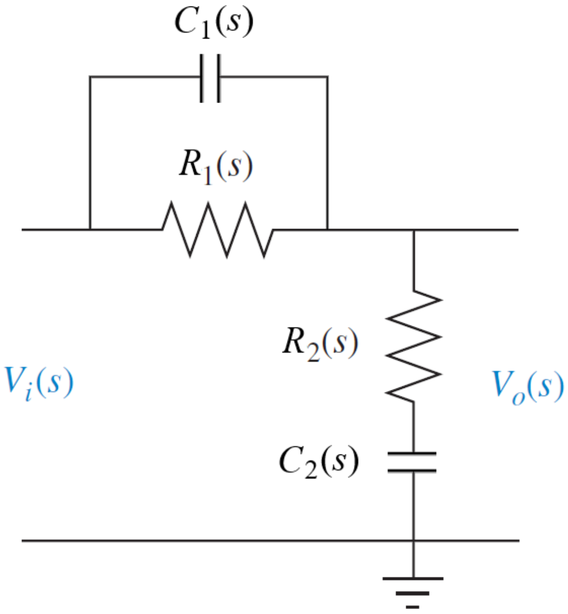


Characteristics of Lead-Lag Compensators

- Lead pole at $-p_{lead}$ and lead zero at $-z_{lead}$ are used to improve transient response.
- Lead zero at $-z_{lead}$ and the lead pole at $-p_{lead}$ are selected to indicate the design point.
- Lead pole at $-p_{lead}$ is more negative than lead zero at $-z_{lead}$.
- Active circuits are not required to implement.

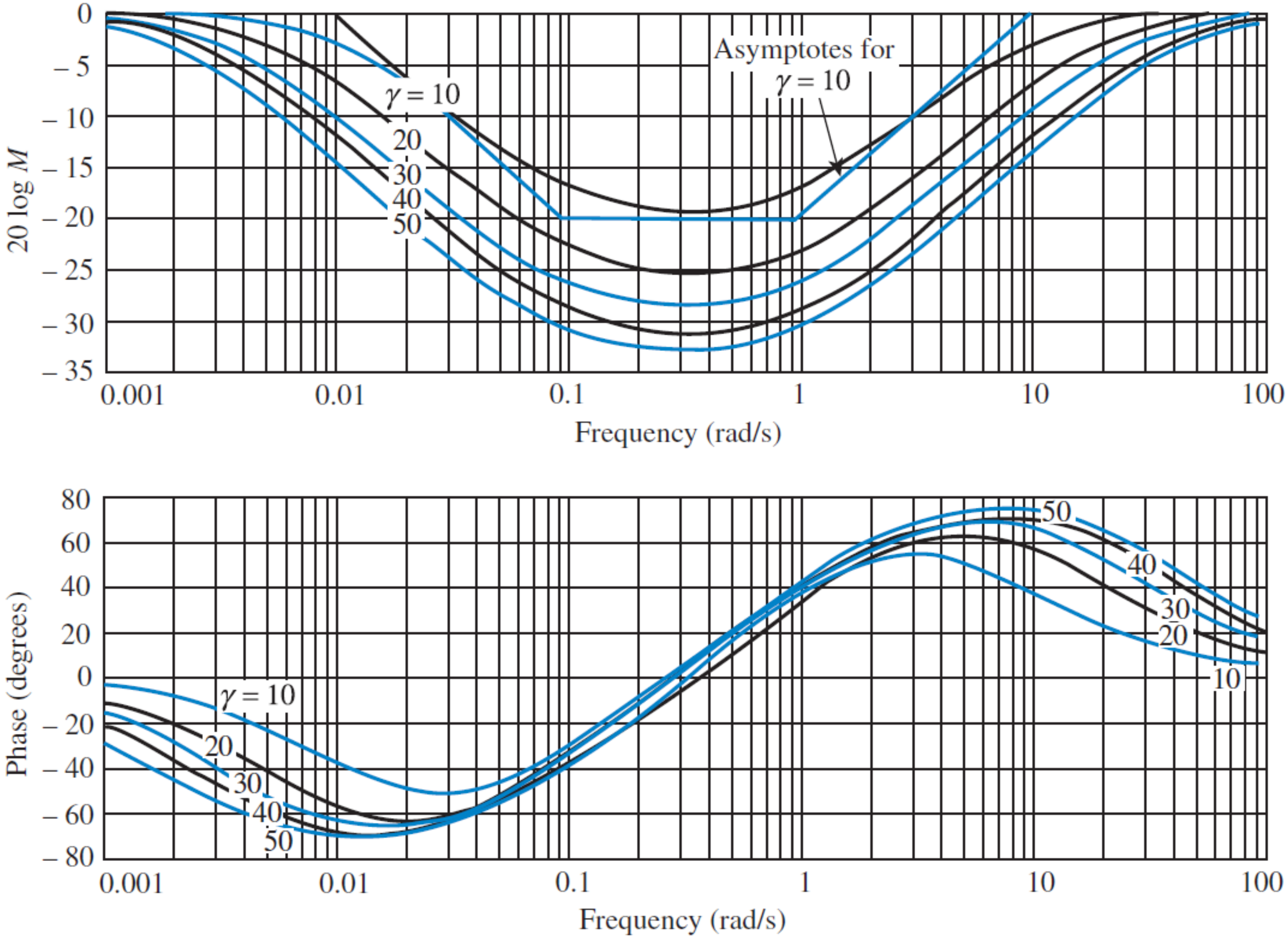
$$G_c(s) = \frac{(s+z_c)}{(s+p_c)}\frac{(s+z_c')}{(s+p_c')}$$





Characteristics of Lead-Lag Compensators

- The frequency response of lead-lag compensator with varied γ:



Applications of Lead-Lag Compensators

- Considering the transfer-function equation of the second-order plant is:

$$G(s) = \frac{K}{(s+T_1)(s+T_2)}$$

- For improving transient response, we can make z_c in the $G_c(s)$ to be equal to largest of T_1 and T_2, say T_2, to speed up the system.
- Then, the open-loop transfer-function equation of the system is:

$$\frac{O(s)}{E(s)} = G_c(s)G(s)$$

$$= \left(\frac{s+z_{c(lag)}}{s+p_{c(lag)}}\right)\left(\frac{s+z_{c(lead)}}{s+p_{c(lead)}}\right)\frac{K}{(s+T_1)(s+T_2)}$$

Applications of Lead-Lag Compensators

- Thus, the transfer-function equation of the closed-loop system is:

$$T(s) = \frac{O(s)}{I(s)} = \frac{\text{Forward}}{1 - \text{Loop}}$$

- This gives the closed-loop transfer-function equation:

$$T(s) = \frac{O(s)}{I(s)} = \frac{\left[\dfrac{K}{\left(s + p_{c(lead)}\right)(s + T_1)}\right]}{1 - (-1)\left(\dfrac{K}{\left(s + p_{c(lead)}\right)(s + T_1)}\right)}$$

$$= \frac{K}{\left(s + p_{c(lead)}\right)(s + T_1) + K}$$

Applications of Lead-Lag Compensators

$$T(s) = \frac{K}{s^2 + s\left(p_{c(lead)} + T_1\right) + \left(p_{c(lead)} T_1\right) + K}$$

- This case is a further example of pole-zero cancellation for system improvement.
- Note: a pole is like $1 + sT$ term on the denominator and a zero is such a term on numerator.
- For improving the steady-state condition of the system, we can make z_c of the lead part in the $G_c(s)$ to be equal to the smaller of T_1 and T_2, say T_1, to remove more dominant pole in the system.

Lead-Lag Compensator Characteristics

- Then, the pole in the lead part is used to cancel the zero of the lag part of the compensator.
- This leaves the pole of the lag part to be varied and assign to be very close to the origin e.g. $p_{c(lag)} \cong 0$ (to simulate an integral function like to the system).

$$T(s) = \frac{K}{\left(s + p_{c(lag)}\right)(s + T_2) + K} = \frac{K}{s(s + T_2) + K}$$

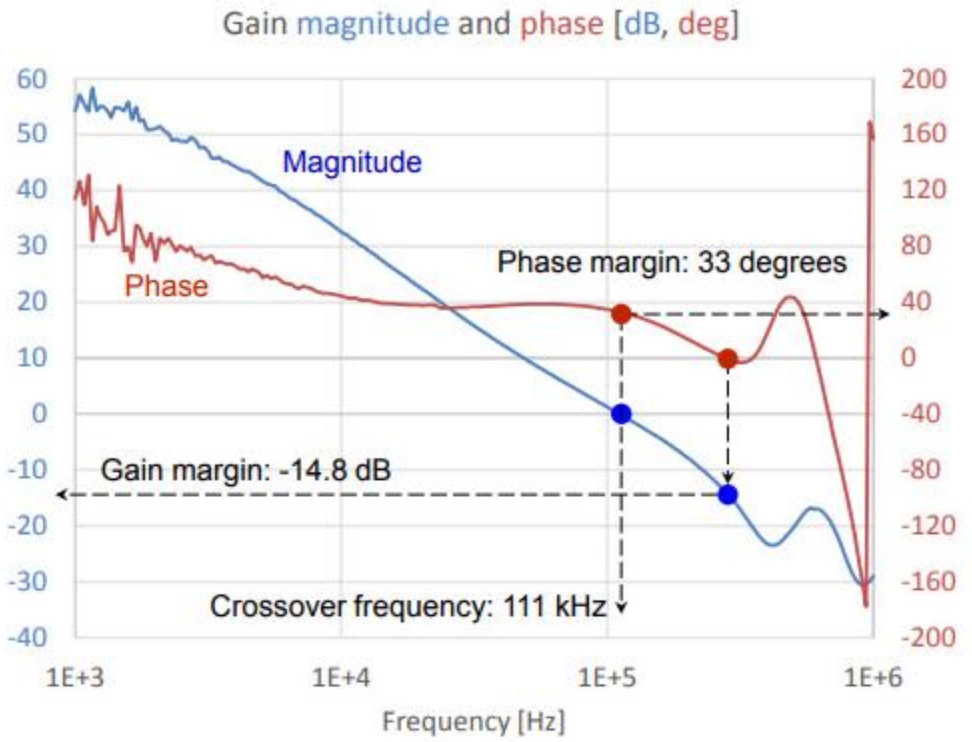
- Thus, this would improve the steady-state condition of the system by reducing or removing the steady-state error in the system.

Intro to Controller/Compensator Design

- We will focus on modifying system characteristics by applying feedback.
- Furthermore, we will be able to tailor the closed-loop transfer function with the addition of a compensator.
- Compensator design is a compromise between two competing goals.
  - Performance: Keeping the open loop gain high reduces system errors and the effects of disturbances.
  - Stability: The closed-loop system must be kept stable by carefully managing the gain where the phase approaches $-180°$.

Intro to Controller/Compensator Design

- Compensator design can often be philosophically reduced to two (interrelated) problems:
  - one operating at low frequencies to achieve the required performance,
  - the other at high frequency to ensure stability.



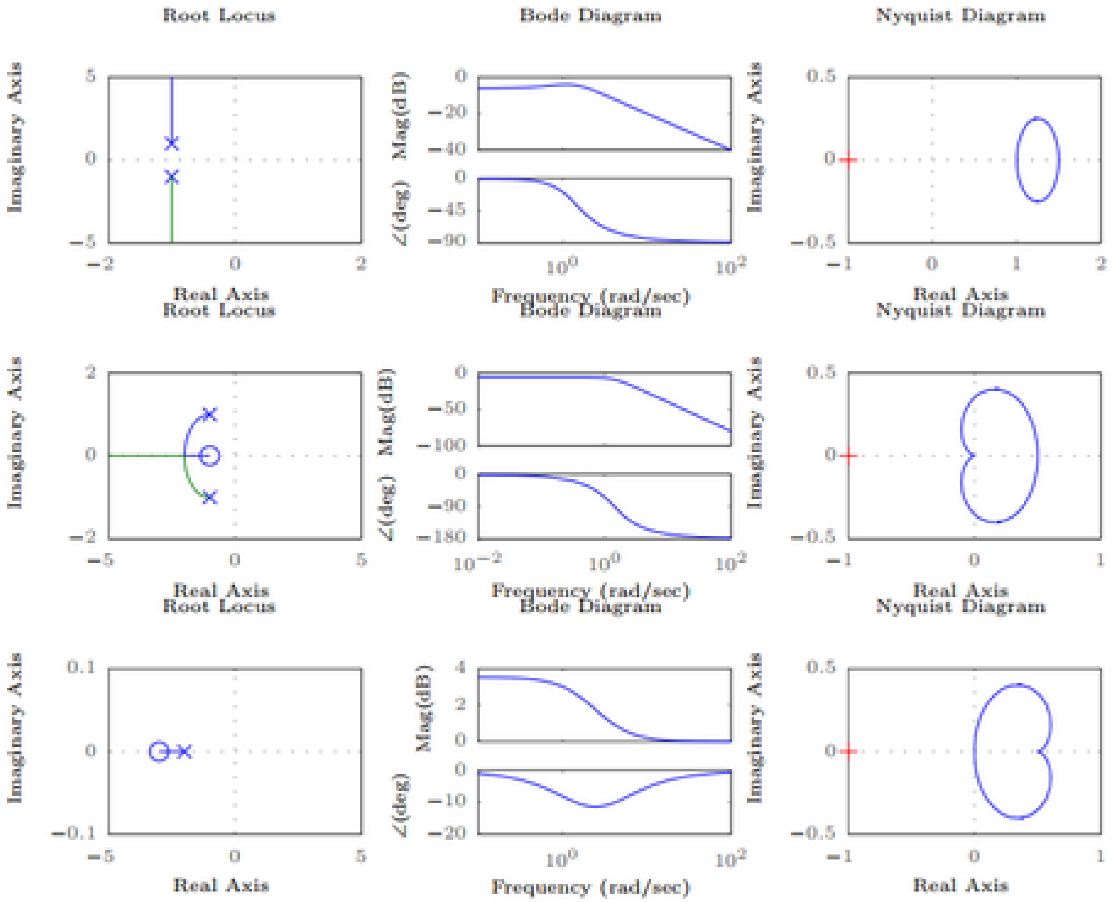
Methods for Controller/Compensator Design

There are a variety of approaches to designing a controller of compensator:

1. Choose a compensator structure and then tune manually.
2. Choose a compensator model and tune using a “recipe” (e.g. Ziegler-Nichols).
3. Use a model and solve for desired pole locations.
4. Measure the system performance and use a graphical technique.
5. Use a mathematical model with a graphical technique.
6. Use mathematical tools to achieve optimal performance (State-Space Analysis).

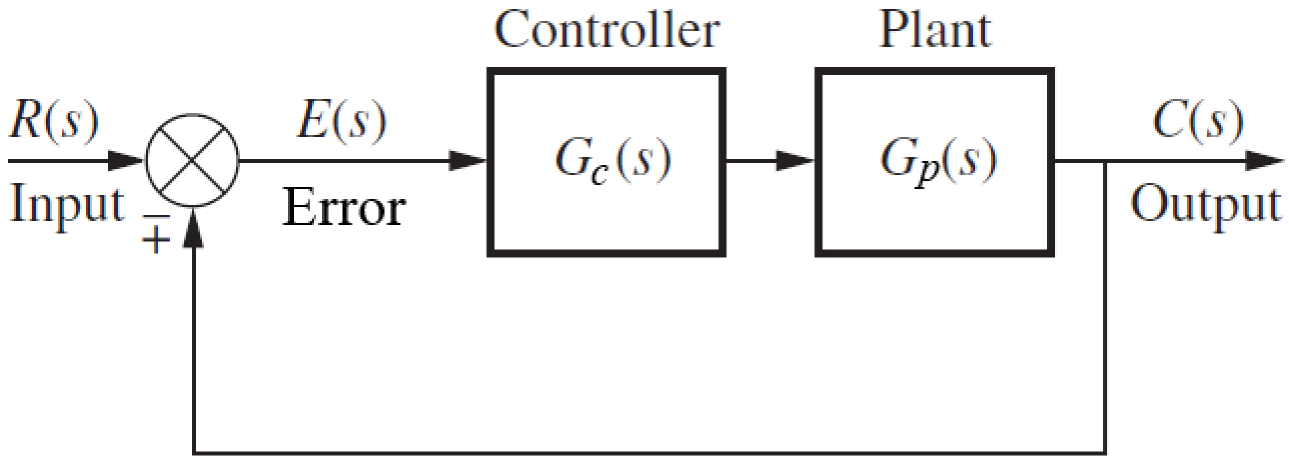
Controller/Compensator Design in the Course

- In the remaining lectures, we will focus on the graphical methods which form the classical control.
- These are mainly about items 3 and 5 in the list.



Convention for System Topology and Notation

- We will generally design out controller or compensators assuming unity gain feedback with the compensator $C(s)$ is placed in the forward path.



- Remember that this is equivalent to a system with the controller or compensator in the feedback path.