Tutorial 2: ECEN321 Engineering Statistics (answers on page 834)

Section 2.1 Basic probability idea

1. The probability that a bearing fails during the first month of use is 0.12. What is the probability that it does not fail during the first month?

3. A section of an exam contains four True-False questions. A completed exam paper is selected at random, and the four answers are recorded.

a. List all 16 outcomes in the sample space.

b. Assuming the outcomes to be equally likely, find the probability that all the answers are the same.

c. Assuming the outcomes to be equally likely, find the probability that exactly one of the four answers is "True."

d. Assuming the outcomes to be equally likely, find the probability that at most one of the four answers is "True."

5. Four candidates are to be interviewed for a job. Two of them, numbered 1 and 2, are qualified, and the other two, numbered 3 and 4, are not. The candidates are interviewed at random, and the first qualified candidate interviewed will be hired. The outcomes are the sequences of candidates that are interviewed. So one outcome is 2, and another is 431.

a. List all the possible outcomes.

b. Let A be the event that only one candidate is interviewed. List the outcomes in A.

c. Let B be the event that three candidates are interviewed. List the outcomes in B.

d. Let C be the event that candidate 3 is interviewed. List the outcomes in C.

e. Let D be the event that candidate 2 is not interviewed. List the outcomes in D.

f. Let E be the event that candidate 4 is interviewed. Are A and E mutually exclusive? How about B and E, C and E, D and E?

9. Among the cast aluminum parts manufactured on a certain day, 80% were flawless, 15% had only minor flaws, and 5% had major flaws. Find the probability that a randomly chosen part

a. has a flaw (major or minor).

b. has no major flaw.

13. Let S be the event that a randomly selected college student has taken a statistics course, and let C be the event that the same student has taken a chemistry course. Suppose $P(S) = 0.4$, $P(C) = 0.3$, and $P(S \cap C) = 0.2$.

a. Find the probability that a student has taken statistics, chemistry, or both.

b. Find the probability that a student has taken neither statistics nor chemistry.

c. Find the probability that a student has taken statistics but not chemistry.

15. All the fourth-graders in a certain elementary school took a standardized test. A total of 85% of the students were found to be proficient in reading, 78% were found to be proficient in mathematics, and 65% were found to be proficient in both reading and mathematics. A student is chosen at random.

a. What is the probability that the student is proficient in mathematics but not in reading?

b. What is the probability that the student is proficient in reading but not in mathematics?

c. What is the probability that the student is proficient in neither reading nor mathematics?

17. A system contains two components, A and B. The system will function only if both components function. The probability that A functions is 0.98, the probability that B functions is 0.95, and the probability that either A or B functions is 0.99. What is the probability that the system functions?

Section 2.2 Counting Methods

1. DNA molecules consist of chemically linked sequences of the bases adenine, guanine, cytosine, and thymine, denoted A, G, C, and T. A sequence of three bases is called a *codon*. A base may appear more than once in a codon.

a. How many different codons are there?

b. The bases A and G are *purines*, while C and T are *pyrimidines*. How many codons are there whose first and third bases are purines and whose second base is a pyrimidine?

c. How many codons consist of three different bases?

3. The article "Improved Bioequivalence Assessment of Topical Dermatological Drug Products Using Dermatopharmacokinetics" (B. N'Dri-Stempfer, W. Navidi, et al., *Pharmaceutical Research*, 2009:316–328) describes a study in which a new type of ointment was applied to forearms of volunteers to study the rates of absorption into the skin. Eight locations on the forearm were designated for ointment application. The new ointment was applied to four locations, and a control was applied to the other four. How many different choices were there for the four locations to apply the new ointment?

5. In horse racing, one can make a trifecta bet by specifying which horse will come in first, which will come in second, and which will come in third, in the correct order. One can make a box trifecta bet by specifying which three horses will come in first, second, and third, without specifying the order.

a. In an eight-horse field, how many different ways can one make a trifecta bet?

b. In an eight-horse field, how many different ways can one make a box trifecta bet?

7. A test consists of 15 questions. Ten are true-false questions, and five are multiple-choice questions that have four choices each. A student must select an answer for each question. In how many ways can this be done?

Section 2.3 Conditional Probability and Independence

3. A box contains 15 resistors. Ten of them are labeled 50 Ω and the other five are labeled 100 Ω.

a. What is the probability that the first resistor is 100 Ω?

b. What is the probability that the second resistor is 100 Ω, given that the first resistor is 50 Ω?

c. What is the probability that the second resistor is 100 Ω, given that the first resistor is 100 Ω?

7. Suppose that start-up companies in the area of biotechnology have probability 0.2 of becoming profitable, and that those in the area of information technology have probability 0.15 of becoming profitable. A venture capitalist invests in one firm of each type. Assume the companies function independently.

a. What is the probability that both companies become profitable?

b. What is the probability that neither company becomes profitable?

c. What is the probability that at least one of the two companies become profitable?

9. Of people in a certain city who bought a new vehicle in the past year, 12% of them bought a hybrid vehicle, and 5% of them bought a hybrid truck. Given that a person bought a hybrid vehicle, what is the probability that it was a truck?

13. A particular automatic sprinkler system has two different types of activation devices for each sprinkler head. One type has a reliability of 0.9; that is, the probability that it will activate the sprinkler when it should is 0.9. The other type, which operates independently of the first type, has a reliability of 0.8. If either device is triggered, the sprinkler will activate. Suppose a fire starts near a sprinkler head.

a. What is the probability that the sprinkler head will be activated?

b. What is the probability that the sprinkler head will not be activated?

c. What is the probability that both activation devices will work properly?

d. What is the probability that only the device with reliability 0.9 will work properly?

Section 2.4 Random Variables

3. A chemical supply company ships a certain solvent in 10-gallon drums. Let X represent the number of drums ordered by a randomly chosen customer. Assume X has the following probability mass function:

x	1	2	3	4	5
$p(x)$	0.4	0.2	0.2	0.1	0.1

a. Find the mean number of drums ordered.

b. Find the variance of the number of drums ordered.

c. Find the standard deviation of the number of drums ordered.

d. Let Y be the number of gallons ordered. Find the probability mass function of Y.

e. Find the mean number of gallons ordered.

5. A survey of cars on a certain stretch of highway during morning commute hours showed that 70% had only one occupant, 15% had 2, 10% had 3, 3% had 4, and 2% had 5. Let X represent the number of occupants in a randomly chosen car.

a. Find the probability mass function of X.

b. Find $P(X \leq 2)$.

c. Find $P(X > 3)$.

d. Find μ_X.

e. Find σ_X.

13. Resistors labeled 100 Ω have true resistances that are between 80 Ω and 120 Ω. Let X be the mass of a randomly chosen resistor. The probability density function of X is given by

$$f(x) = \begin{cases} \dfrac{x - 80}{800} & 80 < x < 120 \\ 0 & \text{otherwise} \end{cases}$$

a. What proportion of resistors have resistances less than 90 Ω?

b. Find the mean resistance.

c. Find the standard deviation of the resistances.

d. Find the cumulative distribution function of the resistances.

15. The lifetime in months of a transistor in a certain application is random with probability density function

$$f(t) = \begin{cases} 0.1e^{-0.1t} & t > 0 \\ 0 & t \le 0 \end{cases}$$

a. Find the mean lifetime.

b. Find the standard deviation of the lifetimes.

c. Find the cumulative distribution function of the lifetime.

d. Find the probability that the lifetime will be less than 12 months.

19. The level of impurity (in percent) in the product of a certain chemical process is a random variable with probability density function

$$\begin{cases} \frac{3}{64}x^2(4-x) & 0 < x < 4 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that the impurity level is greater than 3%?
b. What is the probability that the impurity level is between 2% and 3%?
c. Find the mean impurity level.
d. Find the variance of the impurity levels.
e. Find the cumulative distribution function of the impurity level.

21. The error in the length of a part (absolute value of the difference between the actual length and the target length), in mm, is a random variable with probability density function

$$f(x) = \begin{cases} 12(x^2 - x^3) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

a. What is the probability that the error is less than 0.2 mm?
b. Find the mean error.
c. Find the variance of the error.
d. Find the cumulative distribution function of the error.
e. The specification for the error is 0 to 0.8 mm. What is the probability that the specification is met?

Section 2.5 Linear Function of Random Variables

1. If X and Y are independent random variables with means $\mu_X = 9.5$ and $\mu_Y = 6.8$, and standard deviations $\sigma_X = 0.4$ and $\sigma_Y = 0.1$, find the means and standard deviations of the following:

a. $3X$

b. $Y - X$

c. $X + 4Y$

3. The lifetime of a certain transistor in a certain application has mean 900 hours and standard deviation 30 hours. Find the mean and standard deviation of the length of time that four transistors will last.

5. A laminated item is composed of five layers. The layers are a simple random sample from a population whose thickness has mean 1.2 mm and standard deviation 0.04 mm.

a. Find the mean thickness of an item.

b. Find the standard deviation of the thickness of an item.

7. The molarity of a solute in solution is defined to be the number of moles of solute per liter of solution ($1 \text{ mole} = 6.02 \times 10^{23}$ molecules). If X is the molarity of a solution of magnesium chloride ($MgCl_2$), and Y is the molarity of a solution of ferric chloride ($FeCl_3$), the molarity of chloride ion (Cl^-) in a solution made of equal parts of the solutions of $MgCl_2$ and $FeCl_3$ is given by $M = X + 1.5Y$. Assume that X has mean 0.125 and standard deviation 0.05, and that Y has mean 0.350 and standard deviation 0.10.

a. Find μ_M.

b. Assuming X and Y to be independent, find σ_M.

Section 2.6 Jointly Distributed Random Variables

1. In a certain community, levels of air pollution may exceed federal standards for ozone or for particulate matter on some days. For a particular summer season, let X be the number of days on which the ozone standard is exceeded and let Y be the number of days on which the particulate matter standard is exceeded. Assume that the joint probability mass function of X and Y is given in the following table:

	y		
x	**0**	**1**	**2**
0	0.10	0.11	0.05
1	0.17	0.23	0.08
2	0.06	0.14	0.06

a. Find $P(X = 1 \text{ and } Y = 0)$.

b. Find $P(X \geq 1 \text{ and } Y < 2)$.

c. Find $P(X < 1)$.

d. Find $P(Y \geq 1)$.

e. Find the probability that the standard for ozone is exceeded at least once.

f. Find the probability that the standard for particulate matter is never exceeded.

g. Find the probability that neither standard is ever exceeded.

3. Refer to Exercise 1.

a. Find the conditional probability mass function $p_{Y|X}(y\,|0)$.

b. Find the conditional probability mass function $p_{X|Y}(x\,|1)$.

c. Find the conditional expectation $E(Y \mid X = 0)$.

d. Find the conditional expectation $E(X \mid Y = 1)$.

23. An investor has \$100 to invest, and two investments between which to divide it. If she invests the entire amount in the first investment, her return will be X, while if she invests the entire amount in the second investment, her return will be Y. Both X and Y have mean \$6 and standard deviation (risk) \$3. The correlation between X and Y is 0.3.

a. Express the return in terms of X and Y if she invests \$30 in the first investment and \$70 in the second.

b. Find the mean return and the risk if she invests \$30 in the first investment and \$70 in the second.

c. Find the mean return and the risk, in terms of K, if she invests \$$K$ in the first investment and \$$(100 - K)$ in the second.

d. Find the value of K that minimizes the risk in part (c).

e. Prove that the value of K that minimizes the risk in part (c) is the same for any correlation $\rho_{X,Y}$ with $\rho_{X,Y} \neq 1$.

25. Let R denote the resistance of a resistor that is selected at random from a population of resistors that are labeled 100 Ω. The true population mean resistance is $\mu_R = 100$ Ω, and the population standard deviation is $\sigma_R = 2$ Ω. The resistance is measured twice with an ohmmeter. Let M_1 and M_2 denote the measured values. Then $M_1 = R + E_1$ and $M_2 = R + E_2$, where E_1 and E_2 are the errors in the measurements. Suppose that E_1 and E_2 are random with $\mu_{E_1} = \mu_{E_2} = 0$ and $\sigma_{E_1} = \sigma_{E_2} = 1$ Ω. Further suppose that E_1, E_2, and R are independent.

a. Find σ_{M_1} and σ_{M_2}.

b. Show that $\mu_{M_1 M_2} = \mu_{R^2}$.

c. Show that $\mu_{M_1}\mu_{M_2} = \mu_R^2$.

d. Use the results of (b) and (c) to show that $\mathrm{Cov}(M_1, M_2) = \sigma_R^2$.

e. Find ρ_{M_1, M_2}.