ECEN321 Engineering Statistics

Laboratory Session 3

Noise

For this Lab 3, you will need to submit an individual Lab Report (worth 10% of the overall course grade). Please refer to the Lab marking sheets (and the suggested Lab report format) on the course website for reference.

Check with the Lab instructor (co-teacher) the due date of the Lab report, and how / where to submit the report.

Note: “calculations” in this Lab can be done by Matlab, unless otherwise specified.

1 Effect of Amplification on Noise

An amplifier with a voltage gain equal to 10 is used to amplify a DC signal with noise as shown below in Figure 1. The DC level of the signal is 3 V, and the noise has a variance (i.e. power) of 4 V^2.

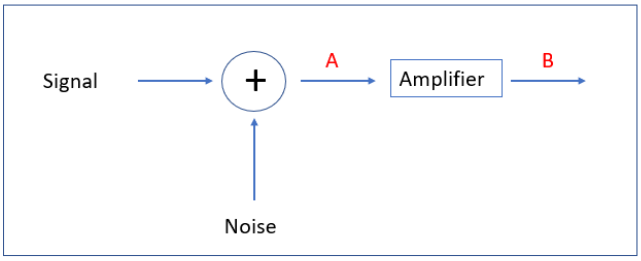


Figure 1: Noise corrupted signal going through an amplifier.

Create 1000 samples of the DC signal + noise, at location A, and then from the signal, estimate the mean and standard deviation.

Then simulate the function of the amplifier amplifying this signal, and again determine the resulting mean and standard deviation, at location B. How does the observation relate to the theories learnt in Lectures? Explain your results.

2 Effect of Averaging on Noise

Signal averaging is a common method used to improve the signal to noise ratio (SNR) and is used extensively in magnetic resonance imaging (MRI) where repeat measurements are performed and the resultant signals averaged.

Create 16 signals as in Part 1 (at location A) and calculate the mean and standard deviation of one of them.

Now sum all 16 signals and divide by 16 to normalise. Recalculate the mean and standard deviation. How does the observation relate to the theories learnt in Lectures? Explain your results.

3 Covariance and Correlation of Noisy Signal

In many experiments the desired signal is often corrupted by noise (e.g. at location D as shown in Figure 2 below) that reduces the correlation between the captured and original parameter being measured.

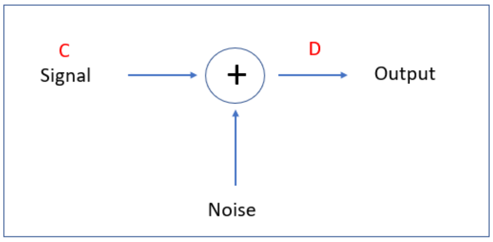


Figure 2: Noise corrupted signal.

Create a sin(x) signal, at location C, using 1000 points of x on a uniform x-axis grid from 0 to 20. Plot the signal.

Then add noise of variance 1 to the signal. Plot this noise corrupted signal, at location D, together on the same plot as above (i.e. the original signal and the noisy signal both appear on the same graph).

And create a scatter plot diagram of the original and the noise corrupted signal.

Calculate the covariance of the original and the noisy signal. How similar are they?

Calculate the correlation coefficient. How similar are they? Explain your results.

4 Probability Distribution of Combined Uniform Uncertainties

In this experiment, we want to investigate the probability distribution of combined uniformly distributed uncertainties, as demonstrated by finding the difference between two voltages with differing levels of uncertainty due to quantisation.

Suppose we measure two voltages V_1 and V_2 and need to calculate $V_2 - V_1$, and its uncertainty.

The two voltages（in volts）are measured on different ranges of a volt meter, and hence have different quantisation errors (assumed to be uniformly distributed).

$V_1 = 1.934 \pm 0.001$ $\qquad$ $V_2 = 2.53 \pm 0.01$

Figure 3 below shows how a uniform distribution looks like.

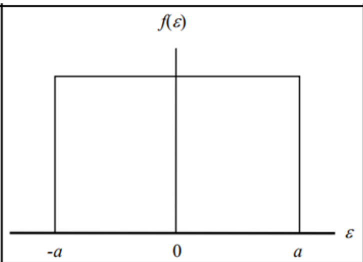
Uniform		$f(\varepsilon) = \begin{cases} \frac{1}{2a}, & -a \le \varepsilon \le a \\ 0, & \text{otherwise} \end{cases}$

Figure 3: Uniform distribution.

We are interested to find $V_2 - V_1$ and the uncertainty.

The difference of $V_2 - V_1$ is straight forward to calculate. However, there are 3 methods to calculate the uncertainty, as explained in the article [1] which is available on the course website; you are advised to read the article to get more detailed information, as well as to train yourself in reading research articles for your postgraduate study.

Method 1: By using calculation (this is known as the GUM method in the article)

Referring to the theories learnt in Lectures, calculate by hands $V_2 - V_1$ and the uncertainty.

Method 2: By using the theory of Convolution

According to the article [1], in theory if two or more uncertainties are statistically independent, then the distribution of the combination of uncertainties can be found by <u>convoluting</u> the distributions of the uncertainties [1]. Therefore, we would expect that the distribution of the voltage difference to be the convolution of the two error distributions, centred on the mean of the voltage difference (i.e. $V_2 - V_1$).

Using **conv ( )** in Matlab where appropriate, plot the probability distributions of V_1, V_2 and $V_2 - V_1$.

Note that because both V_1 and V_2 are both uniformly distributed, the result of a convolution of two uniform distributions will look something like Figure 4 below (see page 13 of article [1]).

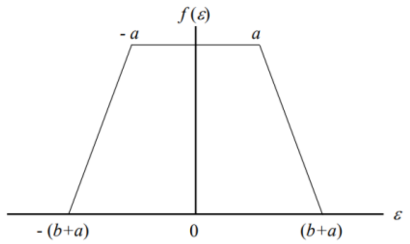


Figure 4: Convolved Distribution for Two Uniformly Distributed Errors, b > a.

Method 3 By using Monte-Carlo Experimental Simulation

Alternatively, we can also derive the above probability distributiuon of $V_2 - V_1$ by simulation; also known as the Monte-Carlo simulation.

First we generate (using Matlab) 100,000 samples of V_1 and V_2, then we derive $V_2 - V_1$ and plot a histrogram of it. This will give a distribution of the probability density of $V_2 - V_1$.

Compare the results of Method 2 and Method 3 (both your results should look somehting like Figure 5 below). Explain your results.

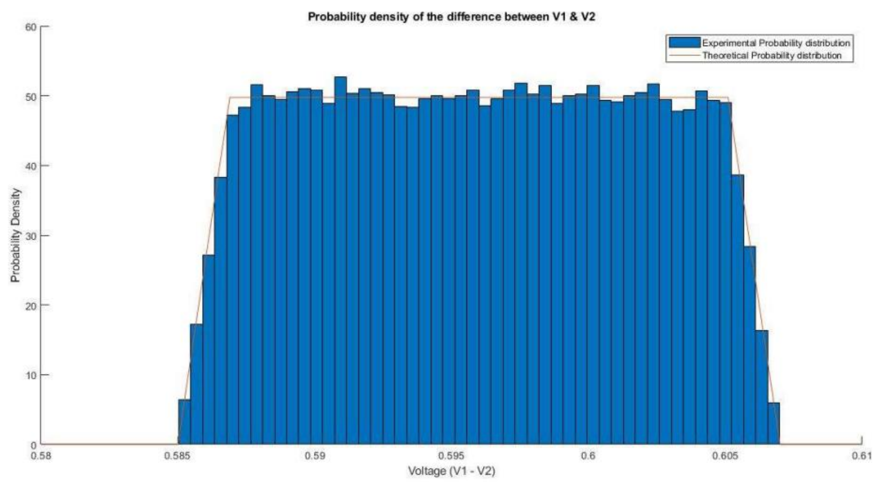


Figure 5: Expected plot of Method 2 (theoretical) and Method 3 (experimental).

[1] S. Castrup, “Comparison of Methods for Establishing Confidence Limits,” in Measurement Science Conference, Pasadena, CA, 2010.

5 Simulation of the Probability Distribution of Rolling 2 Dice

We have learnt in the Lectures that rolling a fair 6-sided die* constitutes a random event, where there is an equal probability of getting a 1, 2, 3, 4, 5 or 6; i.e. the probability distribution of rolling a fair die is represented by a discrete uniformly distributed probability mass function, as shown in Figure 6 below, which is obtained by simulating the rolling of a die randomly for 100,000 times in Matlab.

** "die" is the singular form of "dice", not a typo; it originated from the French word "des".*

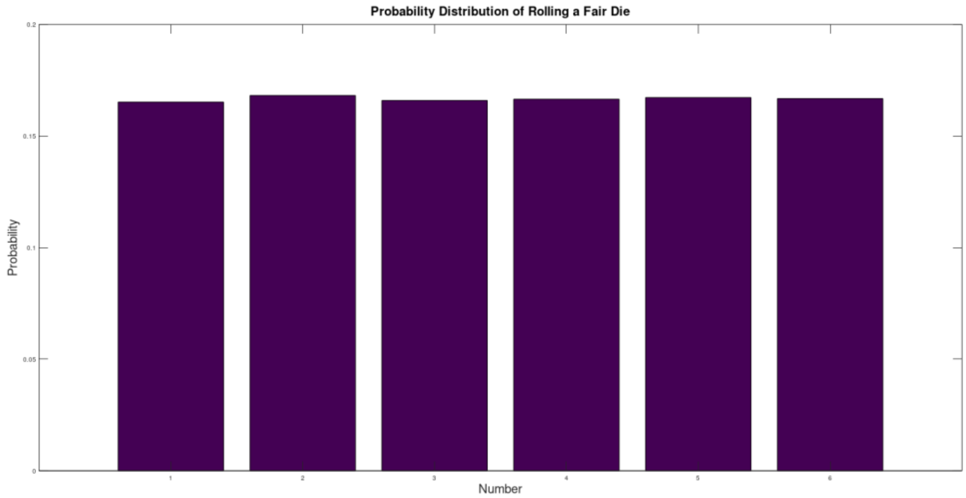


Figure 6: Probability distribution of rolling a fair die.

If we roll 2 fair dice (either simultaneously, or one after the other) and assuming that the outcomes are independent, we would expect to get a sum of the two numbers in between 2, 3, 4, … to 12. Will these numbers occur with equal probability? What is the resulting probability distribution of rolling 2 dice?

We will investigate this interesting issue.

Firstly, think about the scenario of rolling 2 dice in relation to the above questions. Analyse the problem, formulate a strategy and work out your conclusion. Report your analysis.

Secondly, to verify whether your analysis and conclusion above is correct, use Matlab to simulate rolling 2 dice and look at the sum of the 2 dice. Repeat the experiment for 100,000 times, and plot the resulting probability distribution. Explain your results.

~~ End of Lab 3 ~*~*