VICTORIA UNIVERSITY OF WELLINGTON
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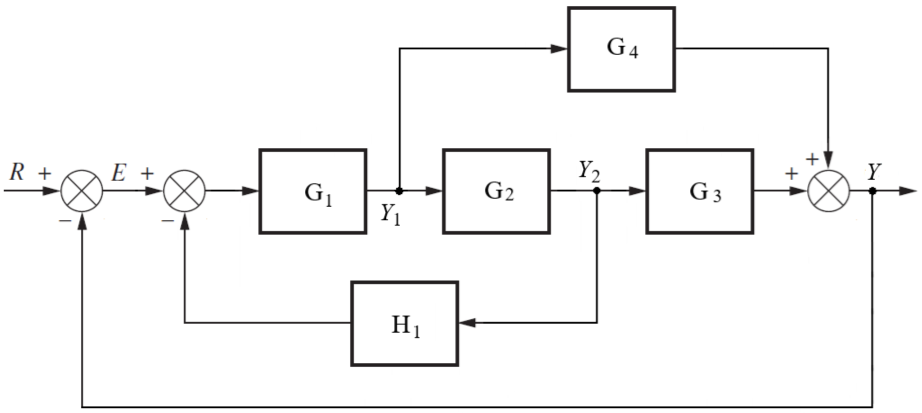
XMUT315 Control Systems Engineering

Tutorial 2: Block Diagram and Feedback (Solution)

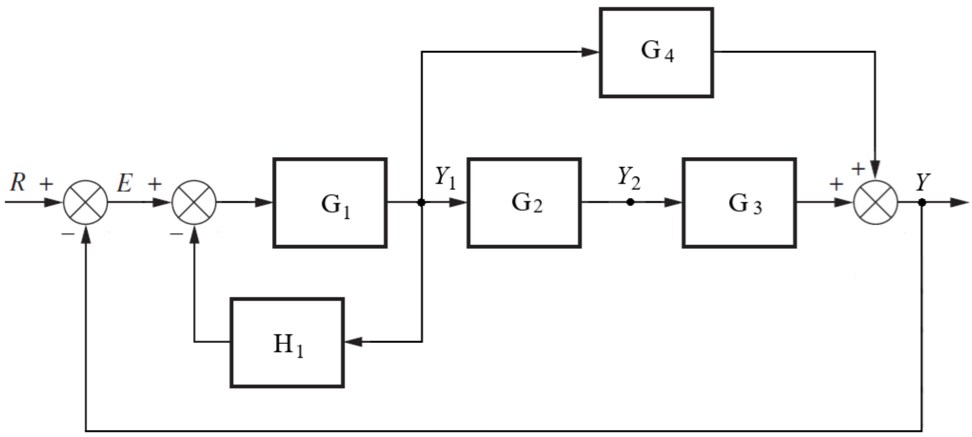
A. Block Diagrams

1. Reduce the following block diagram of a control system into a single block. [8 marks]

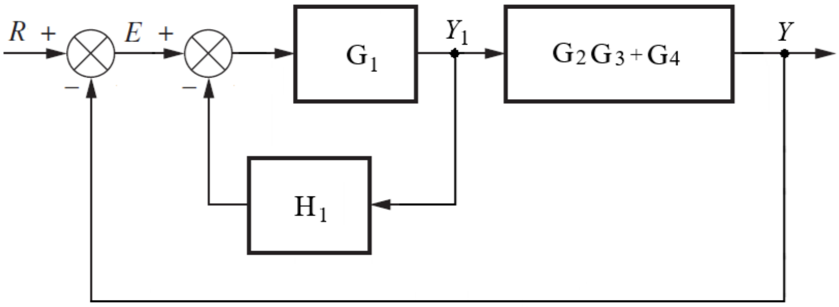


Solution

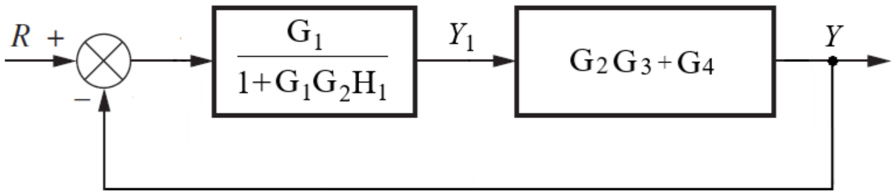
To perform the block diagram reduction, one approach is to move the branch point at Y_2 to the left of block G_2, as shown in the figure below.



After that, the reduction becomes trivial, first by combining the blocks G_2 with G_3, and these blocks form feed forward loop with G_4 block as shown in the figure below.



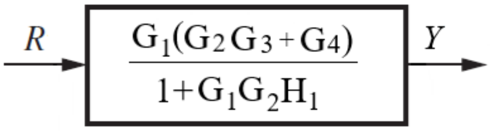
Then, solve G_1 block with G_2H_1 block in the negative feedback loop. In the end, you are able to eliminate the two feedback loops.



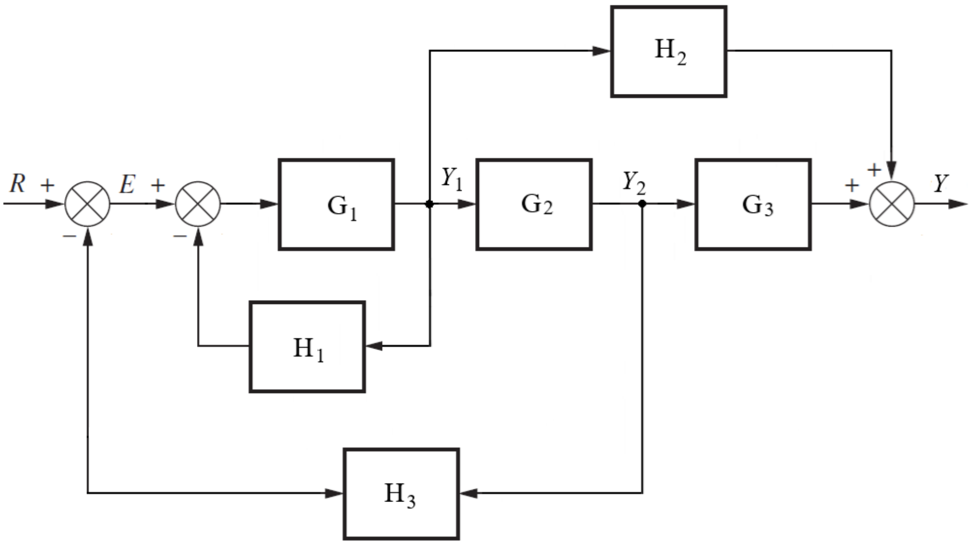
Then, combine the first block with the second block in the forward path. Solve the combined block with the negative unity feedback loop.

$$\frac{Y(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_1 + G_1G_2G_3 + G_1G_4}$$

As a result, the transfer function of the final system after the reduction in the figure above become:

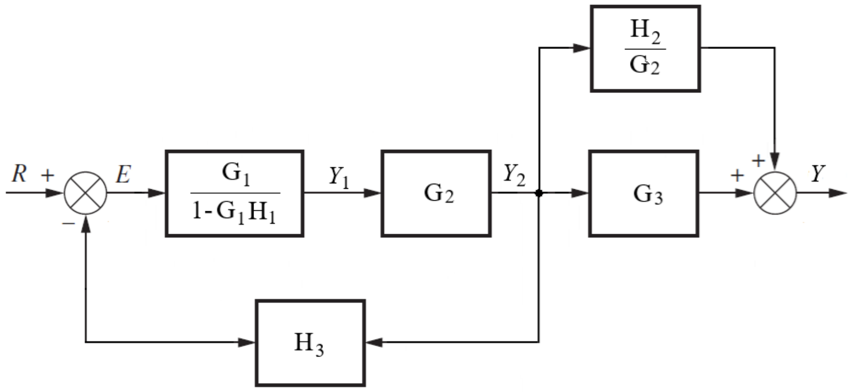


2. Reduce the block diagram shown below and find the Y/R. [10 marks]

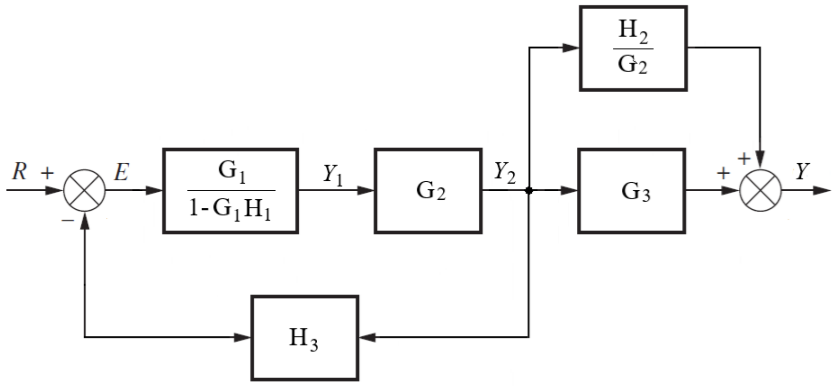


Solution

Solve G_1 block which forms negative feedback loop with H_1 block.

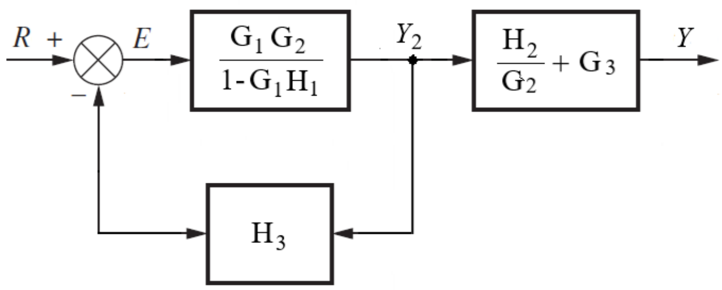


Simplify the model by moving the take-off point from the left of G_2 block to the right of this block.

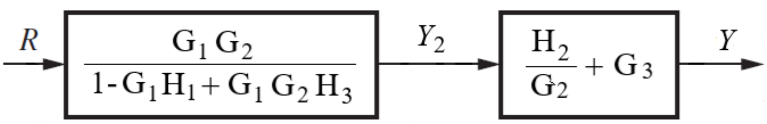


The H_2 block now becomes G_2/H_2 as a result of moving its take-off point to the right.

Combine G_2 block with the block on its left-hand-side and sum G_3 block with H_2/G_2 block.

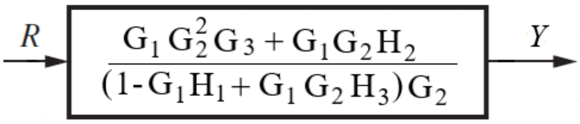


Then, solve H_3 block that forms negative feedback loop with $G_1G_2/(1 - G_1H_1)$ block.

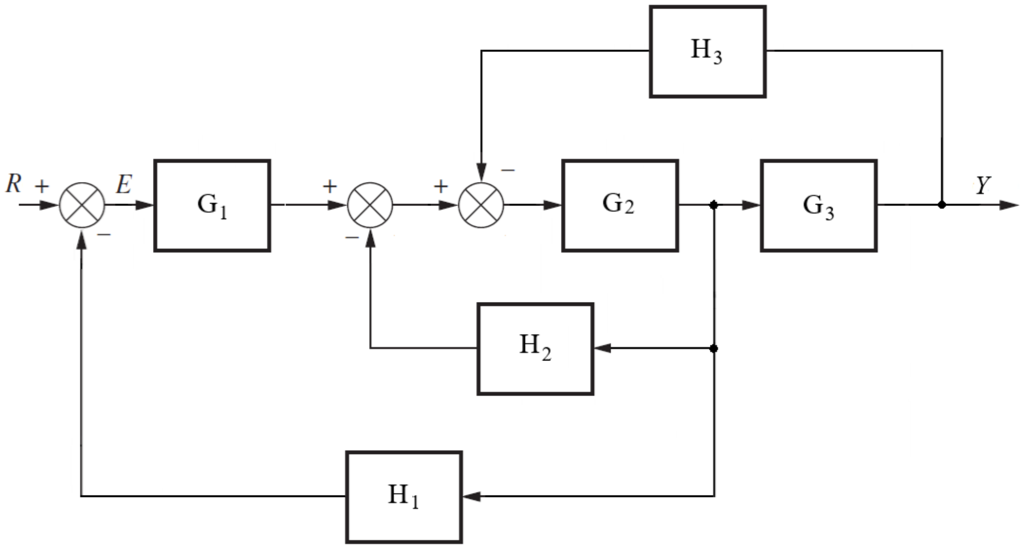


The result of this is then combine with $G_3 + (H_2/G_2)$ block to form a single block as shown below.

$$\frac{Y}{R} = \frac{\left(\frac{G_1 G_2}{1 - G_1 H_1}\right)}{1 + \left(\frac{G_1 G_2 H_3}{1 - G_1 H_1}\right)} \left(G_3 + \frac{H_2}{G_2}\right) = \frac{G_1 G_2^2 G_3 + G_1 G_2 H_2}{(1 - G_1 H_1 + G_1 G_2 H_3) G_2}$$

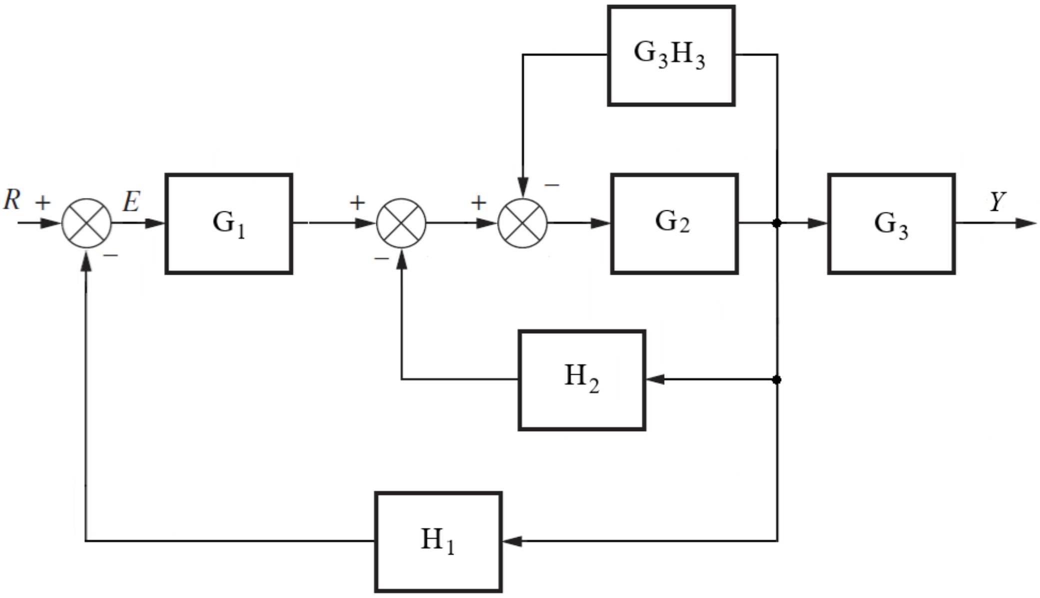


3. Reduce the block diagram shown below to unity feedback form and find the Y/X. [10 marks]

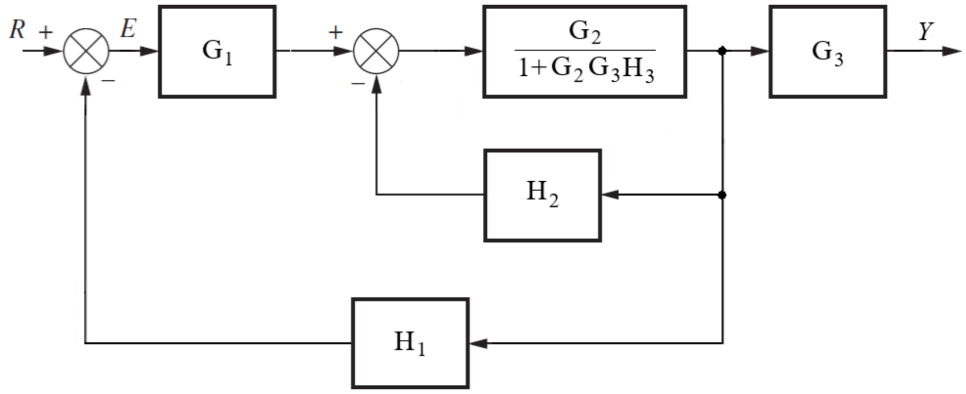


Solution

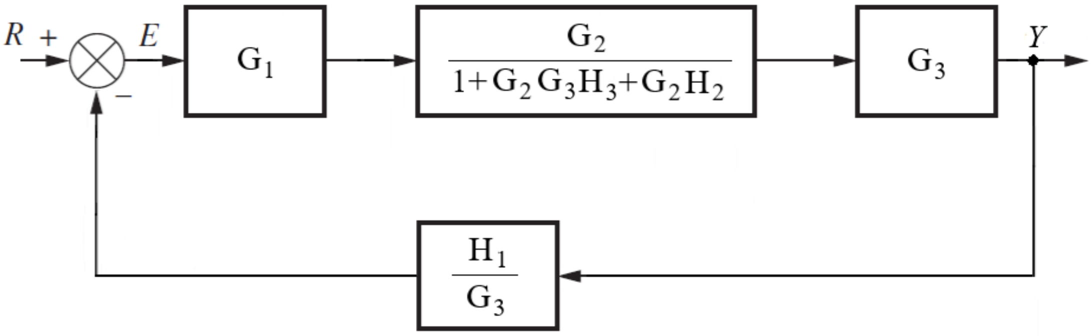
First, move the take-off point of H_3 block from left of G_3 block to its left. As a result, G_3 block is transformed to becoming $G_3 H_3$.



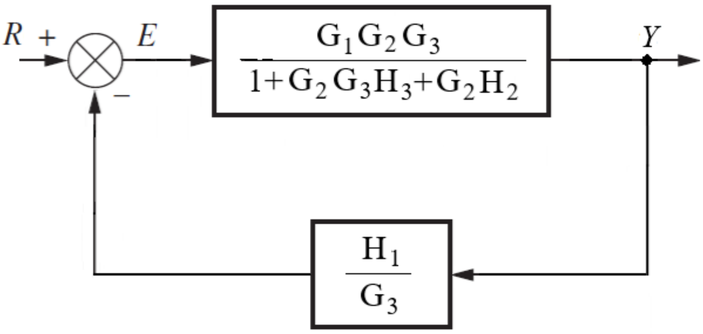
Solve G_3H_3 block which forms a negative feedback loop with G_2 block.



Solve H_2 that forms a negative feedback loop with $G_2/(1+G_2G_3H_3)$ block.

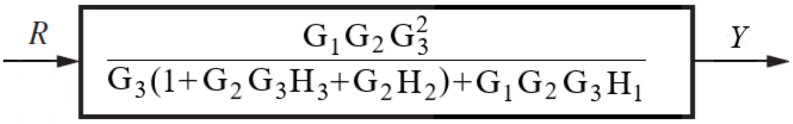


Move the take off-point of H_1 block from left-hand side of G_3 to the right-hand side so the block becomes H_1/G_3. Combine G_1 block in the forward loop which is in series with the $G_2G_3/(1+G_2G_3H_3+G_2H_2)$ block and G_3 block.



Solve H_1/G_3 which forms a negative feedback loop with $G_1G_2G_3/(1+G_2G_3H_3+G_2H_2)$ block. This would give you a single block as shown below.

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_3+G_2H_2}}{1+\left(\dfrac{G_1G_2G_3}{1+G_2G_3H_3+G_2H_2}\right)\left(\dfrac{H_1}{G_3}\right)} = \frac{G_1G_2G_3^2}{G_3(1+G_2G_3H_3+G_2H_2)+G_1G_2G_3H_1}$$



B. Introduction to Feedback Systems

4. In control system engineering, feedback is typically introduced to influence the performance of the control system.

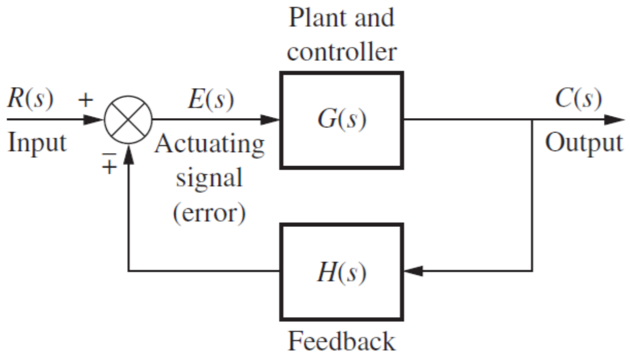
   a. What is feedback? [2 marks]

   b. List and describe types of feedback. [2 marks]

   c. Describe two improvements to the system when a feedback mechanism implemented in the system. [4 marks]

Solution

a. Feedback is system in which the value of some output quantity is controlled by feeding back the value of the controlled quantity and using it to manipulate an input quantity so as to bring the value of the controlled quantity closer to a desired value. Also known as closed-loop control system.



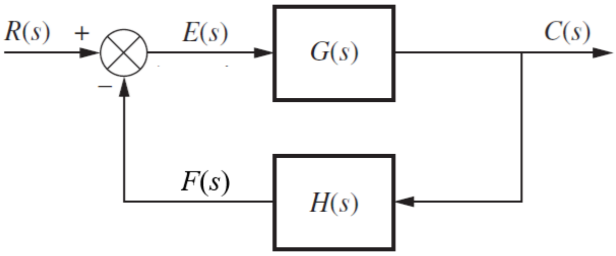
b. There are two main types of feedback control systems: negative feedback and positive feedback.

- In a positive feedback control system, the set point and output values are added.
- In a negative feedback control, the set point and output values are subtracted.

c. Two improvements to the system when feedback is implemented:

- Robustness to component variations – any component tolerances will be normalized in the feedback system (variation of the values of the component would be tolerated in the feedback system, it would not create a major issue as in the open loop system).
- Lower noise and distortion – noise or distortion in the feedback system will be canceled out as the same noise or distortion will be fed back and, in the end, this noise or distortion would be removed from the system.

5. You are given a given feedback system as shown in the following diagram.



a. List components of the feedback control system. [2 marks]

b. Prove that the equation that represent the following feedback system is: [6 marks]

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

Solution

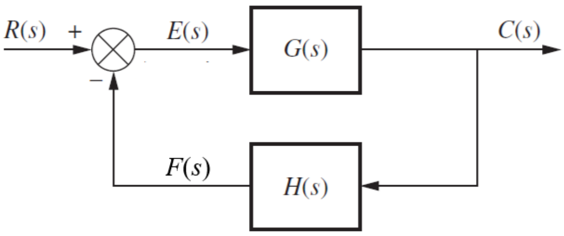
a. A feedback control system consists of five basic components: $R(s)$ input, $G(s)$ process being controlled, $C(s)$ output and sensing elements, $H(s)$ controller and actuating devices, and $F(s)$ feedback signal.

b. The output voltage of the open loop path of the feedback system can be equated as follows:

$$C(s) = G(s)E(s) \qquad (Eq. 1)$$

Taking into account the feedback loop path of the feedback system, the error signal is given as follows:

$$E(s) = R(s) - H(s)C(s) \qquad (Eq. 2)$$



Considering the feedback signal is:

$$F(s) = C(s)H(s) \qquad (Eq. 3)$$

Putting equation (2) into (1) results in the following equation:

$$C(s) = G(s)[R(s) - H(s)C(s)]$$

Rearranging the equation given above will give the following:

$$[1 + G(s)H(s)]C(s) = G(s)R(s)$$

Finally, the transfer function of the feedback system, $X_o(s)/X_i(s)$, is given as follows:

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} = A_{cl}$$

C. Feedback Control Systems

6. Describe the effect of feedback system if implemented in a control system in terms of the following characteristics and behaviour of the system:

   a. Noise or disturbance. [4 marks]

   b. Transient response. [4 marks]

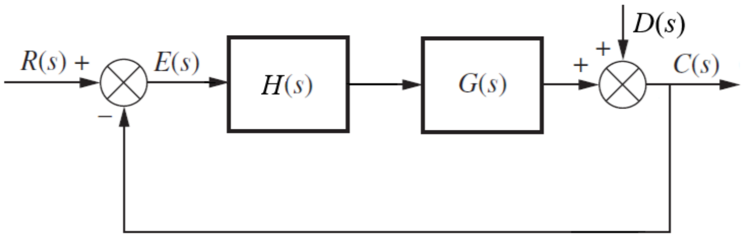
   c. Steady state condition. [4 marks]

Solution

a. Noise or disturbance:

Following equation is for feedback system with disturbance added into the system:

$$O = \left(\frac{CP}{1 + CP}\right) R + \left(\frac{1}{1 + CP}\right) D$$



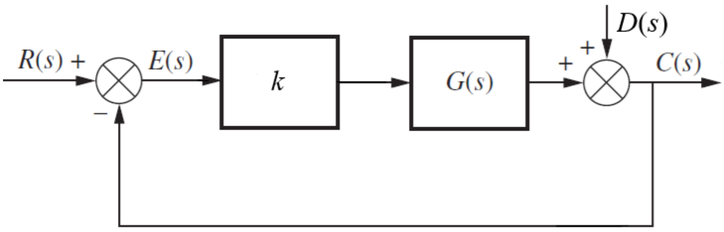
Having feedback system in the control system minimises effects of disturbances.

This is true especially when $G(s)$ and $H(s)$ are large, so $D(s)$ in the equation given above can be minimised.

If $G(s)$ and $H(s)$ are small, then disturbance can still have big influence on the output of the control system.

b. Transient response:

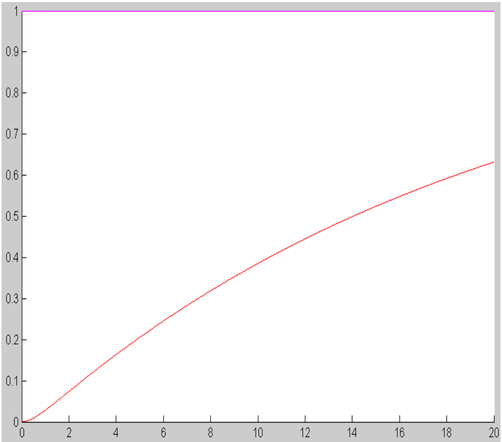
Transient response of the control system can be modified in terms of its gain (k) and time constant affecting system's time constant (T_1, T_2, etc.) and damping factor.

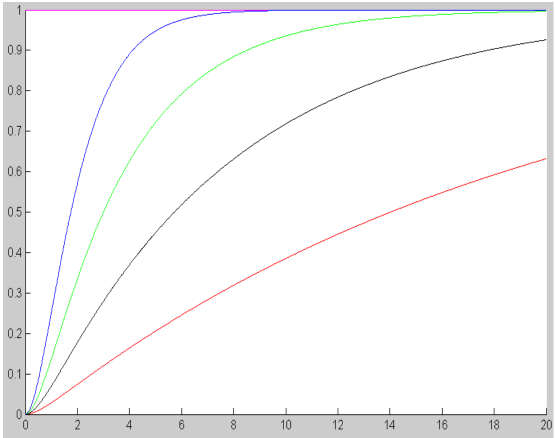


This means the behaviour and characteristics of the control system with added feedback system can be modified and hence managed, so it can meet specific design or improvement criteria.

Furthermore, with careful design, the control system that has some issues or problems can be improved by introducing the feedback system. This is done through changing the parameters of parts of the system e.g. controller or compensator to fix or minimise these issues or problems.

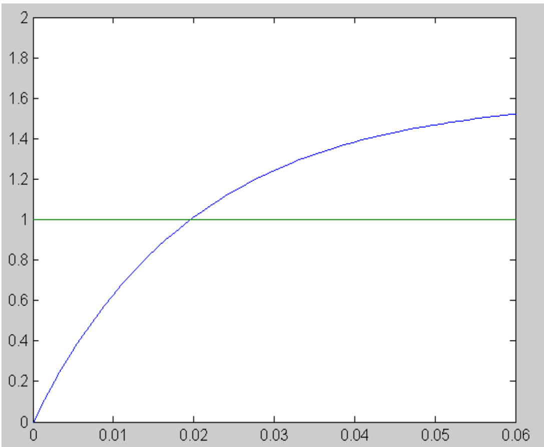
In the figure below, it shows a slow or sluggish system in the left-hand side can be made alive by introducing feedback system and adjusting the parameters of the systems e.g. gain and/or time constant of the system.

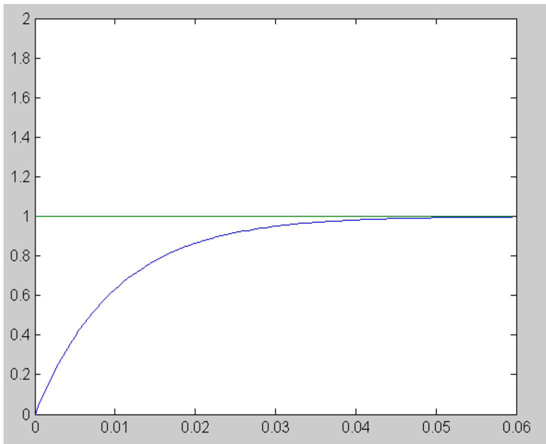




c. Steady-state condition:

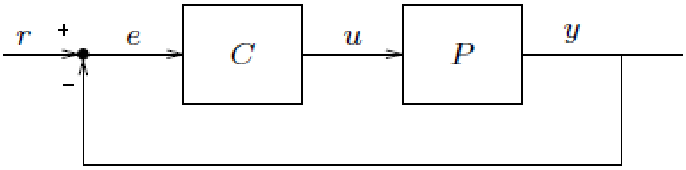
The feedback system whenever it is introduced to the control system will minimise the steady-state error. Just like modifying the parameters of parts of the system e.g. controller or compensator to fix or minimise transient response issues or problems, steady-state error of the system.





In the example given above, a large steady-state error of a control system is minimised with introducing feedback system and modifying the parameters of the system.

7. Given the following feedback control system, perform the following tasks.



a. Find the equivalent open loop of the feedback control system given above. [2 marks]

b. Compare the open- and closed-loop arrangements of the system in terms of:

   i. Changes in P. [2 marks]

   ii. Input and output disturbance. [6 marks]

   iii. Sensor or measurement noise. [8 marks]

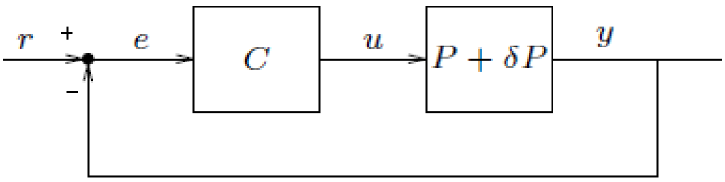
Solution

a. The equivalent open loop of the feedback control system. Note that the equivalent system has the same input and output gain as the closed loop system i.e. $T(s)$.

b. Comparisons between open and closed loop arrangements are:

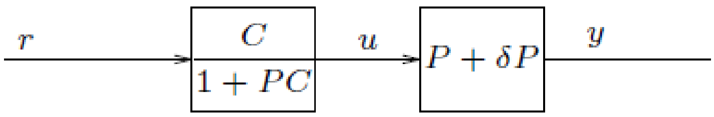
   i. Changes in P.

   Closed-loop system:



$$\delta T_{cl} = \frac{PC + \delta PC}{1 + PC + \delta PC} - \frac{PC}{1 + PC}$$

   Open-loop equivalent system:



$$\delta T_{ol} = \frac{PC + \delta PC}{1 + PC} - \frac{PC}{1 + PC}$$

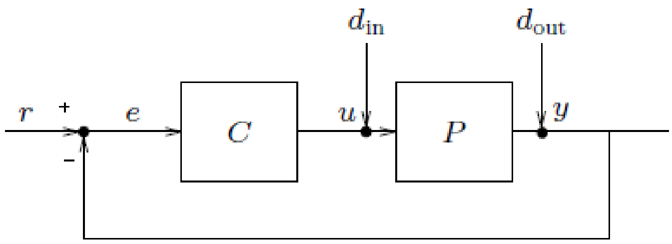
   Hence (after some algebra):

$$\delta T_{cl} = \left(\frac{1}{1 + PC + \delta PC}\right) \delta T_{ole}$$

So, for small δP, $\delta T_{cl} = S\delta T_{ol}$ where $S = 1/(1 + PC)$

ii. Input and output disturbances.

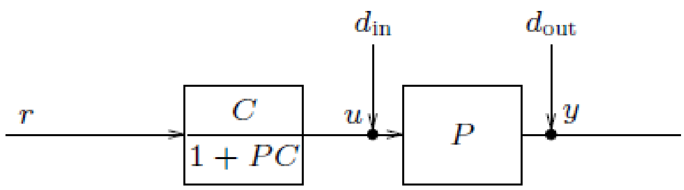
Suppose disturbances d_{in} and d_{out} act on the plant of the closed-loop system:



Effect on y (with $r = 0$)

$$y_{cl} = \left(\frac{P}{1 + PC}\right) d_{in} + \left(\frac{1}{1 + PC}\right) d_{out}$$

Open-loop equivalent system:



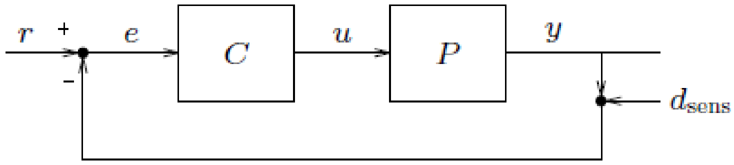
Effect on y (with $r = 0$)

$$y_{ol} = Pd_{in} + d_{out}$$

Hence $y_{cl} = Sy_{ol}$ where $S = 1/(1 + PC)$. In this example, the effects of disturbances are multiplied by S. Large L will lead to small S and hence small effect of disturbance.

iii. Sensor or measurement noise.

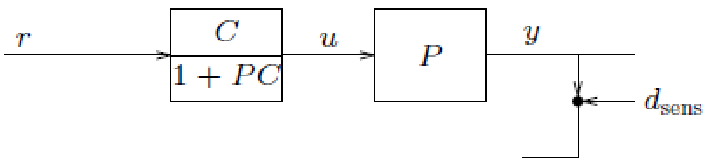
Suppose sensor has noise d_{sens} in the closed-loop system:



Effect on y (with $r = 0$)

$$y_{cl} = \left(\frac{-PC}{1 + PC}\right) d_{sens}$$

Open-loop equivalent system:

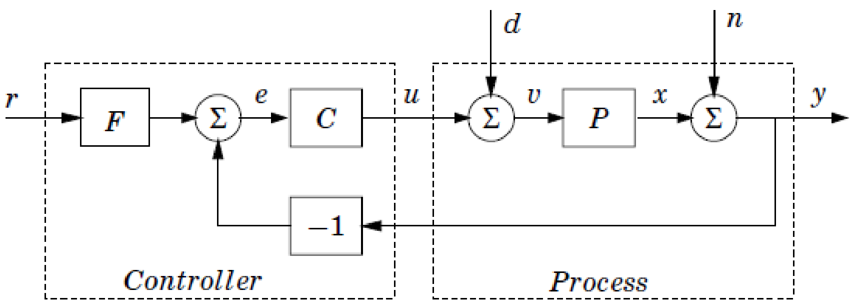


Effect on y (with $r = 0$)

$$y_{ol} = 0$$

This is much better than the closed-loop system. One of the disadvantages of feedback system is the output can be affected by sensor or measurement noise.

8. Given a block diagram model of a feedback control system as shown in the figure below, derive the equations for these signals in the control system.



a. All relevant equations for the derivation process. [10 marks]

b. Process variable signal $X(s)$: [14 marks]

$$X(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right] D(s) - \left[\frac{P(s)C(s)}{1 + P(s)C(s)}\right] N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right] R(s)$$

c. Measured signal $Y(s)$: [14 marks]

$$Y(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right] D(s) + \left[\frac{1}{1 + P(s)C(s)}\right] N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right] R(s)$$

d. Control variable signal $U(s)$: [14 marks]

$$U(s) = -\left[\frac{P(s)C(s)}{1 + P(s)C(s)}\right] D(s) - \left[\frac{C(s)}{1 + P(s)C(s)}\right] N(s) + \left[\frac{C(s)F(s)}{1 + P(s)C(s)}\right] R(s)$$

Solution

a. All equations for the derivation process:

Notice that error $E(s)$ is the reference signal $R(s)$ subtracted with measured signal $Y(s)$. So, the error signal is calculated from:

$$E(s) = R(s)F(s) - Y(s) \qquad (Eq. 1)$$

The control signal $U(s)$ is derived from:

$$U(s) = C(s)E(s) \qquad (Eq.2)$$

Notice that disturbance $D(s)$ is added with the control signal $U(s)$. The variable signal $V(s)$ is derived from:

$$V(s) = U(s) + D(s) \qquad (Eq.3)$$

The process variable signal $X(s)$ is derived from:

$$X(s) = P(s)V(s) \qquad (Eq.4)$$

Notice that noise $N(s)$ is added with the process variable signal $X(s)$. The measure signal $Y(s)$ is derived from:

$$Y(s) = X(s) + N(s) \qquad (Eq.5)$$

b. Equation for the process variable signal $X(s)$:

Substituting $V(s)$ in equation (4) with equation (3):

$$X(s) = P(s)[U(s) + D(s)] \qquad (Eq.6)$$

Substituting $E(s)$ in equation (2) with equation (1):

$$U(s) = C(s)[R(s)F(s) - Y(s)] \qquad (Eq.7)$$

Substituting $Y(s)$ in equation (7) above with equation (5):

$$U(s) = C(s)[R(s)F(s) - \{X(s) + N(s)\}] \qquad (Eq.8)$$

Substituting $U(s)$ in (6) with equation (8)

$$X(s) = P(s)[C(s)[R(s)F(s) - X(s) - N(s)] + D(s)]$$

Expanding the equation above, it becomes:

$$X(s) = P(s)C(s)R(s)F(s) - P(s)C(s)X(s) - P(s)C(s)N(s) + P(s)D(s)$$

Rearranging the equation above for $X(s)$, the equation is now:

$$[1 + P(s)C(s)]X(s) = P(s)C(s)R(s)F(s) - P(s)C(s)N(s) + P(s)D(s)$$

As a result, the equation for process variable signal $X(s)$ is:

$$X(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right]D(s) - \left[\frac{P(s)C(s)}{1 + P(s)C(s)}\right]N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right]R(s)$$

c. Equation for the measured signal $Y(s)$:

Substituting $X(s)$ in equation (5) with equation (4):

$$Y(s) = P(s)V(s) + N(s) \qquad (Eq.9)$$

Substituting $V(s)$ in equation (9) above with equation (3):

$$Y(s) = P(s)[U(s) + D(s)] + N(s) \qquad (Eq.10)$$

Substituting $U(s)$ in equation (10) above with equation (2):

$$Y(s) = P(s)[C(s)E(s) + D(s)] + N(s) \qquad (Eq.11)$$

Substituting $E(s)$ in equation (11) above with equation (1):

$$Y(s) = P(s)[C(s)\{R(s)F(s) - Y(s)\} + D(s)] + N(s) \qquad (Eq. 12)$$

Expanding the equation above, it becomes:

$$Y(s) = P(s)C(s)F(s)R(s) - P(s)C(s)Y(s) + P(s)D(s) + N(s)$$

Rearranging the equation above for $Y(s)$, the equation is now:

$$[1 + P(s)C(s)]Y(s) = P(s)C(s)F(s)R(s) + P(s)D(s) + N(s)$$

As a result, the equation for measured signal $Y(s)$ is:

$$Y(s) = \left[\frac{P(s)}{1 + P(s)C(s)}\right]D(s) + \left[\frac{1}{1 + P(s)C(s)}\right]N(s) + \left[\frac{P(s)C(s)F(s)}{1 + P(s)C(s)}\right]R(s)$$

d. Equation for the control variable signal $U(s)$:

Substituting $E(s)$ in equation (2) with equation (1):

$$U(s) = C(s)[R(s)F(s) - Y(s)] \qquad (Eq. 13)$$

Substituting $Y(s)$ in equation (13) above with equation (5)

$$U(s) = C(s)[R(s)F(s) - \{X(s) + N(s)\}] \qquad (Eq. 14)$$

Substituting $V(s)$ in equation (4) with equation (3):

$$X(s) = P(s)[U(s) + D(s)] \qquad (Eq. 15)$$

Substituting $X(s)$ in the equation (14) with equation (15):

$$U(s) = C(s)[R(s)F(s) - \{P(s)[U(s) + D(s)] + N(s)\}] \qquad (Eq. 15)$$

Expanding the equation above, it becomes:

$$U(s) = C(s)F(s)R(s) - P(s)C(s)U(s) - P(s)C(s)D(s) - C(s)N(s)$$

Rearranging the equation above for $U(s)$, the equation is now:

$$[1 + P(s)C(s)]U(s) = C(s)F(s)R(s) - P(s)C(s)D(s) - C(s)N(s)$$

As a result, the equation for control variable signal $U(s)$ is:

$$U(s) = -\left[\frac{P(s)C(s)}{1 + P(s)C(s)}\right]D(s) - \left[\frac{C(s)}{1 + P(s)C(s)}\right]N(s) + \left[\frac{C(s)F(s)}{1 + P(s)C(s)}\right]R(s)$$

9. What are the inputs typically used for evaluating and testing in the control system? Why do we need to have various input for these matters? [6 marks]

Solution

Inputs in control systems engineering which are used for evaluating, testing and evaluation of the control systems are as follows:

- An *impulse input* is a very high pulse applied to a system over a very short time (i.e. it is not maintained). That is, the magnitude of the input approaches infinity while the time approaches zero.
- A *step input* is instantaneously applied at some time (typically taken as zero) and thereafter held at a constant level.
- A *ramp input* increases linearly with time. However, in practice, there is a physical limit, or the dynamic problem ends before the input gets too large.
- A *sinusoid input* is applying a sinusoid signal into the system.

Why do we need different types of input for testing and evaluation?

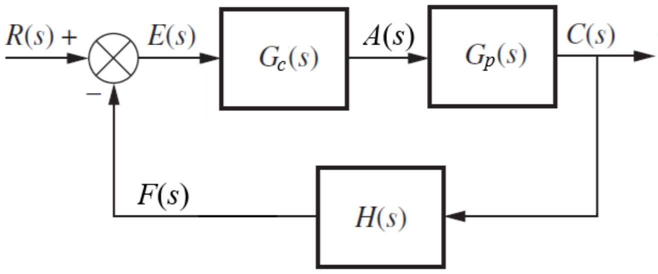
- There various shapes and forms of the control systems, having just one particular type of evaluation or testing input would not be necessarily sufficient to cater for complex tasks of analyzing and designing control systems.
- Furthermore, some systems could respond well with one type of input when they are being tested or evaluated but would poorly react with others.

10. List and describe types of controllers in control systems engineering. [2 marks]

Solution

Types of controllers in control systems engineering:

- Proportional controller (P).
- Integral controller (I).
- Derivative controller (D).
- Any combination of the above, i.e. PI, PD, or PID controllers.
- Fuzzy logic controller.
- Model based controller.



Note: $F(s)$ = feedback signal and $A(s)$ = actuator signal.

11. Why is compensator necessary for improving the performance and/or stability of a given control system? [4 marks]

Solution

Compensator is needed for improving the control systems (at least two reasons for full mark):

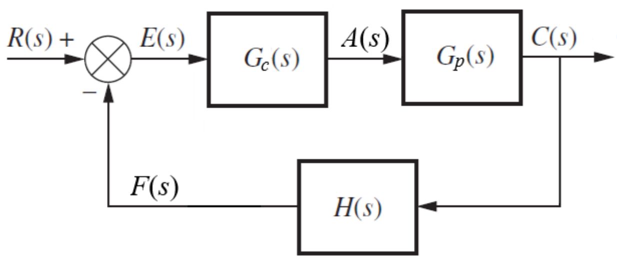
- In order to obtain the desired performance of the system, we use compensators. Compensators are applied to the system in the form of feed forward path gain adjustment.
- Compensate an unstable system to make it stable.
- A compensating network is used to minimise overshot.
- Compensators could increase the steady state accuracy of the system. An important point to be noted here is that the increase in the steady state accuracy brings instability to the system.
- Compensator could also introduce poles and zeros in the system thereby causes changes in the transfer function of the system. Due to this, performance specifications of the system change.

12. List and describe basic compensators in control systems. What are the main differences between the controllers with the compensators? [6 marks]

Solution

The simplest compensating networks used for compensators in control systems engineering are known as:

- Lead compensators.
- Lag compensators.
- Lead-lag compensators.



Compensators are different from the controllers (at least two differences for full mark):

- Compensators are used mainly for fixing or solving control systems issues or problems whereas controllers are used typically for managing control system.
- Controllers are designed to be easily adjusted or modifiable in the first place compared with compensators that tend to be relatively fixed.
- Compensators are less complex to implement practically than the controller.
- Controllers require more tuning-in effort compared with compensators.