Introduction to Julia

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21st October

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What is Julia?

- Julia is a programming language
- Initially created by Jeff Bezanson, Stefan Karpinski, Viral B. Shah, and Alan Edelman
- Aims to have the simplicity of Python, the speed of C, and the functionality of Lisp



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Background and Notable Uses

- First released in 2012.
- Version 1.0 was released in 2018
- Won the 2019 James H. Wilkinson Prize for Numerical Software
- Used both at NASA and at CERN
 - Used at NASA to model spacecraft separation dynamics -15000 times faster than MATLAB
 - Used at CERN for one of the Large Hadron Collider experiments
- Many major changes over the years is finally now in a good place to be used by everyday users!

What makes Julia good?

Built around the concept of multiple dispatch

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- High performance
- Native support for parallelism
- Optional typing/duck typing
- Strong support for metaprogramming
- Support for Unicode
- And more!

Ease of Use

Language Features

Performance Improvements

Useful Libraries for Research

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Downsides

Questions/Discussion

Ease of Use

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REPL

- While Julia is a compiled language, it provides a read-eval-print loop (REPL) to interactively write code
- Similar to Lisp and Python
- Allows for on-the-fly testing of code during development



Documentation: https://docs.julialang.org

Type "?" for help, "]?" for Pkg help.

Version 1.7.0 (2021-11-30) Official https://julialang.org/ release

ulia> helloworld("ECRG") Hello World, hello ECRG

Dynamic Typing

- Similar to Python, Julia supports dynamic (or duck) typing
- Optional static typing can improve computation speed and aid multiple dispatch

- Can write full programs in Julia without using types
- The same name can refer to multiple different types throughout execution of the code

String Interpolation

- ► Julia supports C++ style string interpolation
- Instead of format strings, variables and computations can be directly inserted into the string

'Using dataset {}, seed {}, and the {} algorithm with population size {}, {}-tournament selection, {} elitism, {} crossover and {} mutation'.format(dataset, seed, algorithm, population, tournament, elitism, crossover, mutation)

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'Using dataset {}, seed {}, and the {} algorithm with population size {}, {}-tournament selection, {} elitism, {} crossover and {} mutation'.format(dataset, seed, algorithm, population, tournament, elitism, crossover, mutation)

"Using dataset \$dataset, seed \$seed, and the \$algorithm algorithm with population size \$population, \$(tournament)tournament selection, \$elitism elitism, \$crossover crossover and \$mutation mutation"

Other Usability Bonuses

```
Short circuit evaluation
conflicting(rule, v) && (return true)
best isa Nothing || (fitness = best.fitness)
```

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- 1-indexed
- Single line function definitions addtwo(a) = a + 2
- Easy vectorization of functions addtwo.(somelist)

Language Features

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Multiple Dispatch

- The main concept/core paradigm behind Julia!
- Each function can have an arbitrary number of method implementations, each operating on different types
- Julia decides which method to run as the most specific method based on parameter types
- Methods can be typed as abstract types
- Allows for a huge amount of code reuse/code sharing.

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After some time really changes your coding style

Multiple Dispatch

- Many languages we are used to use single dispatch (OOP) Cat bella = Cat(); cat.meowAt(dog)
- It can take some getting used to the style of multiple dispatch Cat bella = Cat(); meowAt(cat, dog)
- New method definitions can be added at any time all code that already used that function now works with the new method!
- Does not need to be inside the class like it would have to be for OOP
- I highly recommend watching "The Unreasonable Effectiveness of Multiple Dispatch" https://www.youtube.com/watch?v=kc9HwsxE1OY

Interfaces

- Thanks to multiple dispatch, Julia provides some easy to implement interfaces!
- To make a type iterable, only have to implement iterate(iter) and iterate(iter, state)

Interfaces

We can also index a type by implementing getindex(X, i), setindex!(X, v, i), firstindex(X), and lastindex(X)

```
julia> function getindex(S::Squares, i::Int)
           1 <= i <= S.count || throw(BoundsError(S, i))
          return i*i
       end
getindex (generic function with 218 methods)
julia> getindex(S::Squares, I) = [S[i] for i in I]
getindex (generic function with 218 methods)
julia> firstindex(S::Squares) = 1
firstindex (generic function with 17 methods)
julia> lastindex(s::Squares) = s.count
lastindex (generic function with 14 methods)
iulia> Sauares(6)[3:end]
4-element Vector{Int64}:
16
```

Interfaces

```
ulia> struct SquaresVector <: AbstractVector{Int}
           count::Int
       end
julia> size(S::SquaresVector) = (S.count,)
size (generic function with 101 methods)
julia> IndexStyle(::Type{<:SquaresVector}) = IndexLinear()</pre>
IndexStyle
julia> getindex(S::SquaresVector, i::Int) = i*i
getindex (generic function with 217 methods)
julia> s = SquaresVector(4)
4-element SquaresVector:
 4
 9
 ulia> length(s)
julia> s[s .> 8]
2-element Vector{Int64}:
iulia≻ s + s
4-element Vector{Int64}:
```

Metaprogramming

- As with Lisp, Julia represents code as a data structure in the language itself
- This means we can generate and transform code within the code itself!

```
julia> exp = Expr(:call, :+, Expr(:call, :*, 4, 2), Expr(:call, :-, 6, :x))
:(4 * 2 + (6 - x))
julia> x = 5
5
julia> eval(exp)
9
```

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```
julia> exp.args[2]
:(4 * 2)
julia> exp.args[2] = Expr(:call, :/, 10, 5)
:(10 / 5)
julia> exp
(10 / 5 + (6 - x))
julia> eval(exp)
3.0
julia≻ x = 6
julia> eval(exp)
```

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Metaprogramming

- As with Lisp, Julia represents code as a data structure in the language itself
- This means we can generate and transform code within the code itself!
- This looks a lot like genetic programming!
 - This makes sense, with GP's roots in Lisp
- Can also use metaprogramming to hold arbitrary information many libraries use Symbols (eg. :callable) to represent settings in functions

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Parallelism

Julia has inbuilt support for multiple type of parallelism

- LoopVectorization.jl allows for specific lines of code to be parallelised
- Julia base has support for classic multi-threading
 - Loops can be parallelised with @threads
- Built in GPU/CUDA support
- Utilise multiple machines with distributed computing

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Unicode

- Full support in both strings and names for Unicode including UTF and emoji
- This seems like a small feature, but it has a lot of benefits
- Code can directly relate to mathematical expressions it implements - no more spelling out Greek letters! area(c::Circle) = π * c.r²

```
julia> struct Circle
    r::Float64
    end
julia> (c::Circle) = π * c.r^2
    (generic function with 1 method)
julia> (Circle(2))
12.566370614359172
```

Interface with Other Languages

- Julia has functions and packages to easily call code from other languages!
- Call C functions with ccall
- Similar libraries exist for others PyCall.jl, RCall.jl, and JavaCall.jl are all easy to use
- Helps with the infancy of Julia just use complex packages from more mature languages!

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Conversions and Promotions

As with most other languages, Julia automatically converts data types when it can and needs to

- Assigning to a typed field/variable/array
- Returning from a typed function
- Math: 1 + 1.5 -> 1.0 + 1.5 = 2.5
- Unlike other languages, we can define our own conversions! convert(::Type{MyType}, x) = MyType(x)
- For math, types will be promoted to a common type. We can also define these rules:

Performance Improvements

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Performance

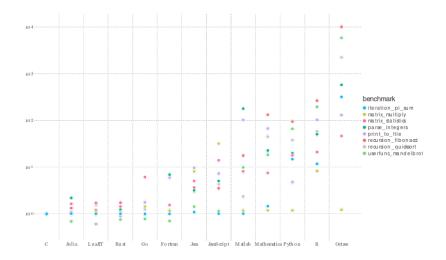
- As a compiled language, Julia can achieve much higher performance than languages it emulates
- No C backend directly compiles itself
- Unlike Python, for loops perform just as well as vectorisations
- Sample benchmark square a list → add 3 to all items → square root → sum:

Single for loop in Julia = 7.059 ms ± 4.517 ms Single for loop in Python = 721.5896 ms

Multiple for loops in Julia = 8.468 ms ± 3.799 ms Multiple for loops in Python = 957.65503 ms

Julia vectorisation = 5.051 ms ± 3.889 ms Python (numpy) vectorisation = 4.814175158 ms

Benchmarks



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Useful Libraries for Research

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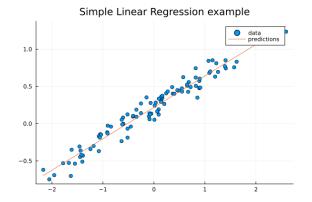
- There are two "main" plotting libraries for Julia
- Plots.jl provides a simpler interface that is more familiar to those used to matplotlib

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```
function sampleplotting(X, y, pred)
    scatter(X[1], y, title = "Simple Linear Regression
        example", label="data")
    plot!(X[1], pred, label="predictions")
end
```

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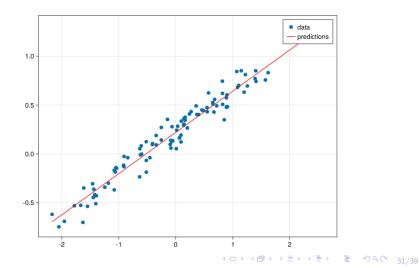
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```
function sampleplotting(X, y, pred)
    Makie.scatter(X[1], y, label="data")
    lines!(X[1], pred, label="predictions", color=:red)
    axislegend()
    current_figure()
end
```

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- Makie.jl provides a more complex interface that is more powerful than Plots



Machine Learning

- Similar to sklearn in Python, Julia has MLJ.jl
- Partially developed at University of Auckland
- Unified interface for many ML packages
- Slightly more complex to use than sklearn, but after a small learning curve works just as well

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Has support for sklearn models!

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- Has support for sklearn models!
- Flux.jl also provides powerful deep learning functionality

Evolutionary Computation

Of particular interest to this group will be Evolutionary.jl

- Implements algorithms for GA, DE, GP, and more
- Works about as well as DEAP problems and all
- Initially strange workflow quick to pick up!
- Few contributors, so not complete in places

Downsides

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Compilation Times

- In order to achieve high performance, the compiler does a lot of work
- This is very slow
- Improved in recent versions of Julia now attempts to compile packages when they are installed through the package manager
- Still very slow for some packages the worst l've found is plotting packages
- In runtime needs to compile each dynamic dispatch method -JIT

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Variable Performance

While Julia can have very good performance, this requires it to be used in a specific way:

- Code is only fast when it is inside a function
- Global variables slow down computation
- Containers slow down with abstract types
 - A Vector{Real} is much slower than a Vector{Float64}!

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- Fields with abstract types are slow
- Essentially the compiler can only do so much!

Very Young Language

- Bugs in core code
- Poor documentation
- Interfaces hard to find information on
- Parts of the language still very subject to change

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${\sf Questions}/{\sf Discussion}$

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