VICTORIA UNIVERSITY OF WELLINGTON TE HERENGA WAKA

XMUT315 Control Systems Engineering

Tutorial 1: Laplace Transform and System Modelling

A. Laplace Transform

1. Determine the complete response of the following model, which has a ramp input. [6 marks]

$$\frac{dx}{dt} + 3x = 5t \qquad \text{and} \qquad x(0) = 10$$

2. Given the following differential equation, solve for $y(t)$ if all initial conditions are zero. Use the Laplace transform.

$$\frac{d^2y}{dt^2} + 12\frac{dy}{dt} + 32y = 32u(t)$$

   Determine also its inverse Laplace transform. [12 marks]

3. Obtain the Laplace transform the following second-order differential equation as shown below. Notice the initial conditions of the equation. [10 marks]

$$\ddot{x}(t) + 4\dot{x}(t) + 53x(t) = 15u(t)$$

   And

$$\dot{x}(0) = 8 \qquad x(0) = -19$$

4. Obtain the inverse Laplace transform of the following transfer function equation. [8 marks]

$$X(s) = \frac{5}{s(s + 3)}$$

5. Inverse Laplace transform the following transfer function equation by representing it as the sum of terms that appear in Laplace transform table. [14 marks]

$$X(s) = \frac{8s + 13}{s^2 + 4s + 53}$$

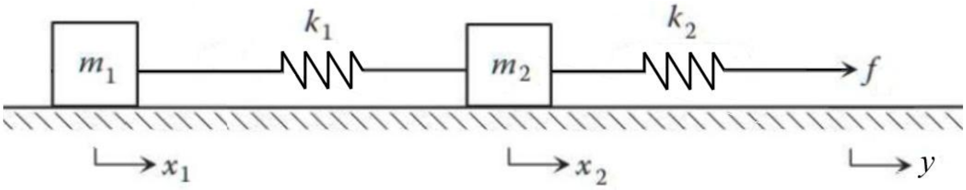
6. Given below is a transfer function $F(s)$ with real and repeated roots in the denominator.

$$F(s) = \frac{2}{(s + 1)(s + 2)^2}$$

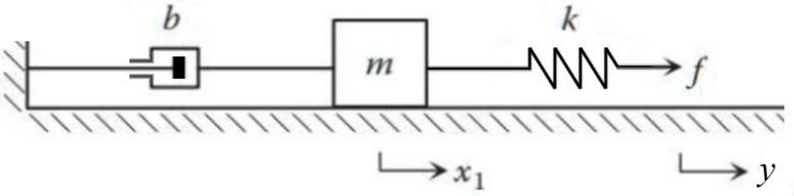
Find $f(t)$ through inverse Laplace transform. [10 marks]

B. Modelling of Physical Systems

7. Figure below shows a two-mass system where the displacement $y(t)$ of the right-hand end of the spring is a given function. The masses slide on a frictionless surface. When $x_1 = x_2 = y = 0$, the springs are at their free lengths. Derive the equations of motion and determine transfer function equation of the system as $X_2(s)/Y(s)$. [10 marks]



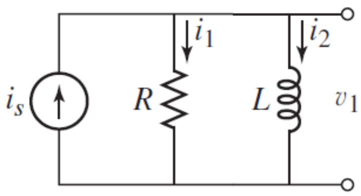
8. For the translational mechanical system shown in the figure below, the input is the displacement y of the right-end of the spring due to a force $f(t)$ is acting on the right most end of the spring to the right. The output is the displacement x of the mass. The spring is at its free length when $x = y$.



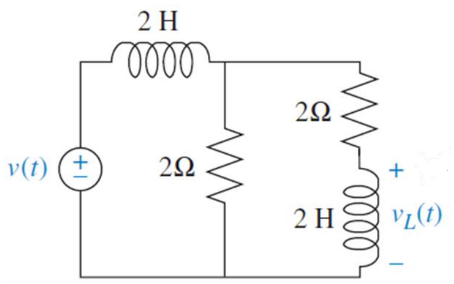
   a. Derive the equation of the motion for the system. [6 marks]

   b. Determine the transfer function equation of the system, $G(s) = X(s)/F(s)$, when c = 4 N-s/m, m = 5 kg, and k = 5 N/m. [8 marks]

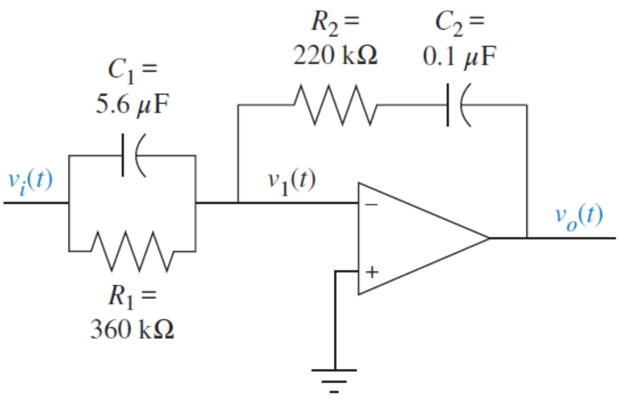
9. The resistor and inductor in the circuit shown in the figure below are said to be in parallel because they have the same voltage v_1 across them. Obtain the model of the current i_2 passing through the inductor. Assume that the supply current i_s is known. Derive Laplace transform of the circuit in terms of ratio of currents $I_2(s)/I_s(s)$. [10 marks]



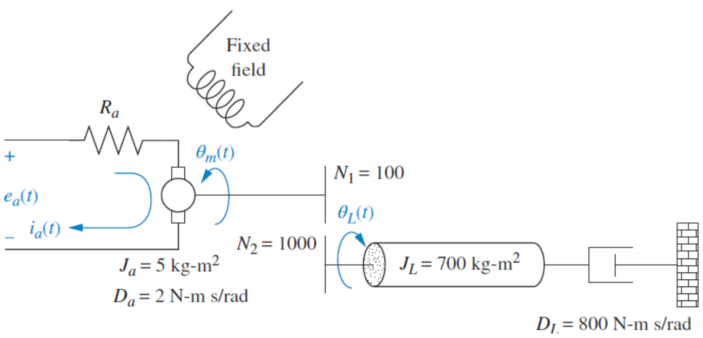
10. Find the transfer function, $G(s) = V_L(s)/V(s)$, for the network shown in the figure below. Solve the problem using mesh analysis. [16 marks]



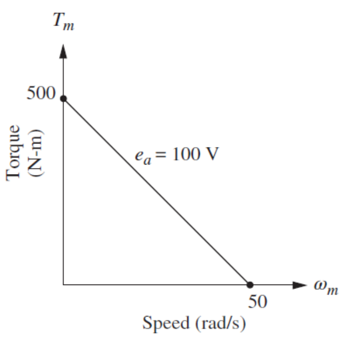
11. Find the transfer function, $V_o(s)/V_i(s)$, for the inverting amplifier circuit given below. [8 marks]



12. Given the electromechanical system and torque-speed curve as shown below, find the transfer function of the system, $\theta_L(s)/E_a(s)$. [20 marks]



(a) Electromechanical system



(b) Torque-speed curve

Appendix – Laplace Transforms

Time Domain	Laplace Domain
$u(t)$	$\frac{1}{s}$
t	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{-at}	$\frac{1}{s+a}$
te^{-at}	$\frac{1}{(s+a)^2}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$e^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at}\cos(\omega t)$	$\frac{(s+a)}{(s+a)^2+\omega^2}$