CIRNO: Leveraging Capacity Interference Relationship for Dense Networks Optimization

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Abstract—To meet the rising data-offloading demands, IEEE 802.11-based WiFi networks have undergone consistent densification. The unlicensed spectrum has also been harnessed through LTE-WiFi coexistence. However, in dense and ultra-dense networks (DNs/UDNs), the network capacity is even more adversely impacted by the endemic interference. Yet, the precise nature of Capacity Interference Relationship (CIR) in DNs/UDNs and LTE-WiFi coexistence remains to be studied. Densification also exacerbates the challenges to network optimization. The conventional approaches to simplify the complex SINR-Capacity constraints lead to high convergence times in DN/UDN optimization. We investigate the CIR in dense and ultra-dense WiFi (IEEE 802.11a) and LTE-WiFi (LTU/LAA) networks through real-time experiments. We then subject the empirical data to linear and polynomial regression to determine the nature of CIR and demonstrate that strong linear correlations may exist. We also study the impact of predictor variables, topology, and radio access technology on CIR. Most importantly, we propose CIRNO, a *CIR-inspired network optimization* approach, wherein the empirically determined CIR equation replaces the theoretically assumed SINR-Capacity constraints in optimization formulations. We evaluate CIRNO by implementing three recent works on optimization. We demonstrate the relevance of CIR and CIRNO in DNs/UDNs through a significant reduction in convergence times (by over 50%) while maintaining high accuracy (over 95%). To the best of our knowledge, this is the first work to statistically analyze CIR in DNs/UDNs and LTE-WiFi heterogeneous networks (HetNets) and to use CIR regression equations in network optimization.

I. INTRODUCTION

Between 2012 and 2017 the world has witnessed a 17-fold increase in mobile data traffic. Over 50% of 4G data traffic was offloaded to WiFi in 2017, and as per *Cisco Global Mobile Data Forecasts*, by the year 2022, 59% of all 4G data, and a staggering 71% of global 5G data will be offloaded to WiFi networks. To cater to this tremendous rise in data demands and offloading requests, WiFi networks have undergone consistent densification. As a result, dense (inter-AP distance $\leq$ 10m) and ultra-dense (inter-AP distance $\leq$ 5m) WiFi networks have proliferated the urban centers of major metropolitan cities.

However, the adverse impact of interference is expected to be more severe in the upcoming dense IEEE 802.11ax deployments than in traditional WiFi networks [1]. Statistical analysis of Capacity Interference Relationship (CIR) in traditional WiFi networks has offered insights on how future dense IEEE 80211.ax-based networks can approach the challenges of endemic interference [2]. An alternate 4G/LTE (Long Term Evolution) based solution to the challenges of densification coupled with data-offloading is to harness the paradigm of LTE-WiFi coexistence in dense networks (DNs) and ultra-dense networks (UDNs).

Even more appealing to mobile operators is the effective utilization of unlicensed spectrum through *LTE in unlicensed spectrum* (LTE-U) and *LTE license assisted access* (LTE-LAA). These variants of the LTE not only offer efficient utilization of the unlicensed spectrum but are also equipped with physical (PHY) and media access control (MAC) layer mechanisms to effectively deal with co-channel interference. Although these features are lacking in WiFi, it is still the primary candidate for data-offloading as it provides high network capacity with minimal capital expenditure (CapEx) and operational investment. In contrast, LTE deployments are CapEx intensive. This makes a strong case for DNs and UDNs based on LTE-WiFi coexistence [3].

A. Network Densification and Interference

Application of pragmatic solutions that work in traditional IEEE 802.11 deployments to LTE-WiFi coexistence DNs and UDNs is unlikely to yield optimal network performance.

Interference-related challenges anticipated in dense LTE-WiFi coexistence networks, vehicular networks, and upcoming IEEE 802.11ax mesh networks are elucidated in [4]. Despite a renewed focus, current studies lack a robust analysis of CIR in DNs/UDNs. They successfully highlight the challenges posed by interference, but do not offer any insights on the impact of network densification on the relationship between network capacity and interference [1], [4].

Earlier studies have tried to theoretically approximate the association between interference and capacity as a non-linear inverse relationship [5]. However, several open questions on CIR remain unanswered, and attempts have been made to address them [2], especially the question of whether CIR is always non-linear in DNs/UDNs. With increased spatial co-location of multiple APs in dense deployments, the convergence time of optimal solutions to network problems will also rise significantly [6]. It remains unexplored whether CIR in DNs/UDNs can be used in optimization formulations to make it less computationally expensive [1].

B. Research Contributions

We study CIR in dense/ultra-dense WiFi and LTE-WiFi coexistence networks through real-time experimental deployments. We focus on the unlicensed spectrum by considering

both LTE-U and LTE-LAA variants of LTE. We conduct Linear and Polynomial regression analysis of the SINR and capacity values observed in the experiments to derive empirical CIR models. Thereafter, we propose *CIRNO*, a CIR-inspired network optimization approach, which makes use of the empirically derived CIR equations as constraints in network optimization formulations. CIRNO demonstrates a significant reduction in computational overhead in terms of convergence time by replacing theoretical CIR equations with real-time CIR models. In addition, we offer empirical evidence that a linear correlation between interference and capacity may exist for some network configurations. We also compare the analysis of our real-time experiments with the ns-3 simulation results from an earlier study [2].

Some recent studies have investigated LTE-WiFi coexistence in dense networks using Markov analysis and stochastic geometry [7]. However, to the best of our knowledge, our work is the first to conduct statistical analysis of CIR for dense/ultra-dense, WiFi and LTE-WiFi coexistence networks and to propose the use of statistically derived CIR equations in network optimization.

II. LTE-WiFi Coexistence in Unlicensed Spectrum

Almost 600 MHz of the spectrum has been allocated for unlicensed operation in the 5GHz Unlicensed National Information Infrastructure (U-NII) radio band. The increasing offloading traffic has sparked a great interest in the spatial coexistence of LTE small cells with IEEE 802.11 WiFi networks in the unlicensed spectrum with the objective of providing high-bandwidth access to the end-users. The LTE-WiFi coexistence is ideal as it blends two complementary technologies.

IEEE 802.11-based WLANs were designed to operate on a distributed carrier sense multiple access collision avoidance (CSMA/CA) mechanism. In sharp contrast, the LTE architecture was conceived with a centralized resource-allocation mechanism in the licensed spectrum. Further, while the CSMA/CA MAC protocol of WiFi makes it technically ideal for spectrum-sharing within coexistence architectures, the LTE architecture was designed to operate as a stand-alone system. Therefore, for the unlicensed spectrum, two variants of LTE with specifications that facilitate spectrum-sharing and coexistence with WiFi were standardized and released, viz., LTE-U and LTE-LAA, where 'U' denotes "Unlicensed" and 'LAA' denotes "License Assisted Access." The former makes use of a load-dependent duty-cycle mechanism enabled by Carrier Sense Adaptive Transmission (CSAT). The LTE-LAA variant employs a Listen-Before-Talk (LBT) approach, which resembles the CSMA mechanism in IEEE 802.11 WLANs. While LTE-LAA is an official 3GPP standard for markets worldwide, LTE-U is an informal variant promoted by a special interest group and is suitable in scenarios where LBT is not required [8].

LTE-WiFi coexistence in the unlicensed spectrum has its own operational challenges which include, energy threshold detection, contention window adjustment, resource allocation,
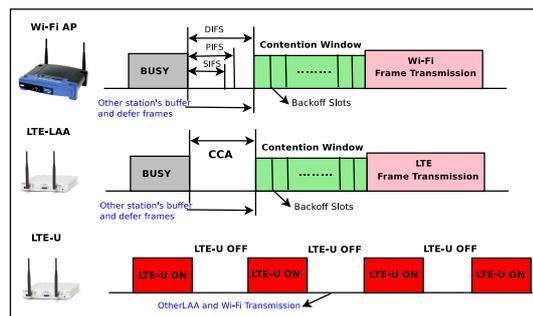

Fig. 1: WiFi, LTE-LAA, and LTE-U: Channel Access Mechanisms

and most importantly, interference management and mitigation. The CIR analysis in LTE-WiFi coexistence is necessary as the 3GPP release notes on the LAA standard are limited to stating that it will "not impact WiFi services more than an additional WiFi network on the same carrier, with respect to throughput and latency." It has also been demonstrated that the fairness assumed in coexistence spectrum sharing in the 3GPP standard does not hold true in several scenarios, and practical approaches to fair-sharing are necessary [3]. Likewise, the assumption in the 3GPP release notes that the coexistence of WiFi WLAN with an LTE-U/LTE-LAA network will have no bearing on the system capacity needs to be validated. Since this work focuses on the analysis and use of CIR in dense LTE-WiFi coexistence networks, the channel access mechanisms of WiFi, LTE-LAA, and LTE-U are explained through the illustration presented in Fig. 1.

III. Leveraging CIR for Network Optimization

A key study on the capacity of wireless networks established a largely non-linear relationship between endemic interference and network performance indicators [5], providing the theoretical foundation for several network optimization solutions to efficient power-control, enhancing throughput, or optimizing inter-nodal distance [9]. However, the direct application of a theoretical non-linear CIR to network optimization presents some issues.

Firstly, CIR is a multivariate relationship influenced by various factors, such as topology, bandwidth, and wireless technology. Besides, recent evidence suggests that a linear correlation between capacity and interference may exist in some network configurations [2]. Second, assuming a non-linear CIR makes an optimization formulation computationally expensive. The increased network density and complexity in DNs/UDNs will lead to an exponential rise in convergence times, rendering this approach to optimal solutions infeasible in dense deployments [6]. Thirdly, the popular approach to circumvent the challenge posed by the assumption of a non-linear CIR is to relax or linearize the resource-intensive constraints, by making other assumptions or considering specific scenarios. Often, simple Interference Distribution Functions are assumed to derive computationally less intensive heuristics [9].

However, arbitrary CIR constraint-linearizations aimed only at *solving* the optimization faster, overlook and undermine the practical viability of the model. Rarely are these relaxations

grounded in empirical results or motivated by real-world measurements. Such reductionist techniques will not be able to offer feasible solutions in dense/ultra-dense coexistence networks and IEEE 802.11ax OBSS WLANs.

A. CIRNO: CIR-Inspired Network Optimization

To address these problems, we propose an alternative approach to network optimization called *CIR-inspired Network Optimization* (CIRNO) which relies on empirically observed SINR-Capacity relationship. CIRNO is suitable for both conventional and dense wireless networks. It does not involve making assumptions about network conditions or parameters and will greatly enhance DN/UDN performance. The four-step CIRNO process is described below :

Step 1. *Empirical data gathering*: Gather SINR and Capacity data from an operational/simulated wireless network.
Step 2. *Statistical Analysis*: Analyze gathered data through statistical tools to generate reliable system models.
Step 3. *Modeling CIR*: Selecting the most suitable system model and identifying the mathematical equation that best explains the CIR.
Step 4. *Network Optimization*: Utilize the CIR relationship equation from Step 3 as a constraint in network optimization.

CIRNO overcomes the three problems described earlier. Since it is based on an empirically derived SINR-Capacity model, theoretical assumptions of non-linearity are not made. Likewise, it replaces arbitrary and impractical constraint-relaxation techniques with statistically generated constraints, whether non-linear or linear. Finally, CIRNO leverages the empirically determined relationship between SINR and capacity, which can be linear for some network configurations, as demonstrated in this work through real-time experiments using different Radio Access Technologies (RATs) in dense/ultra-dense topologies. Another aspect is the *Convergence time and Accuracy Trade-off*, which is a primary concern in the performance optimization of dense networks [6]. Optimization models for DNs/UDNs will require shorter convergence times while maintaining high accuracy. We demonstrate that CIRNO balances the two successfully. It reduces the time-costs and the computational overhead, while maintaining high accuracy, for both non-linear and linear CIR models. CIRNO paves the way for an empirical approach to leverage CIR in optimization formulations for the upcoming LTE-U/LAA and WiFi coexistence DNs/UDNs, IEEE 802.11ax networks, dense vehicular networks, and opportunistic mobile social networks.

IV. STATISTICAL ANALYSIS OF CIR

We conduct regression analysis (RA) to analyze CIR in dense/ultra-dense LTE-WiFi coexistence in real-time deployments. RA is a set of reliable statistical tools to determine if one or more system variables have a *statistically significant* (SS) relationship, which is reflected in the Regression Model (RM) of the system [10]. Variables in a system can be categorized as *predictor variables* (P_{var}) and *response*

variables (R_{var}). RA is most suited for explaining if, and how, variations in one or more P_{var} impacts one or more R_{var}. For a comprehensive analysis, we investigate the CIR from both directions, i.e., by considering interference (SINR) as P_{var} and capacity as R_{var}, and vice-versa.

We apply two popular RA techniques, viz. Linear Regression and Polynomial Regression, on the experimental network data to determine:
1) the statistical significance of the relationship between SINR and network capacity;
2) how the variation in SINR can be explained by the variation in capacity, and vice versa; and,
3) the existence of a linear relationship between SINR and capacity in real dense and ultra-dense LTE-WiFi networks.

The *statistical significance* of the relationship can be validated by the *P-value*, while the *degree of variation* in the response variable, which can be explained by a change in the predictor variable, is given by *R-Squared* (R^2) [10]. Further, the existence and strength of a linear correlation between SINR and capacity is best reflected by *Correlation Coefficient* (CC) of a linear RM.

P-values are measured in terms of the *level of risk*, α, and generally, a value of $\alpha = 0.05$ is considered. The sufficiency of P-values as *proof* for the existence of a relationship is being hotly debated as α does not reflect the accuracy of the model [11]. This concern has been addressed using a *Selection Algorithm for Regression Model* (SAM) [2], by considering a stringent tiered approach to statistical significance. Here, we use SAM to filter out less suitable RMs and select the *Best Regression Model* (BRM) for CIR in each network scenario. To further alleviate the concerns around P-values, we run SAM with the additional caveat that only *Highly Statistically Significant* (HSS) CIR models are selected, restricting the level of risk to less than 0.001, thereby reducing the probable "misinterpretation" of P-values [11] by a factor of 50.

V. EXPERIMENTS, CIR ANALYSIS, & OPTIMIZATION

A. Experimental Set-up

The WiFi-LTE coexistence experimental set-up employs National Instruments USRP 2953 R software-defined radios (SDRs). The NI boards used in the set-up can be configured either as two WiFi APs or two LTE-U/LAA base stations (BS), also known as Evolved Node B (eNB). To create a dense scenario and study the adverse impact of interference in the medium, we increase the number of nodes to six (two NI WiFi/LTE and four Netgear WiFi APs). We consider a dense random topology with nodes placed at a distance of 5m to 10m, and an ultra-dense mesh topology with inter-nodal distance of less than 5m. A representative illustration of the experimental set-up is presented in Fig. 2. Further, we carry out three sets of experiments each for both DN and UDN topologies, viz., WiFi only, WiFi & LTE-U coexistence, and WiFi & LTE-LAA coexistence. All the APs are configured through laptops (via LAN connection). For the WiFi-LTE
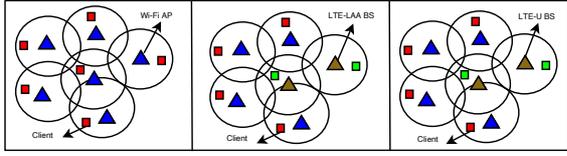
Fig. 2: Experimental Network Topology

(LTE-U/LAA) coexistence experiments, two LTE-LAA/LTE-U, and four WiFi APs are deployed. NI LTE-LAA/LTE-U, NI WiFi AP, and Netgear WiFi APs are provisioned to transmit in the downlink to their clients (one client per AP/eNB) on the same channel (Channel 161). For WiFi, we make use of the IEEE 802.11a standard with 20 MHz bandwidth. In both LTE-U and LTE-LAA, the smallest unit of the spectrum is a Resource Block (RB) of 180 kHz bandwidth allocated to an end-device known as User Equipment (UE) for a transmission time interval (TTI) of one subframe (duration 1ms). Therefore, each LTE-LAA node uses 100 RBs to cover the 20 MHz bandwidth and 1 orthogonal frequency-division multiplexing (OFDM) symbol in a subframe is assigned as the physical downlink control channel (PDCCH) symbol. The transmission characteristics along with other experimental parameters are summarized in Table I.

TABLE I: Simulation Parameters.

Parameter	Value
Number of nodes	6
Transmission Power	23 dBm
Operating Frequency	5 GHz
LTE-U/LAA RF Transmission	Loopback
LTE Transmission Channel	PDSCH, PDCCH
Data Traffic	Full buffer
WiFi Channel Access Protocol	CSMA
LAA Channel Access Protocol	LBT

*PDSCH - Physical Downlink Shared Channel

B. Regression Analysis of CIR

We generate 22 regression Test Scenarios (denoted by TS_i, where $i \in \{1\ldots22\}$) based on the network topology, Radio Access Technology (RAT) used in the experiments, and the predictor-response variables, as listed in Table II. The test scenarios considered in this work enable us to isolate and study the impact of individual factors such as network topology and RAT on CIR. For each TS_i, the BRM is selected using SAM, the regression model selection algorithm [2] that considers several model parameters, such as R^2, outliers, CC, and statistical significance of higher-order terms, to pick the most suited model.

C. Application of CIRNO

We consider three related works that optimize performance in WiFi-LTE coexistence networks. Firstly, a recent study [12] proposes an optimized resource allocation scheme for an enhanced LTE-LAA variant to maximize capacity in a WiFi-LTE HetNet. Likewise, in [3], LTE-U/LAA HetNets are optimized for efficient placement of nodes, improved signal strength, and enhanced end-user throughput. However, the solution did not account for the unlicensed spectrum used by WiFi APs. This has been addressed by a generic HetNet capacity optimization

TABLE II: Regression Analysis of CIR.

TS No.	Nature of CIR	R^2 (%)	CC [-1,1]	BRM Equation representing CIR
TS_1	Linear	70.42	0.84	Y = 10.85 + 0.1083 X
TS_2	Linear	84.20	0.92	Y = 10.53 + 0.1071 X
TS_3	Linear	86.73	0.93	Y = 11.45 + 0.1225 X
TS_4	Linear	85.88	0.93	Y = 10.34 + 0.1065 X
TS_5	Non-Linear	92.86	NA	Y = 16.49 + 0.2961 X + 0.001365 X^2
TS_6	Linear	85.86	0.93	Y = 12.71 + 0.1401 X
TS_7	Linear	70.42	0.84	Y = - 84.20 + 6.501 X
TS_8	Non-Linear	88.26	NA	Y = - 102.9 + 14.12 X - 0.75825 X^2
TS_9	Non-Linear	91.88	NA	Y = - 103.7 + 15.18 X - 0.8139 X^2
TS_{10}	Linear	85.88	0.93	Y = - 89.92 + 8.066 X
TS_{11}	Non-Linear	95.39	NA	Y = - 98.47 + 13.01 X - 0.75376 X^2
TS_{12}	Non-Linear	89.67	NA	Y = - 97.53 + 12.30 X - 0.5790 X^2
TS_{13}	Linear	76.53	0.87	Y = 10.59 + 0.1074 X
TS_{14}	Linear	85.43	0.92	Y = 11.50 + 0.1233 X
TS_{15}	Linear	85.36	0.92	Y = 12.09 + 0.1314 X
TS_{16}	Non-Linear	80.37	NA	Y = - 103.6 + 14.43 X - 0.75711 X^2
TS_{17}	Non-Linear	90.74	NA	Y = - 100.8 + 13.69 X - 0.75774 X^2
TS_{18}	Non-Linear	90.40	NA	Y = - 100.8 + 13.89 X - 0.7119 X^2
TS_{19}	Linear	81.00	0.90	Y = 11.00 + 0.1139 X
TS_{20}	Linear	84.79	0.92	Y = 11.68 + 0.1262 X
TS_{21}	Non-Linear	85.61	NA	Y = - 102.9 + 14.24 X - 0.75928 X^2
TS_{22}	Non-Linear	89.78	NA	Y = - 100.0 + 13.66 X - 0.75779 X^2

model through an efficient placement of nodes [8]. We use them as the basis to demonstrate the versatility of CIRNO. These models are implemented on GAMS [13] as per the DN/UDN network layout and specifications for the real-time experiments. In each optimization model, we replace the constraints representing SINR-Capacity associations with the CIR equations of the BRMs. We then evaluate CIRNO based on two metrics generated from the observed optimization results, viz., *Convergence Time Reduction* (CTR), and *Accuracy*. The latter is defined as the percentage difference between CIRNO-optimal capacity and optimal capacity of the original model.

VI. RESULTS AND ANALYSIS

A. CIR in Dense Networks

BRMs for each scenario, and the relevant regression parameters viz., nature of the relationship (linear/non-linear), R^2, CC, and CIR equations are presented in Table II. We make observations on the variation in CIR as topology and RAT are varied. We also analyze and compare them to the inferences made earlier in [2].

1) Nature of CIR: The most important and counter-intuitive inference of this work is that a strong linear correlation may exist between interference and capacity, and their relationship may not necessarily be non-linear as is theoretically propounded. While earlier observations of linear CIR correlation were based on ns-3 simulations of IEEE 802.11g/n networks [2], we have observed a similar linear pattern in real-time DNs/UDNs using three different RATs, i.e., IEEE 802.11a and LTE (U/LAA). This inference is also significant as over half the scenarios exhibit a strong linear correlation between SINR and capacity. This is reflected in CC which is above 0.8 for all TS_i, and averages to a value of 0.904 signifying a near-perfect linear correlation.

However, a marked departure from the earlier study [2] is that the nature of CIR seems to be largely dependent on the choice of predictor and response variables. In almost all TS_i the linear correlation is observed when SINR is the predictor

and capacity is the response. The reverse is also true, i.e., the non-linearity in CIR is concentrated in TS_i with capacity as P_{var} and except two scenarios (TS_7 & TS_{10}) all exhibit a non-linear CIR. Therefore, in DNs/UDNs the predictor for the CIR analysis must be carefully selected to meet the demands of the optimization model.

As noted earlier, a linear CIR is theoretically improbable [5], and the Shannon limit also defines a logarithmic relationship between the maximum achievable data rate and the noise in the channel. However, regression analysis of the empirically observed network data may offer a different view of CIR. It is possible that the linearity in CIR arises in specific situations. These aspects are beyond the scope of this work and need to be investigated.

We emphasize that CIRNO does not depend upon or influence the nature of the CIR model. Whether CIR is linear or non-linear, the CIRNO approach can be employed to simplify dense network optimization through the use of empirically derived CIR equation.

2) Impact of Topology and RAT: In TS_{19} through TS_{22}, the RAT is kept constant so that the impact of variation in topology can be studied. It has been observed that topology influences the ability of the model to explain the variation in response variable [2], however, no concrete inferences could be made. In contrast, the analysis of experiments on dense random and ultra-dense mesh deployments reveals that the former show higher R^2 and CC values, which implies better explainability of response variables and a stronger linear correlation. This can be attributed to the increasing complexity in ultra-dense deployment, although the differences in R^2 and CC of the two layouts are narrow ($\approx 5\%$). In TS_{13} through TS_{18}, the topologies are kept constant while the RATs are varied. For the two WiFi-LTE deployments, R^2 and CC are almost identical and higher than the WiFi-only network. Surprisingly for WiFi, R^2 and CC values are lowest, despite the added complexity of coexistence in LTE-WiFi HetNets.

As for the nature of CIR, the predictor variable appears to be the determining factor. Neither topology nor RAT seems to have any visible effect, although, as discussed above, a slight variation in the magnitude of CC is observable.

3) Dense vs. Conventional Networks: A regression model can only be as reliable and accurate as the predictor variable used. In this work, we employ experimentally observed values of SINR and capacity as predictor variables, instead of theoretical interference metrics. Further, the dense/ultra-dense nature of the deployments also affects the CIR model parameters. All CIR models of DN/UDN real-time experiments are very reliable HSS RMs. Thus we make comparisons only with HSS models of conventional WiFi mesh networks [2]; a comparison of regression parameters is presented in Fig. 3. The proportion of linear CIR models has increased in DN/UDNs, and the linear correlation (CC) is stronger than conventional networks, which is noteworthy. Also, no negative correlation (CC<0) between SINR and capacity is observed in DN/UDN CIR models. They also offer a more accurate explainability of the response variable, which is reflected in the average R^2 value
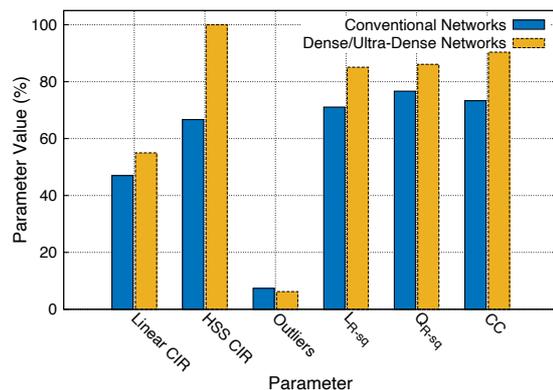

Fig. 3: CIR Parameters and Network Density

of both linear and quadratic models. The number of outliers has also reduced, although marginally. DN/UDN CIR models based on empirical data are unarguably more reliable which makes the CIR more suitable for use in network optimization models.

B. CIRNO: Convergence time and Accuracy

We evaluate the CIRNO approach by analyzing the optimal results generated by three optimization models (OMs) labeled as OM_1 [12], OM_2 [3], and OM_3 [8]. We optimize network capacity, and observe Convergence Times and Accuracy of CIRNO-optimal versions of these OMs. We first demonstrate that the choice of predictor variables has a great bearing on CIR and the subsequent use of the CIR equation in the OMs. Further, we show that regardless of the nature of CIR, an empirically grounded CIR constraint is preferable to non-linear constraint relaxation approaches based on arbitrary or specific assumptions.

1) Choice of Predictor: First, we consider the 11 TS_i where SINR is the P_{var}, which include both linear and non-linear CIR models. We compare the run-times of the optimal and CIRNO-optimal models through the Convergence Time Reduction (CTR) parameter presented in Fig. 4(a). It is evident that the use of statistically generated CIR equations offers a significant reduction in convergence time for all three OMs. The analysis of the Accuracy of CIRNO-optimal values with respect to OM-values is illustrated in Fig. 4(b). There is a negligible difference between CIRNO-optimal values and the respective OM-values, and on an average, CIRNO has an Accuracy of 96.2%.

For the remaining TS_i, where capacity is the P_{var}, the inverse of the CIR equation has to be considered. Ten out of 12 scenarios are non-linear. CTR and Accuracy are illustrated in Fig. 4(c) and Fig. 4(d), respectively. There is a clear dip in CTR values, which underscores the importance of the predictor variable. Since these models optimize capacity, SINR value as the predictor provides the ideal CIR for the model. Yet, despite capacity as the predictor, CIRNO still reduces convergence times while maintaining high Accuracy (97.6% on an average). So, regardless of the predictor, CIRNO performs better than conventional OMs with a minimal trade-off in Accuracy.
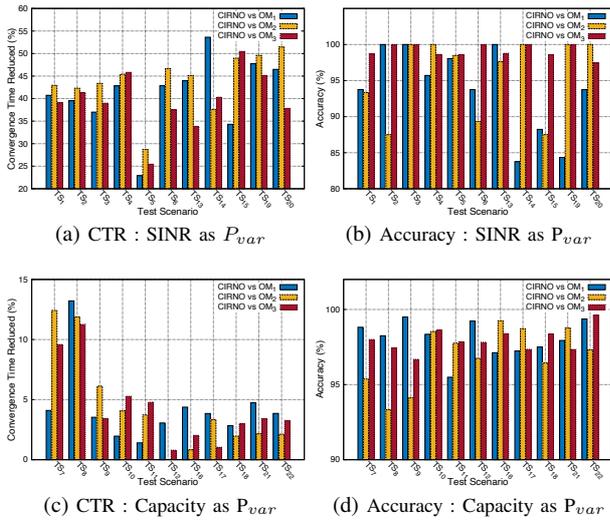
(a) CTR : SINR as P_{var}

(b) Accuracy : SINR as P_{var}

(c) CTR : Capacity as P_{var}

(d) Accuracy : Capacity as P_{var}

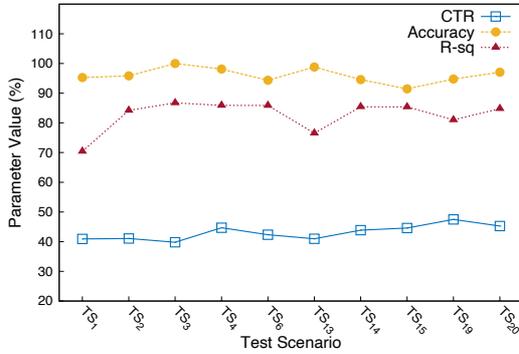Fig. 4: Evaluation of CIRNO

Fig. 5: Relationship of R-sq (R^2) with CTR and Accuracy

2) Nature of CIR: CIRNO is designed to be independent of the nature (linear or non-linear) of the CIR equation and is applicable to most dense network optimization scenarios. We analyze the optimal results and present the average CTR and Accuracy values for linear and non-linear CIRs for the three OMs in Table III.

TABLE III: Performance Evaluation of CIRNO

CIRNO	Linear CIR (%)				Non-Linear CIR (%)			
Parameter	OM_1	OM_2	OM_3	*Avg.*	OM_1	OM_2	OM_3	*Avg.*
CTR	36.25	39.15	35.44	36.95	6.37	6.08	5.85	6.10
Accuracy	94.20	95.76	99.06	96.34	97.96	97.09	97.95	97.67

CIRNO offers a significant reduction in convergence time for linear CIRs. Even for non-linear CIRs, CIRNO lowers the convergence time of OMs while maintaining high Accuracy. This validates our argument that the use of actual CIR equations accelerates network optimization as compared to arbitrary relaxation of theoretical CIR constraints. Thus, CIRNO outperforms conventional constraint relaxation techniques, regardless of the nature of the CIR equation. This is particularly significant for dense and ultra-dense networks, where even a slight reduction in convergence times enhances the quality experienced by the end-user [6].

3) R^2, CTR, and Accuracy: We analyze if the magnitude of R^2 of the CIR model has any bearing on the CTR and Accuracy of CIRNO-optimal values, by keeping the predictor and nature of CIR constant. The three parameters for linear CIRs are presented in Fig. 5. Prima facie, it appears that they may be correlated, but a deeper analysis shows that R^2 and CTR, and R^2 and Accuracy do not have a statistically significant relationship. Therefore, even if CIR data fits the regression line more tightly, it does not necessarily translate into higher Accuracy or reduced run-times. It is likely that the presence of outliers in CIR data-points influences the Accuracy and CTR of CIRNO-optimal values.

VII. CONCLUSIONS AND FUTURE WORK

The proposed 4-step *CIRNO* offers a practical approach to network optimization and demonstrates its efficacy over three state-of-the-art works on network optimization. Further, in several deployments, we observe that the relationship of network capacity and interference in dense and ultra-dense networks is not necessarily non-linear, and a strong linear correlation may exist. *However, the CIRNO approach is independent of the CIR model and makes dense network optimization less resource-intensive for both linear and non-linear CIR models.*

We now plan to analyze CIR through real-time LTE-WiFi coexistence data from network operators, and test CIRNO on real-world network deployments.

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