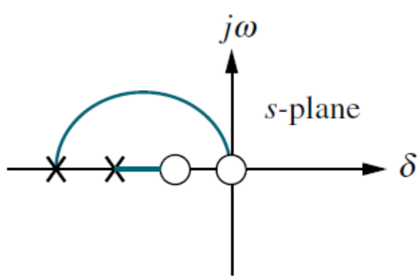
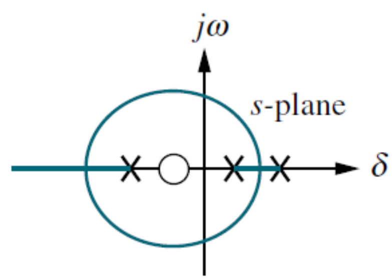
XMUT315 Control Systems Engineering

Tutorial 7: Analysis with Root Locus Diagram (Solution)

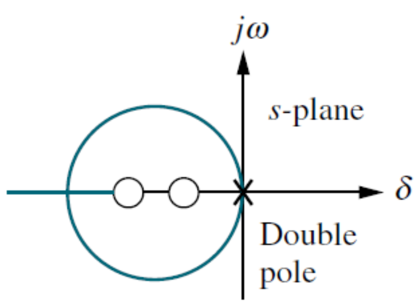
1. For each of the root loci shown in the figure below, describe briefly whether, or not the sketch can be a root locus. If the sketch cannot be a root locus, explain why. Give all reasons. [16 marks]



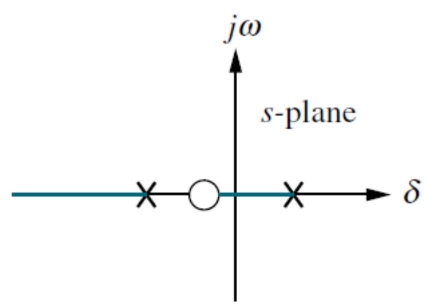
(a)



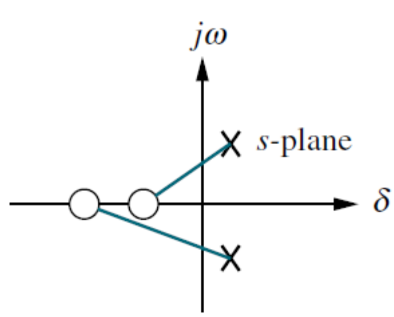
(b)

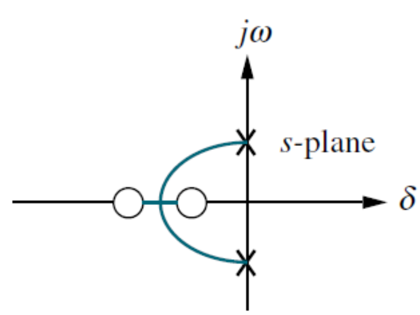


(c)

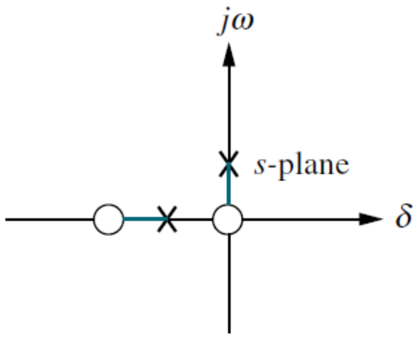


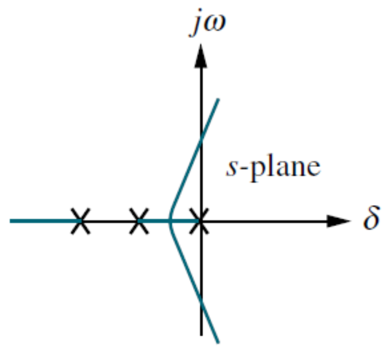
(d)





(e) (f)





(g) (h)

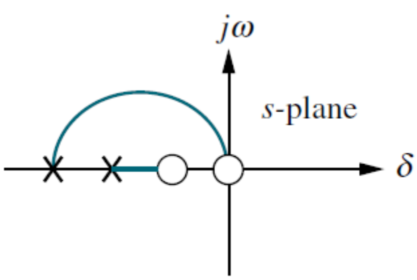
Solution

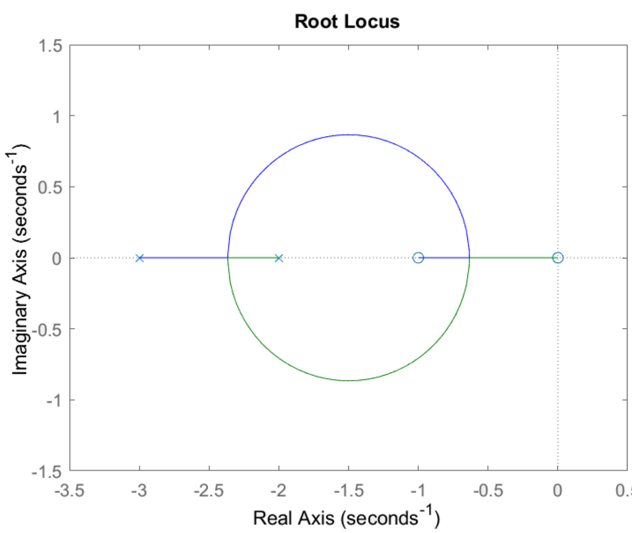
Note that all root locus diagrams actually meet Evan's rule #1 - Root locus is symmetric about the real axis.

a. No:

On real axis to left of an even number of poles and zeros (e.g. Evan's rule #3 – A branch of the root locus will only be on the real axis to the left of an odd number of finite open-loop roots).

The correct root locus diagram as simulated in MATLAB is as shown in the figure below.



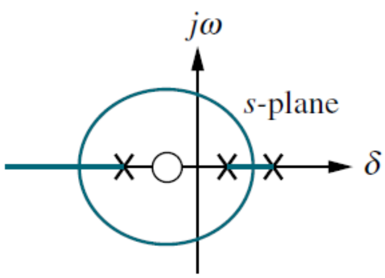


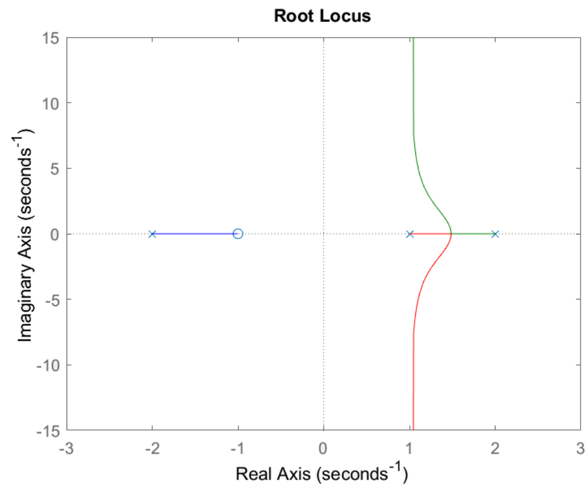
b. No:

On real axis to left of an even number of poles and zeros (e.g. Evan's rule #3 – A branch of the root locus will only be on the real axis to the left of an odd number of finite open-loop roots).

As we have only one zero in the root locus, poles on the R.H.S of the s-plane do not form asymptotes (e.g. Evan's rule #5 – Root locus approaches straight-line asymptotes as the gain approaches infinity. These asymptotes have real axis intercept and angles).

The correct root locus diagram as simulated in MATLAB is as shown in the figure below.



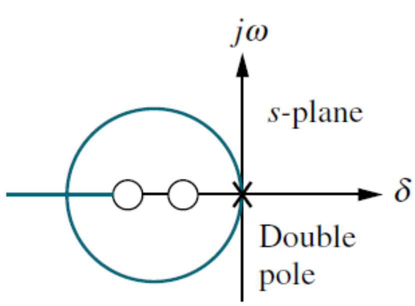


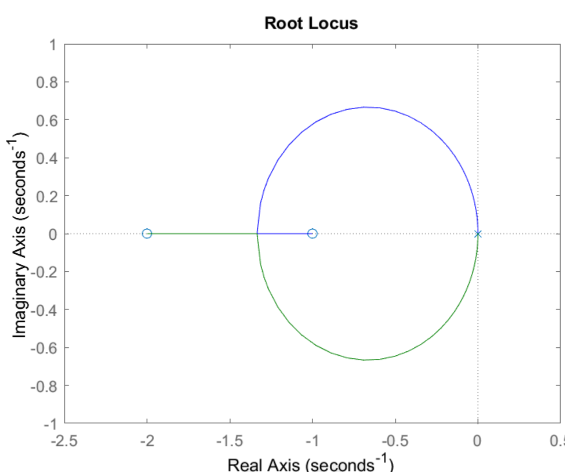
c. No:

On real axis to left of an even number of poles and zeros (e.g. Evan's rule #3 – A branch of the root locus will only be on the real axis to the left of an odd number of finite open-loop roots).

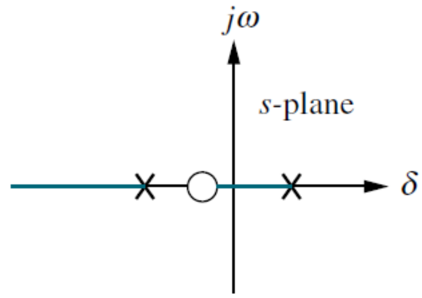
A zero is not connected to any pole (e.g. Evan's rule #4 – Root locus begins at finite and infinite zeros poles of closed-loop system and ends at finite and infinite zeros of closed-loop system).

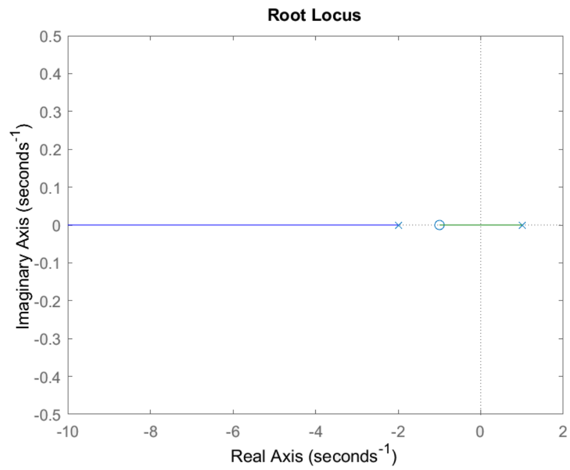
The correct root locus diagram as simulated in MATLAB is as shown in the figure below.





d. Yes. The root locus diagram as simulated in MATLAB is as shown in the figure below.



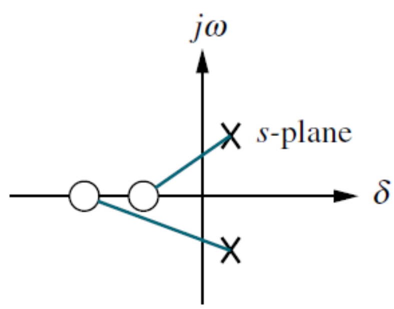


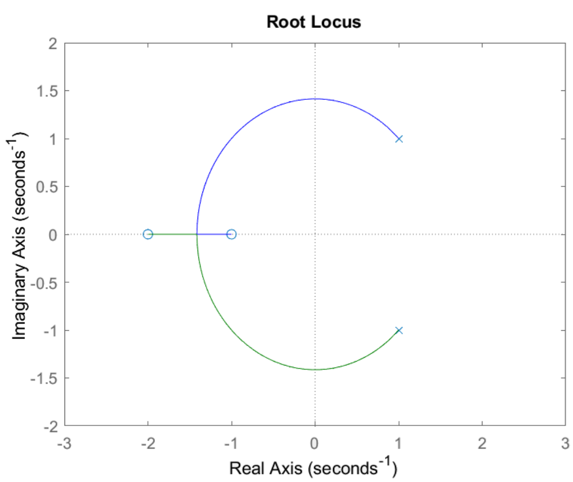
e. No:

Not symmetric (e.g. Evan's rule #1 - Root locus is symmetric about the real axis).

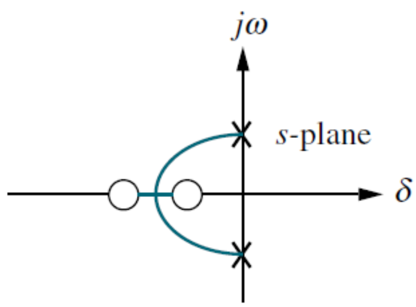
Not on real axis to left of odd number of poles and/or zeros (e.g. Evan's rule #3 – A branch of the root locus will only be on the real axis to the left of an odd number of finite open-loop roots).

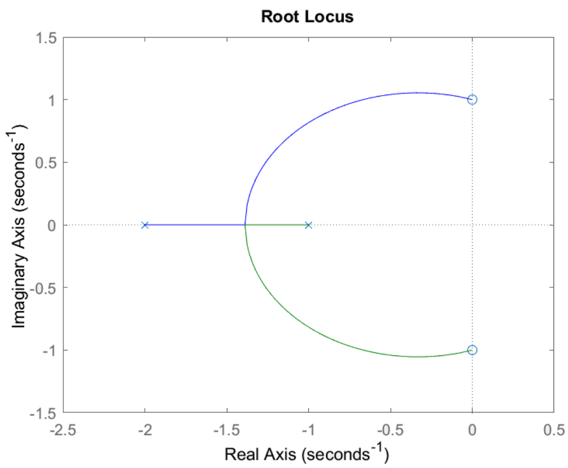
The correct root locus diagram as simulated in MATLAB is as shown in the figure below.





f. Yes. The root locus diagram as simulated in MATLAB is as shown in the figure below.





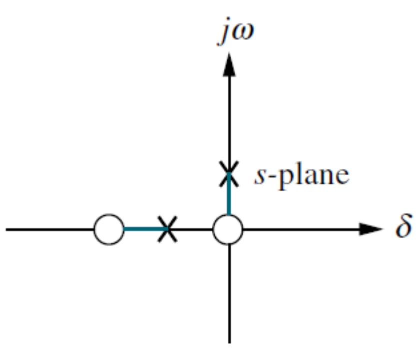
g. No:

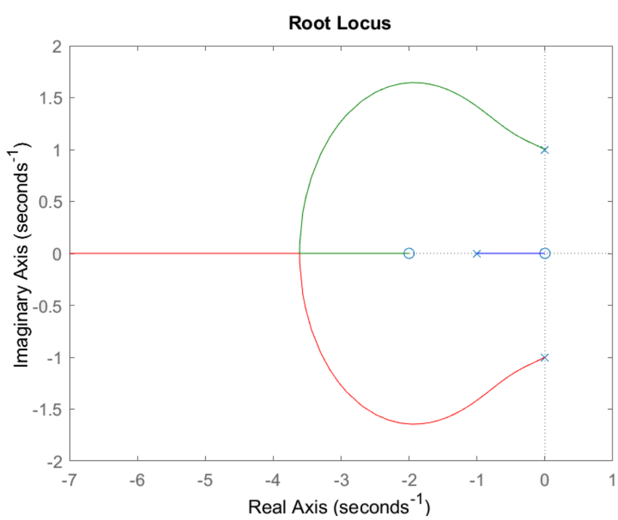
Not symmetric (e.g. Evan's rule #1 - Root locus is symmetric about the real axis);

Real axis segment is not to the left of an odd number of poles (e.g. Evan's rule #3 – A branch of the root locus will only be on the real axis to the left of an odd number of finite open-loop roots).

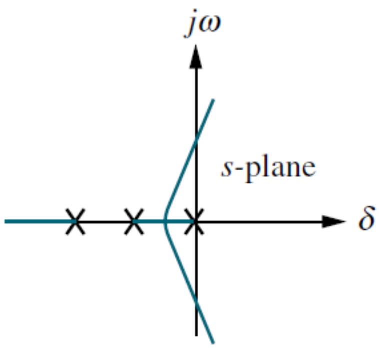
If it is a correct root locus diagram, for a symmetric plot of root locus, there should be another pole on the bottom of the right-hand side of the plot.

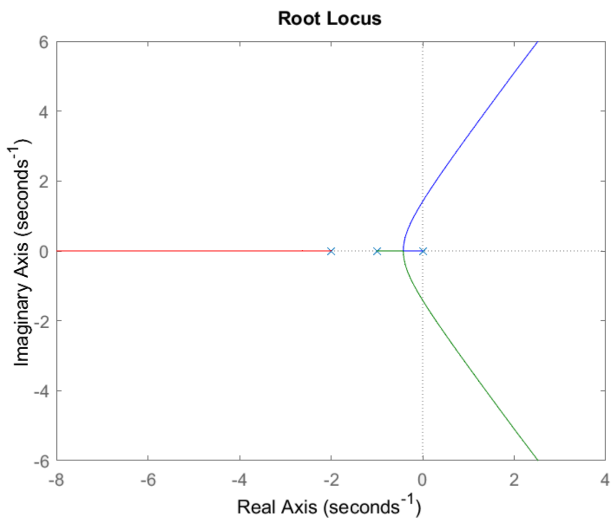
The correct root locus diagram as simulated in MATLAB is as shown in the figure below.



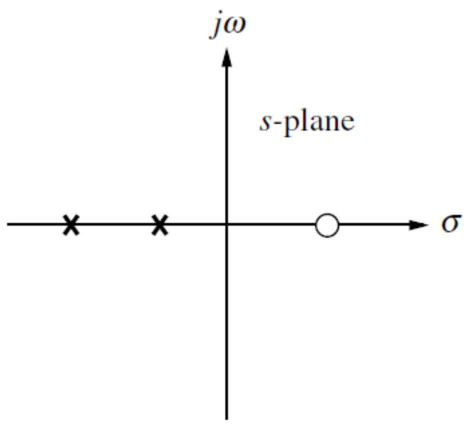


h. Yes. The root locus diagram as simulated in MATLAB is as shown in the figure below.

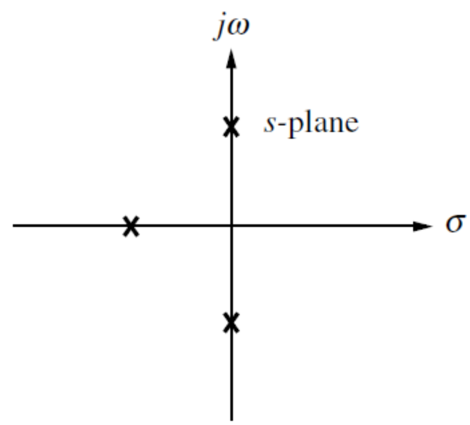




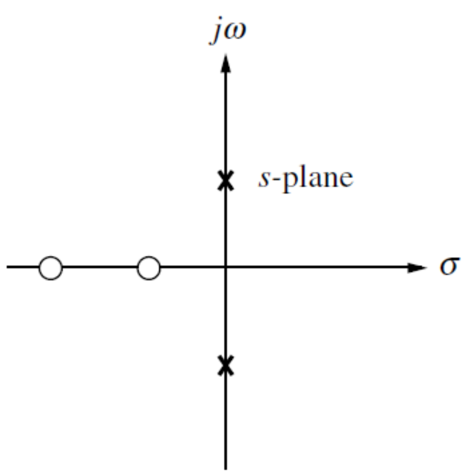
2. Sketch the general shape of the root locus for each of the open-loop pole-zero plots shown in the figure below. [30 marks]

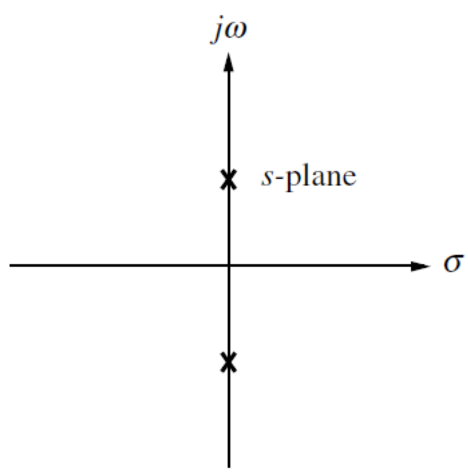


(a)

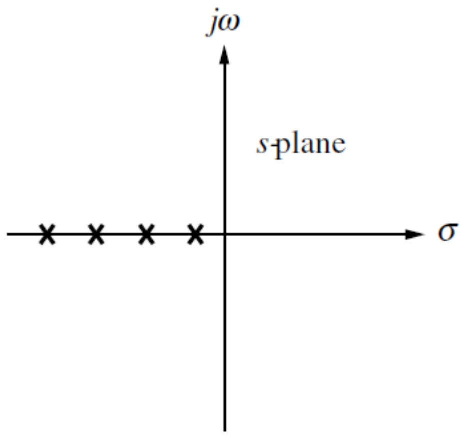


(b)

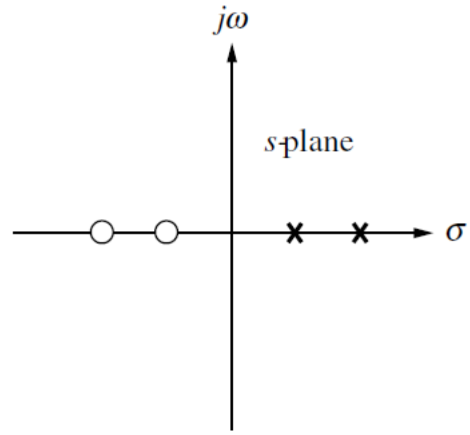




(c)



(d)

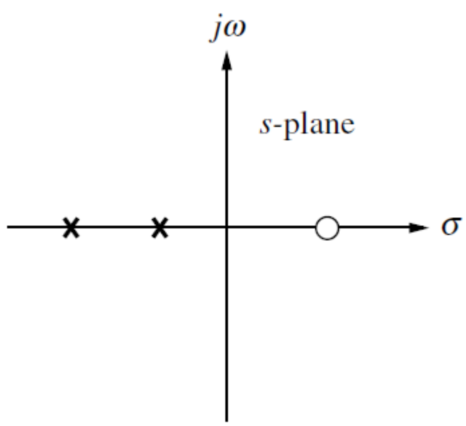


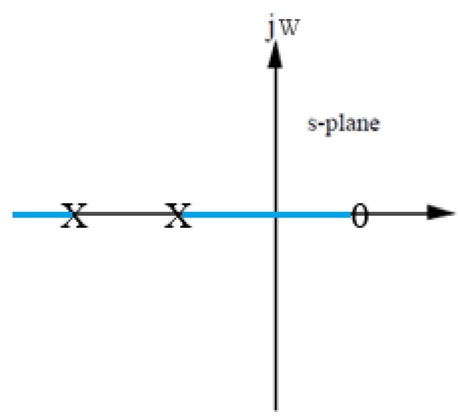
(e)

(f)

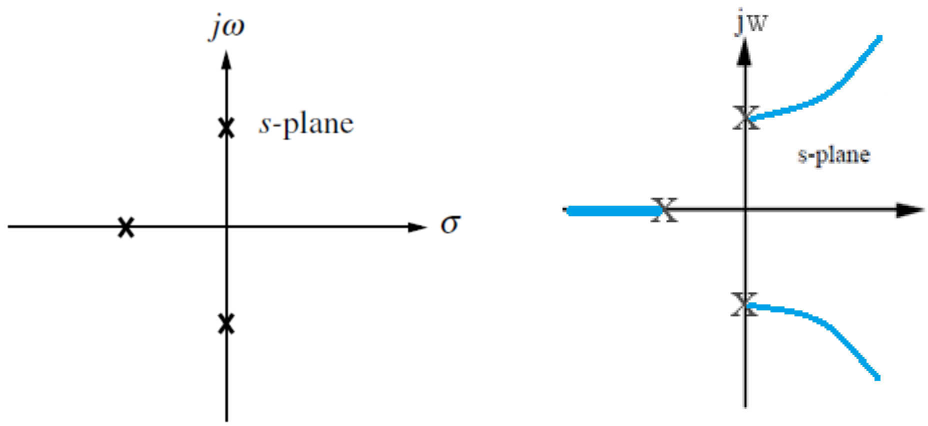
Solution

a. The sketch of the root locus diagram of the first system is as shown in the figure below. There are two loci e.g. one is going from pole on the right-hand side to zero and the other is going as asymptote from the pole on the left-hand side along the negative x-axis.

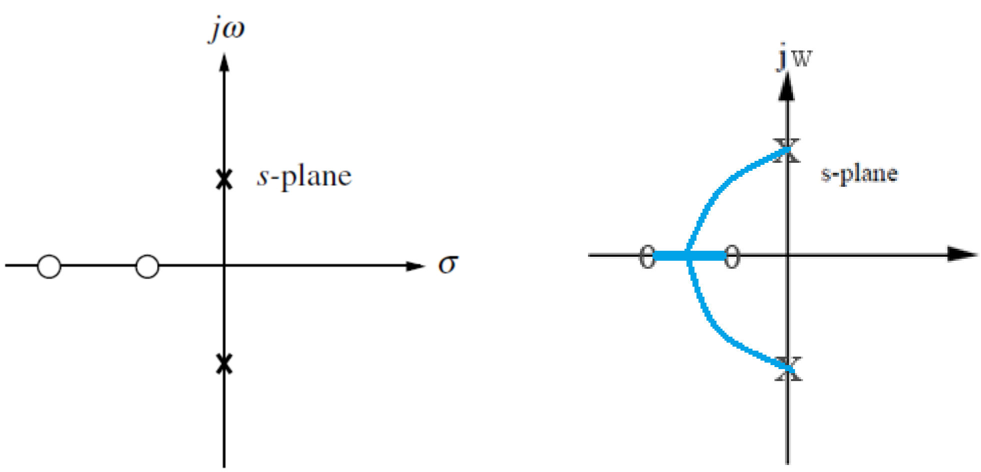




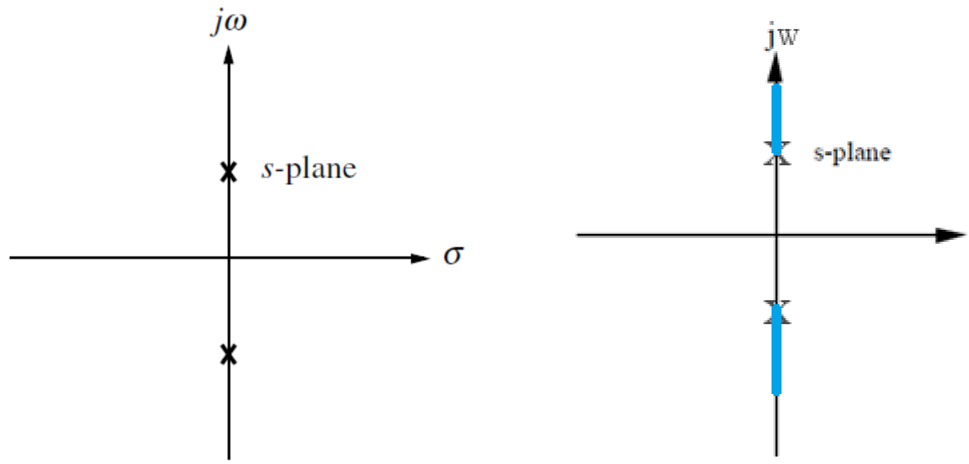
b. The sketch of the root locus diagram of the second system is as shown in the figure below. There are three asymptote loci in the diagram e.g. two from pair of complex poles on the y-axis to the right-hand sides and the other is going along the negative x-axis from the third pole on the x-axis.



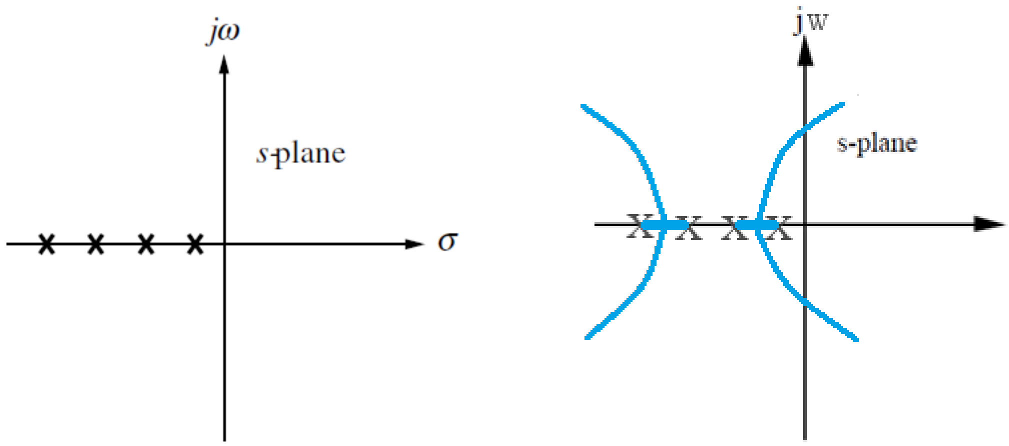
c. The sketch of the root locus diagram of the third system is as shown in the figure below. The two loci from the complex pair of poles on the y-axis end up on the two zeros on the negative x-axis. One locus is going from the top pole to the zero on the right-hand side and the other is going from the bottom pole to the zero on the left-hand side.



d. The sketch of the root locus diagram of the fourth system is as shown in the figure below. There are two asymptote loci in the diagram. One is originally from the top pole to positive infinity and the other is from the bottom pole to negative infinity.

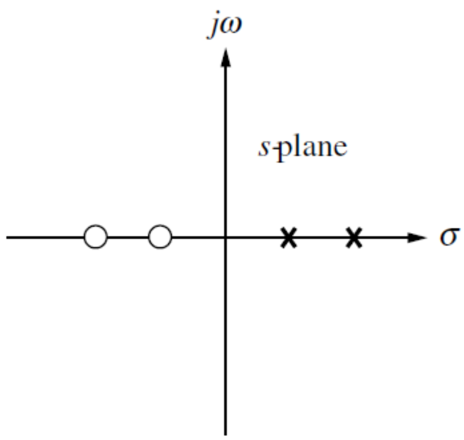


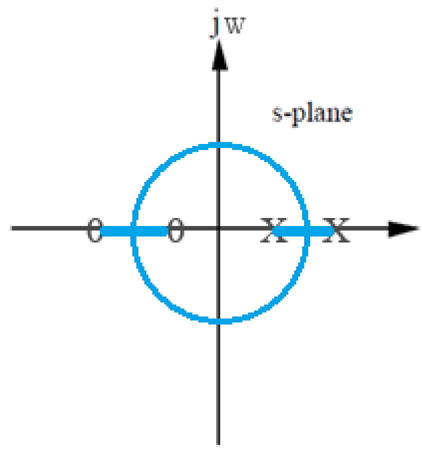
e. The sketch of the root locus diagram of the fifth system is as shown in the figure below.



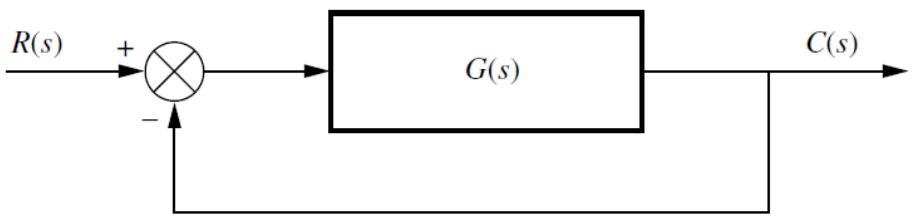
There are four asymptote loci in the diagram e.g. each locus is going from each of the poles to its relevant asymptote respectively. Notice that the break-away points from the poles meet in the location between the poles before settling into their relevant asymptotes.

f. The sketch of the root locus diagram of the sixth system is as shown in the figure below. The two loci in the diagram are originated from the poles on the right-hand side of the s-plane that settle on the two zeros on the left-hand side. Notice that the two loci break-away in the location between the two poles and break-in in the location between the two zeros.





3. Sketch and simulate in MATLAB the root locus for the unity feedback system shown in the figure below for the following transfer functions:



a. System 1 [10 marks]

$$G_1(s) = \frac{K(s+2)(s+6)}{s^2+8s+25}$$

b. System 2 [10 marks]

$$G_2(s) = \frac{K(s^2+4)}{(s^2+1)}$$

c. System 3 [10 marks]

$$G_3(s) = \frac{K(s^2+1)}{s^2}$$

d. System 4 [10 marks]

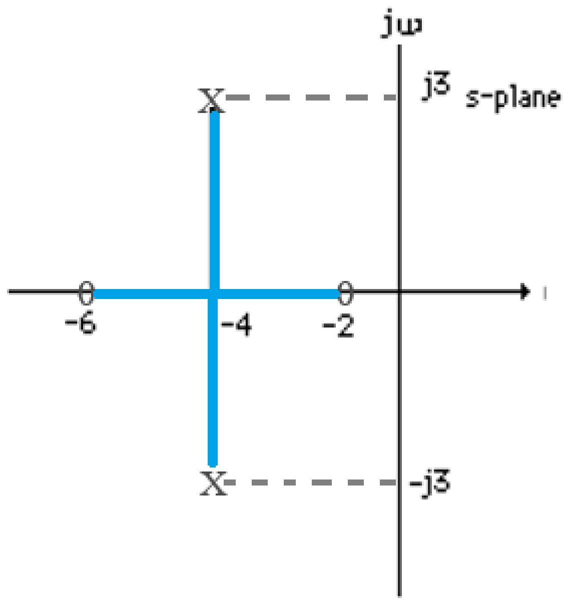
$$G_4(s) = \frac{K}{(s+1)^3(s+4)}$$

Solution

a. The sketch of root locus diagram of the system 1 is as shown in the figure below.

$$G_1(s) = \frac{K(s+2)(s+6)}{s^2+8s+25}$$

The zeros of the system are located on (-2, 0) and (-6, 0). The poles are complex pair at (-4, 3) and (-4, -3).



There are two loci e.g. one is going from one of the complex poles at (-4, 3) to the zero at (-2, 0) and the other is going from the other complex pole at (-4, -3) to the zero at (-6, 0).

Real axis intercept

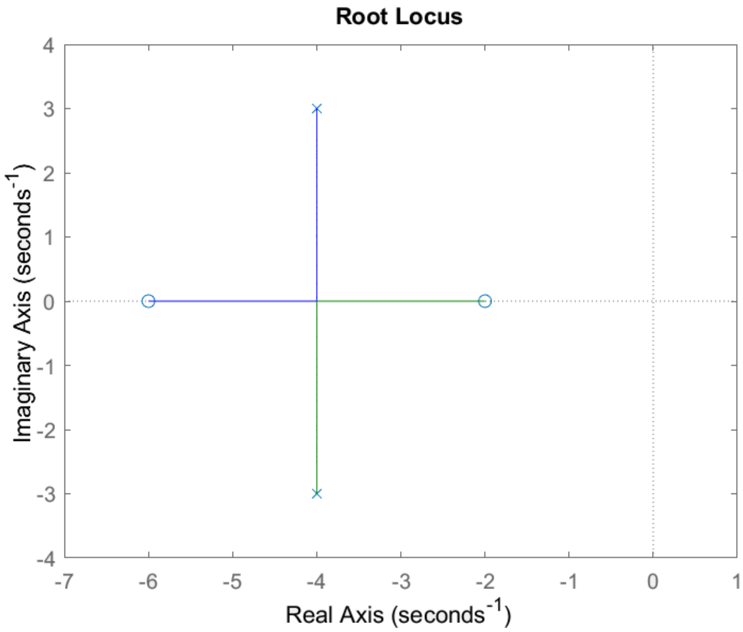
$$\sigma_a = \frac{\sum_i p_i - \sum_i z_i}{P - Z} = \frac{(-4 + -4) - (-2 + -6)}{2 - 2} = \infty$$

Asymptote

$$\theta_a = \frac{(2k + 1)\pi}{P - Z} = \frac{(2k + 1)\pi}{2 - 2} = \infty$$

Thus, there is no real-axis intercept and no infinity asymptote.

The simulation of the root locus diagram in MATLAB is shown in the figure below.



b. The sketch of root locus diagram of the system 2 is as shown in the figure below.

$$G_2(s) = \frac{K(s^2 + 4)}{(s^2 + 1)}$$

The poles of the system are complex pair located at (0, 1) and (0, -1) and the zeros are complex pair at (0, 2) and (0, -2).

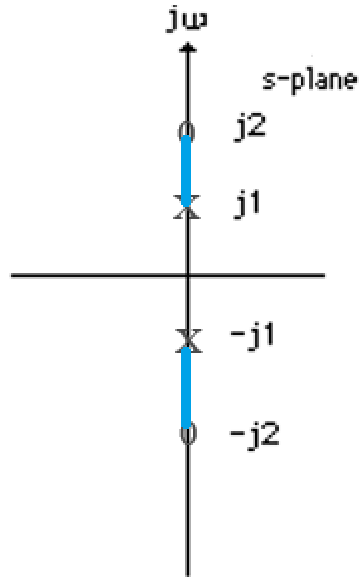
Real axis intercept

$$\sigma_a = \frac{\sum_i p_i - \sum_i z_i}{P - Z} = \frac{(0 + 0) - (0 + 0)}{2 - 2} = \infty$$

Asymptote

$$\theta_a = \frac{(2k + 1)\pi}{P - Z} = \frac{(2k + 1)\pi}{2 - 2} = \infty$$

Thus, there is no real-axis intercept and infinity asymptote.



There are two loci e.g. one is going from the complex pole at (0, 1) to the complex zero at (0, 2) and the other is going from the complex pole at (0, -1) to the complex zero at (0, -2).

Real axis intercept

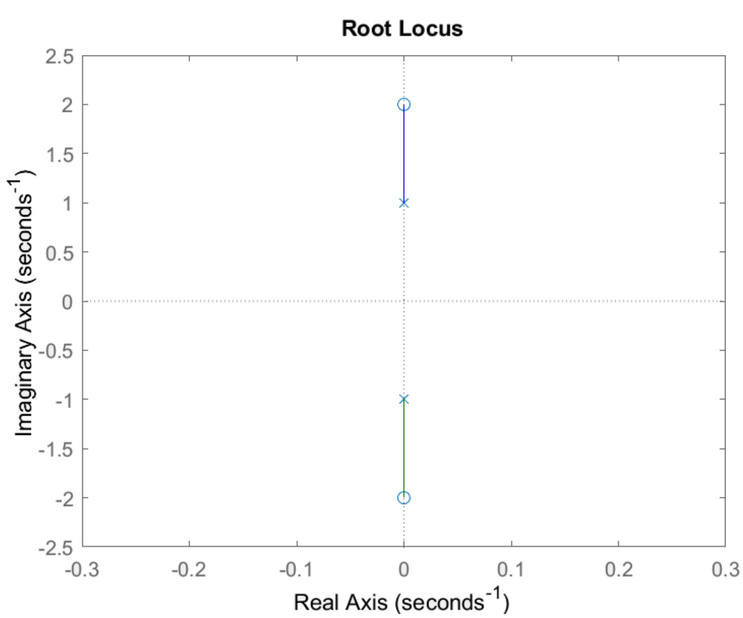
$$\sigma_a = \frac{\sum_i p_i - \sum_i z_i}{P - Z} = \frac{(0+0) - (0+0)}{2-2} = \infty$$

Asymptote

$$\theta_a = \frac{(2k+1)\pi}{P - Z} = \frac{(2k+1)\pi}{2-2} = \infty$$

Thus, there is no real-axis intercept and no infinity asymptote.

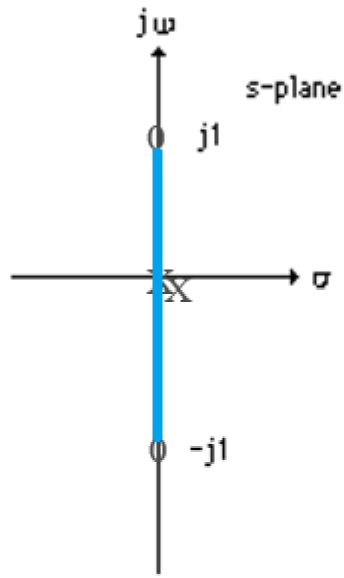
The simulation of the root locus diagram in MATLAB is shown in the figure below.



c. The sketch of root locus diagram of the system 3 is as shown in the figure below.

$$G_3(s) = \frac{K(s^2+1)}{s^2}$$

The poles are double poles located at origin (0, 0) and the zeros are complex pair at (0, 1) and (0, -1).



The first locus is going from the pole at origin (0, 0) to the complex zero at (0, 1) and the other locus is from the pole at origin (0, 0) to the complex zero at (0, -1).

Real axis intercept

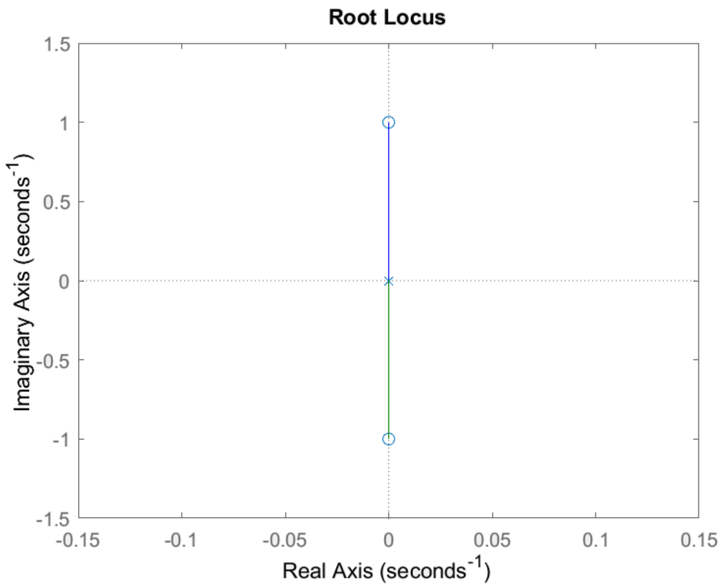
$$\sigma_a = \frac{\sum_i p_i - \sum_i z_i}{P - Z} = \frac{(0+0)-(0+0)}{2-2} = \infty$$

Asymptote

$$\theta_a = \frac{(2k+1)\pi}{P-Z} = \frac{(2k+1)\pi}{2-2} = \infty$$

Thus, there is no real-axis intercept and no infinity asymptote.

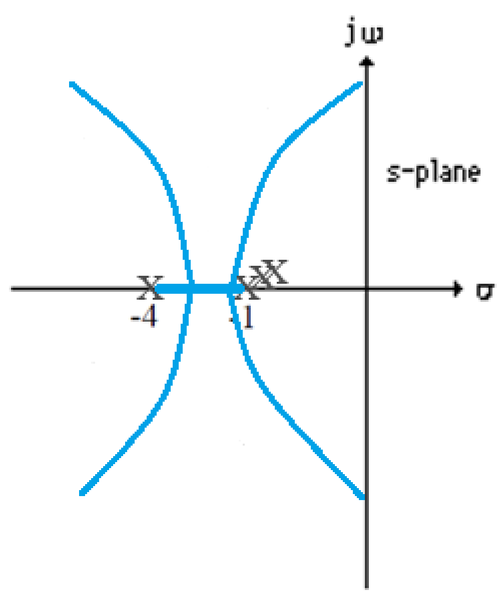
The simulation of the root locus diagram in MATLAB is shown in the figure below.



d. The sketch of root locus diagram of the system 4 is as shown in the figure below.

$$G_4(s) = \frac{K}{(s+1)^3(s+4)}$$

The poles are located as triple real poles at (-1, 0) and a real pole at (-4, 0).



There are four asymptote loci to the infinities originating from these real poles to their respective infinity asymptotes. A pair of asymptotes are from the two poles at (-1, 0) and the pole at (-1, 0) and pole at (-4, 0) meet and form the other pair of asymptotes.

Real axis intercept

$$\sigma_a = \frac{\sum_i p_i - \sum_i z_i}{P - Z} = \frac{(-1 + -1 + -1 + -4) - 0}{4 - 0} = \frac{-7}{4} = -1.75$$

Asymptote

$$\theta_a = \frac{(2k+1)\pi}{P - Z}$$

For $k = -1$, the asymptote is:

$$\theta_a = \frac{(2(-1)+1)\pi}{P - Z} = \frac{-\pi}{4 - 0} = -\frac{\pi}{4}$$

For $k = 0$, the asymptote is:

$$\theta_a = \frac{(2(0)+1)\pi}{P - Z} = \frac{\pi}{4 - 0} = \frac{\pi}{4}$$

For $k = 1$, the asymptote is:

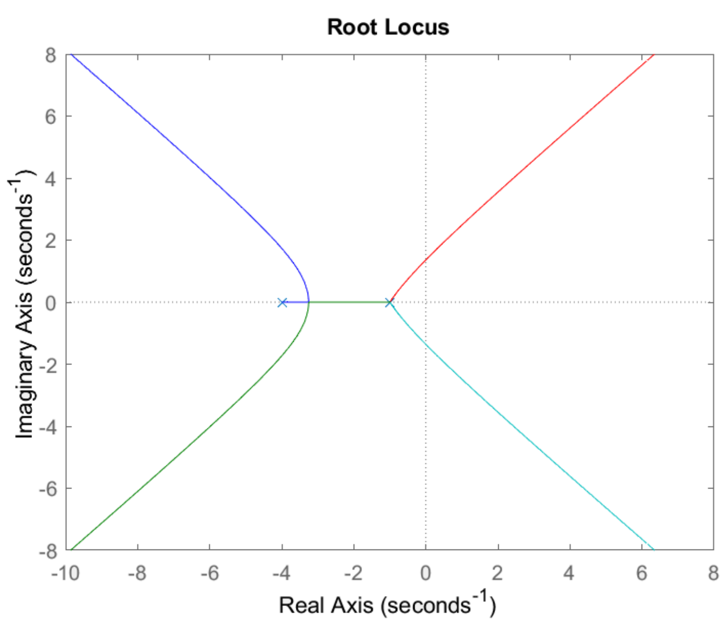
$$\theta_a = \frac{(2(1)+1)\pi}{P - Z} = \frac{3\pi}{4 - 0} = \frac{3\pi}{4}$$

For $k = -2$, the asymptote is:

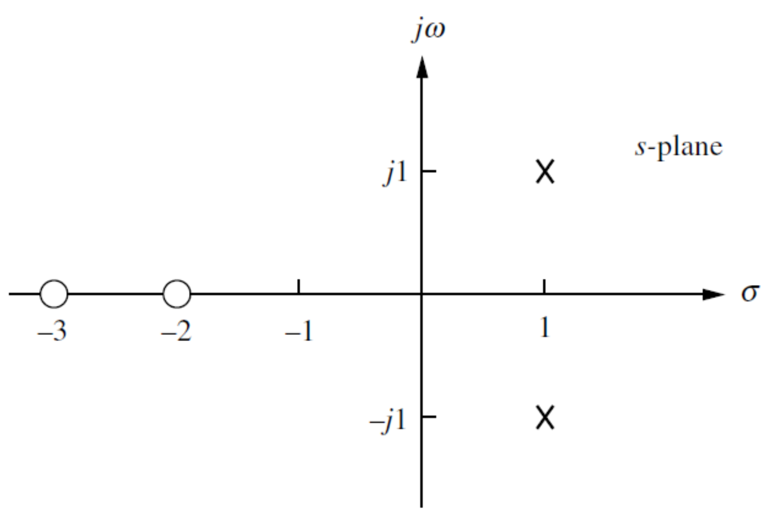
$$\theta_a = \frac{(2(-2)+1)\pi}{P - Z} = \frac{-3\pi}{4 - 0} = -\frac{3\pi}{4}$$

Thus, real axis intercept is -1.75 and infinity asymptotes are $-\pi/4$, $\pi/4$, $3\pi/4$, and $-3\pi/4$.

The simulation of the root locus diagram in MATLAB is shown in the figure below.



4. For the open-loop pole-zero plot shown in the figure below, perform the following tasks.



a. Sketch the root locus diagram of the system. [10 marks]

b. Determine the break-in point and break-away point with differentiation method. [12 marks]

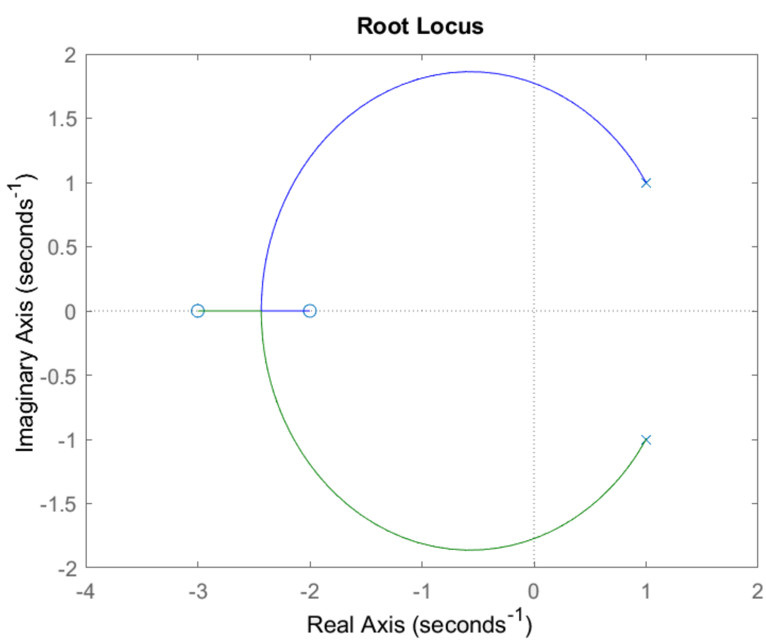
c. Repeat part (b) without differentiation. [12 marks]

Solution

a. The transfer function of the equation is as shown before.

$$G(s) = \frac{(s+2)(s+3)}{(s-1+j)(s-1-j)}$$

The sketch of root-locus diagram of the system is given in the figure below.



b. There is a break-in point in the diagram, but there is no break-away point as the loci are originally coming from two complex pair of poles.

The open-loop transfer function of the system is:

$$G(s) = \frac{(s+2)(s+3)}{(s-1+j)(s-1-j)} = \frac{s^2+5s+6}{s^2-2s+2}$$

The break-in point (σ) is determined from:

$$KG(s)H(s) = \frac{K(\sigma^2+5\sigma+6)}{\sigma^2-2\sigma+2} = -1$$

Thus

$$K = \frac{-(\sigma^2-2\sigma+2)}{(\sigma^2+5\sigma+6)}$$

Differentiate K with respect to σ.

$$\frac{dK}{d\sigma} = -\left[\frac{(2\sigma-2)(\sigma^2+5\sigma+6)-(\sigma^2-2\sigma+2)(2\sigma+5)}{(\sigma^2+5\sigma+6)^2}\right]$$

Set the derivative equal to zero for obtaining values of maxima yields:

$$\frac{dK}{d\sigma} = -\left[\frac{(8-1)\sigma^2+(2-6)\sigma+(-12-10)}{(s^2+5s+6)^2}\right] = 0$$

Simplify the equation above, it is now:

$$7\sigma^2-4\sigma-22=0$$

Finding the roots of the equation above

$$\sigma_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-(-4)\pm\sqrt{(-4)^2-4(7)(-22)}}{2(7)} = \frac{4\pm\sqrt{16+616}}{14}$$

Thus

$$K = \frac{-[(-2.43)^2-2(-2.43)+2]}{(-2.43)^2+5(-2.43)+6} = 53.1$$

The break-in point in the root locus diagram is located at σ = -2.43 for K = 53.1.

c. Without differentiation the break-in and break-away points are calculated from:

$$\sum_{1}^{m}\frac{1}{\sigma+z_i} = \sum_{1}^{n}\frac{1}{\sigma+p_i}$$

Entering the coefficients of the equation for the given system, the equation above becomes:

$$\frac{1}{(\sigma+2)}+\frac{1}{(\sigma+3)}=\frac{1}{(\sigma+1+j)}+\frac{1}{(\sigma+1-j)}$$

Thus

$$\frac{(\sigma+2)+(\sigma+3)}{(\sigma+2)(\sigma+3)}=\frac{(\sigma+1+j)+(\sigma+1-j)}{(\sigma+1+j)(\sigma+1-j)}$$

Solve and set the equation to zero for obtaining values of maxima:

$$\frac{(2\sigma+5)(\sigma+1+j)(\sigma+1-j)-(2\sigma+2)(\sigma+2)(\sigma+3)}{(\sigma+2)(\sigma+3)(\sigma+1+j)(\sigma+1-j)}=0$$

Evaluate only the denominator of the equation and simplify it:

$$\frac{(2\sigma+5)(\sigma^2+2\sigma+2)-(2\sigma+2)(\sigma^2+5\sigma+6)}{(\sigma+2)(\sigma+3)(\sigma+1+j)(\sigma+1-j)}=0$$

Thus

$$(9-12)\sigma^2+(14-22)\sigma+(10-12)$$

Simplify the equation above, it is now:

$$3\sigma^2+8\sigma-2=0$$

Finding the roots of the equation above

$$\sigma_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-8\pm\sqrt{(8)^2-4(3)(-2)}}{2(3)}=\frac{-8\pm\sqrt{64+24}}{6}$$

Thus

$$K=\frac{-[(-2.43)^2-2(-2.43)+2]}{(-2.43)^2+5(-2.43)+6}=53.1$$

The break-in point in the root locus diagram is located at σ = -2.43 for K = 53.1.

5. For a given system, its transfer function is as shown below,

$$G(s)=\frac{K}{s(s+4)(s^2+6s+64)}$$

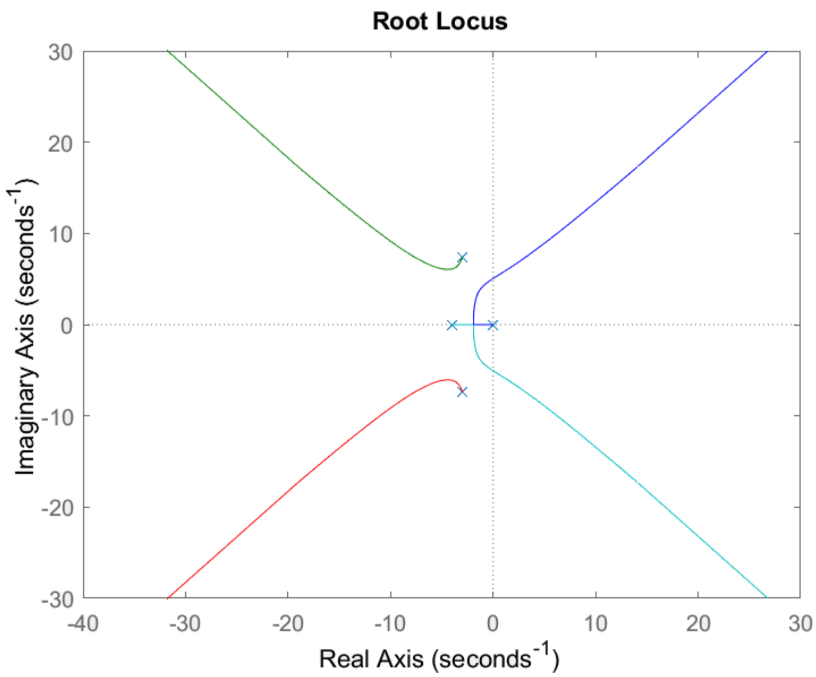
a. Sketch the root locus diagram of the system. [5 marks]

b. Determine the location and the gain of the system when it crosses the y-axis. [20 marks]

c. Determine the location and the gain of the system when it crosses the y-axis without Routh-Hurwitz method. [20 marks]

Solution

a. The root locus diagram of the system is as shown in the figure below.



b. The closed-loop transfer function equation of the system is:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)}$$

Entering the open-loop transfer function equation of the system to the equation above.

$$T(s) = \frac{\frac{K}{s(s+4)(s^2+6s+64)}}{1 + \frac{K}{s(s+4)(s^2+6s+64)}} = \frac{K}{s(s+4)(s^2+6s+64) + K}$$

The characteristic equation of the transfer function of the system is:

$$s(s+4)(s^2+6s+64) + K$$

Expanding the equation given above, the resulting polynomial equation is:

$$s^4 + 10s^3 + 88s^2 + 256s + K$$

Construct the Routh table using the characteristic equation of the transfer function of the system as given above.

s^4	1	88	K
s^3	10	256	
s^2	62.4	K	
s^1	$\frac{15,974 - 10K}{62.4}$		
s^0	K		

Thus, the value of K is determine from:

$$\frac{15{,}974 - 10K}{62.4} = 0$$

So, for the system to be stable, the value of K is found as $0 < K < 1597.4$.

When K = 1597.4, the locus is at the y-axis. At this point, the location of the locus in the root locus diagram is calculated from:

$$s^4 + 10s^3 + 88s^2 + 256s + 1597.4$$

Factorising the equation given above, the roots are:

$$s_{1,2} = -5 \pm j6.12 \qquad \text{and} \qquad s_{3,4} = \pm j5.06$$

So, the crossing location between the locus and the y-axis is $\pm j5.06$.

c. To determine the location and the gain when the pole crosses y-axis without Routh-Hurwitz method, we will be using algebraic derivation that involves equating the characteristic equation to zero.

From part (b), the characteristic equation of the system is:

$$s(s+4)(s^2 + 6s + 64) + K$$

Equate the characteristic equation of the transfer function of the system to zero:

$$s(s+4)(s^2 + 6s + 64) + K = 0$$

Since $s = j\omega$, substituting it into the equation above, it becomes:

$$j\omega(j\omega + 4)((j\omega)^2 + 6(j\omega) + 64) + K = 0$$

Expanding the equation given above, it is now:

$$(4j\omega - \omega^2)(6j\omega - \omega^2 + 64) + K = 0$$

Thus

$$-24\omega^2 - 4j\omega^3 + 256j\omega - 6j\omega^3 + \omega^4 - 64\omega^2 + K = 0$$

Gathering the real and imaginary part from the equation above, we obtain:

$$(\omega^4 - 88\omega^2 + K) + j[\omega(10\omega^2 - 256)] = 0$$

Comparing the complex number on the right and left-hand side of the equation, we obtain two equations:

$$\omega(10\omega^2 - 256) \qquad (Eq.1)$$

And

$$\omega^4 - 88\omega^2 + K = 0 \qquad (Eq.2)$$

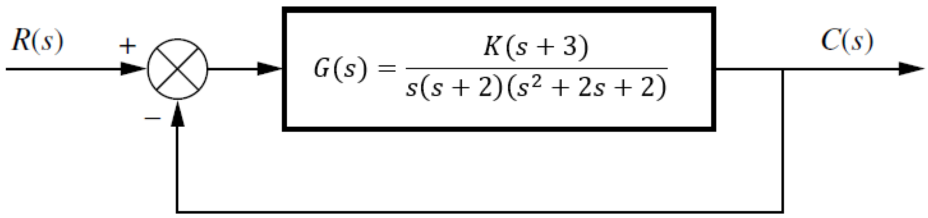
Solving the first equation leads to $\omega = 0$ and $\omega = \pm j8\sqrt{10}/5 = \pm j5.06$. For $\omega = 0$, this leads to solution $K = 0$.

Referring to the second equation, for $\omega = \pm j5.06$, the value of K is:

$$K = \omega^2(88 - \omega^2) = (5.06)^2[88 - (5.06)^2] = 1597.44$$

In the end, the locus crosses the y-axis at $\omega = \pm j5.06$ that happens at $K = 1597.44$.

6. Given the unity feedback system with complex poles of the following figure, perform the following tasks.



a. Find the angle of departure from the complex poles and sketch the root locus. [8 marks]

b. Simulate the root locus diagram of the system in MATLAB. Comment on the results obtained from the simulation. [6 marks]

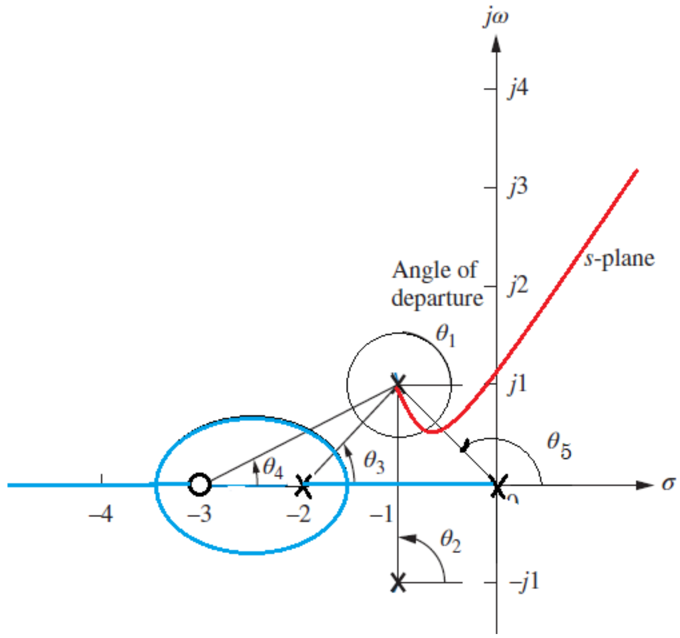
Solution

a. Using the poles and zeros of system $G(s)$ as plotted in the figure below, we calculate the sum of angles drawn to a point ϵ close to the complex pole, -1 + j1, in the second quadrant.

$$G(s) = \frac{s+3}{s(s+2)(s^2+2s+2)}$$

The angles of contribution of all poles and zeros of the system in the s-plane are:

$$-\theta_1 - \theta_2 + \theta_3 - \theta_4 - \theta_5 = 180°$$



For calculating the angle of departure θ_1 of pole at (-1, 1) as identified in the diagram, we need to determine the other angles of contribution of the poles and zeros of the system.

For angle θ_2 of the pole at (-1, -1) it is:

$$\theta_2 = 90°$$

For angle θ_3 of the pole at (-2, 0) it is:

$$\theta_3 = \tan^{-1}\left(\frac{1}{2-1}\right) = \tan^{-1}\left(\frac{1}{1}\right)$$

For angle θ_4 of the zero at (-3, 0) it is:

$$\theta_4 = \tan^{-1}\left(\frac{1}{3-1}\right) = \tan^{-1}\left(\frac{1}{2}\right)$$

For angle θ_5 of the pole at origin (0, 0) it is:

$$\theta_4 = \tan^{-1}\left(\frac{1}{0-1}\right) = \tan^{-1}\left(\frac{1}{-1}\right)$$

Thus

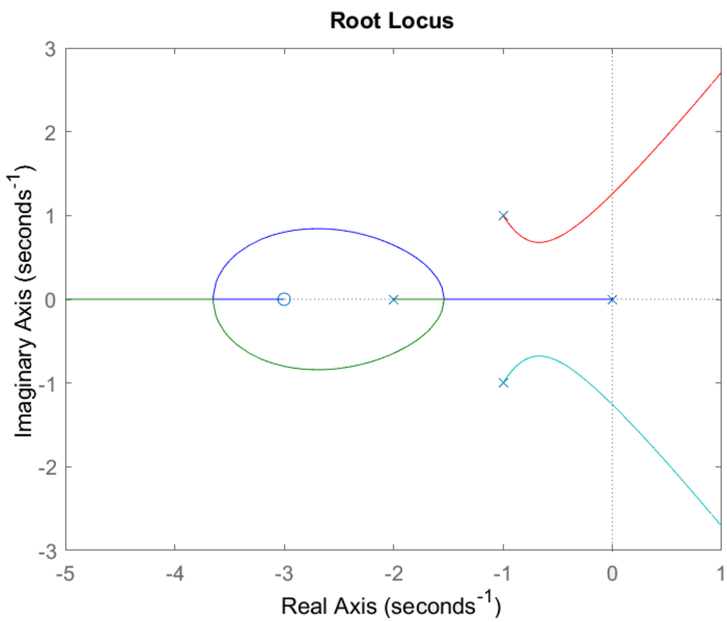
$$-\theta_1 - 90° + \tan^{-1}\left(\frac{1}{1}\right) - \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{1}{-1}\right) = 180°$$

Or

$$\theta_1 = -90° - 45 + 26.56° - 135° - 180$$

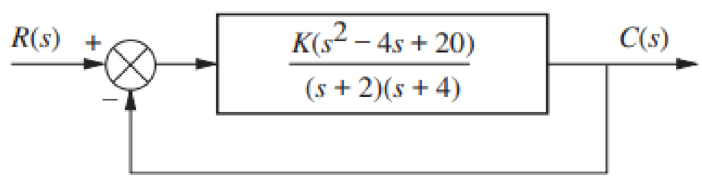
From which the angle of departure θ_1 = -423.44° = 296.56°. Notice how the departure angle from the complex poles helps us to refine the shape.

b. The root locus diagram as simulated in the MATLAB is as shown in the figure below.



The simulation result shows that the angle of departure resembles the sketch result.

7. For the system shown in the following figure, perform the following tasks:



a. Sketch the root locus for the system. [10 marks]

b. Determine the exact point and gain where the locus crosses the 0.45 damping ratio line. [6 marks]

c. Determine the exact point and gain where the locus crosses the $j\omega$-axis. [6 marks]

d. Determine the breakaway point on the real axis. [12 marks]

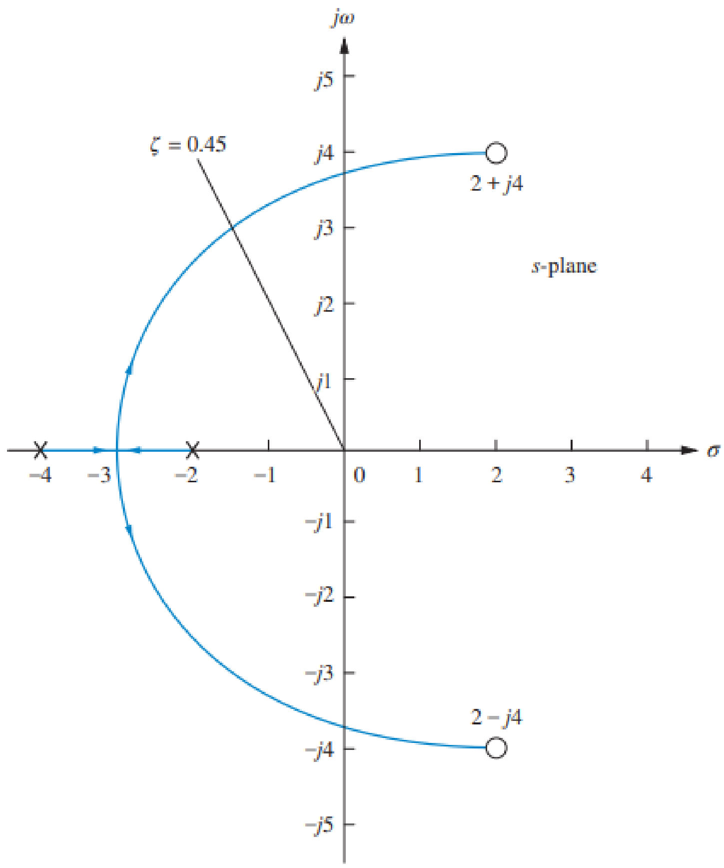
e. Determine the range of K within which the system is stable. [2 marks]

Solution

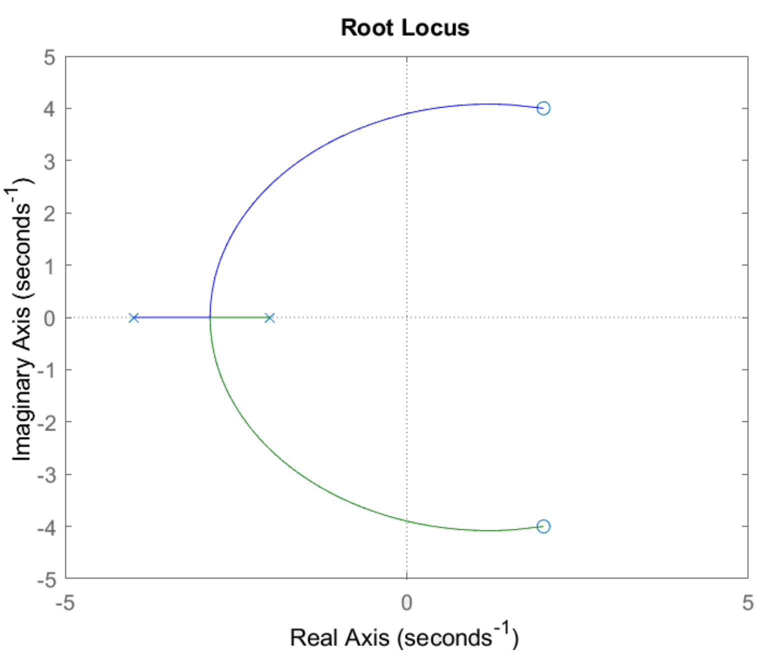
a. The solution is shown, in part, in the figure below. First, sketch the root locus diagram. Using root locus design rule, the real-axis segment is found to be between -2 and -4.

Design rule also tells us that the root locus starts at the open-loop poles and ends at the open-loop zeros. These two rules alone give us the general shape of the root locus.

Then, with the damping ratio of 0.45 line included in the s-plane diagram, the root locus diagram looks like as shown in the figure below.



The following diagram shows the root locus diagram as it is simulated in the MATLAB.



b. To find the exact point where the locus crosses the ζ = 0.45 line, we can use Matlab to search along the line for the point where the angles add up to an odd multiple of 180°.

$$\theta = 180° - \cos^{-1} 0.45 = 116.7°$$

Searching in polar coordinates, we find that the root locus crosses the ζ = 0.45 line at 1.5 ± j3.8 or 3.4 ∠ 116.7° with a gain, K, of 0.417.

Analytically, we can calculate the gain of the system at the intersection point as follows.

The gain of the system is:

$$K = \frac{s^2 - 4s + 20}{(s+2)(s+4)}$$

Entering the value of s = 1.5 ± j3.8, the gain of the system is:

$$K = \frac{(1.5 + j3.8)^2 - 4(1.5 + j3.8) + 20}{(1.5 + j3.8 + 2)(1.5 + j3.8 + 4)}$$

Calculating the magnitudes in the equation given above, the gain of the system is found to be:

$$K = \frac{\sqrt{(1.7)^2 + (15.4)^2}}{\left[\sqrt{(3.5)^2 + (3.8)^2}\right]\left[\sqrt{(5.5)^2 + (3.8)^2}\right]} = 0.4$$

c. To find the exact point where the locus crosses the $j\omega$-axis, use Matlab to search along the line for the point where the angles add up to an odd multiple of 180°.

$$\theta = 90°$$

Searching in polar coordinates, we find that the root locus crosses the $j\omega$-axis at ±j3.9 with a gain of K = 1.5.

Analytically we can determine the gain of the system when the locus crosses the imaginary axis as follows.

The open-loop transfer function of the system:

$$G(s) = \frac{K(s^2 - 4s + 20)}{(s+2)(s+4)}$$

The closed-loop transfer function of the system.

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{\left[\frac{K(s^2 - 4s + 20)}{(s+2)(s+4)}\right]}{1 + \left[\frac{K(s^2 - 4s + 20)}{(s+2)(s+4)}\right]}$$

$$= \frac{K(s^2 - 4s + 20)}{K(s^2 - 4s + 20) + (s+2)(s+4)}$$

$$= \frac{K(s^2 - 4s + 20)}{(K+1)s^2 + (6 - 4K)s + 28}$$

The gain of the system (K) when the root locus across the imaginary axis is determined below.

s^2	$(K+1)$	28
s^1	$(6-4K)$	
s^0	28	

From the Routh table, $K = -1$ and $K = 6/4 = 1.5$.

So, as negative gain is not possible, when $K = 1.5$, the locus crosses the imaginary axis.

d. To find the breakaway point, use MATLAB to search the real axis between -2 and -4 for the point that yields maximum gain. Naturally, all points will have the sum of their angles equal to an odd multiple of 180°. Therefore, the breakaway point is between the open-loop poles on the real axis at -2.88. Using simulation, the maximum gain at the point -2.88 is 0.0248.

e. From the answer to part (b), as obtained from the Routh table, for avoiding change of sign in the first column in the table:

$$K > -1 \text{ and } K < \frac{6}{4}$$

With negative gain is not possible, as a result, the system is stable for K between 0 and 1.5.