“Multi-objective Differential Evolution for Feature Selection in Classification” *Online Supplementary Materials*

Peng Wang, *Student Member, IEEE*, Bing Xue, *Senior Member, IEEE*, Jing Liang, *Senior Member, IEEE*, and Mengjie Zhang, *Fellow, IEEE*

I. Introduction

This is the Online Supplementary Materials of Multi-objective Differential Evolution for Feature Selection in Classification.

Tables I and II show the average test classification accuracy and the average number of selected features of the solution with the best training accuracy under the results of 30 runs. Table III presents the average PV and AN results of Omni-optimizer, DN-NSGAII, MO_Ring_PSO, and MOCDE. Table IV gives some examples of different feature subsets with the same objective values on the Multiple, Arrhythmia, SRBCT, Leukemia, and DLBCL datasets.

To readers' convenience, an example of the representation of individuals is given in Fig. 1. Meanwhile, Fig. 2 presents the performance of the proposed initialization method (PIM) on two multi-objective optimization methods, i.e., NSGA-II and SPEA2.

To illustrate the effect of the frequency of grouping m on the algorithm performance, the average results of I_H and I_{IGD} according to the different values of m are shown in Fig. S.3. Five situations under four datasets (Sonar, Musk1, SRBCT and Leukemia datasets) where $m = 1, 5, 10, 15, 20$ (scale value in horizontal axes) are explored, respectively. To obtain the I_{IGD} results on the test sets, the true PF is still estimated by the non-dominated solutions obtained from the eight algorithms with 30 independent runs.

II. The Representation of Individuals

In Fig. 1, suppose that a dataset includes n features, F_1 to F_n, and the individuals S_1 to S_P form a population with size P. Each individual in DE is a n-dimensional vector, and one dimension corresponds to one feature. Each element in an individual is a real number between 0 and 1. If a value is not less than a predefined threshold θ, e.g., $\theta = 0.6$, its corresponding feature will be selected. Otherwise, it will not be selected. For example, the first feature F_1 is not selected in S_1, but selected in S_2.

Manuscript received March 26, 2021; revised September 19, 2021; accepted November 2, 2021. (*Corresponding author:Jing Liang*)

M. Zhang, B. Xue and P. Wang are with the Evolutionary Computation Research Group, Victoria University of Wellington, Wellington 6140, New Zealand.

J. Liang is with the School of Electrical Engineering, Zhengzhou University, Zhengzhou 450001, China.

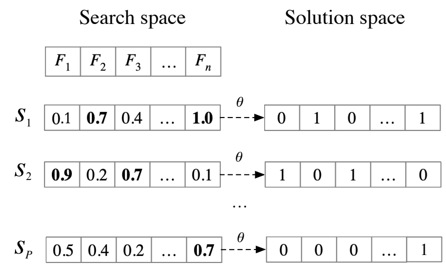


Fig. 1. An example of the encoding scheme for the individuals in different spaces.

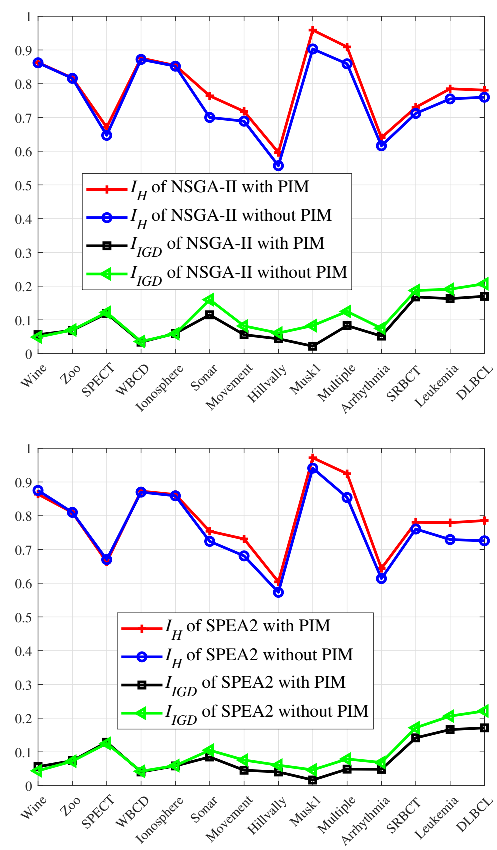
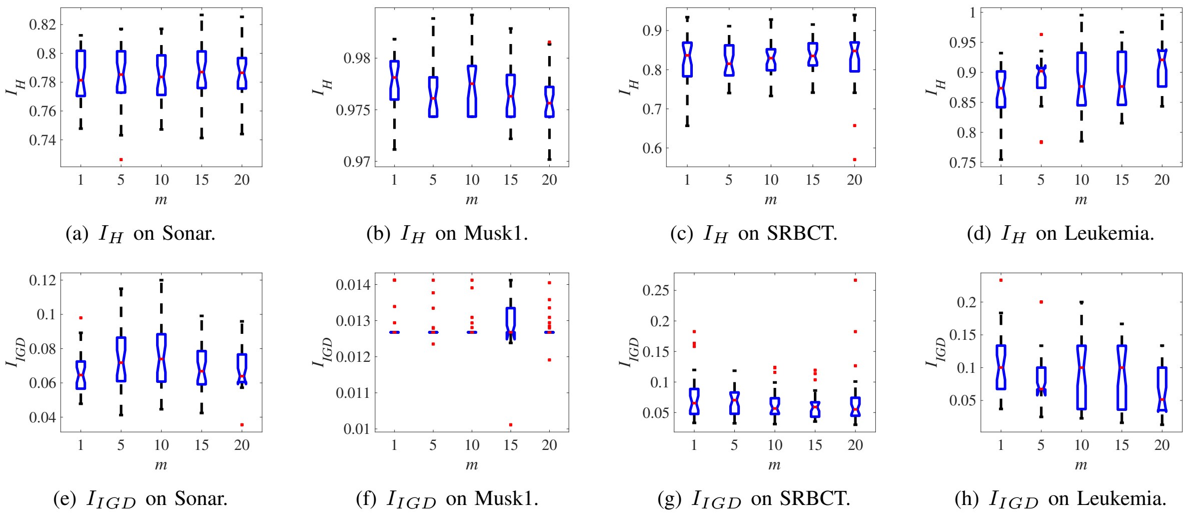


Fig. 2. The I_H and I_{IGD} of NSGA-II and SPEA2 with and without the proposed PIM initialization method.

III. Analysis on the Initialization and Clustering Interval

A. *Performance of the proposed initialization method*

In Fig. 2, NSGA-II or SPEA2 without PIM means NSGA-II or SPEA2 uses the traditional random initialization method,



(a) I_H on Sonar. (b) I_H on Musk1. (c) I_H on SRBCT. (d) I_H on Leukemia.

(e) I_{IGD} on Sonar. (f) I_{IGD} on Musk1. (g) I_{IGD} on SRBCT. (h) I_{IGD} on Leukemia.

Fig. 3. The boxplots of two indicators on the test sets based on the different values of m.

where all features have the same probability to be selected in the initialization step. Noted that the smaller I_{IGD} values mean the better algorithm performance, which is opposite for I_H. It can be observed that NSGA-II with PIM achieves larger I_H values and smaller I_{IGD} values on almost all the 14 datasets compared with NSGA-II with the traditional initialization method.

As for SPEA2, by using the proposed PIM initialization method, SPEA2 obtains a higher I_H value than the traditional initialization method apart from the Wine dataset. The superiority is particularly obvious on the last nine datasets. A similar trend can be seen on the I_{IGD} results.

The results indicate that on almost all the 14 datasets, applying the MIC-based initialization method can help EMO-based feature selection algorithms to achieve better classification performance.

B. Analysis on the effect of the clustering interval m

In Fig. 3, the average I_H and I_{IGD} on the test sets have little difference when the value of m of a dataset is different. The average difference does not exceed 0.1 for both I_H and I_{IGD}. It stays almost unchanged, e.g., the average I_{IGD} on the Musk1 dataset and the average I_H on the Sonar dataset. But there are still some fluctuations, which are more obvious when the number of features is large. For example, for the Leukemia dataset, the largest value of average I_{IGD} is 0.267 while the smallest value is 0.012.

As mentioned above, the larger the I_H and the lower the I_{IGD}, the better the algorithm. When $m = 1$, the upper limit of I_H and the lower limit of I_{IGD} are not the highest and lowest compared with other values of m in any of those four datasets. Also, there are more large outliers for average I_{IGD} on the SRBCT and Leukemia datasets than that of $m = 5, 10, 15$.

According to the average values of I_H and I_{IGD} in Fig. 3, the proposed MOCDE method is not very sensitive to the values of the parameter m. Cluster interval $m = 1$ means that MOCDE will perform clustering at each generation, which increases the computational complexity of the proposed MOCDE algorithm. When m is too big, e.g., 20, the performance is slightly decreased in general. Therefore, a smaller value of m is not recommended. Based on Fig. 3, $m = 10$ is a good starting point.

IV. COMPUTATIONAL COMPLEXITY ANALYSIS

In this subsection, the computation complexity of the used dynamic SCD and the hypervolume contribution indicator is analyzed. Then, the total computation complexity of MOCDE is discussed.

Assume using a population with its size of N to solve a problem with M objectives and n decision variables. The computational complexity of traditional crowding distance is $O(MN \log N)$ [1]. The difference in computation complexity between SCD and the traditional crowding distance lies in the calculation of crowding distance in the search space. Then, the computation complexity of SCD is $O((M + n)N \log N)$. Suppose that the number of solutions that need to be removed is t in Algorithm 2 of Section III.F, the computational complexity of the used dynamic SCD is $O((M+n+t)N \log N)$. Similarly, the computational complexity of the used dynamic hypervolume contribution indicator is $O((m + N)^M)$ (m is the number of solutions that need to be removed in Algorithm 3 of Section III.G), and that of the original hypervolume contribution indicator is $O(N^M)$ [2]. In this work, M is 2, and the minimum and the maximum values of t and m are 0 and N. Therefore, the computation complexity of the dynamic SCD is in the range of $[O(nN \log N), O(nN \log N^2)]$. Similarly, the computational complexity of the used dynamic hypervolume contribution indicator in the proposed MOCDE algorithm is between $O(N^2)$ and $O(4N^2)$.

The complexity of the proposed MOCDE algorithm mainly includes three aspects: the main iteration loop of DE, clustering, and the calculations of SCD and C_H. The complexity of the main loop and clustering are $O(N)$ and $O(N^2)$. Therefore, the total computational complexity of MOCDE is between $O(N^2)$ and $O(N^3 + nN^2 \log N)$.

TABLE I: The average classification accuracy results on the test sets

Dataset	NSGA-II	SPEA2	MOEA/D	SparseEA	Omni-optimizer	DN-NSGAII	MO_Ring_PSO	MOCDE
Wine	95.83%	96.42%	96.09%	**96.79%**	94.99%	94.94%	93.10%	95.05%
Zoo	89.19%	86.43%	88.72%	84.79%	86.77%	86.37%	82.69%	**90.03%**
SPECT	65.72%	63.33%	65.76%	61.89%	66.66%	64.76%	63.83%	**69.24%**
WBCD	88.22%	88.20%	88.49%	87.92%	88.49%	88.66%	87.91%	**90.94%**
Ionosphere	85.94%	86.53%	86.01%	86.06%	86.04%	85.63%	85.73%	**86.96%**
Sonar	67.66%	65.00%	67.74%	67.14%	66.83%	66.63%	68.65%	**76.98%**
Movement	61.96%	63.42%	62.33%	61.85%	61.22%	60.30%	61.51%	**70.45%**
Hillvally	53.14%	53.25%	53.25%	54.34%	51.39%	51.87%	51.88%	**55.72%**
Musk1	96.74%	96.92%	97.21%	96.57%	95.16%	95.45%	95.79%	**97.48%**
Multiple	92.88%	93.46%	92.88%	93.69%	93.96%	93.70%	94.22%	**95.09%**
Arrhythmia	**57.55%**	56.92%	57.00%	50.35%	55.14%	55.75%	55.89%	56.01%
SRBCT	75.59%	78.73%	77.35%	72.55%	76.77%	77.35%	79.31%	**85.83%**
Leukemia	82.63%	81.75%	83.57%	85.86%	87.37%	86.93%	86.93%	**89.56%**
DLBCL	87.41%	87.65%	86.46%	75.69%	81.10%	81.74%	81.83%	**90.03%**

TABLE II: The average size results on the test sets

Dataset	NSGA-II	SPEA2	MOEA/D	SparseEA	Omni-optimizer	DN-NSGAII	MO_Ring_PSO	MOCDE
Wine	**3.53**	4.00	3.98	4.30	4.97	4.93	4.37	4.03
Zoo	6.33	6.27	6.32	**5.73**	6.97	7.23	6.87	7.43
SPECT	7.07	7.07	6.88	**4.57**	6.87	6.93	7.10	6.83
WBCD	4.83	4.53	**4.26**	7.13	10.23	10.43	8.37	8.17
Ionosphere	5.97	**5.07**	5.12	6.43	8.53	9.93	6.73	7.13
Sonar	10.50	**10.20**	11.00	14.87	20.00	19.80	18.23	17.33
Movement	**14.47**	14.87	14.68	20.10	29.57	32.60	29.40	22.67
Hillvally	13.77	12.23	12.11	**11.40**	34.00	33.50	30.00	19.57
Musk1	17.27	15.83	16.57	**13.93**	57.03	55.83	51.77	38.10
Multiple	**47.07**	58.73	66.35	76.57	89.40	86.93	96.43	101.60
Arrhythmia	27.73	28.00	26.52	**11.17**	95.87	97.47	97.33	63.47
SRBCT	100.20	117.40	105.24	**11.80**	826.63	831.53	832.37	51.23
Leukemia	565.57	594.70	535.87	**4.00**	1912.57	1918.60	1871.30	35.93
DLBCL	1031.20	1107.53	857.25	**8.90**	2701.23	2707.30	2648.17	125.40

TABLE III: The PV and AN of the four algorithms (Omni-optimizer, DN-NSGAII, MO_Ring_PSO, and MOCDE)

Dataset	Omni-optimizer		DN-NSGAII		MO_Ring_PSO		MOCDE	
	PV	AN	PV	AN	PV	AN	PV	AN
Wine	0%	0	0%	0	6.7%	0.1	**96.7%**	**2.3**
Zoo	13.3%	0.4	20%	0.4	80%	4.0	**100%**	**6.1**
SPECT	16.7%	0.4	6.7%	0.1	86.7%	3.3	**100%**	**14.7**
WBCD	6.7%	0.1	20%	0.5	26.7%	0.6	**100%**	**6.0**
Ionosphere	3.3%	0.1	10%	0.2	36.7%	1.0	**93.3%**	**7.5**
Sonar	3.3%	0.1	6.7%	0.1	20%	0.4	**96.7%**	**10.1**
Movement	13.3%	0.3	16.7%	0.3	3.3%	0.1	**93.3%**	**6.3**
Hillvally	3.3%	**0.1**	**6.7%**	**0.1**	3.3%	**0.1**	0%	0
Musk1	80%	3.3	83.3%	3.4	10%	0.2	**93.3%**	**16.4**
Multiple	**100%**	10.8	**100%**	12.4	0%	0	**100%**	**27.9**
Arrhythmia	90%	6.3	76.7%	**6.4**	0%	0	**100%**	2.9
SRBCT	90%	25.3	93.3%	**36.5**	23.3%	0.5	**100%**	8.7
Leukemia	93.3%	38.6	**100%**	**78.4**	13.3%	0.3	**100%**	10.3
DLBCL	**100%**	80.6	**100%**	**91.6**	3.3%	0.1	93.3%	15.2

References

[1] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan, "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. Evol. Comput.*, vol. 6, no. 2, pp. 182–197, 2002.

[2] A. P. Guerreiro, C. M. Fonseca, and L. Paquete, "The hypervolume indicator: Problems and algorithms," *arXiv preprint arXiv:2005.00515*, 2020.

TABLE IV: The obtained different feature subsets with the same training accuracy on the five datasets

Dataset	Feature subsets	Accuracy
Multiple	$\{F_{1}, F_{3}, F_{15}, F_{19}, F_{28}, F_{30}, F_{32}, F_{34}, F_{45}, F_{48}, F_{51}, F_{55}, F_{56}, F_{64}, F_{68}, F_{73}, F_{75}, F_{80}, F_{87}, F_{103}, F_{105}, F_{109}, F_{110}, F_{112}, F_{115}, F_{122}, F_{139}, F_{141}, F_{150}, F_{155}, F_{158}, F_{163}, F_{168}, F_{180}, F_{185}, F_{188}, F_{190}, F_{192}, F_{193}, F_{197}, F_{202}, F_{206}, F_{213}, F_{218}, F_{221}, F_{224}, F_{229}, F_{233}, F_{238}, F_{240}\}$, $\{F_{12}, F_{19}, F_{25}, F_{29}, F_{30}, F_{39}, F_{48}, F_{55}, F_{56}, F_{58}, F_{65}, F_{66}, F_{68}, F_{70}, F_{72}, F_{79}, F_{83}, F_{86}, F_{88}, F_{91}, F_{94}, F_{97}, F_{101}, F_{103}, F_{106}, F_{110}, F_{112}, F_{116}, F_{125}, F_{126}, F_{129}, F_{134}, F_{138}, F_{139}, F_{146}, F_{152}, F_{159}, F_{161}, F_{163}, F_{169}, F_{170}, F_{172}, F_{183}, F_{193}, F_{202}, F_{206}, F_{218}, F_{221}, F_{226}, F_{231}, F_{233}\}$	97.0%
	$\{F_{2}, F_{3}, F_{4}, F_{8}, F_{12}, F_{15}, F_{16}, F_{18}, F_{29}, F_{30}, F_{38}, F_{40}, F_{41}, F_{44}, F_{48}, F_{51}, F_{56}, F_{59}, F_{61}, F_{63}, F_{65}, F_{67}, F_{71}, F_{72}, F_{73}, F_{75}, F_{80}, F_{81}, F_{82}, F_{94}, F_{96}, F_{97}, F_{100}, F_{102}, F_{103}, F_{108}, F_{109}, F_{110}, F_{113}, F_{115}, F_{119}, F_{122}, F_{123}, F_{125}, F_{127}, F_{128}, F_{129}, F_{132}, F_{133}, F_{137}, F_{140}, F_{141}, F_{142}, F_{146}, F_{147}, F_{151}, F_{152}, F_{154}, F_{155}, F_{157}, F_{166}, F_{168}, F_{169}, F_{171}, F_{172}, F_{175}, F_{177}, F_{183}, F_{195}, F_{197}, F_{200}, F_{204}, F_{206}, F_{215}, F_{219}, F_{221}, F_{225}, F_{230}, F_{233}, F_{238}\}$, $\{F_{3}, F_{7}, F_{12}, F_{14}, F_{15}, F_{18}, F_{19}, F_{20}, F_{21}, F_{26}, F_{30}, F_{34}, F_{35}, F_{39}, F_{42}, F_{44}, F_{49}, F_{51}, F_{53}, F_{56}, F_{57}, F_{66}, F_{70}, F_{75}, F_{76}, F_{77}, F_{79}, F_{80}, F_{82}, F_{86}, F_{88}, F_{91}, F_{96}, F_{97}, F_{100}, F_{103}, F_{105}, F_{108}, F_{111}, F_{112}, F_{117}, F_{119}, F_{120}, F_{124}, F_{125}, F_{127}, F_{129}, F_{131}, F_{140}, F_{141}, F_{148}, F_{151}, F_{153}, F_{155}, F_{156}, F_{157}, F_{158}, F_{164}, F_{168}, F_{171}, F_{172}, F_{176}, F_{183}, F_{194}, F_{195}, F_{202}, F_{203}, F_{208}, F_{210}, F_{212}, F_{214}, F_{218}, F_{219}, F_{223}, F_{225}, F_{226}, F_{227}, F_{230}, F_{231}, F_{239}\}$	98.7%
Arrhythmia	$\{F_{50}, F_{95}, F_{107}, F_{112}, F_{131}, F_{160}, F_{171}, F_{231}, F_{241}, F_{254}\}$, $\{F_{25}, F_{50}, F_{107}, F_{112}, F_{115}, F_{123}, F_{131}, F_{160}, F_{231}, F_{241}\}$	56.5%
	$\{F_{4}, F_{12}, F_{15}, F_{25}, F_{32}, F_{33}, F_{34}, F_{39}, F_{41}, F_{46}, F_{49}, F_{56}, F_{59}, F_{80}, F_{81}, F_{91}, F_{96}, F_{97}, F_{105}, F_{109}, F_{115}, F_{120}, F_{123}, F_{128}, F_{137}, F_{151}, F_{168}, F_{171}, F_{174}, F_{178}, F_{181}, F_{207}, F_{211}, F_{212}, F_{215}, F_{227}, F_{228}, F_{238}, F_{245}, F_{248}, F_{259}, F_{260}, F_{275}\}$ $\{F_{15}, F_{32}, F_{36}, F_{37}, F_{39}, F_{49}, F_{54}, F_{59}, F_{62}, F_{74}, F_{75}, F_{84}, F_{87}, F_{89}, F_{91}, F_{94}, F_{103}, F_{107}, F_{112}, F_{114}, F_{122}, F_{125}, F_{129}, F_{137}, F_{138}, F_{143}, F_{149}, F_{155}, F_{156}, F_{157}, F_{165}, F_{167}, F_{175}, F_{178}, F_{179}, F_{192}, F_{198}, F_{205}, F_{235}, F_{237}, F_{241}, F_{250}, F_{256}, F_{270}\}$	63.5%
SRBCT	$\{F_{159}, F_{169}, F_{236}, F_{246}, F_{256}, F_{337}, F_{391}, F_{392}, F_{509}, F_{512}, F_{577}, F_{622}, F_{685}, F_{718}, F_{754}, F_{816}, F_{870}, F_{926}, F_{975}, F_{1008}, F_{1044}, F_{1071}, F_{1232}, F_{1256}, F_{1305}, F_{1370}, F_{1455}, F_{1475}, F_{1575}, F_{1611}, F_{1635}, F_{1731}, F_{1747}, F_{1949}, F_{2009}, F_{2078}, F_{2079}, F_{2121}\}$ $\{F_{5}, F_{59}, F_{151}, F_{169}, F_{237}, F_{244}, F_{246}, F_{375}, F_{391}, F_{411}, F_{509}, F_{521}, F_{572}, F_{621}, F_{754}, F_{778}, F_{815}, F_{816}, F_{964}, F_{988}, F_{1001}, F_{1358}, F_{1381}, F_{1443}, F_{1475}, F_{1559}, F_{1578}, F_{1603}, F_{1611}, F_{1809}, F_{1953}, F_{2008}, F_{2014}, F_{2084}, F_{2140}, F_{2175}, F_{2210}, F_{2252}\}$	96.1%
	$\{F_{93}, F_{157}, F_{182}, F_{238}, F_{257}, F_{261}, F_{276}, F_{335}, F_{349}, F_{361}, F_{482}, F_{509}, F_{545}, F_{566}, F_{738}, F_{744}, F_{745}, F_{782}, F_{791}, F_{829}, F_{830}, F_{869}, F_{937}, F_{951}, F_{975}, F_{978}, F_{1057}, F_{1074}, F_{1269}, F_{1274}, F_{1370}, F_{1410}, F_{1431}, F_{1503}, F_{1596}, F_{1601}, F_{1694}, F_{1778}, F_{1835}, F_{1856}, F_{1935}, F_{1955}, F_{1957}, F_{2053}, F_{2069}, F_{2084}, F_{2292},\}$ $\{F_{93}, F_{136}, F_{149}, F_{238}, F_{248}, F_{261}, F_{361}, F_{363}, F_{499}, F_{509}, F_{545}, F_{566}, F_{623}, F_{705}, F_{744}, F_{745}, F_{830}, F_{869}, F_{937}, F_{951}, F_{975}, F_{978}, F_{1057}, F_{1067}, F_{1254}, F_{1410}, F_{1431}, F_{1441}, F_{1524}, F_{1551}, F_{1595}, F_{1601}, F_{1650}, F_{1670}, F_{1675}, F_{1679}, F_{1850}, F_{1856}, F_{1939}, F_{2047}, F_{2055}, F_{2084}, F_{2170}, F_{2208}, F_{2229}, F_{2230}, F_{2281}\}$	100%
Leukemia	$\{F_{17}, F_{84}, F_{102}, F_{546}, F_{558}, F_{1011}, F_{1293}, F_{1333}, F_{1450}, F_{1726}, F_{1753}, F_{1819}, F_{2014}, F_{2728}, F_{2906}, F_{2913}, F_{2915}, F_{3031}, F_{3069}, F_{3071}, F_{3184}, F_{3200}, F_{3582}, F_{3718}, F_{3726}, F_{3760}, F_{3872}, F_{4026}, F_{4033}, F_{4185}, F_{4226}, F_{4231}, F_{4306}, F_{4484}, F_{4593}, F_{4626}, F_{4645}, F_{4834}, F_{4880}, F_{4983}, F_{5087}\}$, $\{F_{678}, F_{740}, F_{985}, F_{1232}, F_{1280}, F_{1304}, F_{1349}, F_{1375}, F_{1413}, F_{1437}, F_{1478}, F_{1628}, F_{1682}, F_{1824}, F_{1904}, F_{1960}, F_{2452}, F_{2756}, F_{2816}, F_{3069}, F_{3195}, F_{3200}, F_{3395}, F_{3465}, F_{3481}, F_{3550}, F_{3638}, F_{3789}, F_{3800}, F_{3818}, F_{3821}, F_{3859}, F_{3967}, F_{4026}, F_{4068}, F_{4119}, F_{4306}, F_{4664}, F_{4715}, F_{4779}, F_{4947}\}$, $\{F_{160}, F_{219}, F_{266}, F_{461}, F_{530}, F_{852}, F_{1122}, F_{1184}, F_{1293}, F_{1468}, F_{1646}, F_{1732}, F_{1922}, F_{1927}, F_{1934}, F_{1960}, F_{1980}, F_{2080}, F_{2138}, F_{2158}, F_{2502}, F_{2722}, F_{2735}, F_{2930}, F_{3045}, F_{3365}, F_{3378}, F_{3517}, F_{3728}, F_{3886}, F_{4290}, F_{4305}, F_{4497}, F_{4513}, F_{4669}, F_{4696}, F_{4713}, F_{4715}, F_{5021}, F_{5037}, F_{5117}\}$	100%
DLBCL	$\{F_{321}, F_{357}, F_{480}, F_{715}, F_{755}, F_{777}, F_{1247}, F_{1275}, F_{1313}, F_{1372}, F_{1705}, F_{1917}, F_{2071}, F_{2161}, F_{2452}, F_{2458}, F_{3102}, F_{3179}, F_{3187}, F_{3293}, F_{3758}, F_{3772}, F_{3943}, F_{3949}, F_{4112}, F_{4252}, F_{4292}, F_{4298}, F_{4302}, F_{4379}, F_{4539}, F_{4621}, F_{5081}, F_{5389}, F_{5418}, F_{5437}, F_{5452}, F_{5668}, F_{5838}, F_{5936}, F_{6043}, F_{6244}, F_{6261}, F_{6643}, F_{6835}, F_{6868}, F_{6910}, F_{6913}, F_{6948}, F_{7023}\}$ $\{F_{25}, F_{41}, F_{476}, F_{544}, F_{755}, F_{760}, F_{871}, F_{957}, F_{1013}, F_{1134}, F_{1372}, F_{1393}, F_{1505}, F_{1562}, F_{1914}, F_{2245}, F_{2280}, F_{2351}, F_{2460}, F_{2575}, F_{2591}, F_{2778}, F_{2915}, F_{3794}, F_{3813}, F_{3837}, F_{3852}, F_{3870}, F_{4018}, F_{4214}, F_{4314}, F_{4367}, F_{4460}, F_{4574}, F_{4593}, F_{4621}, F_{4907}, F_{4949}, F_{4984}, F_{5011}, F_{5064}, F_{5105}, F_{5308}, F_{5321}, F_{5547}, F_{6074}, F_{6135}, F_{6191}, F_{6995}, F_{7037}\}$	96.1%
	$\{F_{5}, F_{82}, F_{476}, F_{869}, F_{1134}, F_{1405}, F_{1562}, F_{1611}, F_{1670}, F_{1676}, F_{1990}, F_{2043}, F_{2081}, F_{2193}, F_{2213}, F_{2419}, F_{2460}, F_{2497}, F_{2542}, F_{2595}, F_{3008}, F_{3102}, F_{3604}, F_{3704}, F_{3716}, F_{3897}, F_{3949}, F_{3987}, F_{4314}, F_{4344}, F_{4504}, F_{4557}, F_{4564}, F_{4621}, F_{4838}, F_{5002}, F_{5169}, F_{5342}, F_{5389}, F_{5577}, F_{5781}, F_{5904}, F_{6259}, F_{6277}, F_{6510}, F_{6547}, F_{6668}, F_{6941}, F_{7003}, F_{7017}, F_{7023}, F_{7046}\}$ $\{F_{15}, F_{321}, F_{740}, F_{777}, F_{957}, F_{1016}, F_{1059}, F_{1218}, F_{1344}, F_{1621}, F_{1768}, F_{1847}, F_{2158}, F_{2213}, F'_{2419}, F_{2441}, F_{2460}, F_{2470}, F_{2513}, F_{2753}, F_{2863}, F_{3198}, F_{3363}, F_{3656}, F_{3779}, F_{3949}, F_{3990}, F_{4273}, F_{4460}, F_{4504}, F_{4737}, F_{4777}, F_{5002}, F_{5112}, F_{5236}, F_{5418}, F_{5644}, F_{5777}, F_{5781}, F_{5867}, F_{5888}, F_{6068}, F_{6074}, F_{6091}, F_{6259}, F_{6469}, F_{6556}, F_{6648}, F_{6910}, F_{6940}, F_{6990}, F_{7023}\}$	98.1%
	$\{F_{61}, F_{621}, F_{760}, F_{1013}, F_{1016}, F_{1134}, F_{1250}, F_{1405}, F_{1407}, F_{1421}, F_{1440}, F_{1454}, F_{1492}, F_{1496}, F_{1504}, F_{1561}, F_{1626}, F_{1768}, F_{1891}, F_{2094}, F_{2100}, F_{2277}, F_{2370}, F_{2460}, F_{2871}, F_{3020}, F_{3039}, F_{3102}, F_{3136}, F_{3268}, F_{3781}, F_{3927}, F_{3955}, F_{4179}, F_{4230}, F_{4280}, F_{4379}, F_{4504}, F_{4667}, F_{5055}, F_{5157}, F_{5169}, F_{5236}, F_{5496}, F_{5576}, F_{5615}, F_{5703}, F_{6256}, F_{6259}, F_{6272}, F_{6358}, F_{6736}, F_{6941}, F_{7023}\}$ $\{F_{7}, F_{79}, F_{82}, F_{286}, F_{476}, F_{799}, F_{800}, F_{974}, F_{1562}, F_{1704}, F_{2006}, F_{2081}, F_{2158}, F_{2245}, F_{2459}, F_{2460}, F_{2795}, F_{2943}, F_{3079}, F_{3267}, F_{3388}, F_{3511}, F_{3596}, F_{3617}, F_{3704}, F_{3717}, F_{3781}, F_{3990}, F_{4160}, F_{4165}, F_{4219}, F_{4295}, F_{4352}, F_{4581}, F_{4621}, F_{4835}, F_{4891}, F_{4938}, F_{4949}, F_{5055}, F_{5092}, F_{5320}, F_{5361}, F_{5736}, F_{5897}, F_{5975}, F_{6280}, F_{6732}, F_{6736}, F_{6771}, F_{6802}, F_{6884}, F_{6892}, F_{7023}\}$ $\{F_{47}, F_{79}, F_{357}, F_{504}, F_{649}, F_{727}, F_{735}, F_{755}, F_{1013}, F_{1381}, F_{1407}, F_{1568}, F_{1612}, F_{1768}, F_{1823}, F_{1825}, F_{2163}, F_{2164}, F_{2460}, F_{2466}, F_{2591}, F_{2726,} F_{2833}, F_{2875}, F_{2956}, F_{2974}, F_{3102}, F_{3155}, F_{3394}, F_{3596}, F_{3897}, F_{4186}, F_{4220}, F_{4228}, F_{4298}, F_{4302}, F_{4460}, F_{4495}, F_{4982}, F_{5170}, F_{5336}, F_{5686}, F_{5747}, F_{5762}, F_{5907}, F_{6051}, F_{6091}, F_{6132}, F_{6457}, F_{6736}, F_{6913}, F_{6948}, F_{7023}, F_{7040}\}$	100%