Online Supplementary Materials of the Paper "Simplified Bacterial Foraging Optimization With Quorum Sensing for Global Optimization"

These supplementary materials are used for the paper "Simplified Bacterial Foraging Optimization With Quorum Sensing for Global Optimization".

Table I describes some basic properties of 30 test functions used in the experiments. The experimental results on 6 unimodal and 24 multimodal functions are summarized in Table II, and Table III, respectively. In both Tables II and III, the mean (ME) and standard deviation (SD) of the final results obtained by all algorithms investigated in the paper are listed. For each function, the best result(s) is marked in bold font.

Fig. 10 provides the graphical comparisons of execution time (averaged over 30 trails) between BFO and SBFO on 30 test functions. Fig. 11 depicts the convergence curves of these algorithms with increasing FEs on unimodal functions, where the Y-coordinate is the average best cost obtained over 30 runs. Fig. 12 shows the convergence procedures of these algorithms with increasing FEs on multimodal functions.

Please see the next pages for more details.

TABLE I
DESCRIPTIONS OF 30 BENCHMARK FUNCTIONS SELECTED IN THE EXPERIMENTS

	Function name	Mathematical formula (the dimension $n = 50$ in the experiments)	Domain	Minimum
f_1	Sphere [36]	$f_1 = \sum_{i=1}^{n} x_i^2$	(-100,100)	0
f_2	Quartic [54]	$f_2 = \sum_{i=1}^{n} i x_i^2$	(-50,50)	0
f_3	Schwefel2.21 [54]	$f_3 = \max(\|x_i\|)$	(-10000,10000)	0
f_4	Step [54]	$f_4 = \sum_{i=1}^{n} (\lfloor x_i + 0.5 \rfloor)^2$	(-100,100)	0
f_5	Schwefel2.22 [54]	$f_5 = \sum_{i=1}^{n} \|x_i\| + \prod_{i=1}^{n} \|x_i\|$	(-50,50)	0
f_6	Shifted Sphere [56]	$f_6 = \sum_{i=1}^{n} z_i^2 + f^*,\ z = x - o$	(-100,100)	-1400
f_7	Rastrigin [54]	$f_7 = \sum_{i=1}^{n} (x_i^2 - 10\cos(2\pi x_i) + 10)$	(-5.12,5.12)	0
f_8	Griewank [54]	$f_8 = \sum_{i=1}^{n} x_i^2/4000 - \prod_{i=1}^{n} \cos(x_i/\sqrt{i}) + 1$	(-600,600)	0
f_9	Ackley [54]	$f_9 = -20\exp\left(-0.2\sqrt{\sum_{i=1}^{n} x_i^2/n}\right) - \exp(\sum_{i=1}^{n} \cos(2\pi x_i)/n) + 20 + \mathrm{e}$	(-32.768,32.768)	0
f_{10}	Michalewicz10 [81]	$f_{10} = -\sum_{i=1}^{n} \sin(x_i)\sin^{10}(i x_i^2/\pi)$	$(-10\pi, 10\pi)$	-50
f_{11}	Michalewicz20 [81]	$f_{11} = -\sum_{i=1}^{n} \sin(x_i)\sin^{20}(i x_i^2/\pi)$	$(-10\pi, 10\pi)$	-50
f_{12}	Levy [81]	$f_{12} = -\sum_{i=1}^{n-1} (x_i - 1)^2\left(1 + \sin^2(3\pi x_{i+1})\right) + \sin^2(3\pi x_1) + \|x_n - 1\|(1 + \sin^2(3\pi x_n))$	(-100,100)	0
f_{13}	Penalized1 [54]	$f_{13} = \pi/n(10\sin^2(\pi y_1) + \sum_{i=1}^{n-1} (y_1 - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_n - 1)^2) + \sum_{i=1}^{n} u_i$	(-50,50)	0
f_{14}	Alpine [81]	$f_{14} = \sum_{i=1}^{n} \|x_i \sin(x_i) + 0.1 x_i\|$	(-10,10)	0
f_{15}	NCRastrigin [81]	$f_{15} = \sum_{i=1}^{n} (y_i^2 - 10\cos(2\pi y_i) + 10)$	(-5.12,5.12)	0
f_{16}	Weierstrass [81]	$f_{16} = \sum_{i=1}^{n} (\sum_{k=0}^{m} a^k \cos(2\pi b^k (x_i + 0.5))) - n\sum_{k=0}^{m} a^k \cos(2\pi b^k (x_i + 0.5))$	(-0.5,0.5)	0
f_{17}	Himmelblau [81]	$f_{17} = \sum_{i=1}^{n} ({x_i}^4 - 16{x_i}^2 + 5x_i)/\,n$	(-10,10)	-78.33
f_{18}	Rotated Ackley [56]	f_{18} corresponds to the rotated multimodal function 8 in [56]	(-100,100)	-700
f_{19}	Rotated Weierstrass [56]	f_{19} corresponds to the rotated multimodal function 9 in [56]	(-100,100)	-600
f_{20}	Rotated Rastrigin [56]	$f_{20} = \sum_{i=1}^{n} (z_i^2 - 10\cos(2\pi z_i) + 10) + f^*,\ z = x - o$	(-100,100)	-300
f_{21}	Rotated NCRastrigin [56]	f_{21} corresponds to the multimodal function 13 in [56]	(-100,100)	-200
f_{22}	Schwefel [56]	$f_{22} = 418.9829 * \mathrm{n} - \sum_{i=1}^{n} g(z_i) + f^*,\ z = x - o$	(-100,100)	-100
f_{23}	Rotated Katsuura [56]	f_{23} corresponds to the multimodal function 16 in [56]	(-100,100)	200
f_{24}	Bi_rastrigin [56]	f_{24} corresponds to the multimodal function 17 in [56]	(-100,100)	300
f_{25}	Rotated Bi_rastrigin [56]	f_{25} corresponds to the rotated multimodal function 18 in [56]	(-100,100)	400
f_{26}	Expanded Scaffer [56]	f_{26} corresponds to the rotated multimodal function 20 in [56]	(-100,100)	600
f_{27}	Composition [56]	f_{27} corresponds to the unrotated composition function 22 in [56]	(-100,100)	800
f_{28}	Composition [56]	f_{28} corresponds to the rotated composition function 23 in [56]	(-100,100)	900
f_{29}	Composition [56]	f_{29} corresponds to the rotated composition function 24 in [56]	(-100,100)	1000
f_{30}	Composition [56]	f_{30} corresponds to the rotated composition function 27 in [56]	(-100,100)	1300

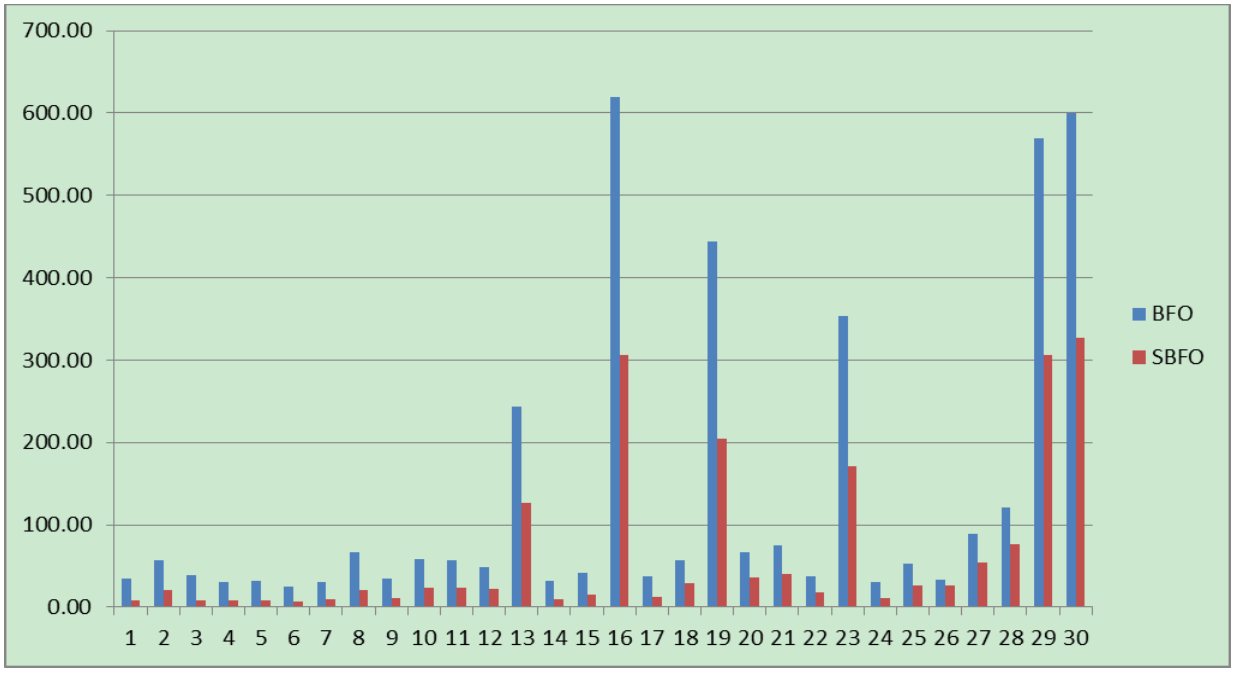
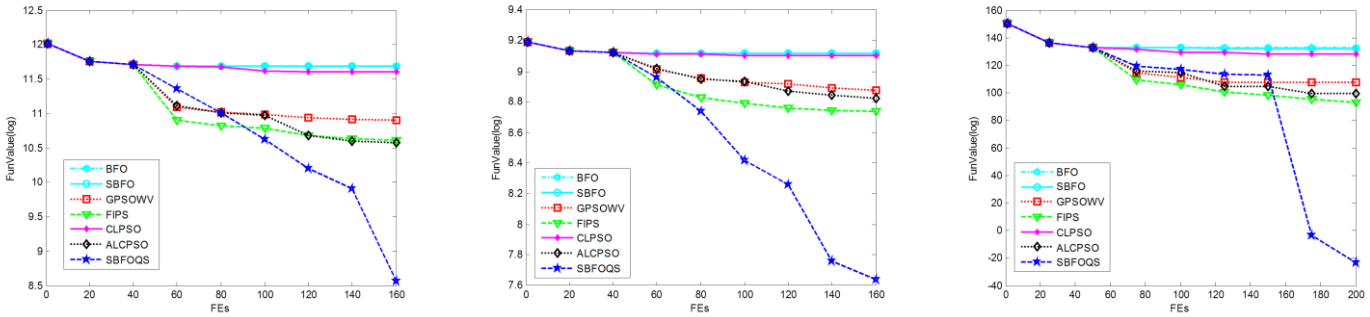


Fig. 10 Comparisons of execution time on 30 test functions. X-axis is the function number varying from 1 to 30 while Y-axis indicates the average CPU time (Seconds) spent on each function over 30 independent runs.

TABLE II
COMPARISON RESULTS ON SIX 50-DIMENSIONAL UNIMODAL FUNCTIONS

		SBFOQS	GPSOWV	FIPS	CLPSO	ALCPSO	BFO	SBFO
f_1	ME	**0.00E+00**	2.03E-32	6.39E-12	8.50E-19	1.18E-110	1.20E+03	9.14E+01
	SD	**0.00E+00**	5.29E-32	2.37E-12	3.54E-19	4.59E-110	5.43E+02	1.52E+01
	SR	**30**	0	0	0	**30**	0	0
	TE	---	1	1	1	0	1	1
f_2	ME	**0.00E+00**	9.31E-34	3.33E-15	2.64E-28	8.67E-130	1.22E+03	1.57E+04
	SD	**0.00E+00**	2.41E-33	1.78E-15	1.85E-28	2.09E-129	2.36E+02	3.99E+03
	SR	**30**	0	0	0	**30**	0	0
	TE	---	1	1	1	1	1	1
f_3	ME	**0.00E+00**	7.38E+02	3.33E+01	3.66E+02	2.81E+02	8.81E+03	7.54E+03
	SD	**0.00E+00**	1.48E+02	4.95E+00	4.62E+01	1.12E+02	1.64E+02	3.21E+02
	SR	**30**	0	0	0	0	0	0
	TE	---	1	1	1	1	1	1
f_4	ME	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.33E-01	4.66E+04	2.88E+02
	SD	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	5.04E-01	5.95E+03	4.80E+01
	SR	**30**	**30**	**30**	**30**	24	0	0
	TE	---	0	0	0	0	1	1
f_5	ME	**0.00E+00**	2.19E-16	3.94E-07	1.47E-10	5.24E-05	3.88E+36	1.03E+37
	SD	**0.00E+00**	1.18E-15	1.15E-07	4.73E-11	2.85E-04	1.83E+37	3.69E+37
	SR	**30**	**30**	0	**30**	28	0	0
	TE	---	0	1	1	0	1	1
f_6	ME	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**	**-1.40E+03**	-1.17E+03	-1.30E+03
	SD	1.47E+00	**0.00E+00**	**0.00E+00**	**0.00E+00**	**0.00E+00**	2.03E+01	1.38E+01
	SR	25	**30**	**30**	**30**	**30**	0	0
	TE	---	0	0	0	0	1	1

ME: mean value of 30 runs, SD: standard deviation of 30 runs, SR: success rates in 30 runs,
TE: indicator of the t-test between SBFOQS and the algorithm in the corresponding column.
(Note that TE =1 or 0 indicates the significant and no significant difference, respectively.)

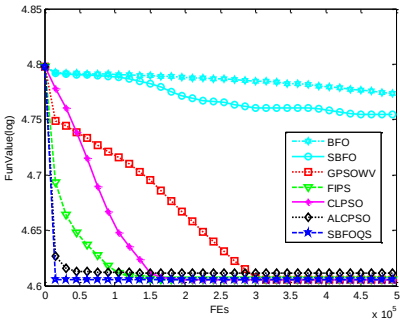


(a) f_1 (b) f_3 (c) f_5

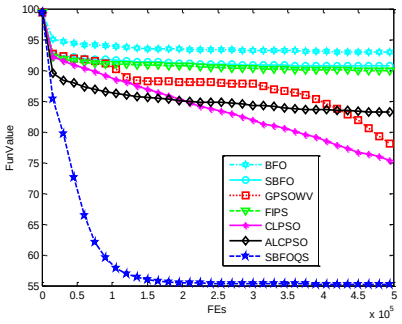
Fig. 11 Comparisons of convergence rate on 50-dimensional unimodal functions.

TABLE III
COMPARISON RESULTS ON TWENTY-FOUR 50-DIMENSIONAL MULTIMODAL FUNCTIONS

		SBFOQS	GPSOWV	FIPS	CLPSO	ALCPSO	BFO	SBFO
f_7	ME	**0.00E+00**	5.87E+01	1.49E+02	2.33E-04	8.66E+01	3.21E+02	6.19E+02
	SD	0.00E+00	1.22E+01	2.23E+01	1.55E-04	1.53E+01	2.07E+01	4.26E+01
	TE	---	1	1	1	1	1	1
f_8	ME	**0.00E+00**	9.10E-03	2.40E-08	2.85E-14	1.09E-02	7.57E+02	1.68E+01
	SD	0.00E+00	1.04E-02	8.21E-08	6.83E-14	1.75E-02	5.78E+01	1.50E+01
	TE	---	1	0	1	1	1	1
f_9	ME	**2.19E-15**	2.56E-14	4.66E-07	2.80E-10	6.33E-01	1.83E+01	1.62E+01
	SD	1.74E-15	1.32E-14	8.96E-08	7.37E-11	7.38E-01	4.71E-01	5.53E+00
	TE	---	1	1	1	1	1	1
f_{10}	ME	**-4.52E+01**	-3.08E+01	-1.21E+01	-2.78E+01	-2.59E+01	-8.66E+00	-1.07E+01
	SD	9.91E-01	5.88E+00	4.25E-01	8.16E-01	6.57E+00	5.12E-01	5.74E-01
	TE	---	1	1	1	1	1	1
f_{11}	ME	**-4.48E+01**	-2.23E+01	-1.01E+01	-2.47E+01	-1.68E+01	-7.06E+00	-9.30E+00
	SD	1.14E+00	6.07E+00	6.92E-01	9.28E-01	2.70E+00	5.92E-01	5.34E-01
	TE	---	1	1	1	1	1	1
f_{12}	ME	3.72E-02	3.30E-02	2.58E-11	**2.34E-17**	8.95E-02	1.18E+05	1.09E+05
	SD	6.04E-02	5.12E-02	1.01E-11	8.46E-18	3.48E-01	8.70E+03	2.11E+04
	TE	---	0	1	1	0	1	1
f_{13}	ME	7.79E-03	1.04E-02	4.99E-14	**9.54E-19**	5.66E+00	6.62E+00	1.19E+01
	SD	1.31E-02	2.36E-02	2.52E-14	4.43E-19	1.89E+01	1.34E+00	1.93E+00
	TE	---	0	1	1	0	1	1
f_{14}	ME	**0.00E+00**	7.73E-14	1.74E-02	1.36E-03	1.13E-06	2.45E+01	5.59E+01
	SD	0.00E+00	2.35E-13	3.33E-03	3.21E-04	6.17E-06	1.71E+00	6.31E+00
	TE	---	0	1	1	0	1	1
f_{15}	ME	**0.00E+00**	3.72E+01	1.42E+02	3.54E-02	5.41E+01	3.59E+02	5.76E+02
	SD	0.00E+00	1.39E+01	2.02E+01	1.82E-01	2.45E+01	3.52E+01	4.23E+01
	TE	---	1	1	0	1	1	1
f_{16}	ME	**2.28E-08**	1.01E+00	8.64E-03	1.94E-01	6.09E+00	6.73E+01	7.11E+01
	SD	1.25E-07	9.08E-01	2.09E-02	9.26E-02	3.24E+00	2.07E+00	9.96E-01
	TE	---	1	1	1	1	1	1
f_{17}	ME	-7.70E+01	-7.13E+01	**-7.83E+01**	**-7.83E+01**	-6.82E+01	-6.60E+01	-6.06E+01
	SD	8.64E-01	1.71E+00	5.28E-12	0.00E+00	2.19E+00	1.48E+00	1.27E+00
	TE	---	1	1	1	1	1	1
f_{18}	ME	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**	**-6.79E+02**
	SD	4.11E-02	4.16E-02	3.73E-02	3.72E-02	3.79E-02	3.42E-02	3.53E-02
	TE	---	0	0	0	0	1	0
f_{19}	ME	**-5.62E+02**	-5.59E+02	-5.51E+02	-5.46E+02	-5.51E+02	-5.49E+02	-5.48E+02
	SD	7.13E+00	5.13E+00	4.79E+00	1.95E+00	4.22E+00	2.03E+00	4.98E+00
	TE	---	0	1	1	1	1	1
f_{20}	ME	**-4.40E+01**	-4.24E+01	6.11E+01	-1.01E+01	-4.15E+01	3.08E+02	2.25E+02
	SD	4.32E+01	1.08E+02	1.67E+01	2.71E+01	6.63E+01	5.67E+01	6.28E+01
	TE	---	0	1	1	0	1	1
f_{21}	ME	1.84E+02	1.83E+02	1.65E+02	**1.43E+02**	2.24E+02	9.18E+02	8.42E+02
	SD	3.65E+01	6.10E+01	1.57E+01	2.47E+01	6.22E+01	1.37E+02	1.46E+02
	TE	---	0	1	1	1	1	1
f_{22}	ME	**7.26E+02**	1.76E+03	9.64E+03	1.17E+03	2.78E+03	1.01E+04	8.84E+03
	SD	1.99E+02	4.11E+02	6.50E+02	2.46E+02	6.28E+02	4.93E+02	6.18E+02
	TE	---	1	1	1	1	1	1
f_{23}	ME	**2.02E+02**	2.11E+02	2.03E+02	2.03E+02	2.03E+02	2.03E+02	2.11E+02
	SD	3.62E-01	4.91E-01	3.38E-01	3.19E-01	4.30E-01	4.26E-01	2.49E-01
	TE	---	1	1	1	1	1	1
f_{24}	ME	4.80E+02	4.33E+02	6.66E+02	**3.85E+02**	4.80E+02	2.16E+03	1.50E+03
	SD	3.56E+01	1.85E+01	1.90E+01	3.88E+00	2.67E+01	1.59E+02	1.46E+02
	TE	---	1	1	1	0	1	1
f_{25}	ME	**7.21E+02**	8.82E+02	8.15E+02	8.10E+02	8.56E+02	2.29E+03	1.60E+03
	SD	4.39E+01	3.23E+01	1.63E+01	1.96E+01	7.18E+01	1.38E+02	1.33E+02
	TE	---	1	1	1	1	1	1
f_{26}	ME	**6.20E+02**	6.23E+02	6.21E+02	6.23E+02	6.22E+02	6.25E+02	6.25E+02
	SD	6.59E-01	4.49E-01	2.99E-01	3.42E-01	9.46E-01	8.96E-03	2.82E-02
	TE	---	1	1	1	1	1	1
f_{27}	ME	**1.93E+03**	2.92E+03	1.02E+04	3.15E+03	4.11E+03	1.11E+04	1.21E+04
	SD	4.62E+02	4.54E+02	7.99E+02	4.63E+02	7.14E+02	4.54E+02	8.19E+02
	TE	---	1	1	1	1	1	1
f_{28}	ME	1.18E+04	1.39E+04	1.52E+04	1.39E+04	**1.09E+04**	1.23E+04	1.21E+04
	SD	9.88E+02	1.26E+03	3.90E+02	5.08E+02	1.43E+03	6.21E+02	7.60E+02
	TE	---	1	1	1	1	1	1
f_{29}	ME	**1.30E+03**	1.31E+03	1.33E+03	1.34E+03	1.35E+03	1.36E+03	1.36E+03
	SD	1.11E+01	1.10E+01	9.58E+00	6.23E+00	1.72E+01	5.96E+00	1.24E+01
	TE	---	0	1	1	1	1	1
f_{30}	ME	**2.58E+03**	2.66E+03	2.91E+03	3.02E+03	2.92E+03	3.08E+03	3.13E+03
	SD	1.42E+02	1.20E+02	1.01E+02	5.81E+01	1.39E+02	6.80E+01	1.16E+02
	TE	---	1	1	1	1	1	1



(1) f_9



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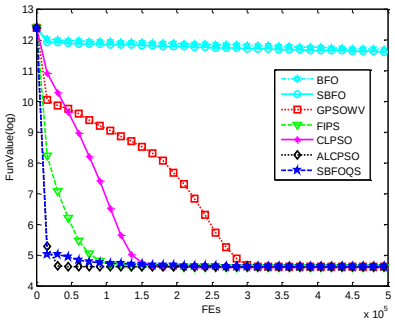

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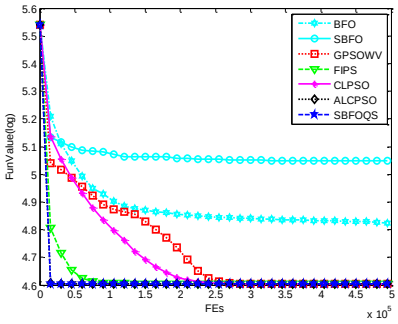
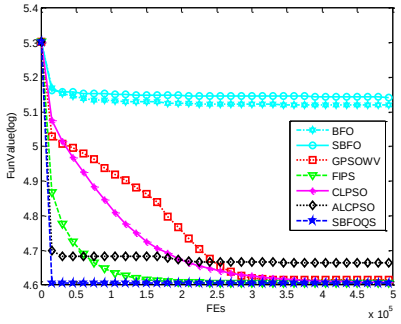


(4) f_{14}



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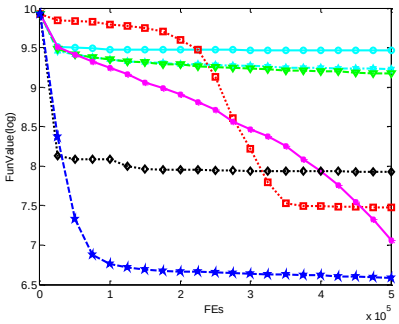

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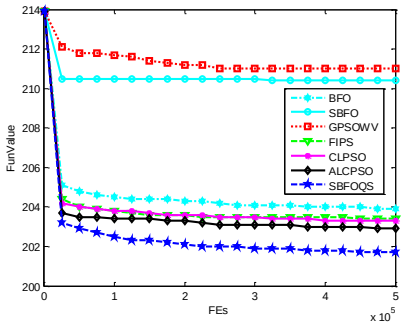

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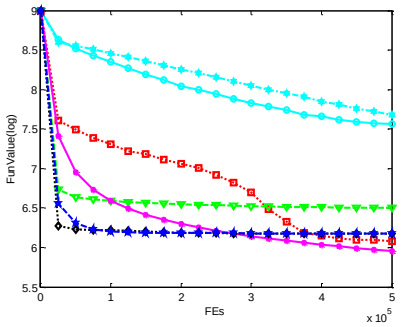

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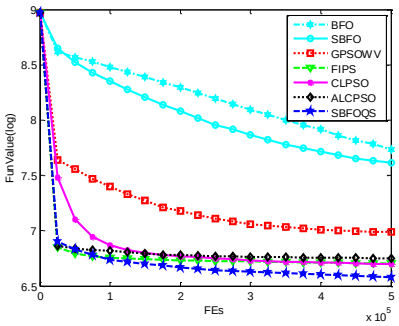

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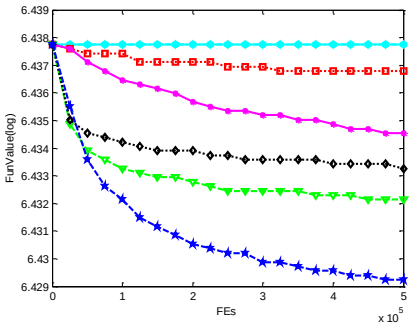

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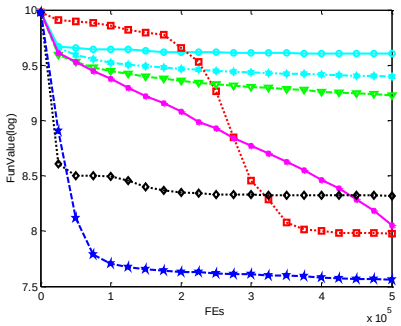
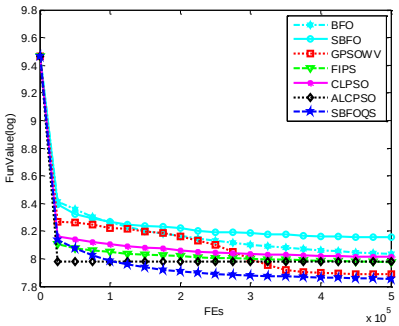


(11) f_{27}



(12) f_{30}

Fig. 12 Comparisons of convergence rate on 50-dimensional multimodal functions.