XMUT315 Control Systems Engineering

Tutorial 8: Analysis with Nyquist Diagram (Solution)

A. Construction of Nyquist Diagram

1. Given a first-order system with its transfer function equation:

$$G(s) = \frac{1}{s+4}$$

Derive the real and imaginary equations needed for sketching the Nyquist diagram. List the points required for sketching the Nyquist diagram. Sketch the Nyquist diagram of the system.

[20 marks]

Solution

Equations needed from sketching the Nyquist diagram are derived as follows. For the given transfer function equation of the system,

$$G(s) = \frac{1}{s+4}$$

substituting $s = j\omega$, the equation above becomes:

$$G(j\omega) = \frac{1}{j\omega + 4} = \left(\frac{1}{j\omega + 4}\right)\left(\frac{j\omega - 4}{j\omega - 4}\right) = \frac{j\omega - 4}{-16 - \omega^2}$$

For sketching the Nyquist diagram, we need the following equations for labelling the points in the Nyquist diagram:

The real part of the complex equation:

$$\mathrm{Re}\{G(j\omega)\} = \frac{-4}{-16 - \omega^2}$$

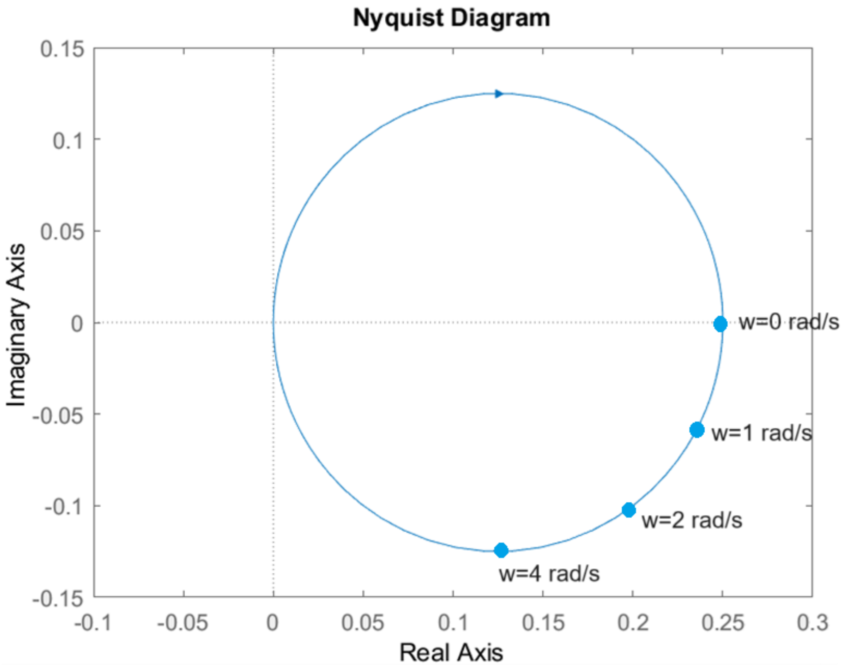
The imaginary part of the complex equation:

$$\mathrm{Im}\{G(j\omega)\} = j\left(\frac{\omega}{-16 - \omega^2}\right)$$

The points for sketching the Nyquist diagram are calculated and tabulated in the following table.

$\boldsymbol{\omega}$	$\mathbf{Re}\{\boldsymbol{G(j\omega)}\}$	$\mathbf{Im}\{\boldsymbol{G(j\omega)}\}$
0	$\frac{-4}{-16-(0)^2}=\frac{1}{4}=0.25$	$\left(\frac{(0)}{-16-(0)^2}\right)=0$
1	$\frac{-4}{-16-(1)^2}=\frac{4}{17}=0.235$	$\left(\frac{1}{-16-(1)^2}\right)=-\frac{1}{17}=-0.0588$
2	$\frac{-4}{-16-(2)^2}=\frac{1}{5}=0.2$	$\left(\frac{2}{-16-(2)^2}\right)=-\frac{1}{10}=-0.1$
4	$\frac{-4}{-16-(4)^2}=\frac{1}{8}=0.125$	$\left(\frac{4}{-16-(4)^2}\right)=-\frac{1}{8}=-0.125$

Based on the points listed in the table, the following diagram shows the Nyquist diagram of the given system.



2. The Nyquist diagram can be created directly from the transfer function equation of the control system.

   a. Rather than using the real and imaginary equations, determine the steps for creating a Nyquist diagram for a second order system by deriving the gain and phase angle of the frequency response of the system. [12 marks]

   b. Using the equations obtained in part (a), sketch the Nyquist diagram of the following control system given below. [24 marks]

$$G(s)=\frac{1}{s^2+2s+10}$$

Solution

a. Consider the magnitude/gain and phase shift contributions of a second order control system with the transfer function given below.

$$G(s) = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

As we move from zero to infinite frequency the phase shift of the given second-order system would move from 0° to -180°.

$$\angle \mathrm{G}(j\omega) = \theta^\circ = \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega^2}\right)$$

At the same time, the gain will drop from DC gain to the gain at high frequency.

$$|G(j\omega)| = \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n)^2}}$$

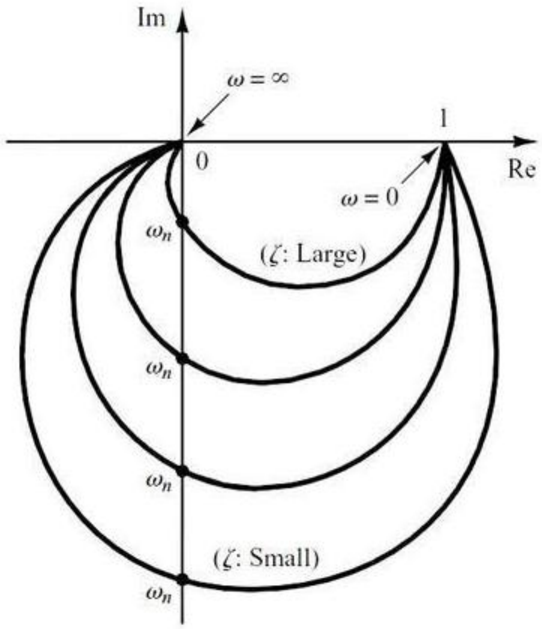
Calculate the gain and phase shift of the control system at various values of frequencies.

$\boldsymbol{\omega}$	$\boldsymbol{\lvert G(j\omega)\rvert}$	$\boldsymbol{\angle G(j\omega)}$
ω_1	$\frac{1}{(\omega_n^2 - \omega_1^2)^2 + (2\zeta\omega_n)^2} = M(\omega_1)$	$\tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega_1^2}\right) = \theta(\omega_1)$
ω_2	$\frac{1}{(\omega_n^2 - \omega_2^2)^2 + (2\zeta\omega_n)^2} = M(\omega_2)$	$\tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega_2^2}\right) = \theta(\omega_2)$
ω_3	$\frac{1}{(\omega_n^2 - \omega_3^2)^2 + (2\zeta\omega_n)^2} = M(\omega_3)$	$\tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega_3^2}\right) = \theta(\omega_3)$
…		

We can therefore sketch the Nyquist diagram from the gain and phase shift results given in the table above. The Nyquist diagram of the system is as shown in the figure below.

- Point A, at ω = ω_1 rad/s: Gain = $M(\omega_1)$ and Angle = $\theta(\omega_1)^\circ$.
- Point B, at ω= ω_2 rad/s: Gain = $M(\omega_2)$ and Angle = $\theta(\omega_2)^\circ$.
- Point C, at ω= ω_3 rad/s: Gain = $M(\omega_3)$ and Angle = $\theta(\omega_3)^\circ$.
- …

The Nyquist diagram is as shown in the figure below.



b. The Nyquist diagram of the given control system is worked out as follows:

$$G(s) = \frac{1}{s^2 + 2s + 10}$$

The phase shifts of the system.

$$\angle \mathrm{G}(j\omega) \ = \theta° = \tan^{-1}\left(\frac{2\zeta\omega_n}{\omega_n^2 - \omega^2}\right)$$

The gain or magnitude of the system.

$$|G(j\omega)| = \frac{1}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n)^2}}$$

Calculate the gain and phase shift of the control system at various values of frequencies.

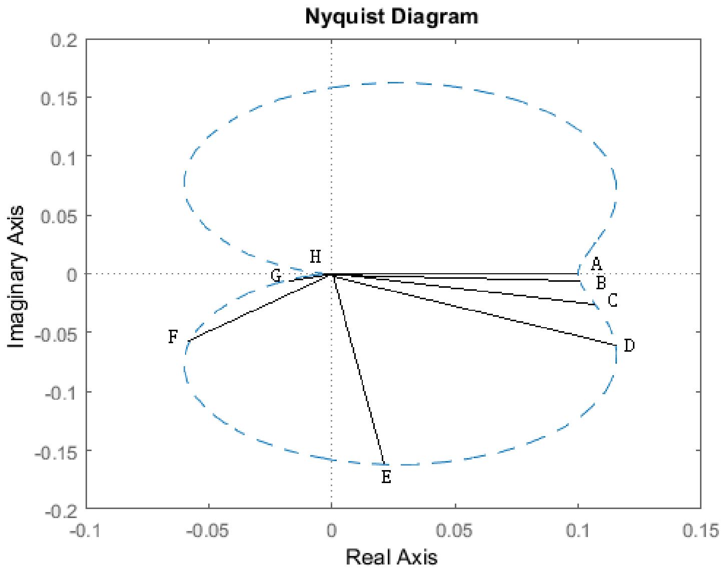
$\boldsymbol{\omega}$	$\boldsymbol{|G(j\omega)|}$	$\boldsymbol{\angle G(j\omega)}$
0	$\frac{1}{\sqrt{(10-0^2)^2 + (2(0))^2}} = 0.1$	$-\tan^{-1}\left(\frac{2(0)}{10-0^2}\right) = 0°$
0.1	$\frac{1}{\sqrt{(10-(0.1)^2)^2 + (2(0.1))^2}} = 0.1$	$-\tan^{-1}\left(\frac{2(0.1)}{10-(0.1)^2}\right) = -1.15°$
1	$\frac{1}{\sqrt{(10-(1)^2)^2 + (2(1))^2}} = 0.108$	$-\tan^{-1}\left(\frac{2(1)}{10-(1)^2}\right) = -12.5°$

2	$\frac{1}{\sqrt{(10-(2)^2)^2+(2(2))^2}} = 0.139$	$-\tan^{-1}\left(\frac{2(2)}{10-(2)^2}\right) = -33.69°$
3	$\frac{1}{\sqrt{(10-(3)^2)^2+(2(3))^2}} = 0.164$	$-\tan^{-1}\left(\frac{2(3)}{10-(3)^2}\right) = -80.53°$
5	$\frac{1}{\sqrt{(10-(5)^2)^2+(2(5))^2}} = 0.055$	$-\tan^{-1}\left(\frac{2(5)}{10-(5)^2}\right) = -146.3°$
10	$\frac{1}{\sqrt{(10-(10)^2)^2+(2(10))^2}} = 0.0108$	$-\tan^{-1}\left(\frac{2(10)}{10-(10)^2}\right) = -167.5°$
100	$\frac{1}{\sqrt{(10-(100)^2)^2+(2(100))^2}} = 0.000108$	$-\tan^{-1}\left(\frac{2(100)}{10-(100)^2}\right) = -178.85°$

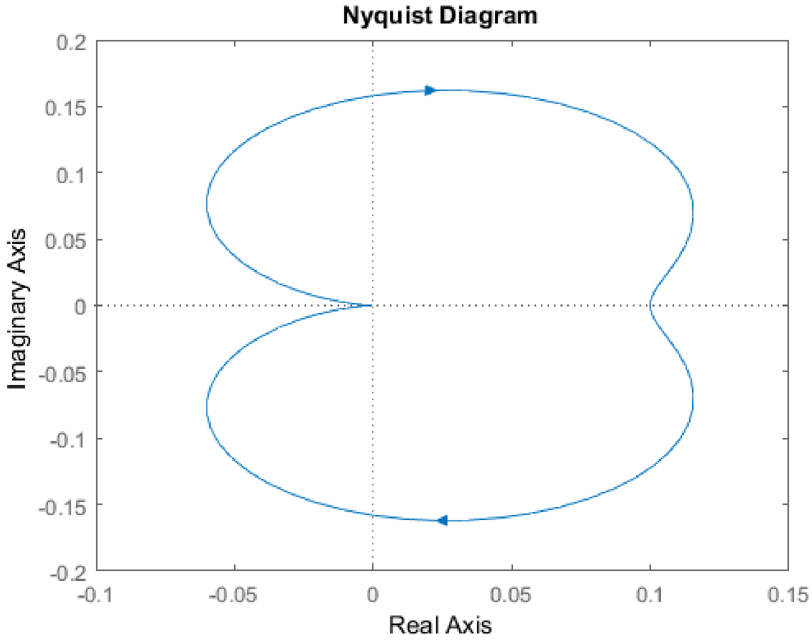
Based on the data in the table above, the points in the Nyquist diagram of the control system are indicated as listed below and the contour is sketched as shown in the figure given below.

- Point A, at $\omega = 0$ rad/s: Gain = 0.1 and Angle = 0°.
- Point B, at $\omega = 0.1$ rad/s: Gain = 0.1 and Angle = -1.15°.
- Point C, at $\omega = 1$ rad/s: Gain = 0.108 and Angle = -12.5°.
- Point D, at $\omega = 2$ rad/s: Gain = 0.139 and Angle = -33.69°.
- Point E, at $\omega = 3$ rad/s: Gain = 0.164 and Angle = -80.53°.
- Point F, at $\omega = 5$ rad/s: Gain = 0.055 and Angle = -146.3°.
- Point G, at $\omega = 10$ rad/s: Gain = 0.0108 and Angle = -167.5°.
- Point H, at $\omega = 100$ rad/s: Gain = 0.000108 and Angle = -178.85°.

Based on the gain and phase results in the table above, the Nyquist diagram of the system is:



The Nyquist diagram from the MATLAB simulation is as shown in the figure below (hint: use `nyquist()` function in MATLAB to plot Nyquist diagram). The plot confirms the likeness of the sketch.



3. Given a second-order system with the following transfer function equation:

$$G(s) = \frac{K}{(s+1)^2}$$

   a. Assume $K = 1$, plot the root locus diagram of the system in MATLAB. [6 marks]

b. Convert from the root locus diagram to Bode plot and Nyquist diagram. [24 marks]

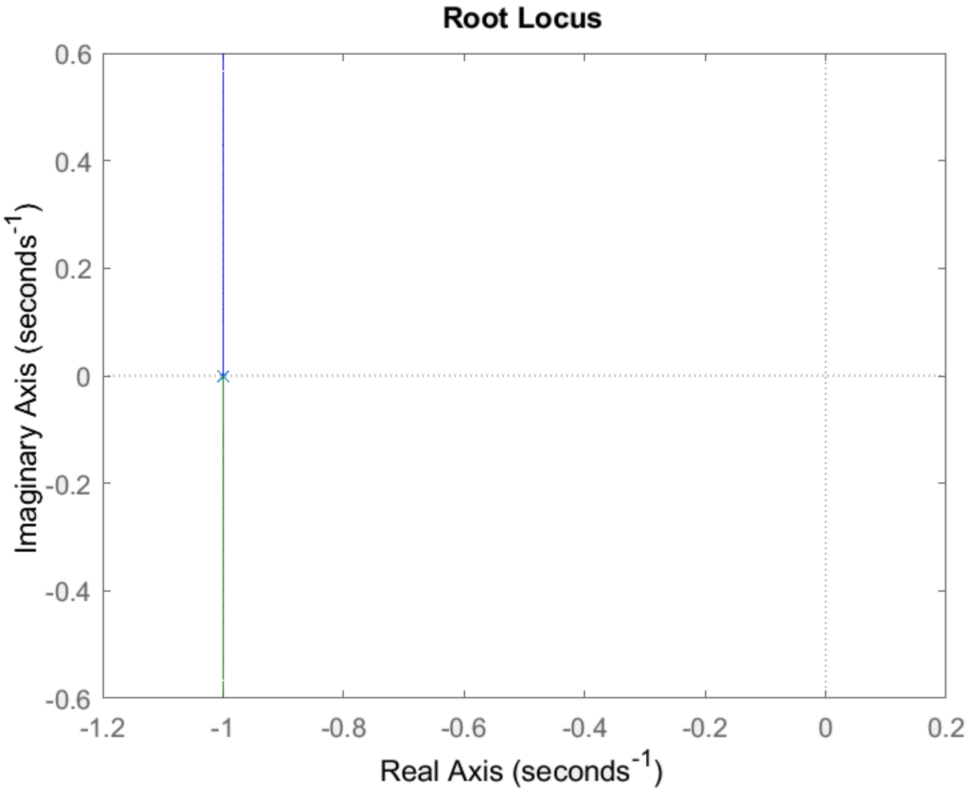
Solution

a. For $K = 1$, the transfer function equation of the system is:

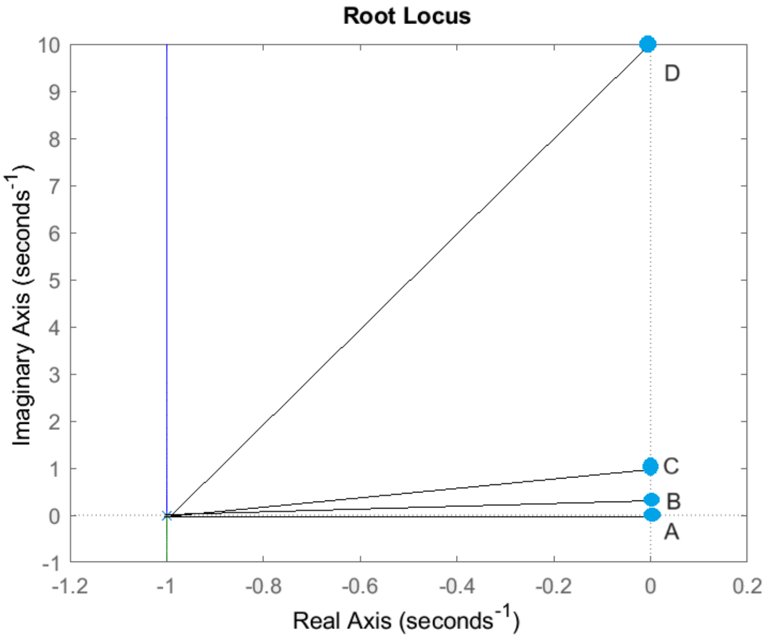
$$G(s) = \frac{1}{(s+1)^2}$$

Using MATLAB to do the simulation, the root locus diagram of the system is as shown in the figure below.

Notice on the diagram that the two loci start from the double poles at (-1, 0) and settles into positive and negative asymptotes.



b. First, for helping us to sketch the Bode plot and Nyquist diagram, choose several points from 0 rad/s to +∞ rad/s along the y-axis (i.e. imaginary axis) in the root locus diagram e.g. A = 0j, B = 0.1j, C = 1j, and D = 10j. The root locus diagram of the system with selected points indicated on the graph is as shown in the figure below.



Notice that we have several vectors originated from the double poles to the selected points. We need to determine the magnitudes and phase angles of these vectors and then use these points for sketching the contour in the Nyquist diagram.

Before working on that, we need to derive the equations needed to calculate the magnitude and phase of the vectors to be drawn in the Nyquist diagram.

Knowing that the transfer function equation of the system is:

$$G(s) = \frac{1}{(s+1)^2}$$

Converting the transfer function equation of the system into its pole and zero constituents, the magnitude equation becomes:

$$|G(j\omega)| = \left(\frac{1}{|j\omega + p_1|}\right)\left(\frac{1}{|j\omega + p_2|}\right)$$

Since $p_1 = p_2$, thus:

$$|G(j\omega)| = \left[\frac{1}{|j\omega + p|}\right]^2 = \left[\frac{1}{\sqrt{(j\omega)^2 + (1)^2}}\right]^2$$

For the phase shift, the equation is:

$$\angle G(j\omega) = -\angle(j\omega + p_1) - \angle(j\omega + p_2)$$

Since $p_1 = p_2$, thus:

$$\angle G(j\omega) = -2\angle(j\omega + p) = -2\tan^{-1}\left(\frac{j\omega}{1}\right)$$

Where:

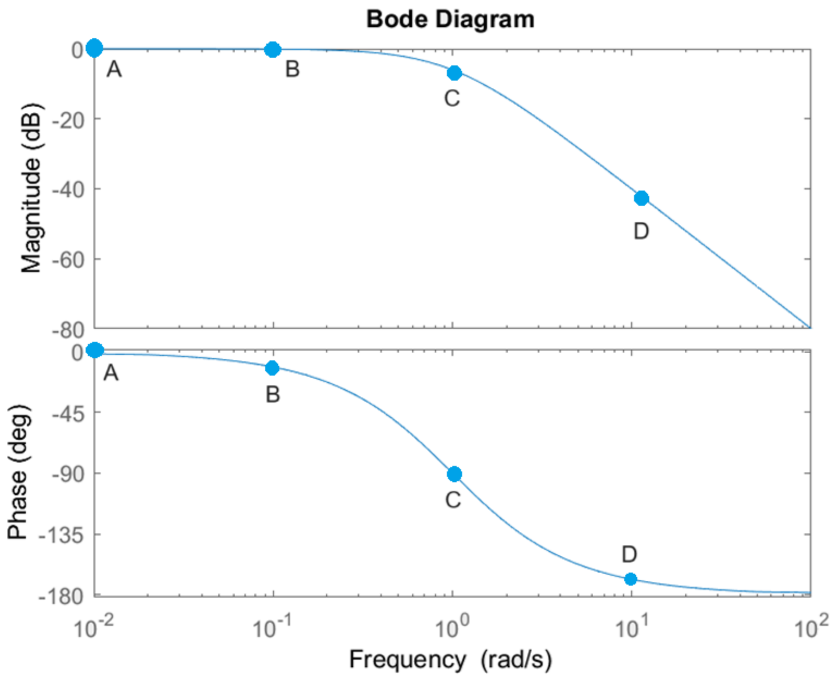
$|G(j\omega)|$ = magnitude of the system.

$\angle G(j\omega)$ = phase shift of the system.

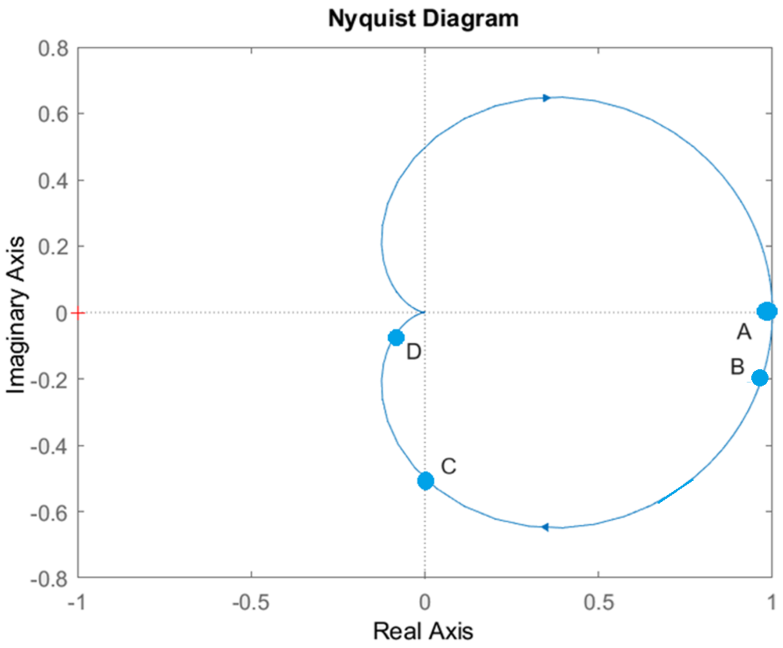
The points to be plotted in the diagrams are tabulated in the table below.

Point	$\boldsymbol{j\omega}$	$\boldsymbol{\lvert G(j\omega)\rvert}$	$\boldsymbol{\lvert G(j\omega)\rvert}$ **in dB**	$\boldsymbol{\angle G(j\omega)}$
A	0	$\left(\frac{1}{\sqrt{(1)^2+(0)^2}}\right)^2 = (1)^2 = 1$	0	$-2\tan^{-1}\left(\frac{0}{1}\right) = 0$
B	0.1	$\left(\frac{1}{\sqrt{(1)^2+(0.1)^2}}\right)^2 = \left(\frac{1}{\sqrt{1.01}}\right)^2 = 0.99$	-0.09	$-2\tan^{-1}\left(\frac{0.1}{1}\right) = -11.42°$
C	1	$\left(\frac{1}{\sqrt{(1)^2+(1)^2}}\right)^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 0.5$	-3	$-2\tan^{-1}\left(\frac{1}{1}\right) = -90°$
D	10	$\left(\frac{1}{\sqrt{(1)^2+(10)^2}}\right)^2 = \left(\frac{1}{10}\right)^2 = 0.01$	-40	$-2\tan^{-1}\left(\frac{10}{1}\right) = -168.58°$

Using the values of magnitude and phase shift from the table, the Bode plot of the system is sketched with the selected points as shown in the figure below.



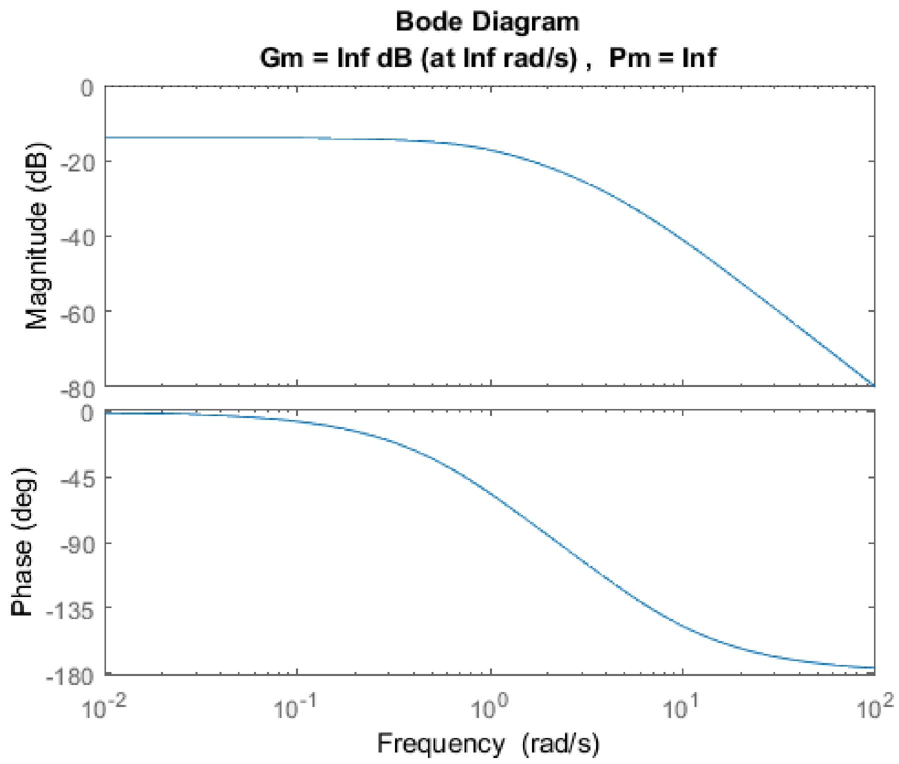
Using the same magnitude and phase shift from the table, the Nyquist diagram of the system is sketched with the selected points as shown in the figure below.



4. Nyquist diagram can be created from the results of the frequency response (gain and phase shift) of the Bode plots. To save time in performing this conversion, typically we pick only several interesting points from the Bode plots.

$$G(s) = \frac{1}{(s+1)(s+5)}$$

Given in the figure below is the Bode plots of a control system with the transfer function as shown above.



a. Create the Nyquist diagram of the control system from the Bode plots. [12 marks]

b. Although both approaches are frequency response analysis methods, describe at least two differences between Bode plots and Nyquist diagram for determining the stability of the control systems. [4 marks]

Solution

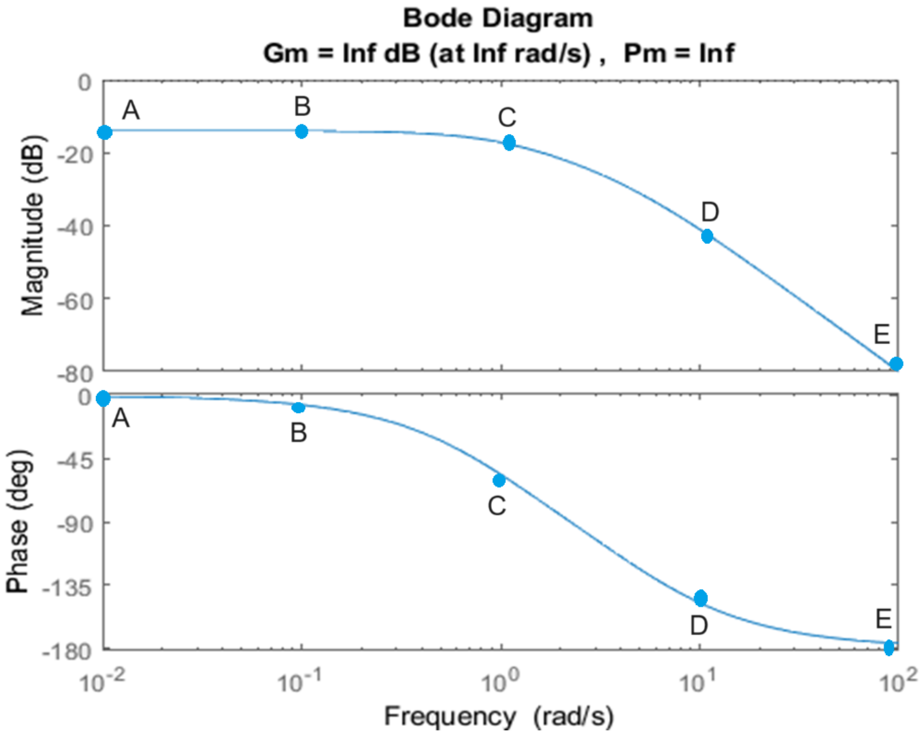
a. Nyquist diagram of the control system is created from the Bode plots by determining several interesting points in the Bode plots.

These points are decided to be at 0.01, 0.1, 1, 10, and 100 rad/s.

The points for plotting the Nyquist diagram are listed as tabulated below.

ω	$\lvert G(\omega)\rvert$	$\angle G(\omega)$
0.01	-14 dB (0.2)	0°
0.1	-14 dB (0.2)	-7°
1	-17.2 dB (0.138)	-56.3°

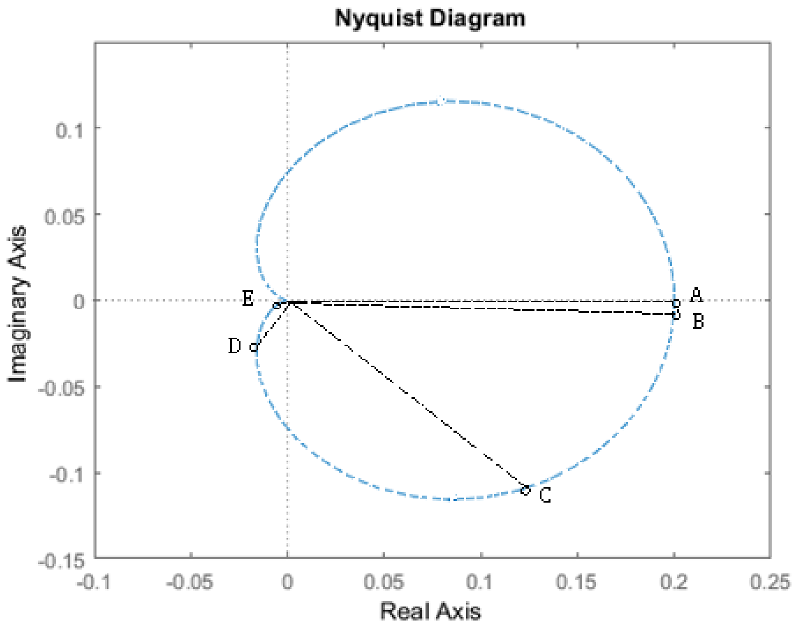
10	-41.1 dB (0.0088)	-148°
100	-80 dB (0.0001)	-178°



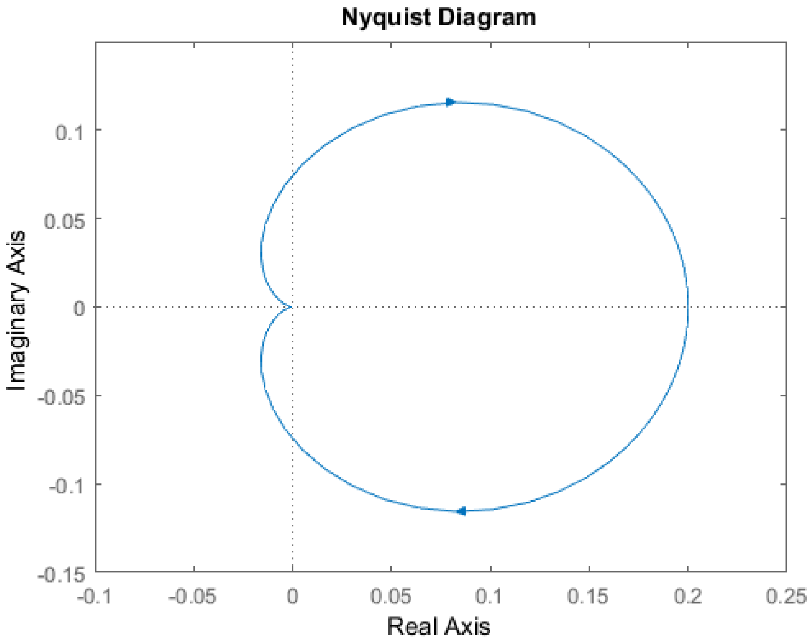
Based on the data in the table above, the points in the Nyquist diagram of the control system are indicated as listed below and the contour on the Nyquist diagram is to be sketched based on these points.

- Point A, at ω = 0.01 rad/s: Gain = 0.2 and Angle = 0°.
- Point B, at ω = 0.1 rad/s: Gain = 0.2 and Angle = -7°.
- Point C, at ω = 1 rad/s: Gain = 0.138 and Angle = -56.3°.
- Point D, at ω = 10 rad/s: Gain = 0.0088 and Angle = -148°.
- Point E, at ω = 100 rad/s: Gain = 0.00001 and Angle = -178°.

The figure shown below is the Nyquist diagram of the given control system.



The Nyquist diagram from MATLAB simulation. This confirms the likeness of the sketching results.



b. Two differences between Bode plots and Nyquist diagram for stability analysis of control systems.

- Bode plots show the frequency response of a system in terms of gain (or magnitude) and phase shift on two separate plots. The Nyquist plot combines gain and phase into one plot in the complex plane.

- Since Bode gain plot is in decibel (dB) whereas Nyquist is a linear gain, determining high frequency gain would be a challenge to do that in Nyquist compared in Bode plots.
- The stability of the closed loop control system can be intuitively determined from the number of encirclements of the plot contour around the test point (-1,0) in Nyquist diagram. This would need to be determined from calculating the gain and phase margins if it is performed in Bode plots.

5. Given a second-order system with the following transfer function equation:

$$G(s) = \frac{K}{(s+1)^2}$$

Sketch the Nyquist diagram of the system for $K = 1$ and $K = 2$. [8 marks]

Solution

The magnitude or gain and phase angle equations of the given system are:

$$|G(j\omega)| = \frac{|K|}{|(j\omega+1)^2|} = |K|\left[\frac{1}{\sqrt{(j\omega)^2+(1)^2}}\right]^2 \quad \text{and} \quad \angle G(j\omega) - tan^{-1}\left(\frac{j\omega}{1}\right) - tan^{-1}\left(\frac{j\omega}{1}\right)$$

For $K = 1$:

$$|G(j\omega)| = (1)\left[\frac{1}{\sqrt{(j\omega)^2+(1)^2}}\right]^2$$

And

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{j\omega}{1}\right) - \tan^{-1}\left(\frac{j\omega}{1}\right) = -2\tan^{-1}\left(\frac{j\omega}{1}\right)$$

Point	$\boldsymbol{j\omega}$	$\boldsymbol{\lvert G(j\omega)\rvert}$	$\boldsymbol{\angle G(j\omega)}$
A	0	$\left[\frac{1}{\sqrt{(0)^2+(1)^2}}\right]^2 = \frac{1}{1} = 1$	$-2\tan^{-1}\left(\frac{0}{1}\right) = 0$
B	0.1	$\left[\frac{1}{\sqrt{(0.1)^2+(1)^2}}\right]^2 = 0.99$	$-2\tan^{-1}\left(\frac{0.5}{1}\right) = -11.42°$
C	1	$\left[\frac{1}{\sqrt{(1)^2+(1)^2}}\right]^2 = 0.5$	$-2\tan^{-1}\left(\frac{1}{1}\right) = -90°$
D	10	$\left[\frac{1}{\sqrt{(10)^2+(1)^2}}\right]^2 = 0.01$	$-2\tan^{-1}\left(\frac{10}{1}\right) = -168.58°$

For $K = 2$:

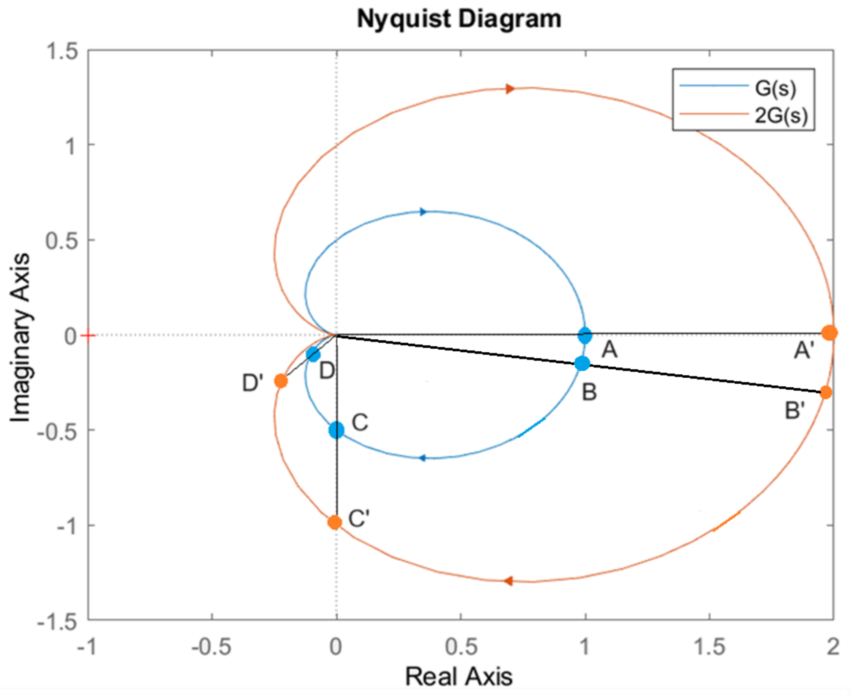
$$|G(j\omega)| = (2)\left[\frac{1}{\sqrt{(j\omega)^2 + (1)^2}}\right]^2$$

And

$$\angle G(j\omega) = -\tan^{-1}\left(\frac{j\omega}{1}\right) - \tan^{-1}\left(\frac{j\omega}{1}\right) = -2\tan^{-1}\left(\frac{j\omega}{1}\right)$$

Point	$j\omega$	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
A	0	$2\left[\frac{1}{\sqrt{(0)^2+(1)^2}}\right]^2 = 2$	$-2\tan^{-1}\left(\frac{0}{1}\right) = 0$
B	0.1	$2\left[\frac{1}{\sqrt{(0.1)^2+(1)^2}}\right]^2 = 1.98$	$-2\tan^{-1}\left(\frac{0.1}{1}\right) = -22.84°$
C	1	$2\left[\frac{1}{\sqrt{(1)^2+(1)^2}}\right]^2 = 1$	$-2\tan^{-1}\left(\frac{1}{1}\right) = -90°$
D	10	$2\left[\frac{1}{\sqrt{(10)^2+(1)^2}}\right]^2 = 0.02$	$-2\tan^{-1}\left(\frac{10}{1}\right) = -168.58°$

The Nyquist diagram of the system with specified values of gain K is shown in the figure below.



B. Analysis with Nyquist Diagram

6. Using MATLAB simulation, perform stability analysis of the following control systems with Nyquist diagram method: [18 marks]

   a. System 1

   $$G(s) = \frac{1}{s(s+3)(s+5)}$$

   b. System 2

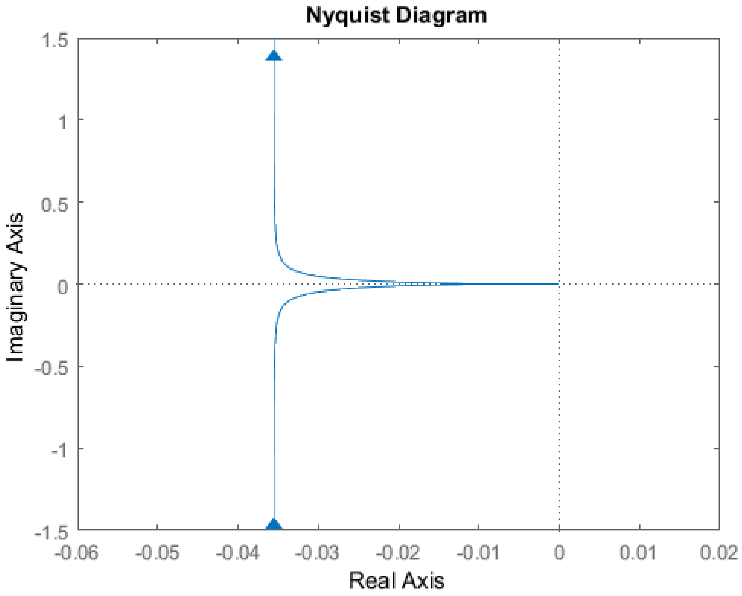
   $$G(s) = \frac{(s+2)}{s^2}$$

   c. System 2

   $$G(s) = \frac{(s+3)(s+5)}{(s-2)(s-4)}$$

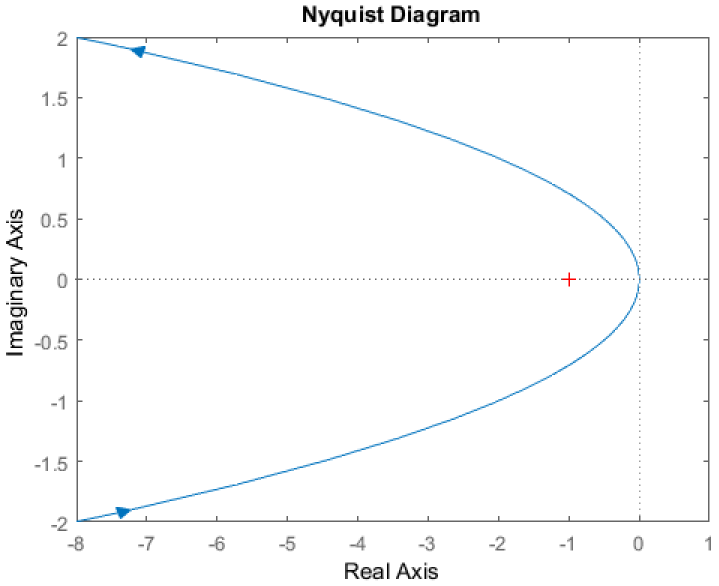
Solution

a. The Nyquist diagram of System 1 from simulation in MATLAB is given in the figure below.



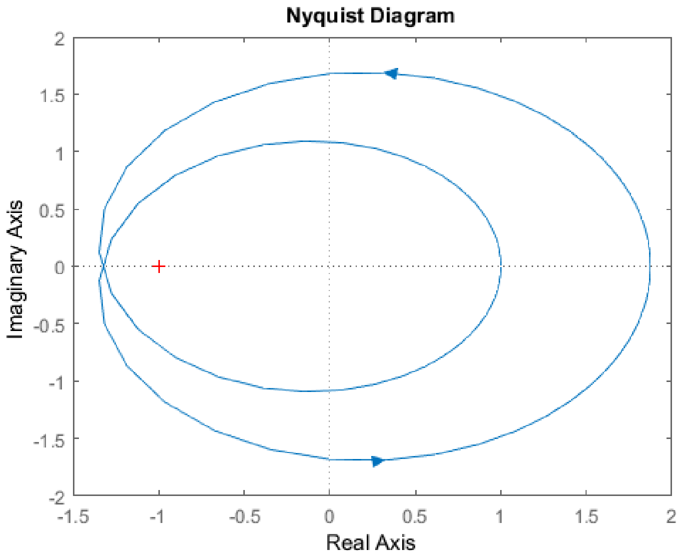
Based on the Nyquist diagram, System 1 is found to be stable as there are no encirclement of the contour about the test point (-1, 0).

b. The Nyquist diagram of System 2 from simulation in MATLAB is given in the figure below.



Based on the Nyquist diagram, System 2 is found to be stable as there are no encirclement of the contour about the test point (-1, 0).

c. The Nyquist diagram of System 3 from simulation in MATLAB is given in the figure below.

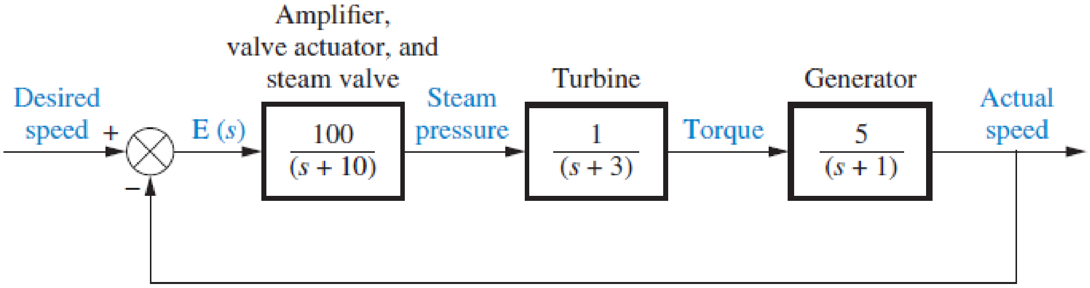


Based on the Nyquist diagram, System 3 is found to be unstable as there are two encirclements of the contour about the test point (-1, 0). This means that the closed loop system has two unstable poles in addition to the two RHS poles of open loop system.

7. Beside the encirclement analysis in Nyquist diagram, determining gain and phase margins from the Nyquist diagram would allow us to analyse the stability of the given control systems.

a. Describe how you determine the gain and phase margins in Nyquist diagram. [4 marks]

b. Using MATLAB simulation, indicate the gain and phase margins in the Nyquist diagram of the following control system below. Determine the gain and phase margins of the system.
[8 marks]

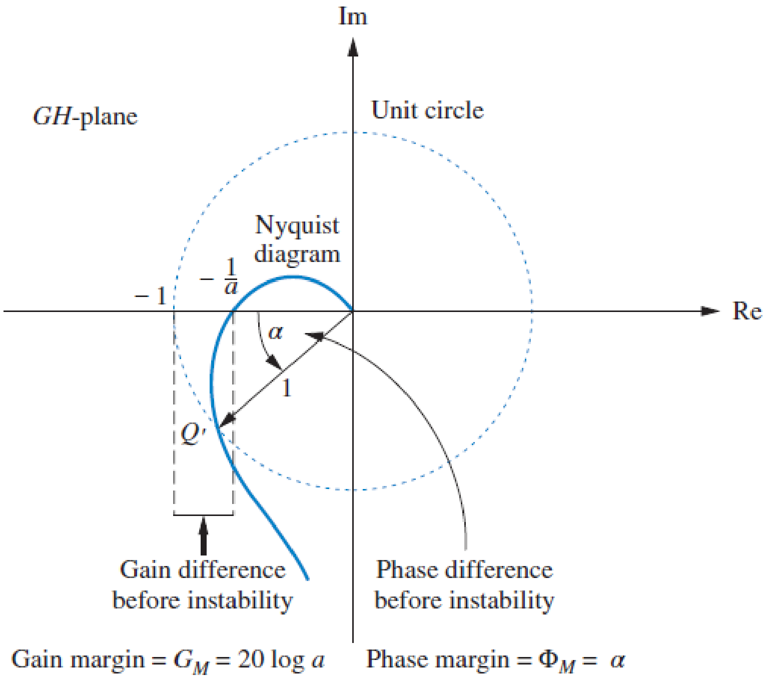


Solution

a. We can examine the Nyquist diagram to determine the gain and phase margins.

- The phase margin can simply be read as the difference between the phase = −180° line and the point where the curve crosses the unit circle.
- The gain margin is the inverse of the distance to the point where the curve crosses the negative real axis.

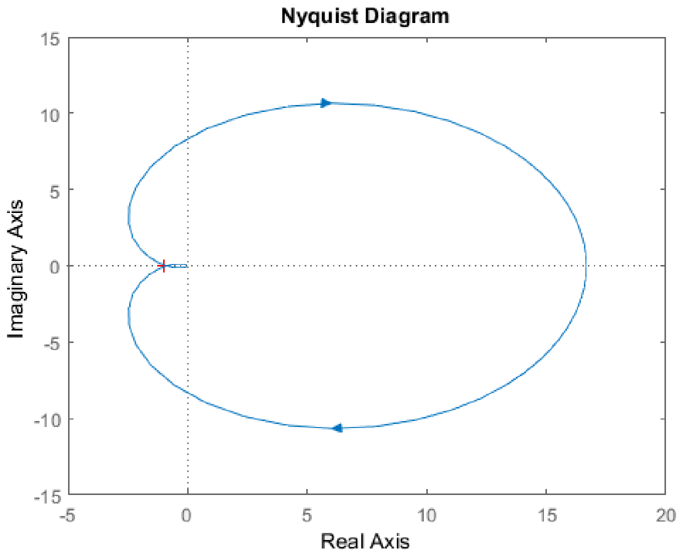
Note: that we can again have multiple gain and phase margins if the curve crosses the negative x-axis multiple times. The figure below illustrates how to find the gain margin (GM) and phase margin (PM) in the Nyquist diagram.



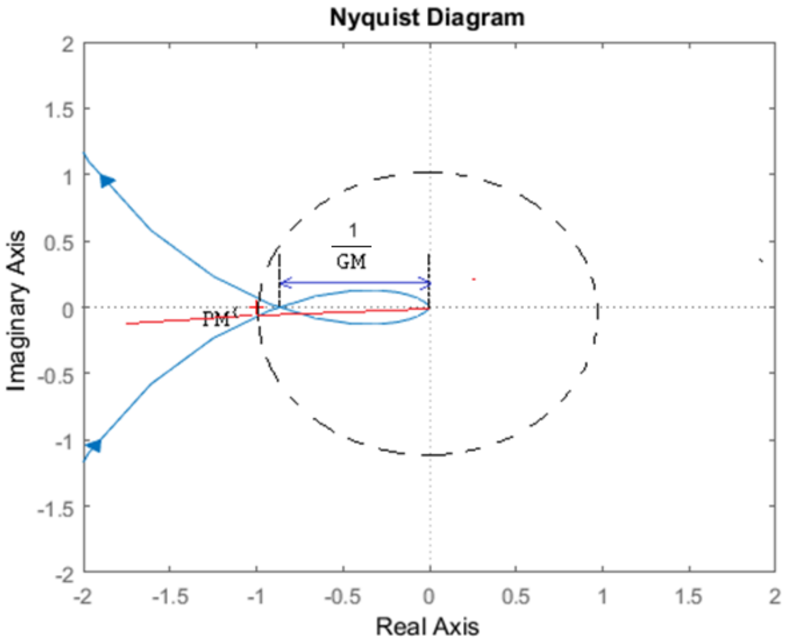
b. For the following control system, the gain and phase margins of the given system is worked out as follows.

$$G(s) = \left(\frac{100}{s+10}\right)\left(\frac{1}{s+3}\right)\left(\frac{5}{s+1}\right) = \frac{500}{(s+1)(s+3)(s+10)}$$

From MATLAB simulation, the Nyquist diagram of the given control system is as shown in the figure shown below.



Zooming out in the region of test point (-1, 0), the gain margin (GM) and phase margin (PM) of the given control system are determined as follow.



The gain margin of the system is found from:

$$a = 1/GM = 0.87, \text{ so } GM = 1.144$$

Or

$$20 \log 1.144 = 1.17 \text{ dB}$$

The phase margin of the system is found from:

$$PM = 3.68°$$

The system is found to be stable as its gain margin and phase margin are positive. As these margins are small, the system is reactive in nature and could easily turn into unstable system.

C. Nichols Chart

8. For example, assume the transfer function of a control system is given below:

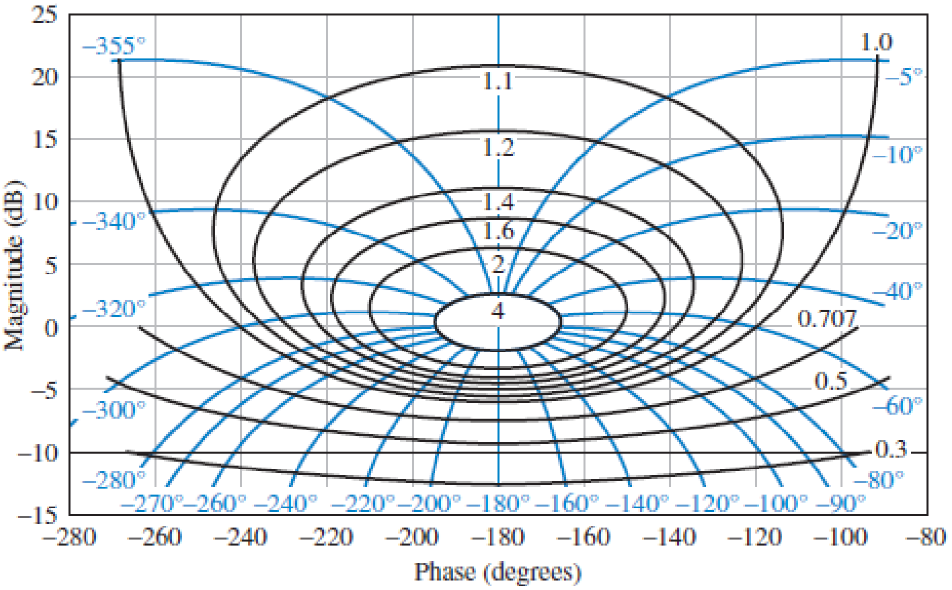
$$G(s) = \frac{K}{s(s+1)(s+2)}$$

a. Describe what is a Nichols chart. [2 marks]

b. Draw a Nichols chart of the system given above when $K = 1$. [20 marks]

Solution

a. Nichols chart is another variant of Nyquist diagram and it displays the gain in dB and phase in degrees, so that changes in gain are as simple to handle as in the Bode plot. A Nichols chart is shown in the figure below.



The chart is a plot of open-loop gain in dB vs. open-loop phase angle in degrees. Every point on the Nyquist plot can be transferred to the Nichols chart as each point on the plot is represented by gain and angle (polar coordinates).

Converting the gain to dB, we can transfer the point in the Nyquist plot to the Nichols chart, using the polar coordinates with gain in dB plotted as the ordinate, and the phase angle plotted as the abscissa.

b. Superimposing the frequency response of $G(s)$ on the Nichols chart by plotting gain in dB vs. phase angle for a range of frequencies from 0.1 to 1 rad/s, we obtain the plot in the figure below for K = 1.

Knowing the transfer function of the control system:

$$G(s) = \frac{K}{(s+1)(s+2)} = \frac{K}{s^2+3s+2}$$

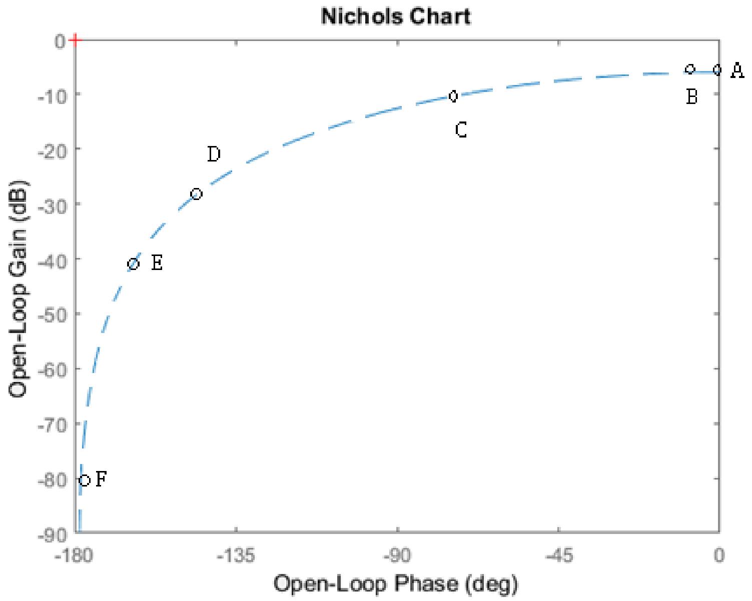
Calculate the gain and phase shift of the control system at various values of frequencies. The following table shows these values.

$\boldsymbol{\omega}$	$\lvert \boldsymbol{G(s)} \rvert$	$\angle \boldsymbol{G(s)}$
0	$\frac{1}{\sqrt{(2-0^2)^2+(3(0))^2}} = 0.5 = -6\text{ dB}$	$-\tan^{-1}\left(\frac{3(0)}{2-0^2}\right) = 0°$
0.1	$\frac{1}{\sqrt{(2-(0.1)^2)^2+(3(0.1))^2}} = 0.5 = -6\text{ dB}$	$-\tan^{-1}\left(\frac{3(0.1)}{2-(0.1)^2}\right) = -8.57°$
1	$\frac{1}{\sqrt{(2-(1)^2)^2+(3(1))^2}} = 0.316 = -10\text{ dB}$	$-\tan^{-1}\left(\frac{3(1)}{2-(1)^2}\right) = -71.56°$
5	$\frac{1}{\sqrt{(2-(5)^2)^2+(3(5))^2}} = 0.0364 = -28.77\text{ dB}$	$-\tan^{-1}\left(\frac{3(5)}{2-(5)^2}\right) = -146.89°$
10	$\frac{1}{\sqrt{(2-(10)^2)^2+(3(10))^2}} = 0.00976 = -40.2\text{ dB}$	$-\tan^{-1}\left(\frac{3(10)}{2-(10)^2}\right) = -162.98°$
100	$\frac{1}{\sqrt{(2-(100)^2)^2+(3(100))^2}} = 0.000099$ $= -80\text{ dB}$	$-\tan^{-1}\left(\frac{3(100)}{2-(100)^2}\right) = -178.28°$

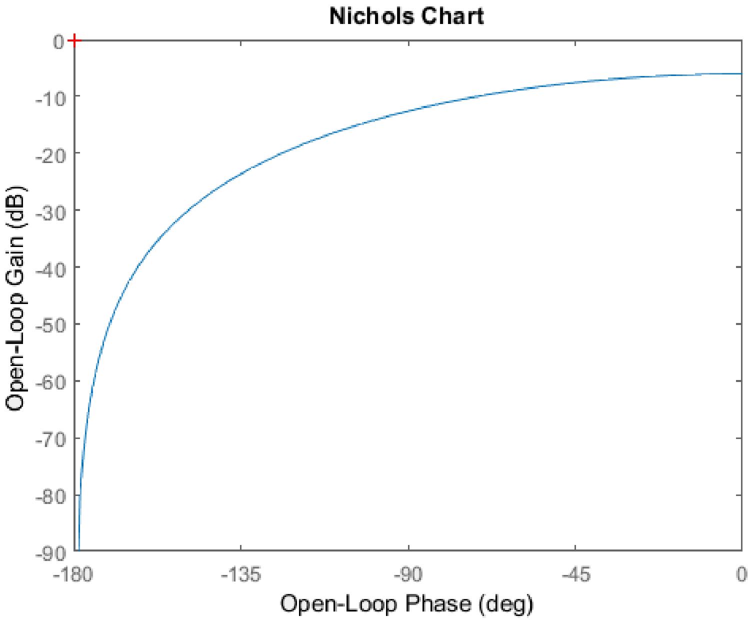
Based on the data in the table above, the points in the Nichols chart of the control system are indicated as listed below and the contour is sketched based on these points.

- Point A, at ω = 0 rad/s: Gain = -6 dB and Angle = 0°.
- Point B, at ω = 0.1 rad/s: Gain = -6 dB and Angle = -8.57°.
- Point C, at ω = 1 rad/s: Gain = -10 dB and Angle = -71.56°.
- Point D, at ω = 2 rad/s: Gain = -28.77 dB and Angle = -146.89°.
- Point E, at ω = 3 rad/s: Gain = -40.2 dB and Angle = -162.98°.
- Point F, at ω = 5 rad/s: Gain = -80 dB and Angle = -178.28°.

The Nichols chart sketch of the given control system is given in the figure below.

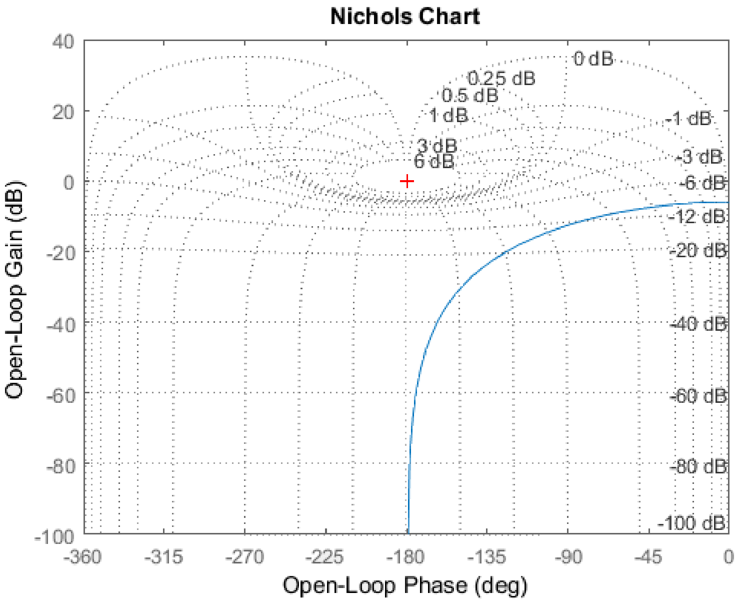


Notice the points indicated in the Nichols chart. The contour of the Nichols chart is sketched based on these points.



Looking into the Nichols chart, the system is found to be stable as the contour is not encircling the test point (0 dB, -180). Because the gain margin (GM) is +∞ and the phase margin (PM) is +∞, the system is deemed also to be stable because of these positive margin results.

From the MATLAB simulation of the Nichols chart of the given system (without the grid enabled). The simulation results confirm the likeness of the sketched Nichols chart with the simulation results. Result of MATLAB simulation of the Nichols chart of the given system (with the grid enabled)



9. Using simulation in MATLAB (hint: use `nichols()` function for plotting Nichols chart in MATLAB), plot the Nichols charts and also determine the stability of the following control systems.

a. System 1 [8 marks]

$$G(s) = \frac{5}{s(s+2)(s+3)}$$

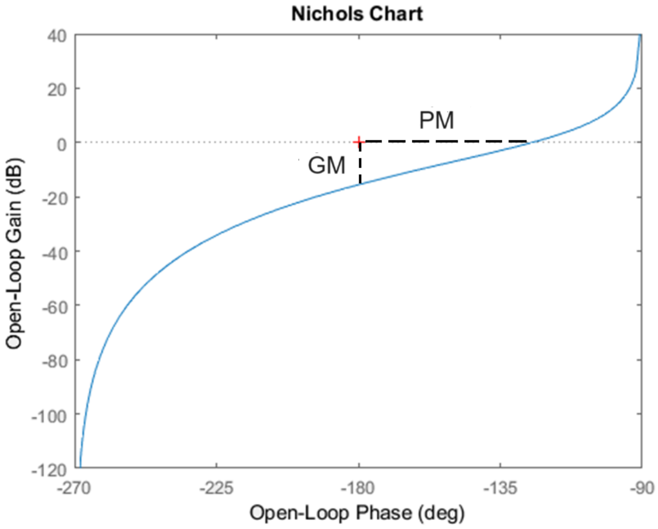
b. System 2 [8 marks]

$$G(s) = \frac{5}{(s+0.5)(s+1)(s+1.5)}$$

Solution

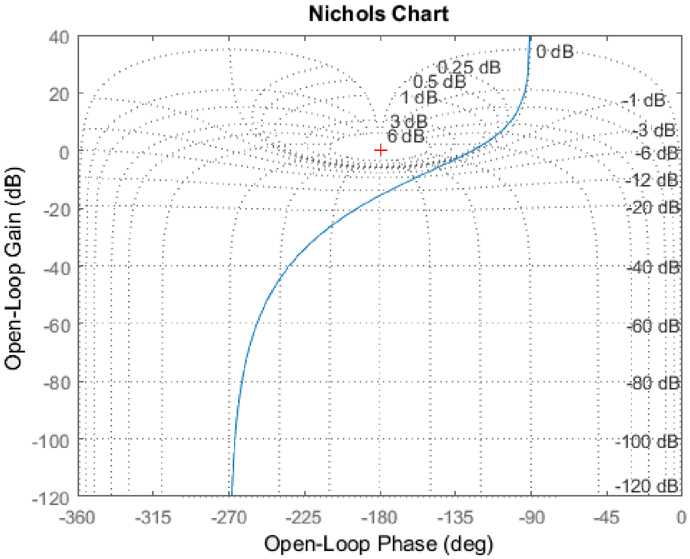
a. The Nichols chart for System 1 from MATLAB simulation is given in the figure below.

Without the grid enabled:



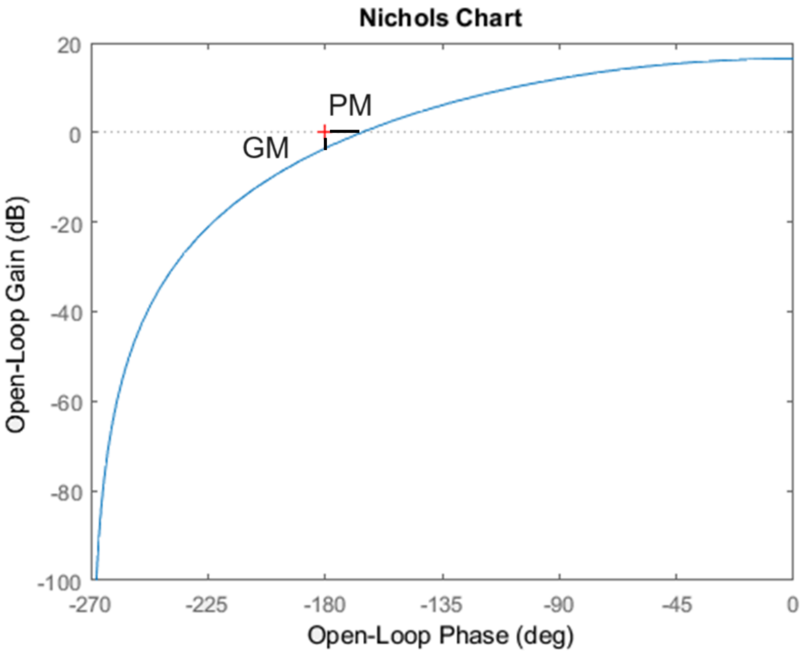
The phase margin is +54° and the gain margin is +16 dB. Since both of the gain and phase margins are positive, the system is found to be stable.

With the grid enabled:



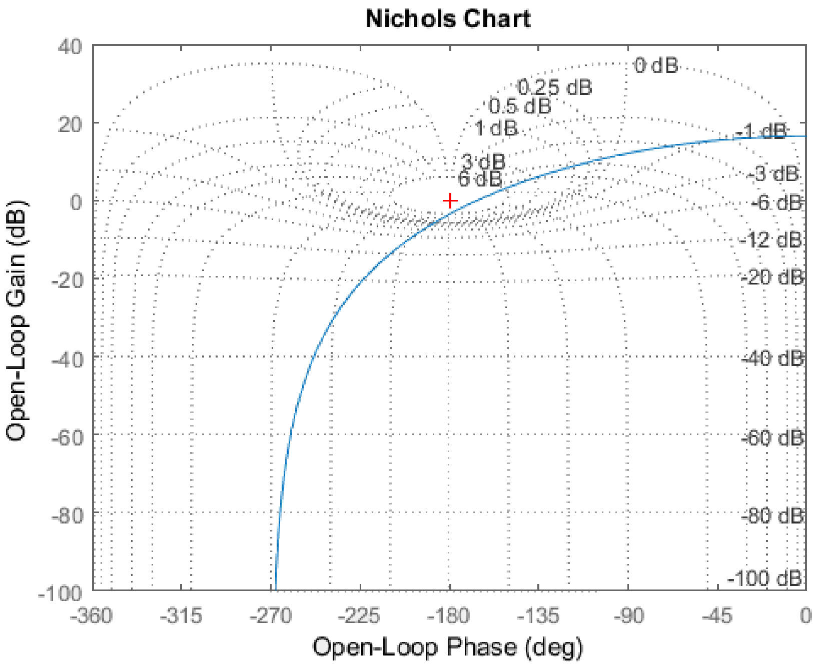
b. The Nichols chart for System 2 from MATLAB simulation is given in the figure below.

Without grid enabled:



The phase margin is +17° and gain margin is +4 dB. Since both gain and phase margins are positive, the system is found to be stable.

With grid enabled:



D. Design with Nyquist Diagram

10. Describe the following methods if they are used for designing the control systems and outline their step-by-step design procedures.
    a. Nyquist diagram. [6 marks]
    b. Nichols chart. [6 marks]

Solution

a. Nyquist diagram is primarily used for analysing and designing the stability of the control systems.

   Procedure for designing of control systems using Nyquist diagram.

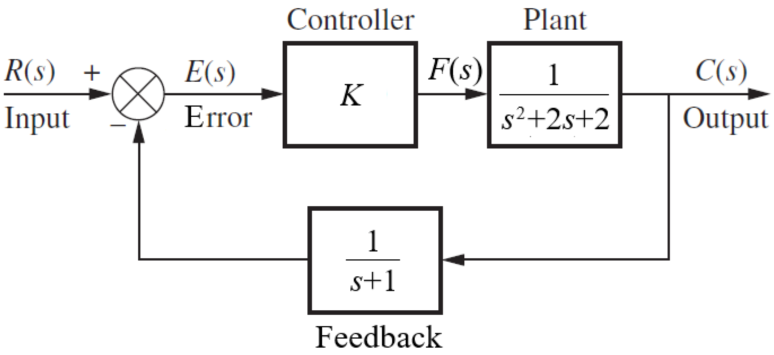
   i. Construct the Nyquist diagram for the given control system.
   ii. Determine the stability of the system by evaluating the position of the curve relative to the critical test point of Nyquist diagram (-1,0).
   iii. Determine the gain and phase margins of the system.
   iv. Increase or decrease the gain of the system to meet the required steady-state condition and transient response of the system.
   v. If previous step is not successful, introduce compensator or controller to meet the required steady-state condition and transient response of the system.
   vi. Readjust, if necessary, the gain of the system to meet the desired design specification.

b. Like Nyquist diagram, Nichols chart could provide alternative for analysing and designing the stability of control systems.

   Procedure for designing of control systems using Nichols chart:

   i. Construct the Nichols chart for the given control system.
   ii. Determine the stability of the system by evaluating the position of the curve relative to the critical test point of Nichols chart (-180°,0).
   iii. Determine the gain and phase margins of the system.
   iv. Increase or decrease the gain of the system to meet the required steady-state condition and transient response of the system.
   v. If previous step is not successful, introduce compensator or controller to meet the required steady-state condition and transient response of the system.
   vi. Readjust, if necessary, the gain of the system to meet the desired design specification.

11. Draw the Nyquist plot for the system in the figure below. Using the Nyquist stability criterion, determine the range of K for which the system is stable. Consider both positive and negative values of K. [12 marks]



Solution

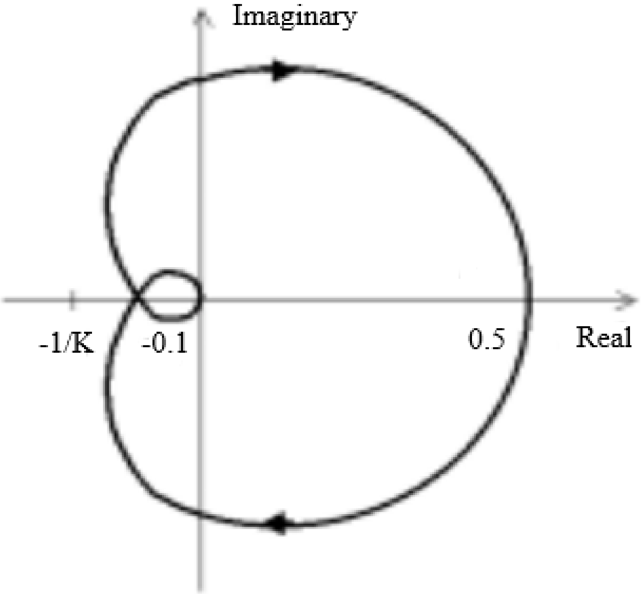
The transfer function equation of the open-loop system is given below:

$$G(s) = \frac{1}{(s+1)(s^2+2s+2)}$$

The characteristic equation of the given system is:

$$1 + K\left(\frac{1}{s^2+2s+2}\right)\left(\frac{1}{s+1}\right) = 0$$

The Nyquist diagram of the given control system is given in the figure below.

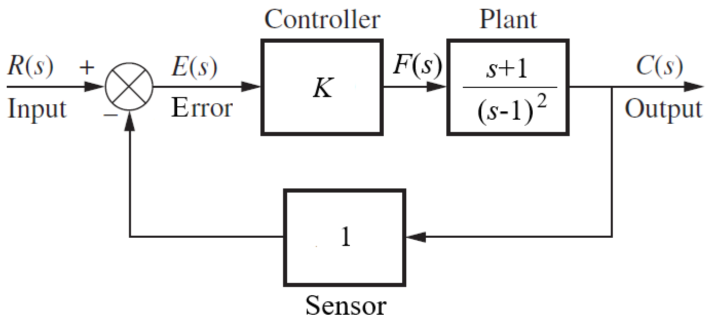


For positive K, note that the gain of the Nyquist plot as it crosses the negative real axis is 0.1, hence $K < 10$ for stability.

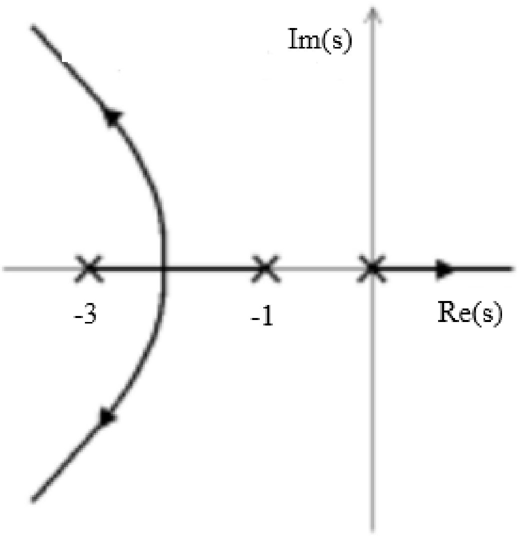
For negative K, the entire Nyquist plot is essentially flipped about the imaginary axis, thus the gain where it crosses the negative real axis will be 0.5 and the stability limit is that $|K| < 2$.

Therefore, the range of K for stability is $-2 < K < 10$.

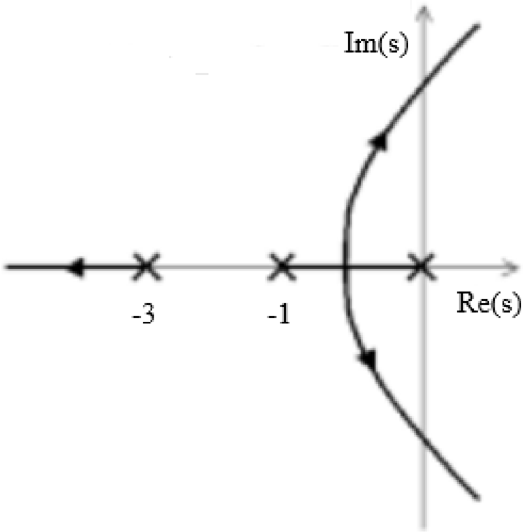
12. For a given control system as shown in the block diagram below, its root locus diagrams are also outlined below for two conditions of the gain of the system (e.g. negative and positive gains).



The root locus diagram of the system is as shown in the figures below for $K < 0$ and $K > 0$.



Root Locus (K<0)



Root Locus (K>0)

a. Determine the Nyquist plot and apply the Nyquist criterion. [6 marks]

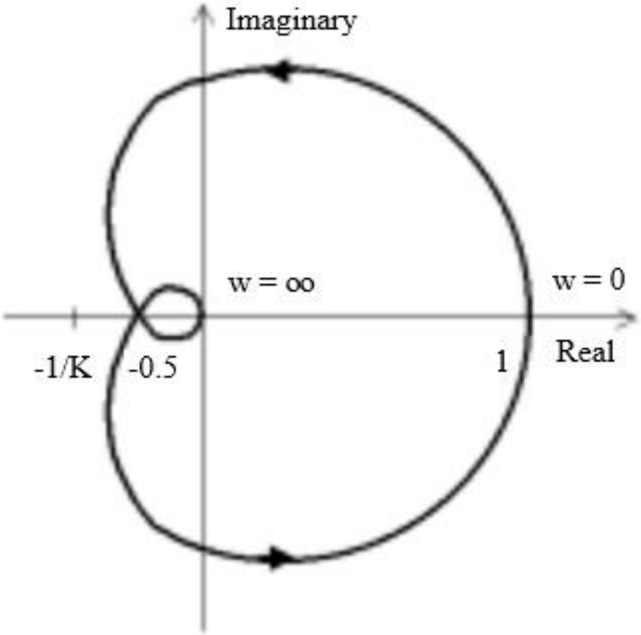
b. Calculate the range of values of K (e.g. positive and negative) for which the system will be stable. Evaluate the number of roots in the RHP for those values of K for which the system is unstable. [16 marks]

Solution

a. The transfer function equation of the given system is:

$$KG(s) = K\frac{s+1}{(s-1)^2}$$

The Nyquist diagram of the system is as shown in the figure given below:



Following Nyquist criterion, it seems that the stability of the system is conditionally depended on the value of the gain of the system (K).

b. From the Nyquist diagram as shown in part (a), we can see that:

- For gain margin of the system: $-\infty < 1/GM < -0.5$

  $$-\infty < -\frac{1}{K} < -\frac{1}{2}$$

  Thus, the gain of the system is:

  $$0 < K < 2$$

  The number of the unstable roots are evaluated as follow:

  $$N = 0, P = 2$$

  So

  $$Z = 2$$

  There are two closed-loop roots in RHP.

- For gain margin of the system: $-0.5 < 1/GM < 0$

$$-\frac{1}{2} < -\frac{1}{K} < 0$$

Thus, the gain of the system is:

$$K > 2$$

The number of the unstable roots are evaluated as follow:

$$N = -2, P = 2$$

So

$$Z = 0$$

The closed-loop system is stable.

- For gain margin of the system: $0 < 1/GM < 1$

$$0 < -\frac{1}{K} < 1$$

Thus, the gain of the system is:

$$K < -1$$

The number of the unstable roots are evaluated as follow:

$$N = -1, P = 2$$

So

$$Z = 1$$

There is one closed-loop roots in RHP.

- For gain margin of the system: $1 < 1/GM < \infty$

$$1 < -\frac{1}{K} < \infty$$

Thus, the gain of the system is:

$$-1 < K < 0$$

The number of the unstable roots are evaluated as follow:

$$N = 0, P = 2$$

So

$$Z = 2$$

There are two closed-loop roots in RHP.