ECEN321 Engineering Statistics

Homework 2 (5%)

Submit your individual work, clearly written, ONLINE to VUW by the due date.

Random Variables

1. (Navidi 2.4.2) Computer chips often contain surface imperfections. For a certain type of computer chip, the probability mass function of the number of defects X is presented in the following table.

x	0	1	2	3	4
$p(x)$	0.4	0.3	0.15	0.10	0.05

   (a) Find $P(X \leq 2)$

   [1 mark]

   (b) Find $P(X > 1)$

   [1 mark]

   (c) Find μ_x

   [1 mark]

   (d) Find σ_X^2

   [2 marks]

2. (Navidi 2.4.14) Elongation (in percent) of steel plates treated with aluminium are random with probability density function

$$f(x) = \begin{cases} \frac{x}{250} & 20 < x < 30 \\ 0 & \text{otherwise} \end{cases}$$

   (a) What proportion of steel plates have elongations greater than 25%?

   [1 mark]

   (b) Find the mean elongation.

   [1 mark]

   (c) Find the variance of the elongations.

   [2 marks]

   (d) Find the standard deviation of the elongations.

   [1 mark]

   (e) Find the cumulative distribution function of the elongations.

   [2 marks]

   (f) A particular plate elongates 28%. What proportion of plates elongate more than this?

   [2 marks]

Linear Functions of Random Variables

3. (Navidi 2.5.6) Two independent measurements are made of the lifetime of a charmed strange meson. Each measurement has a standard deviation of 7×10^{-15} seconds. The lifetime of the meson is estimated by averaging the two measurements. What is the standard deviation of this estimate?

   [1 mark]

Jointly Distributed Random Variables

4. (Navidi 2.6.4,6) In a piston assembly, the specifications for the clearance between piston rings and the cylinder wall are very tight. In a lot (batch) of assemblies, let X be the number with too little clearance and let Y be the number with too much clearance. The joint probability mass function of X and Y is given in the table below:

	y			
x	0	1	2	3
0	0.15	0.12	0.11	0.10
1	0.09	0.07	0.05	0.04
2	0.06	0.05	0.04	0.02
3	0.04	0.03	0.02	0.01

(a) Find the marginal probability mass function of X.

[1 mark]

(b) Find the marginal probability mass function of Y.

[1 mark]

(c) Are X and Y independent? Explain.

[1 mark]

(d) Find μ_x and μ_Y

[2 marks]

(e) Find σ_X and σ_Y

[2 marks]

(f) Find $\text{Cov}(X, Y)$

[3 marks]

(g) Find $\rho(X, Y)$

[1 mark]

(h) Find the conditional probability mass function $p_{Y|X}(y|1)$

[2 marks]

(i) Find the conditional probability mass function $p_{X|Y}(x|2)$

[2 marks]

(j) Find the conditional expectation $E(Y|X = 1)$

[1 mark]

(k) Find the conditional expectation $E(X|Y = 2)$

[1 mark]

5. (Navidi 2.6.16) For continuous random variables X and Y with joint probability density function

$$f(x,y) = \begin{cases} xe^{-(x+xy)} & x > 0 \text{ and } y > 0 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find $P(X > 1 \text{ and } Y > 1)$

[2 marks]

(b) Find the marginal probability density function $f_X(x)$

[2 marks]

(c) Find the marginal probability density function $f_Y(y)$

[2 marks]

(d) Are X and Y independent? Explain.

[1 mark]