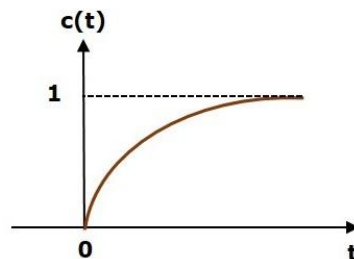


## XMUT315 Control Systems Engineering

### Tutorial 3: Stability Analysis

#### A. Stability Analysis

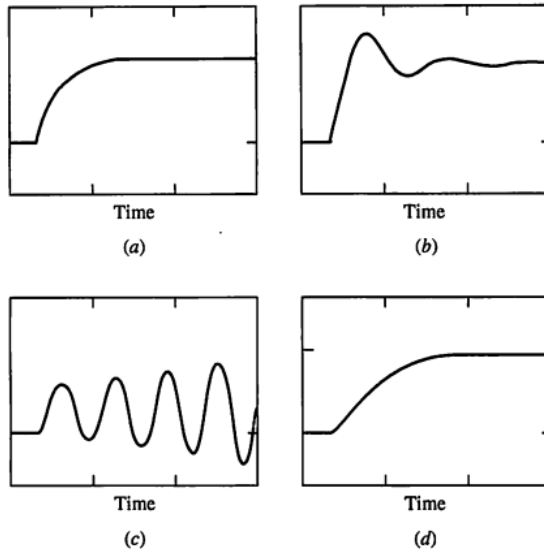
1. In control system engineering, stability is determined to be one of the criteria of performance of a given control system.
  - a. What is stability? [2 marks]
  - b. Describe and compare bounded signal and unbounded signal. [4 marks]
  - c. Describe how a system that has a unit-step response as shown in the diagram below is stable. [2 marks]



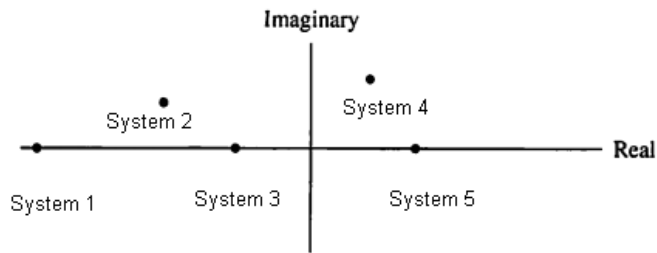
2. Before performing more specific analysis and design for a given control system, stability analysis is typically performed first.
  - a. Explain how you determine stability of a system. [2 marks]
  - b. Describe types of systems based on their stabilities. [6 marks]
3. Describe whether the following systems are stable or not based on their transient responses given in the following figures. [8 marks]

Note:

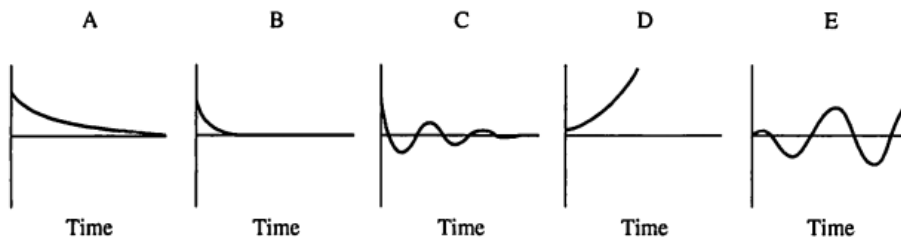
System (a) has an damped exponential response, system (b) has a damped oscillatory response, system (c) has a growing oscillatory response, and system (d) has a damped exponential response with its time constant is longer than system (a).



4. Referring to a (pole zero) s-plane diagram of a number of systems as shown below, answer the following questions.



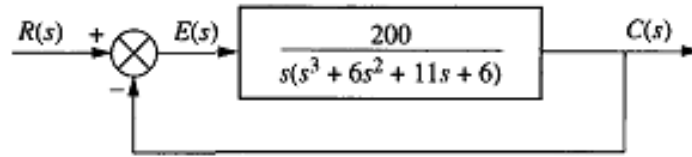
Which of the unit-step responses given below correspond to which systems as above. [5 marks]



**B. Routh Hurwitz Stability Criterion**

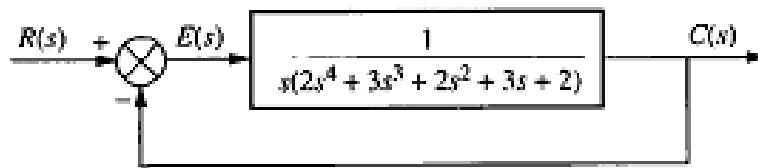
*Standard Routh-Hurwitz*

5. Find the number of poles in the left half-plane, the right half-plane, and on the y-axis for the system of figure below. [12 marks]



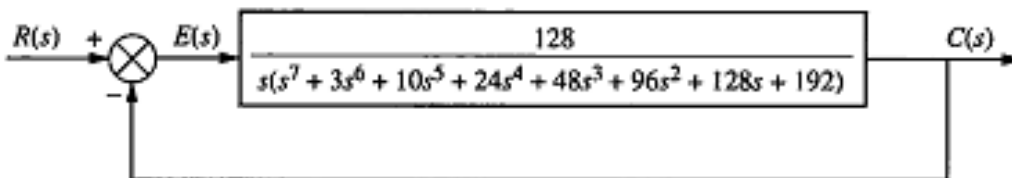
*Routh-Hurwitz with Zero in First Column*

6. Find the number of poles in the left half-plane, the right half-plane, and on the y-axis for the system of figure below. [12 marks]



*Routh-Hurwitz with Row of Zeros*

7. For a control system given in the following block diagram, attempt the following tasks.



- a. Find the number of poles in the left half-plane, the right half-plane, and on the y-axis for the system of figure below. [20 marks]  
 b. Draw conclusions about the stability of the closed-loop system. [6 marks]

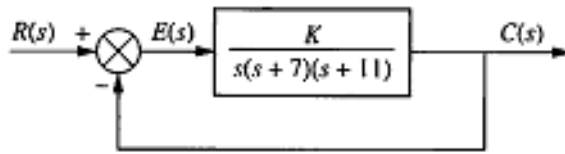
*Pole Distribution via Routh Table with Row of Zeros*

8. For the transfer function given below, tell how many poles are in the right half-plane, in the left half-plane, and on the y-axis. [24 marks]

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

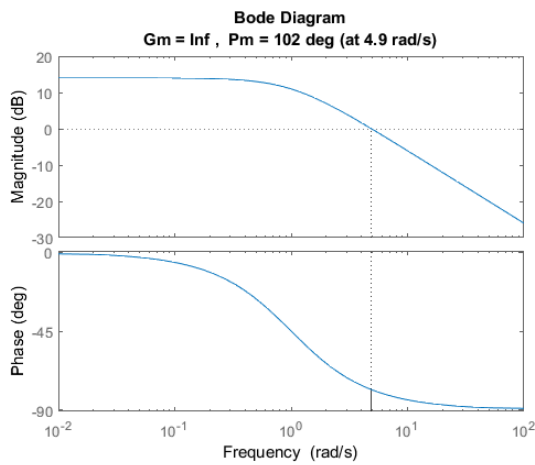
*Stability Design via Routh-Hurwitz*

9. Find the range of gain,  $K$ , for the system of figure below that will cause the system to be stable, unstable, and marginally stable. Assume  $K > 0$ . [20 marks]

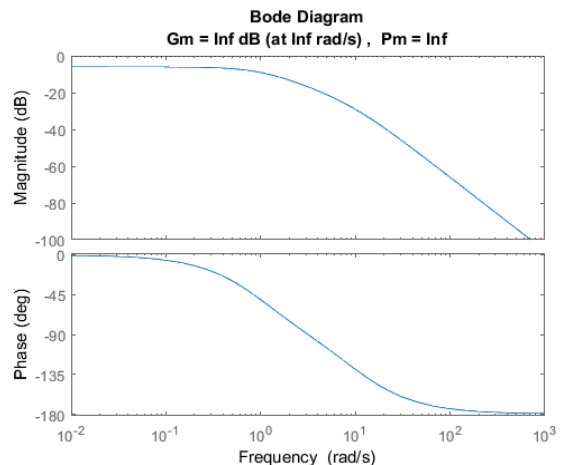


**C. Other Stability Analysis**

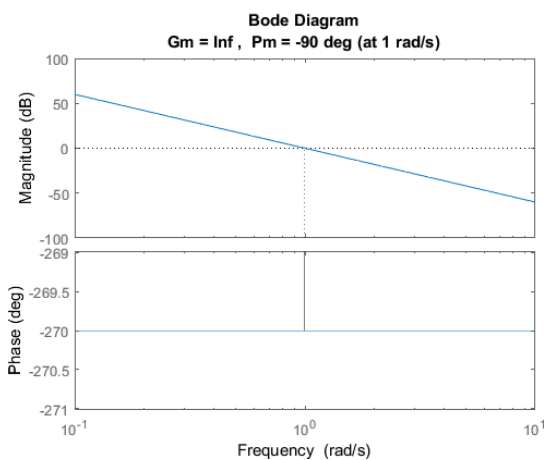
10. Determine the stability of the system using Bode plot if the responses of the system are given in the figures below. [8 marks]



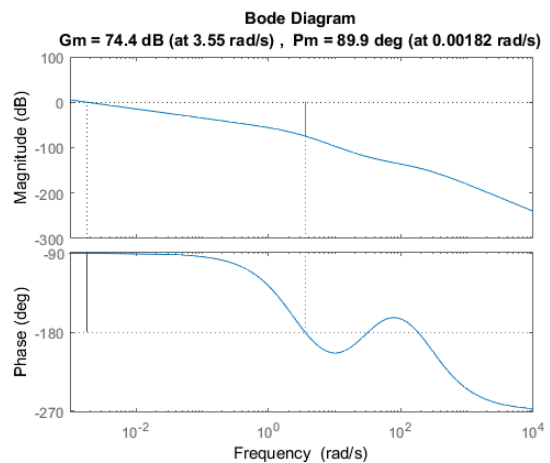
(a) System 1  $G(s) = \frac{5}{s+1}$



(b) System 2  $G(s) = \frac{5}{(s+1)(s+10)}$

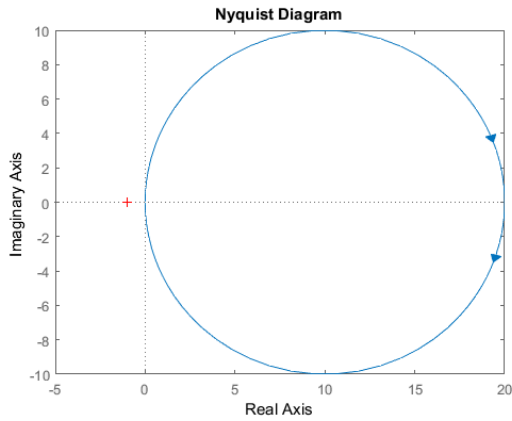


(c) System 3  $G(s) = \frac{1}{s^3}$

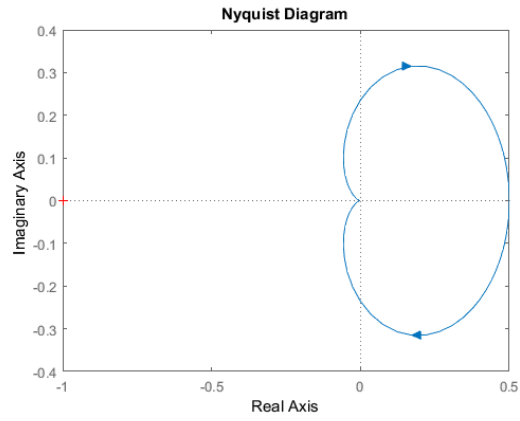


(d) System 4  $G(s) = \frac{(s+25)(s+35)}{s(s+2)(s+4)(s+200)(s+300)}$

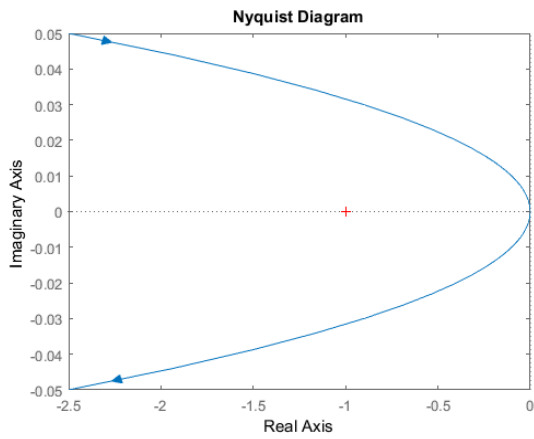
11. Determine the stability of the system using Nyquist plot if the responses of the system are given in the figures below. [8 marks]



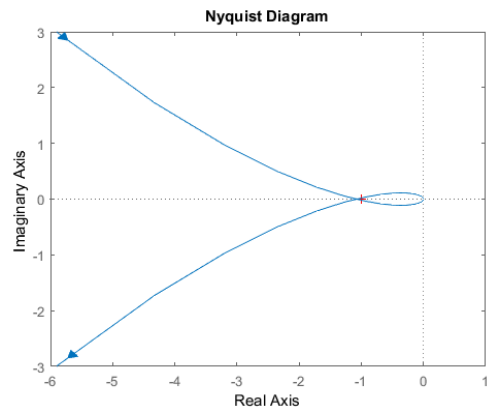
(a) System 1  $G(s) = \frac{100}{s+5}$



(b) System 2  $G(s) = \frac{1}{(s+1)(s+2)}$

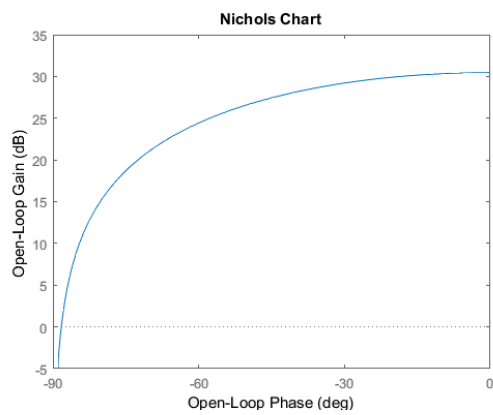


(c) System 3  $G(s) = \frac{1}{s^2(s+10)}$

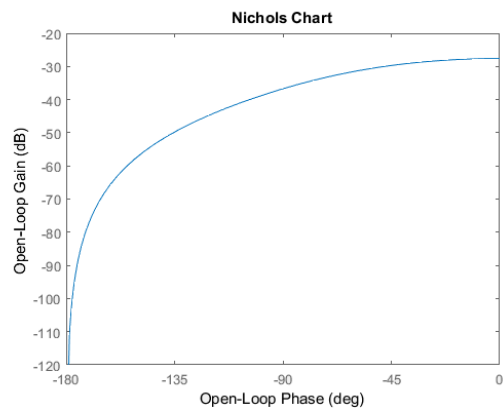


(d) System 4  $G(s) = \frac{(s+1)(s+12)}{s^3}$

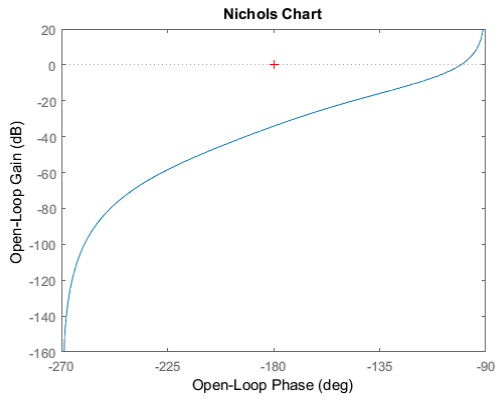
12. Determine the stability of the system using Nichols plot if the responses of the system are given in the figures below. [8 marks]



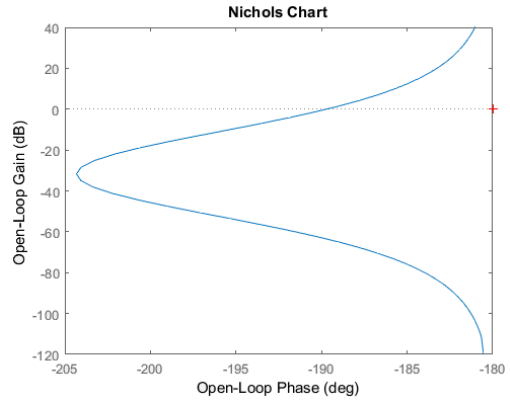
(a) System 1  $G(s) = \frac{100}{s+3}$



(b) System 2  $G(s) = \frac{1}{(s+2)(s+12)}$



(c) System 3  $G(s) = \frac{10}{s(s+2)(s+15)}$



(d) System 4  $G(s) = \frac{(s+12)}{s^2(s+5)}$   $\tau_a > \tau_1$