XMUT315 Control Systems Engineering

Tutorial 4: Time Domain Analysis

A. Time Response Analysis

1. The transient response of a second-order system can be determined from its transfer function equation. Depending on the type of roots in the equation, the response can be categorised as underdamped, critically damped, and overdamped.

   a. Prove the roots of the equation for second-order system are: [10 marks]

   $$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2} \quad \text{or} \quad s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$$

   b. Prove the time domain equation of the underdamped response of a second-order system when it is given a step input is as shown below. [20 marks]

   $$c(t) = 1 - \left(\frac{e^{-\zeta\omega_t}}{\sqrt{1-\zeta^2}}\right)\sin[\omega_d t + \phi]$$

   Where: $\omega_d = \omega_n\sqrt{1-\zeta^2}$ and $\phi = \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right)$

2. Prove that the time-domain equations of the transient responses of the given second-order system when it is given a step input are as shown below.

   a. For the critically damped response second-order system, the time-domain equation is:

   [20 marks]

   $$c(t) = 1 - e^{-\omega_n t}(1 + \omega_n t) \quad \text{for} \quad t \geq 0$$

   b. For the overdamped response second-order system, the time-domain equation is:

   [20 marks]

   $$c(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2 - 1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right) \quad \text{for} \quad t \geq 0$$

   Where: $s_1 = \left(\zeta + \sqrt{\zeta^2 - 1}\right)\omega_n$ and $s_2 = \left(\zeta - \sqrt{\zeta^2 - 1}\right)\omega_n$

3. You are given the following first-order systems:

    i. System 1:

$$G(s) = \frac{5}{s+5}$$

    ii. System 2:

$$G(s) = \frac{20}{s+20}$$

    a. For both systems, calculate the time constant, rise time, and settling time for each system. [12 marks]

    b. Based on the results in part (a), comment on the differences in transient and steady-state responses of the first system with the second system. [4 marks]

    c. Simulate and describe the transient responses of the systems when each of the systems is subjected to a step input. [10 marks]

    d. Simulate and describe the transient responses of the systems when each of the systems is subjected to an impulse input. [10 marks]

4. Given the transfer function below, find damping factor (ζ) and natural frequency (ω_n) of the system. [4 marks]

$$G(s) = \frac{36}{s^2 + 4.2s + 36}$$

5. Given the transfer function of a given control system.

$$G(s) = \frac{100}{s^2 + 15s + 100}$$

Determine the parameters of the time response of the system as follow:

    a. Natural frequency (ω_n) and damping ratio (ζ). [4 marks]

    b. Time-to-peak (T_p), percentage overshoot ($\%OS$) and settling time (T_s). [6 marks]

    c. Rise time (T_r) using the following methods: derived equation, alternative equation, and graph of normalised damping ratio. Simulate the system in MATLAB for rise time and determine which method gives the most accurate result. [8 marks]

6. Find the step response of each of three systems described as the transfer function equations given below and compare them. [12 marks]

- System 1:

$$T_1(s) = \frac{24.542}{s^2 + 4s + 24.542}$$

- System 2:

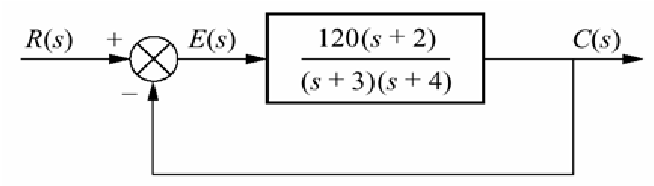
$$T_2(s) = \frac{245.42}{(s + 10)(s^2 + 4s + 24.542)}$$

- System 3:

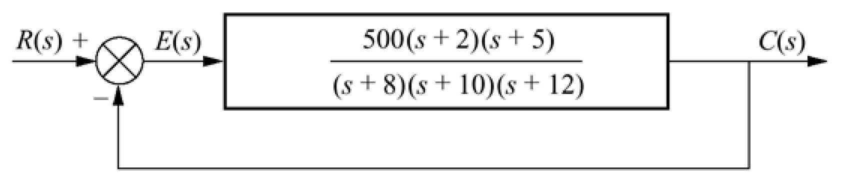
$$T_3(s) = \frac{73.626}{(s + 3)(s^2 + 4s + 24.542)}$$

B. Steady-State Analysis

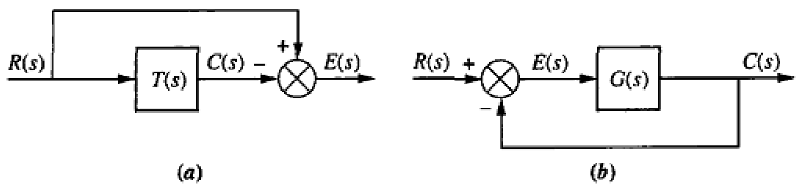
7. For the system shown below, find the steady-state errors for the inputs:
   a. Step input, $5u(t)$. [2 marks]
   b. Ramp input, $5tu(t)$. [2 marks]
   c. Parabolic input, $5t^2u(t)$. [2 marks]



8. For the system below, evaluate the static-error constants and find the steady-state errors for:
   a. Step input, $5u(t)$. [4 marks]
   b. Ramp input, $5tu(t)$. [4 marks]
   c. Parabolic input, $5t^2u(t)$. [4 marks]

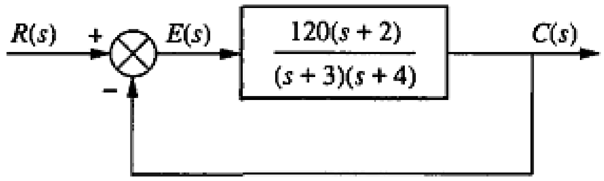


9. Find the steady-state error for the system of the figure given below if $T(s) = 5/(s2 + 7s + 10)$ and the input is a unit step. [6 marks]

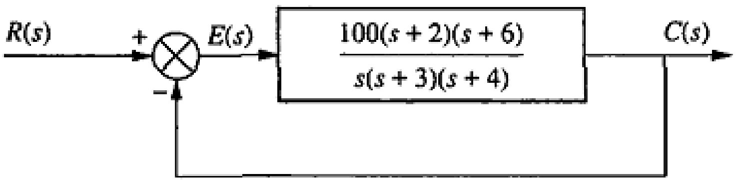


Shown in the diagram above is closed-loop control system error, as a general representation of the system given in (a) and a representation for unity feedback system in (b).

10. Match up the steady-state conditions for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$ to the system shown in the figure below with the simulation results. The function $u(t)$ is the unit step.
[12 marks]



11. For the control system shown in the figure below, attempt the following tasks.

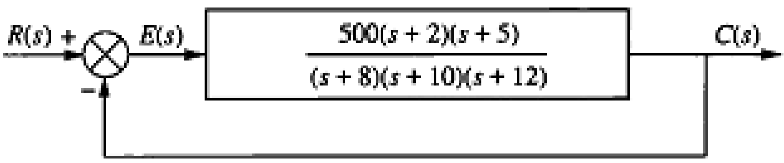


a. Determine the steady-state errors for inputs of $5u(t)$, $5tu(t)$, and $5t^2u(t)$ to the system shown in the figure below and match them up with simulation results. [6 marks]

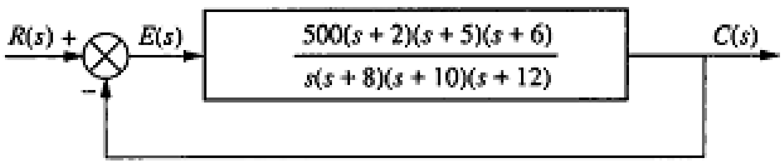
b. Compare the steady-state conditions of the system for the inputs given. [3 marks]

c. Describe the role of integral component towards steady-state characteristics of the system. The function $u(t)$ is the unit step. [5 marks]

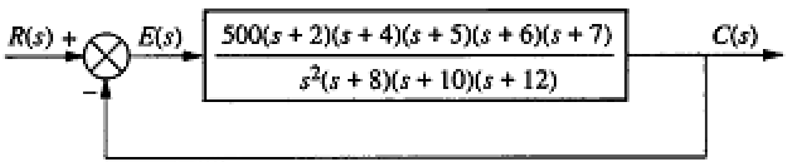
12. For each system in the figure given below, evaluate the static-error constants and find the expected error for the standard step, ramp, and parabolic inputs. [18 marks]



(a)

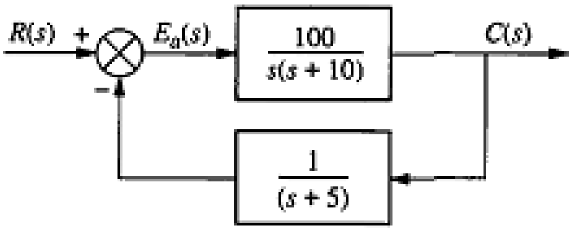


(b)

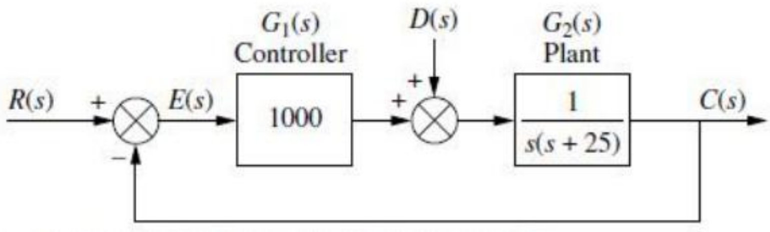


(c)

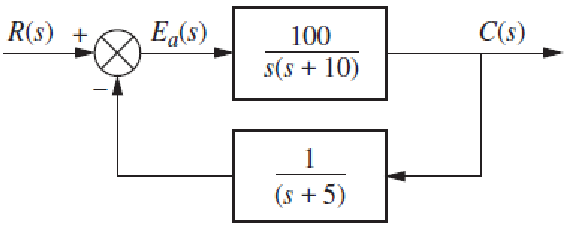
13. For the system shown in the figure given below, find the system type, the appropriate error constant associated with the system type, and the steady-state error for a unit step input. Assume input and output units are the same. [12 marks]



14. Find the steady-state errors component due to a step disturbance for the system given below. [4 marks]



15. For the system shown below, perform the following tasks.



a. Find the system type, the appropriate error constant associated with the system type, and the steady-state errors for a unit step input. Assume input and output units are the same. [8 marks]

b. Find the steady-state actuating signal for the system ($E_a(s)$) for a unit-step input. Repeat for a unit ramp input. [8 marks]

16. Given the system of the figure below, calculate the sensitivity of the closed-loop transfer function to changes in the parameter a. How would you reduce the sensitivity? [6 marks]