Tutorial 3: ECEN321 Engineering Statistics

Section 3.1 Measurement Error

1. The boiling point of water is measured four times. The results are 110.01°C, 110.02°C, 109.99°C, and 110.01°C. Which of the following statements best describes this measuring process?

i. Accurate but not precise
ii. Precise but not accurate
iii. Neither accurate nor precise
iv. Both accurate and precise

3. The weight of an object is given as 67.2 ± 0.3 g. True or false:

a. The weight was measured to be 67.2 g.
b. The true weight of the object is 67.2 g.
c. The bias in the measurement is 0.3 g.
d. The uncertainty in the measurement is 0.3 g.

5. A person stands on a bathroom scale. The reading is 150 lb. After the person gets off the scale, the reading is 2 lb.

a. Is it possible to estimate the uncertainty in this measurement? If so, estimate it. If not, explain why not.

b. Is it possible to estimate the bias in this measurement? If so, estimate it. If not, explain why not.

11. The length of a rod was measured eight times. The measurements in centimeters, in the order they were taken, were 21.20, 21.22, 21.25, 21.26, 21.28, 21.30, 21.32, 21.35.

a. Do these measurements appear to be a random sample from a population of possible measurements? Why or why not?

b. Is it possible to estimate the uncertainty in these measurements? Explain.

Section 3.2 Linear Combinations of Measurements

1. Assume that X and Y are independent measurements with uncertainties $\sigma_X = 0.3$ and $\sigma_Y = 0.2$. Find the uncertainties in the following quantities:

a. $4X$

b. $X + 2Y$

c. $2X - 3Y$

7. A force of $F = 2.2 \pm 0.1$ N is applied to a block for a period of time, during which the block moves a distance $d = 3$ m, which is measured with negligible uncertainty. The work W is given by $W = Fd$. Estimate W, and find the uncertainty in the estimate.

9. The specific gravity of a substance is given by $G = D_S/D_W$, where D_S is the density of the substance in kg/m^3 and D_W is the density of water, which is known to be 1000 kg/m^3. The density of a particular substance is measured to be $D_S = 500 \pm 5$ kg/m^3. Estimate the specific gravity, and find the uncertainty in the estimate.

11. According to Newton's law of cooling, the temperature T of a body at time t is given by $T = T_a + (T_0 - T_a)e^{-kt}$, where T_a is the ambient temperature, T_0 is the initial temperature, and k is the cooling rate constant. For a certain type of beverage container, the value of k is known to be 0.025 min^{-1}.

a. Assume that $T_a = 36°$F exactly and that $T_0 = 72.0 \pm 0.5°$F. Estimate the temperature T at time $t = 10$ min, and find the uncertainty in the estimate.

b. Assume that $T_0 = 72°$F exactly and that $T_a = 36.0 \pm 0.5°$F. Estimate the temperature T at time $t = 10$ min, and find the uncertainty in the estimate.

Section 3.3 Uncertainties for Functions of One Measurement

1. Find the uncertainty in Y, given that $X = 2.0 \pm 0.3$ and

a. $Y = X^3$

b. $Y = \sqrt{2X}$

c. $Y = 3/X$

d. $Y = \ln X$

e. $Y = e^X$

f. $Y = \cos X$ (X is in units of radians)

3. The volume of a cone is given by $V = \pi r^2 h/3$, where r is the radius of the base and h is the height. Assume the height is 6 cm, measured with negligible uncertainty, and the radius is $r = 5.00 \pm 0.02$ cm. Estimate the volume of the cone, and find the uncertainty in the estimate.

9. The density of a rock will be measured by placing it into a graduated cylinder partially filled with water, and then measuring the volume of water displaced. The density D is given by $D = m/(V_1 - V_0)$, where m is the mass of the rock, V_0 is the initial volume of water, and V_1 is the volume of water plus rock. Assume the mass of the rock is 750 g, with negligible uncertainty, and that $V_0 = 500.0 \pm 0.1$ mL and $V_1 = 813.2 \pm 0.1$ mL. Estimate the density of the rock, and find the uncertainty in the estimate.

13. The acceleration g due to gravity is estimated by dropping an object and measuring the time it takes to travel a certain distance. Assume the distance s is known to be exactly 2.2 m. The time is measured to be $t = 0.67 \pm 0.02$ s. Estimate g, and find the relative uncertainty in the estimate. (Note that $g = 2s/t^2$.)

Section 3.4 Uncertainties for Functions of Several Measurements

1. Find the uncertainty in U, assuming that $X = 10.0 \pm 0.5$, $Y = 5.0 \pm 0.1$, and

a. $U = XY^2$

b. $U = X^2 + Y^2$

c. $U = (X + Y^2)/2$

3. From a fixed point on the ground, the distance to a certain tree is measured to be $s = 55.2 \pm 0.1$ m and the angle from the point to the top of the tree is measured to be $\theta = 0.50 \pm 0.02$ radians. The height of the tree is given by $h = s \tan \theta$.

a. Estimate h, and find the uncertainty in the estimate.

b. Which would provide a greater reduction in the uncertainty in h: reducing the uncertainty in s to 0.05 m or reducing the uncertainty in θ to 0.01 radians?

5. When air enters a compressor at pressure P_1 and leaves at pressure P_2, the intermediate pressure is given by $P_3 = \sqrt{P_1 P_2}$. Assume that $P_1 = 10.1 \pm 0.3$ MPa and $P_2 = 20.1 \pm 0.4$ MPa.

a. Estimate P_3, and find the uncertainty in the estimate.

b. Which would provide a greater reduction in the uncertainty in P_3: reducing the uncertainty in P_1 to 0.2 MPa or reducing the uncertainty in P_2 to 0.2 MPa?

7. The lens equation says that if an object is placed at a distance p from a lens, and an image is formed at a distance q from the lens, then the focal length f satisfies the equation $1/f = 1/p + 1/q$. Assume that $p = 2.3 \pm 0.2$ cm and $q = 3.1 \pm 0.2$ cm.

a. Estimate f, and find the uncertainty in the estimate.

b. Which would provide a greater reduction in the uncertainty in f: reducing the uncertainty in p to 0.1 cm or reducing the uncertainty in q to 0.1 cm?